Quantum Astrophysics

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Contents

1	Inti	roducti	ion	5				
2	Basic objections against planetary Bohr orbitology							
	2.1	Also e	xoplanets obey Bohr rules	7				
	2.2	How (General Coordinate Invariance and Lorentz invariance	-				
		are ac	hieved?	8				
		2.2.1	Quantization condition for simple systems	8				
		2.2.2	Generalization of the quantization condition	9				
		2.2.3	Rings and spokes as the basic building blocks of dark					
			matter structures	10				
3	Ger	ieral q	uantum vision about formation of structures	11				
	3.1	Simple	e quantitative model	12				
		3.1.1	The equation for gravitational acceleration	12				
		3.1.2	Quantization of angular momentum	12				
		3.1.3	Combination of the conditions	14				
	3.2	Could	one understand the different values of gravitational					
		Plancl	c constant for inner and outer planetary systems?	14				
		3.2.1	The idea	15				
		3.2.2	The failure of the idea in its simplest form	15				
		3.2.3	How could one modify the idea?	16				
	3.3	Forma	tion of rings like structures	16				
		3.3.1	Rings of Saturn and Jupiter	17				
		3.3.2	NASA Hubble Space Telescope Detects Ring of Dark	-				
			Matter	17				

	3.4	A qua	ntum model for the dark part of the central mass and	
		rings		19
		3.4.1	Four options	20
		3.4.2	Only the p-adic variant works as a model for central	
			mass	21
4	Qua	antum	chaos in astrophysical length scales	22
	4.1	Brief s	summary about quantum chaos	22
	4.2	What	does the transition to quantum chaos mean?	23
		4.2.1	The level of stationary states	23
		4.2.2	The importance of classical periodic orbits in chaotic	
			scattering	24
		4.2.3	In what sense complex systems can be called chaotic?	24
	4.3	Quant	um chaos in astrophysical scales?	25
		4.3.1	Quantum criticality	25
		4.3.2	Dark matter structures as generalization of periodic	
			orbits	26
		4.3.3	Quantal chaos in gravitational scattering?	27
5	Gra	vitatio	onal radiation and large value of gravitational Planc	k
	con	stant		27
	con 5.1	stant Standa	ard view about gravitational radiation	27 28
	con 5.1	stant Standa 5.1.1	ard view about gravitational radiation Gravitational radiation and the sources of gravita-	27 28
	con 5.1	stant Standa 5.1.1	ard view about gravitational radiation	272828
	con : 5.1	stant Standa 5.1.1 5.1.2	ard view about gravitational radiation Gravitational radiation and the sources of gravita- tional waves	 27 28 28 28 28
	con 5.1 5.2	stant Standa 5.1.1 5.1.2 Model	ard view about gravitational radiation Gravitational radiation and the sources of gravita- tional waves	 27 28 28 28 29
	con 5.1 5.2	stant Stands 5.1.1 5.1.2 Model 5.2.1	ard view about gravitational radiation Gravitational radiation and the sources of gravita- tional waves	 27 28 28 29 29
	con 5.1 5.2	stant Standa 5.1.1 5.1.2 Model 5.2.1 5.2.2	ard view about gravitational radiationGravitational radiation and the sources of gravita-tional wavesHow to detect gravitational radiation?for dark gravitonsGravitons in TGDEmission of dark gravitons	 27 28 28 28 29 29 30
	con 5.1 5.2	stant Stand 5.1.1 5.1.2 Model 5.2.1 5.2.2 5.2.3	ard view about gravitational radiation	 27 28 28 29 30 32
	con 5.1 5.2	stant Stand: 5.1.1 5.1.2 Model 5.2.1 5.2.2 5.2.3 5.2.4	ard view about gravitational radiation and the sources of gravitational waves Gravitational waves How to detect gravitational radiation? for dark gravitons Gravitons in TGD Emission of dark gravitons Model for the giant graviton Dark graviton as topological light ray	27 28 28 29 29 30 32 33
