

The Beth experiment is under review

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It is shown that the modern Maxwell electrodynamics cannot explain the result of the Beth experiment. So, the modern electrodynamics is not complete. A spin tensor is used for an explanation of the experiment. It is shown that this tensor completes the Maxwell electrodynamics. A theory of the Beth's experiment is presented.

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1. The Beth experiment

The classical Beth experiment [1] was made 70 years ago. A beam of circularly polarized light exerts a torque on a doubly refracting plate which changes the state of polarization of the light beam. The apparatus used involves a torsional pendulum with about a ten minute period consisting of a round quartz half-wave plate one inch in diameter suspended with its plane horizontal from a quartz fiber about 25 centimeters long. About 4 millimeters above this is mounted a fixed quartz quarter-wave plate. The *top* side of the upper plate was coated by evaporation with a reflecting layer of aluminum. The rotation of the pendulum is observed by a telescope.

A circularly polarized light beam (power $P = 80 \text{ mW}$, $\lambda = 1.2 \mu\text{m}$, $\omega = 1.6 \cdot 10^{15} \text{ s}^{-1}$) passes through the half-wave plate, then it is reflected and passes twice through the quarter-wave plate, and then returns through the half-wave plate. The torque exerting on the half-wave plate is 20 dyne cm. This result is in accordance with the formula

$$\tau = 4P/\omega. \quad (1.1)$$

2. The standard explanation of the Beth's result

According to the Maxwell theory (see, for example, [2]) the Poynting vector $\mathbf{E} \times \mathbf{B}$ is interpreted as the density of momentum of the field. We can then also define an angular momentum relative to a given point or to a given axis,

$$\mathbf{J} = \int \mathbf{r} \times (\mathbf{E} \times \mathbf{B}) dV \quad (2.1)$$

A circularly polarized plane wave without an azimuthal phase structure traveling in the z -direction and with infinite extension in the xy -directions can have no angular momentum about the z -axis, because $\mathbf{E} \times \mathbf{B}$ is in the z -direction and $[\mathbf{r} \times (\mathbf{E} \times \mathbf{B})]_z = 0$. However, this is no longer the case for a beam. Consider a cylindrical beam. At the wall of the cylinder we let the field drop to zero. It can then be shown that the wall of a beam gives a finite contribution to J_z [3–8].

For example, Ohanian [6] wrote, “In a wave of finite transverse extent, the \mathbf{E} and \mathbf{B} fields have a component parallel to the wave vector (the field lines are closed loops) and the energy flow has components perpendicular to the wave vector The circulating energy flow in the wave implies the existence of angular momentum, whose direction is along the direction of propagation. This angular momentum is the spin of the wave.”

The angular momentum (2.1) and the energy U of a piece of the beam was repeatedly calculated,

$$J = \int E_0^2 dV / \omega, \quad U = \int E_0^2 dV \quad (2.2)$$

where E_0 is the amplitude of the \mathbf{E} field in the inner region of the beam. So, the ratio $U/J = \omega$ appears as the ratio U/S , i.e. energy/spin, for a photon.

A circularly polarization of the Beth's beam is being reversed when the beam passes through the half-wave plate. So, according to the paradigm, the plate gets

$$J = 2U/\omega \quad (2.3)$$

when the beam passes through the plate. The plate gets the same angular momentum from the reflected beam. So, having divided by time we arrive to Eq. (1.1).

3. The Beth's experiment is a puzzle

At the same time, it is evident that the Poynting vector $\mathbf{E} \times \mathbf{B}$ equals to zero in the Beth experiment because the passed beam interferes with the reflected one. Indeed, let us start from the expressions [9] for a circularly polarized beam:

$$\overset{\circ}{\mathbf{E}}_1 = \exp[i(z-t+\phi)](\rho + i\rho \dot{\phi} + z i \partial_\rho) u(\rho), \quad \overset{\circ}{\mathbf{B}}_1 = -i \overset{\circ}{\mathbf{E}}_1, \quad u = \frac{\sqrt{2/\pi}}{w} \exp\left\{-\frac{\rho^2}{w^2}\right\}, \quad (3.1)$$

The symbol ‘breve’ marks complex vectors; for short we set $\omega = c = 1$. The arrow placed under a symbol means a covariant vector, or a covariant coordinate vector. We use the cylindrical coordinates ρ, ϕ, z .

$$x = \rho \cos \phi, \quad y = \rho \sin \phi, \quad \rho = \sqrt{x^2 + y^2} \quad (3.2)$$

с метрикой

$$dl^2 = d\rho^2 + \rho^2 d\phi^2 + dz^2, \quad g_{\rho\rho} = 1, \quad g_{\phi\phi} = \rho^2, \quad g_{zz} = 1, \quad \sqrt{g}_\wedge = \rho, \quad g^{\phi\phi} = 1/\rho^2$$

Square root of determinant of the metric tensor is a scalar density of weight +1. Gothic symbols are usually applied to denote tensor densities. We shall, instead, mark the density with the symbol ‘wedge’ at the level of bottom indices for a density of weight +1 and at the level of top indices for a density of weight -1. Volume element is a density of weight -1, $dV^\wedge = d\rho d\phi dz$, as well as the absolute antisymmetric density e_{ijk}^\wedge , which equals to ± 1 , or 0.

When a mirror reflects the beam (3.1), signs preceding z and the sign in the formula for $\overset{\circ}{\mathbf{B}}$ are changed. But because of the quarter-wave plate a helicity of the beam is conserved, and so signs preceding ϕ is changed and the sign in the formula for $\overset{\circ}{\mathbf{B}}$ is changed once more. Thus the reflected beam in the Beth experiment is expressed by the formula (we use index 4 for the reflected beam)

$$\overset{\circ}{\mathbf{E}}_4 = \exp[i(-z-t-\phi)](\rho - i\rho \dot{\phi} - z i \partial_\rho) u(\rho), \quad \overset{\circ}{\mathbf{B}}_4 = -i \overset{\circ}{\mathbf{E}}_4. \quad (3.3)$$

Let us calculate the Maxwell energy-momentum tensor

$$T^{\lambda\mu} = -g^{\lambda\alpha} F_{\alpha\nu} F^{\mu\nu} + g^{\lambda\mu} F_{\alpha\beta} F^{\alpha\beta} / 4 \quad (3.4)$$

for the total field $\overset{\circ}{\mathbf{E}} = \overset{\circ}{\mathbf{E}}_1 + \overset{\circ}{\mathbf{E}}_4$, $\overset{\circ}{\mathbf{B}} = \overset{\circ}{\mathbf{B}}_1 + \overset{\circ}{\mathbf{B}}_4$.

A signature of the metric tensor $g^{\lambda\alpha}$ in Eq. (3.4) is (+---). $F^{\mu\nu} = -F^{\nu\mu}$, $F_{\mu\nu} = F^{\alpha\beta} g_{\mu\alpha} g_{\nu\beta}$ is the field strength tensor. The sense of its components is

$$F^{ti} = -E^i, \quad F_{ti} = E_i, \quad F^{ij} = -B^{ij}, \quad F_{ij} = -B_{ij}, \quad B_k = B_{\wedge}^{ij} e_{ijk}^\wedge, \quad B^k = B_{ij}^\wedge e_{\wedge}^{ijk}, \quad i, j, \dots = \rho, \phi, z. \quad (3.5)$$

For example,

$$F^{\phi t} = -g^{\phi\phi} F_{\phi t} = E^\phi = g^{\phi\phi} E_\phi, \quad F^{\rho\phi} = g^{\phi\phi} F_{\rho\phi} = -B^{\rho\phi} = -g^{\phi\phi} B_{\rho\phi}. \quad (3.6)$$

The component $T_\wedge^{\phi t}$ is the density of an orbital mass-energy flux, i.e. the ϕ -component of the Poynting vector; infinitesimal time averaged mass

$$dp^t = dm = \langle T_\wedge^{\phi t} \rangle da_\phi^\wedge dt = \langle T_\wedge^{\phi t} \rangle dz d\rho dt \quad (3.7)$$

passes through the surface element $da_\phi^\wedge = dz d\rho$ during dt . The component $T_\wedge^{\phi t} = T_\wedge^{\phi t}$ is the volume density of an orbital momentum; infinitesimal time averaged momentum