	con 5.1 5.2 5.3	stant Standa 5.1.1 5.1.2 Model 5.2.1 5.2.2 5.2.3 5.2.4 Detect	ard view about gravitational radiation and the sources of gravitational waves Gravitational radiation and the sources of gravitational waves How to detect gravitational radiation? How to detect gravitational radiation? Gravitons in TGD Emission of dark gravitons Model for the giant graviton Dark graviton as topological light ray tion of gravitational radiation	27 28 28 29 29 30 32 33 33
	con 5.1 5.2 5.3	stant Stand 5.1.1 5.1.2 Model 5.2.1 5.2.2 5.2.3 5.2.4 Detect 5.3.1	ard view about gravitational radiation and the sources of gravitational waves Gravitational radiation and the sources of gravitational waves How to detect gravitational radiation? How to detect gravitational radiation? Gravitons in TGD Emission of dark gravitons Model for the giant graviton Dark graviton as topological light ray TGD counterpart for the classical description of de-	 27 28 28 29 29 30 32 33 33
	con 5.1 5.2 5.3	stant Stand: 5.1.1 5.1.2 Model 5.2.1 5.2.2 5.2.3 5.2.4 Detect 5.3.1	ard view about gravitational radiation Gravitational radiation and the sources of gravita- tional waves	 27 28 28 29 29 30 32 33 33 34
	con 5.1 5.2 5.3	stant Stand 5.1.1 5.1.2 Model 5.2.1 5.2.2 5.2.3 5.2.4 Detect 5.3.1 5.3.2	ard view about gravitational radiation	27 28 28 29 29 30 32 33 33 33 33 34 34
	con 5.1 5.2 5.3	stant Stand 5.1.1 5.1.2 Model 5.2.1 5.2.2 5.2.3 5.2.4 Detect 5.3.1 5.3.2 5.3.3	ard view about gravitational radiationGravitational radiation and the sources of gravita-tional wavesHow to detect gravitational radiation?for dark gravitonsGravitons in TGDEmission of dark gravitonsModel for the giant gravitonDark graviton as topological light raytion of gravitational radiationTGD counterpart for the classical description of de-tection processSequential de-coherenceInformation theoretic aspects	27 28 28 29 29 30 32 33 33 33 33 34 34 35
	con 5.1 5.2 5.3	stant Stand: 5.1.1 5.1.2 Model 5.2.1 5.2.2 5.2.3 5.2.4 Detect 5.3.1 5.3.2 5.3.3 5.3.4	ard view about gravitational radiation Gravitational radiation and the sources of gravita- tional waves	 27 28 28 29 29 30 32 33 33 34 34 35
	con 5.1 5.2 5.3	stant Stand: 5.1.1 5.1.2 Model 5.2.1 5.2.2 5.2.3 5.2.4 Detect 5.3.1 5.3.2 5.3.3 5.3.4	ard view about gravitational radiation Gravitational radiation and the sources of gravita- tional waves	27 28 28 29 29 30 32 33 33 33 34 34 35 36
	con 5.1 5.2 5.3	stant Stand 5.1.1 5.1.2 Model 5.2.1 5.2.2 5.2.3 5.2.4 Detect 5.3.1 5.3.2 5.3.3 5.3.4 Quant	ard view about gravitational radiation Gravitational radiation and the sources of gravita- tional waves	27 28 28 29 29 30 32 33 33 33 34 34 35 36 36
	con 5.1 5.2 5.3	stant Stand: 5.1.1 5.1.2 Model 5.2.1 5.2.2 5.2.3 5.2.4 Detect 5.3.1 5.3.2 5.3.3 5.3.4 Quant 5.4.1	ard view about gravitational radiation Gravitational radiation and the sources of gravita- tional waves	27 28 28 29 29 30 32 33 33 33 33 34 34 34 35 36 36

	5.4.2	The direct detection of giant graviton is not possible						
		for long distances	37					
	5.4.3	Secondary detection	38					
	5.4.4	Some quantitative estimates for gravitational quan-						
		tum transitions in planetary system	39					
5.5	Gener	alization to gauge interactions	41					
	5.5.1	Some applications	41					
	5.5.2	In what sense dark matter is dark	42					
Appendix: Orbital radii of exoplanets as a test for the the-								
ory			43					

6

Abstract

The vision that the quantum dynamics for dark matter is behind the formation of the visible structures suggests that the formation of the astrophysical structures could be understood as a consequence of Bohr rules.