$$dp^\phi = \langle T_\wedge^{\phi t} \rangle d\rho d\phi dz \quad (3.8)$$

is contained in the volume element $d\rho d\phi dz$. Using (3.4) yields zero (the over line marks the complex conjugation):

$$\langle T_\wedge^{\phi t} \rangle = \langle T_\wedge^{\phi t} \rangle = \Re[(\overset{\circ}{\mathbf{E}}_{1z} + \overset{\circ}{\mathbf{E}}_{4z})(\overline{\mathbf{B}}_{1\rho} + \overline{\mathbf{B}}_{4\rho}) - (\overset{\circ}{\mathbf{E}}_{1\rho} + \overset{\circ}{\mathbf{E}}_{4\rho})(\overline{\mathbf{B}}_{1z} + \overline{\mathbf{B}}_{4z})] / 2 = 0. \quad (3.9)$$

This means that no mass rotates in the Beth experiment.

The component $T_\wedge^{\phi z}$ is the flux density of an orbital momentum; infinitesimal time averaged momentum

$$dp^\phi = \langle T_\wedge^{\phi z} \rangle da_z^\wedge dt = \langle T_\wedge^{\phi z} \rangle d\rho d\phi dt \quad (3.10)$$

passes through the surface element $da_z^\wedge = d\rho d\phi$ during dt . This mean that an infinitesimal torque

$$d\tau_z = dL_z / dt = e_{z\rho\phi}^\wedge dL_\wedge^{\rho\phi} / dt = e_{z\rho\phi}^\wedge dL^{\rho\phi} \sqrt{g}_\wedge / dt = \rho dp^\phi \sqrt{g}_\wedge / dt = \langle T_\wedge^{\phi z} \rangle \rho^2 d\rho d\phi \quad (3.11)$$

acts on the surface element $da_z^\wedge = d\rho d\phi$. But

$$\langle T_{\wedge}^{\phi z} \rangle = -\Re[(\check{E}_{1\phi} + \check{E}_{4\phi})(\bar{E}_{1z} + \bar{E}_{4z}) + (\check{B}_{1\phi} + \check{B}_{4\phi})(\bar{B}_{1z} + \bar{B}_{4z})]/2\rho = 0. \quad (3.12)$$

So, no torque acts on the Beth plate according to the standard electrodynamics. Why then the plate experience the torque (1.1)?

4. Defects of the general field theory

The point is that the modern Maxwell electrodynamics cannot explain the result of the Beth experiment. So, the modern electrodynamics is not complete. We must introduce a spin tensor to explain the Beth experiment. Really, the standard classical electrodynamics starts from the free field canonical Lagrangian, which is independent on coordinates explicitly [10]

$$L_c = -F_{\mu\nu} F^{\mu\nu}/4, \quad F_{\mu\nu} = 2\partial_{[\mu} A_{\nu]}. \quad (4.1)$$

Using this Lagrangian, by the Lagrange formalism physicists obtain the canonical energy-momentum tensor

$$T_c^{\lambda\mu} = \partial^\lambda A_\alpha \frac{\partial L_c}{\partial(\partial_\mu A_\alpha)} - g^{\lambda\mu} L_c = -\partial^\lambda A_\alpha F^{\mu\alpha} + g^{\lambda\mu} F_{\alpha\beta} F^{\alpha\beta}/4, \quad (4.2)$$

and the canonical total angular momentum tensor

$$J_c^{\lambda\mu\nu} = 2x^{[\lambda} T_c^{\mu]\nu} + Y_c^{\lambda\mu\nu} \quad (4.3)$$

where

$$Y_c^{\lambda\mu\nu} = -2A^{[\lambda} \delta_\alpha^{\mu]} \frac{\partial L_c}{\partial(\partial_\nu A_\alpha)} = -2A^{[\lambda} F^{\mu]\nu}, \quad Y_c^{ij0} = \mathbf{E} \times \mathbf{A} \quad (4.4)$$

is the canonical spin tensor.