Since space-time surfaces are 4-surfaces in $H = M^4 \times CP_2$, Bohr rules can be formulated in a manner which is general coordinate invariant and Lorentz invariant. The rules are actually for dark matter structures obeying Z_n symmetry for very large *n* characterizing the symmetry of field bodies associated with the structure in question. One can say that orbit becomes particle at the level of dark matter. Circles and spokes representing the dark matter structures, gravi-electric flux quanta, and also circles representing gravi-magnetic flux tubes orthogonal to the quantization plane become basic building blocks of dark matter structures. Simplest of them are rings and cart-wheel like structures. The subgroups of Z_n can act as approximate symmetries of visible matter and if one accepts ruler-and-compass hypothesis very powerful predictions follow.

Concerning Bohr orbitology in astrophysical length scales, the basic observation is that in the case of a straight cosmic string creating a gravitational potential of form v_1^2/ρ Bohr quantization does not pose any conditions on the radii of the circular orbits so that a continuous mass distribution is possible. This situation is obviously exceptional. If one however accepts the TGD based vision that the very early cosmology was cosmic string dominated and that elementary particles were generated in the decay of cosmic strings, this situation might have prevailed at very early times. If so, the differentiation of a continuous density of ordinary matter to form the observed astrophysical structures would correspond to an approach to a stationary situation governed by Bohr rules for dark matter and in the first approximation one could neglect the intermediate stages.

This general picture is applied by considering some simple models for astrophysical systems involving planar structures. There are several universal predictions. Velocity spectrum is universal and only the Bohr radii depend on the choice of mass distribution. The inclusion of cosmic string implies that the system associated with the central mass is finite. Quite generally dark parts of astrophysical objects have shell like structure like atoms as do also ring like structures.

p-Adic length scale hypothesis provides a manner to obtain a realistic model for the central objects meaning a structure consisting of shells coming as half octaves of the basic radius: this obviously relates to Titius-Bode law. Also a simple model for planetary rings is obtained. Bohr orbits do not follow cosmic expansion which is obtained only in the average sense if phase transitions reducing the value of basic parameter v_0 occur at preferred values of cosmic time. This explains why v_0 has different values and also the decomposition of planetary system to outer and inner planets with different values of v_0 .

TGD Universe is guantum critical and guantum criticality corresponds very naturally to what has been identified as the transition region to quantum chaos. The basic formulation of quantum TGD is indeed consistent with what has been learned from the properties of quantum chaotic systems and quantum chaotic scattering. Wave functions are concentrated around Bohr orbits in the limit of quantum chaos, which is just what dark matter picture assumes. In this framework the chaotic motion of astrophysical object becomes the counterpart of quantum chaotic scattering and the description in terms of classical chaos is predicted to fail. By Equivalence Principle the value of the mass of the object does not matter at all so that the motion of sufficiently light objects in solar system might be understandable only as quantum chaotic scattering. The motion of gravitationally unbound comets and rings of Saturn and Jupiter and the collisions of galactic structures known to exhibit the presence of cart-wheel like structures define possible applications.

The description of gravitational radiation provides a stringent test for the idea about dark matter hierarchy with arbitrary large values of Planck constants. In accordance with quantum classical correspondence, one can take the consistency with classical formulas as a constraint allowing to deduce information about how dark gravitons interact with ordinary matter. The standard facts about gravitational radiation are discussed first and then TGD based view about the situation is sketched.

1 Introduction

The mechanisms behind the formation of planetary systems, galaxies and larger systems are poorly understood but planar structures seem to define a common denominator and the recent discovery of dark matter ring in a galactic cluster in Mly scale [7] suggest that dark matter rings might define a universal step in the formation of astrophysical structures.