As is well known, these tensors are not electrodynamics tensors. They obviously contradict experiments, $T_c^{\lambda\mu}$ has a wrong divergence

$$\partial_\mu T_c^{\lambda\mu} = \partial^\lambda A_\sigma \partial_\kappa F^{\sigma\kappa}. \quad (4.5)$$

Physicists are forced to modify these tensors. They add specific terms [11, 12] to the canonical tensors and arrive to the standard energy-momentum tensor $\Theta^{\lambda\mu}$, the standard total angular momentum tensor $J_{st}^{\lambda\mu\nu}$, and the standard spin tensor $Y_{st}^{\lambda\mu\nu}$, which is zero,

$$\begin{aligned} \Theta^{\lambda\mu} &= T_c^{\lambda\mu} - \partial_\nu \tilde{Y}_c^{\lambda\mu\nu}/2 = -\partial^\lambda A_\nu F^{\mu\nu} + g^{\lambda\mu} F_{\alpha\beta} F^{\alpha\beta}/4 + \partial_\nu (A^\lambda F^{\mu\nu}), \\ \tilde{Y}_c^{\lambda\mu\nu} &\stackrel{\text{def}}{=} Y_c^{\lambda\mu\nu} - Y_c^{\mu\nu\lambda} + Y_c^{\nu\lambda\mu} = -2A^\lambda F^{\mu\nu}, \end{aligned} \quad (4.6)$$

$$J_{st}^{\lambda\mu\nu} = J_c^{\lambda\mu\nu} - \partial_\kappa (x^{[\lambda} \tilde{Y}_c^{\mu]\nu\kappa}), \quad (4.7)$$

$$Y_{st}^{\lambda\mu\nu} = J_{st}^{\lambda\mu\nu} - 2x^{[\lambda} \Theta^{\mu]\nu} = Y_c^{\lambda\mu\nu} + 2A^{[\lambda} F^{\mu]\nu} = 0. \quad (4.8)$$

But we all must recognize that the standard tensors have serious defects as well. These defects are:

1. $\Theta^{\lambda\mu}$ obviously contradicts experiments. It is non-symmetrical. It has wrong divergence as well

$$\partial_\mu \Theta^{\lambda\mu} = \partial_\mu T_c^{\lambda\mu} = \partial^\lambda A_\sigma \partial_\kappa F^{\sigma\kappa}. \quad (4.9)$$

Tensor Θ is never used. The Maxwell tensor (3.4),

$$T^{\lambda\mu} = -g^{\lambda\alpha} F_{\alpha\nu} F^{\mu\nu} + g^{\lambda\mu} F_{\sigma\kappa} F^{\sigma\kappa}/4, \quad (4.10)$$

is used in the electrodynamics instead of $\Theta^{\lambda\mu}$. For example, it is the Maxwell tensor that is used in the standard expression for the total angular momentum of electromagnetic field (2.1),

$$J_{st}^{\mu\nu} = 2 \int x^{[\mu} T^{\nu]\alpha} dV_\alpha, \quad \text{i.e.} \quad \mathbf{J}_{st} = \int \mathbf{r} \times (\mathbf{E} \times \mathbf{B}) dV, \quad (4.11)$$

$$\text{rather than } J_{\Theta}^{\mu\nu} = 2 \int x^{[\mu} \Theta^{\nu]\alpha} dV_\alpha, \quad \text{i.e.} \quad \mathbf{J}_\Theta = \int \mathbf{r} \times (\mathbf{E} \times \mathbf{B} - \mathbf{A} \cdot \mathbf{j}) dV. \quad (4.12)$$

2. The main defect is the absence of spin, $\underset{st}{Y}^{\lambda\mu\nu}=0$. Neither Eq. (4.11), nor Eq. (4.12) contains a spin term. In contrast to the canonical pair, $\underset{c}{T}^{\lambda\mu}$, $\underset{c}{Y}^{\lambda\mu\nu}$, the standard pair, $\Theta^{\lambda\mu}$, $\underset{st}{Y}^{\lambda\mu\nu}=0$, is defective. Standard energy-momentum tensor is not accompanied by a spin tensor.

Because of zero spin, the standard theory is not satisfactory, for example, in respects of circularly polarized light. Eqs. (1.11), (1.12) do not explain, in particular, the classical Beth experiment. Because of zero spin, a circularly polarized plane wave has no angular momentum at all in direct contradiction to quantum theory.