Also the dynamics in planet scale is poorly understood. In particular, the rings of Saturn and Jupiter are very intricate structures and far from wellunderstood. Assuming spherical symmetry it is far from obvious why the matter ends up to form thin rings in a preferred plane. The latest surprise [2] is that Saturn's largest, most compact ring consist of clumps of matter separated by almost empty gaps. The clumps are continually colliding with each other, highly organized, and heavier than thought previously.

The situation suggests that some very important piece might be missing

from the existing models, and the vision about dark matter as a quantum phase with a gigantic Planck constant [A9] is an excellent candidate for this piece. The vision that the quantum dynamics for dark matter is behind the formation of the visible structures suggests that the formation of the astrophysical structures could be understood as a consequence of Bohr rules [D6].

Since space-time surfaces are 4-surfaces in $H = M^4 \times CP_2$, Bohr rules can be formulated in a manner which is general coordinate invariant and Lorentz invariant. The rules are actually for dark matter structures obeying Z_n symmetry for very large *n* characterizing the symmetry of field bodies associated with the structure in question. One can say that orbit becomes particle at the level of dark matter. Circles and spokes representing the dark matter structures, gravi-electric flux quanta, and also circles representing gravi-magnetic flux tubes orthogonal to the quantization plane become basic building blocks of dark matter structures. Simplest of them are rings and cart-wheel like structures. The subgroups of Z_n can act as approximate symmetries of visible matter and if one accepts ruler-and-compass hypothesis powerful predictions follow.

TGD Universe is quantum critical and quantum criticality corresponds very naturally to what has been identified as the transition region to quantum chaos. The basic formulation of quantum TGD is indeed consistent with what has been learned from the properties of quantum chaotic systems and quantum chaotic scattering [9]. Wave functions are concentrated around Bohr orbits in the limit of quantum chaos, which is just what dark matter picture assumes. In this framework the chaotic motion of astrophysical object becomes the counterpart of quantum chaotic scattering and classical description is predicted to fail. By Equivalence Principle the value of the mass of the object does not matter at all so that the motion of sufficiently light objects in solar system might be understandable only as quantum chaotic scattering. The motion of gravitationally unbound comets and rings of Saturn and Jupiter and the collisions of galactic structures known to exhibit the presence of cart-wheel like structures define possible applications.

The description of gravitational radiation provides a stringent test for the idea about dark matter hierarchy with arbitrary large values of Planck constants. In accordance with quantum classical correspondence, one can take the consistency with classical formulas as a constraint allowing to deduce information about how dark gravitons interact with ordinary matter. The standard facts about gravitational radiation are discussed first and then TGD based view about the situation is sketched.

The planetary Bohr orbitology has been already discussed in the chap-

ter "TGD and Astrophysics" [D6] with applications solar system and exoplanets. Instead of repeating this discussion, a formulation of these rules which is general coordinate invariant and Lorentz invariant is proposed.

2 Basic objections against planetary Bohr orbitology

There are two objections against planetary Bohr orbitology.

a) The success of this approach in the case solar system [D6] is not enough. In particular, it requires different values of v_0 for inner and outer planets.

b) The basic objection of General Relativist against the planetary Bohr orbitology model is the lack of the manifest General Goordinate and Lorentz invariances. In GRT context this objection would be fatal. In TGD framework the lack of these invariances is only apparent.

2.1 Also exoplanets obey Bohr rules

I have discussed a simple model explaining why inner and outer planets must have different values of v_0 by taking into account cosmic string contribution to the gravitational potential which is negligible nowadays but was not so in primordial times. Among other things this implies that planetary system has a finite size, at least about 1 ly in case of Sun (nearest star is at distance of 4 light years).

Quantization rules have been applied to exoplanets in the case that the central mass and orbital radius are known (the discussion is moved from the chapter "Astrophysics" to the the Appendix of this chapter). Errors are around 10 per cent for the most favored value of $v_0 = 2^{-11}$. The "anomalous" planets with very small orbital radius correspond to n = 1 Bohr orbit (n = 3 is the lowest orbit in solar system). The universal velocity spectrum $v = v_0/n$ in simple systems perhaps the most remarkable prediction and certainly testable: this alone implies that the Bohr radius GM/v_0^2 defines the universal size scale for systems involving central mass. Obviously this is something new and highly non-trivial.

The recently observed dark ring in MLy scale is a further success and also the rings and Moons of Saturn and Jupiter obey the same universal length scale $(n \ge 5 \text{ and } v_0 \rightarrow (16/15) \times v_0 \text{ and } v_0 \rightarrow 2 \times v_0)$.

There is a further objection. For our own Moon orbital radius is much larger than Bohr radius for $v_0 = 2^{-11}$: one would have $n \simeq 138$. $n \simeq 7$

results for $v_0 \rightarrow v_0/20$ giving $r_0 \simeq 1.2R_E$. The small value of v_0 could be understood to result from a sequence of phase transitions reducing the value of v_0 to guarantee that solar system participates in the average sense to the cosmic expansion and from the fact inner planets are older than outer ones in the proposed scenario. The findings of Masreliez [6] discussed in the last section of [D6] support the prediction that planetary system does not participate cosmic expansion in a smooth manner.

2.2 How General Coordinate Invariance and Lorentz invariance are achieved?

One can use Minkowski coordinates of the M^4 factor of the imbedding space $H = M^4 \times CP_2$ as preferred space-time coordinates. The basic aspect of dark matter hierarchy is that it realizes quantum classical correspondence at space-time level by fixing preferred M^4 coordinates as a rest system. This guarantees preferred time coordinate and quantization axis of angular momentum. The physical process of fixing quantization axes thus selects preferred coordinates and affects the system itself at the level of space-time, imbedding space, and configuration space (world of classical worlds). This is definitely something totally new aspect of observer-system interaction.

One can identify in this system gravitational potential Φ_{gr} as the g_{tt} component of metric and define gravi-electric field E_{gr} uniquely as its gradient. Also gravi-magnetic vector potential A_{gr} and and gravi-magnetic field B_{qr} can be identified uniquely.

2.2.1 Quantization condition for simple systems

Consider now the quantization condition for angular momentum with Planck constant replaced by gravitational Planck constant $\hbar_{gr} = GMm/v_0$ in the simple case of point like central mass. The condition is

$$m \oint v \bullet dl = n \times h bar_{gr}$$
 (1)

The condition reduces to the condition on velocity circulation

$$\oint v \bullet dl = n \times \frac{GM}{v_0} . \tag{2}$$

In simple systems with circular rings forced by Z_n symmetry the condition reduces to a universal velocity spectrum $v = v_0/n$ so that only the radii of orbits depend on mass distribution. For systems for which cosmic string dominates only n = 1 is possible. This is the case in the case of stars in galactic halo if primordial cosmic string going through the center of galaxy in direction of jet dominates the gravitational potential. The velocity of distant stars is correctly predicted.

 Z_n symmetry seems to imply that only circular orbits need to be considered and there is no need to apply the condition for other canonical momenta (radial canonical momentum in Kepler problem). The nearly circular orbits of visible matter objects would be naturally associated with dark matter rings or more complex structures with Z_n symmetry and dark matter rings could suffer partial or complete phase transition to visible matter. Note however that radial Z_n symmetry allows also cart-wheel like structures with radial spokes which correspond to n = 0 Bohr orbits.

2.2.2 Generalization of the quantization condition

By Equivalence Principle dark ring mass disappears from the quantization conditions and the left hand side of the quantization condition equals to a generalized velocity circulation applying when central system rotates

$$\oint (v - A_{gr}) \bullet dl. \tag{3}$$

Here one must notice that dark matter ring is Z_n symmetric and closed so that the geodesic motion of visible matter cannot correspond strictly to the dark matter ring (perihelion shift of Mercury). Just by passing notice that the presence of dark matter ring can explain also the complex braidings associated with the planetary rings.

The right hand side of the quantization condition would be the generalization of GM by the replacement

$$GM \to \oint e \bullet r^2 E_{gr} \times dl.$$
 (4)

e is a unit vector in direction of quantization axis of angular momentum, \times denotes cross product, and r is the radial M^4 coordinate in the preferred system. Everything is Lorentz and General Coordinate Invariant and for Schwartschild metric this reduces to the expected form and reproduces also the contribution of cosmic string to the quantization condition correctly.