5. Electrodynamics' spin tensor

The Belinfante-Rosenfeld procedure [11, 12] is an attempt to derive the Maxwell tensor by using the Lagrange formalism. But now we must recognize that it is impossible, and the procedure is not fit for this purpose. The procedure gives the zero spin, $\underset{st}{Y}^{\lambda\mu\nu}=0$, and the standard energy-momentum tensor $\Theta^{\lambda\mu}$, which is even not symmetric. The standard procedure (4.6) – (4.8) is

$$\Theta^{\lambda\mu} = \underset{c}{T}^{\lambda\mu} + \underset{st}{t}^{\lambda\mu}, \quad \underset{st}{t}^{\lambda\mu} = -\partial_\nu \underset{c}{Y}^{\lambda\mu\nu}/2 = \partial_\nu (A^\lambda F^{\mu\nu}), \quad (5.1)$$

$$\underset{st}{Y}^{\lambda\mu\nu} = \underset{c}{Y}^{\lambda\mu\nu} + \underset{st}{s}^{\lambda\mu\nu} = 0, \quad \underset{st}{s}^{\lambda\mu\nu} = -\underset{c}{Y}^{\lambda\mu\nu} = 2A^{[\lambda} F^{\mu]\nu}. \quad (5.2)$$

Another way of using the canonical pair $\underset{c}{T}^{\lambda\mu}$, $\underset{c}{Y}^{\lambda\mu\nu}$ is presented in [13 - 16]. Note that the Maxwell tensor can be gained by adding a term

$$\underset{c}{t}^{\lambda\mu} = \underset{c}{T}^{\lambda\mu} - \underset{c}{T}^{\lambda\mu} = \partial_\nu A^\lambda F^{\mu\nu} \quad (5.3)$$

to the canonical energy-momentum tensor $\underset{c}{T}^{\lambda\mu}$. Here a question arises, what term $\underset{c}{s}^{\lambda\mu\nu}$, instead of $\underset{st}{s}^{\lambda\mu\nu}$, must be added to the canonical spin tensor $\underset{c}{Y}^{\lambda\mu\nu} = -2A^{[\lambda} F^{\mu]\nu}$ for changing it from the canonical spin tensor to an unknown electrodynamics spin tensor $\underset{c}{Y}^{\lambda\mu\nu} = \underset{c}{Y}^{\lambda\mu\nu} + \underset{c}{s}^{\lambda\mu\nu}$? Our answer is [13 - 16]: the addends $\underset{c}{t}^{\lambda\mu}$, $\underset{c}{s}^{\lambda\mu\nu}$ must satisfy a relationship

$$\partial_\nu \underset{c}{s}^{\lambda\mu\nu} - 2\underset{c}{t}^{[\lambda\mu]} = 0, \text{ i.e. } \partial_\nu \underset{c}{s}^{\lambda\mu\nu} - 2\partial_\alpha A^{[\lambda} F^{\mu]\alpha} = 0. \quad (5.4)$$

A simple expression

$$\underset{c}{s}^{\lambda\mu\nu} = 2A^{[\lambda} \partial^\mu] A^\nu \quad (5.5)$$

satisfies Eq. (5.4). So, the suggested electrodynamics spin tensor is

$$2\underset{e}{Y}^{\lambda\mu\nu} = \underset{c}{Y}^{\lambda\mu\nu} + \underset{c}{s}^{\lambda\mu\nu} = -2A^{[\lambda} F^{\mu]\nu} + 2A^{[\lambda} \partial^\mu] A^\nu = 2A^{[\lambda} \partial^{|\nu|} A^{\mu]}. \quad (5.6)$$

The expression (5.6) was obtained heuristically. It is not final one. Spin tensor (5.6) is obvious not symmetric in the sense of electric - magnetic symmetry. It represents only the electric field, \mathbf{E} , $\mathbf{A} = -\int \mathbf{E} dt$. A true spin tensor of electromagnetic waves must depend symmetrically on the magnetic vector potential A_a and on an electric vector potential

$$\Pi_\alpha = e_{\alpha\lambda\mu\nu} \Pi^{\lambda\mu\nu}, \quad \partial_\nu \Pi^{\lambda\mu\nu} = F^{\lambda\mu}. \quad (5.7)$$