2.2.3 Rings and spokes as the basic building blocks of dark matter structures

The Bohr orbit model for the planetary orbits based on the hierarchy of dark matter relies in an essential manner on the idea that macroscopic quantum phases of dark matter dictate to a high degree the behavior of the visible matter. Dark matter is concentrated on closed classical orbits in the simple rotationally symmetric gravitational potentials involved. Orbits become basic structures instead of points at the level of dark matter. A discrete subgroup Z_n of rotational group with very large n characterizes dark matter structures quite generally. At the level of visible matter this symmetry can be broken to approximate symmetry defined by some subgroup of Z_n .

Circles and radial spokes are the basic Platonic building blocks of dark matter structures. The interpretation of spokes would be as (gravi-)electric flux tubes. Radial spokes correspond to n = 0 states in Bohr quantization for hydrogen atom and orbits ending into atom. Spokes have been observed in planetary rings besides decomposition to narrow rings and also in the galactic scale [10]. Also flux tubes of (gravi-)magnetic fields with Z_n symmetry define rotational symmetric structures analogous to quantized dipole fields.

Gravi-magnetic flux tubes indeed correspond to circles rather than field lines of a dipole field for the simplest model of gravi-magnetic field, which means deviation from GRT predictions for gravi-magnetic torque on gyroscope outside equator: unfortunately the recent experiments are performed at equator. The flux tubes be seen only as circles orthogonal to the preferred plane and planetary Bohr rules apply automatically also now.

A word of worry is in order here. Ellipses are very natural objects in Bohr orbitology and for a given value of n would give $n^2 - 1$ additional orbits. In planetary situation they would have very large eccentricities and are not realized. Comets can have closed highly eccentric orbits and correspond to large values of n. In any case, one is forced to ask whether the exactly Z_n symmetric objects are too Platonic creatures to live in the harsh real world. Should one at least generalize the definition of the action of Z_n as symmetry so that it could rotate the points of ellipse to each other. This might make sense. In the case of dark matter ellipses the radial spokes with Z_n symmetry representing radial gravito-electric flux quanta would still connect dark matter ellipse to the central object and the rotation of the spoke structure induces a unique rotation of points at ellipse.

3 General quantum vision about formation of structures

The basic observation is that in the case of a straight cosmic string creating a gravitational potential of form v_1^2/ρ Bohr quantization does not pose any conditions on the radii of the circular orbits so that a continuous mass distribution is possible.

This situation is obviously exceptional. If one however accepts the TGD based vision [D5] that the very early cosmology was cosmic string dominated and that elementary particles were generated in the decay of cosmic strings, this situation might have prevailed at very early times. If so, the differentiation of a continuous density of ordinary matter to form the observed astrophysical structures would correspond to an approach to a stationary situation governed by Bohr rules and in the first approximation one could neglect the intermediate stages.

Cosmic string need not be infinitely long: it could branch into n return flux tubes, n very large in accordance with the Z_n symmetry for the dark matter but also in this case the situation in the nearby region remains the same. For large distances the whole structure would behave as a single mass point creating ordinary Newtonian gravitational potential. Also phase transitions in which the system emits magnetic flux tubes so that the contribution of the cosmic string to the gravitational force is reduced, are possible.

What is of utmost importance is that the cosmic string induces the breaking of the rotational symmetry down to a discrete Z_n symmetry and in the presence of the central mass selects a unique preferred orbital plane in which gravitational acceleration is parallel to the plane. This is just what is observed in astrophysical systems and not easily explained in the Newtonian picture. In TGD framework this relates directly to the choice of quantization axis of angular momentum at the level of dark matter. This mechanism could be behind the formation of planar systems in all length scales including planets and their moons, planetary systems, galaxies, galaxy clusters in the scale of Mly, and even the concentration of matter at the walls of large voids in the scale of 100 Mly.