So the spin tensor of electromagnetic waves has the form

$$\underset{e}{Y}^{\lambda\mu\nu} = \underset{e}{Y}^{\lambda\mu\nu} + \underset{m}{Y}^{\lambda\mu\nu} = A^{[\lambda} \partial^{|\nu|} A^{\mu]} + \Pi^{[\lambda} \partial^{|\nu|} \Pi^{\mu]}, \quad (5.8)$$

and the total angular momentum has the form

$$\underset{e}{J}^{\lambda\mu} = \int (2x^{[\lambda} T^{\mu]\nu} + \underset{e}{Y}^{\lambda\mu\nu}) dV_v, \text{ or } \mathbf{J} = \int \mathbf{r} \times (\mathbf{E} \times \mathbf{B}) dV + \int \underset{e}{Y}^{ij0} dV, \quad (5.9)$$

instead of (1.11), and the angular momentum (1.11) is an orbital angular momentum rather than spin.

6. Theory of the Beth's experiment

We apply the spin tensor (5.8) for an explanation of the classical Beth experiment. For short, we consider the Beth light beams as plane waves because the Poynting vector is zero, and the wall effects are of no importance. Between the half-wave plate and the quarter-wave plate we have from (3.1), (3.3) when $u = 1$,

$$\begin{aligned} \vec{E} = \vec{E}_1 + \vec{E}_4 &= \Re\{\exp[i(z-t+\phi)](\rho+i\rho\phi) + \exp[i(-z-t-\phi)](\rho-i\rho\phi)\} \\ &= 2[\rho\cos(z+\phi) - \rho\phi\sin(z+\phi)]\cos t \end{aligned} \quad (6.1)$$

$$\vec{A} = -\int \vec{E} dt = 2[\rho\cos(z+\phi) - \rho\phi\sin(z+\phi)](-\sin t) \quad (6.2)$$

So, the electrical part of the spin tensor density (5.8) is uniform, but pulses,

$$Y_e^{\rho\phi z} = g^{\phi\phi} \sqrt{g} \wedge (A_{[\rho} \partial_{|z|} A_{\phi]}) = 2\sin^2 t. \quad (6.3)$$

Calculating the magnetic part yields:

$$B = \Re\{-i(\vec{E}_1 + \vec{E}_4)\} = \Im\{\vec{E}_1 + \vec{E}_4\} = 2[-\rho\cos(z+\phi) + \rho\phi\sin(z+\phi)]\sin t, \quad (6.4)$$

$$\Pi = \int B dt = 2[\rho\cos(z+\phi) - \rho\phi\sin(z+\phi)]\cos t, \quad Y_m^{\rho\phi z} = g^{\phi\phi} \sqrt{g} \wedge (\Pi_{[\rho} \partial_{|z|} \Pi_{\phi]}) = 2\cos^2 t. \quad (6.5)$$

So, the total spin flux density is constant with t and z ,

$$Y_{\wedge}^{\rho\phi z} = Y_e^{\rho\phi z} + Y_m^{\rho\phi z} = 2. \quad (6.6)$$

The same calculation for the domain before the plate gives

$$Y_{\wedge}^{\rho\phi z}|_{\text{before}} = -2. \quad (6.7)$$

This means that, of the given handedness of polarization, total spin flux density onto the plate equals to -4 with the absence of an energy flux! This result is in accordance with (1.1).

Another applications of the spin tensor (5.8) are presented in [14 - 16] and at the web sites www.mai.ru/projects/mai_works/, www.sciprint.org. Absorption and reflection of a circularly polarized beam is calculated there, and a radiation of a rotating electrical dipole is considered in these works.

The expression (5.6) for the spin tensor was submitted to JETP Letters on May 14, 1998. This result was rejected more than 300 times by scientific journals. For example (I show an approximate number of the rejections in parentheses): JETP Lett. (8), JETP (13), TMP (10), UFN (9), RPJ (70), AJP (16), EJP (4), EPL (5), PRA (3), PRD (4), PRE (2), APP (5), FP (6), PLA (8), OC (2), JPA (4), JPB (1), JMP (4), JOPA (1), JMO (1), CJP (1), OL (1), NJP (2), arXiv (70). In particular, PLA rejected a paper 'Inner incompleteness of the Maxwell electrodynamics' submitted on Mon, 22 Jul 2002 15:52:07

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