For the visible matter Z_n symmetry can break down to an approximate symmetry corresponding to a subgroup of $Z_m \subset Z_n$, and if one accepts the ruler-and-compass hypothesis for favored values of n, very strong prediction that the subgroup corresponds to m which is product of different Fermat primes and power of 2, follows. Simplest Z_m symmetric visible structures would be cart-wheel like structures consisting of rings with spokes.

3.1 Simple quantitative model

The following elementary model allows to see how the addition of central mass forces the matter to quantized Bohr orbits via the formation of dark matter rings.

3.1.1 The equation for gravitational acceleration

The elementary model for circular orbits involves two equations: the identification radial kinetic acceleration with the acceleration due to the gravitational force and the condition stating quantization of the angular momentum, which requires some additional thought when cosmic string has infinite length.

In cylindrical coordinates the gravitational acceleration due to cosmic string is given by

$$a = \frac{v_1^2}{\rho} ,$$

$$v_1^2 = G \frac{dM}{dL} .$$
(5)

Here v_1 is the rotational velocity of the matter around cosmic string neglecting its own gravitational effects.

The condition for the radial acceleration gives

$$u = \frac{1}{\rho} = \frac{v^2 - v_1^2}{GM} \ . \tag{6}$$

3.1.2 Quantization of angular momentum

The condition for the quantization of angular momentum is not quite obvious since taking into account the mass of entire cosmic string would give an infinite Planck constant. The resolution of the problem relies on the effective 2-dimensionality and Z_n symmetry of the dark matter meaning that it forms rings.

Consider first the situation when only cosmic is present. For dark matter rings it is angular momentum per unit length which is quantized so that Planck constant is replaced with Planck constant per unit length. Hence one has

$$\frac{d\hbar}{dl} = G \times \frac{m}{2\pi} \times \frac{dM}{dL} \times \frac{1}{v_0} = \frac{m}{2\pi} \times \frac{v_1^2}{v_0} .$$
(7)

where m is the mass of dark matter ring. The inclusion of 2π is necessary in order to obtain internal consistency.

The quantization condition for the circular orbits in the presence of only cosmic string would read as

$$\frac{dm}{dl} \times v\rho = n \times \frac{d\hbar}{dl} = n \times \frac{m}{2\pi} \times \frac{v_1^2}{v_0} .$$
(8)

By using $dm/dl = m/2\pi\rho$, one obtains

$$v = n \frac{v_1^2}{v_0}$$
 . (9)

Only n = 1 is consistent with $v = v_1^2/v_0$ resulting from the condition for the radial acceleration and there is no condition on ρ .

The contribution of the cosmic string to the Planck constant can be identified as

$$\hbar(string) = m \times \frac{v_1^2}{v_0} \rho . \qquad (10)$$

One can say that a length ρ of cosmic string contributes to the Planck constant, and that the active part of that cosmic string and point on ring define an equilateral triangle with sides 1 and $\sqrt{5}$ so that Golden Mean emerges.

The generalization of this equation to the case when also central mass is present reads as

$$v\rho = n \frac{GM + \frac{v_1^2}{v_0}\rho}{v_0} .$$
 (11)

This gives the quantization condition

$$u = \frac{vv_0 - nv_1^2}{nGM} . (12)$$

3.1.3 Combination of the conditions

The two equations for $u = 1/\rho$ fix the spectrum of velocities and orbital radii. By introducing the parameter $v_1/v_0 = \epsilon$ and the variable $x = v/v_0$ one can write the basic equation as

$$x^2 - \frac{x}{n} = 0 . (13)$$

The solutions are x = 0 and x = 1/n. Only the latter solution corresponds to u > 0. The same spectrum $v = v_0/n$ of velocities is obtained as in the case of hydrogen atom model so that only the radii are modified. The universality of the velocity spectrum corresponds to the reduction of the quantization of angular momentum to that of circulation implied by the Equivalence Principle.

The radii of the orbits are given by

$$\rho(n) = \frac{n^2}{1 - n^2 \epsilon^2} \times r_0 ,$$

$$r_0 = \frac{GM}{v_0^2} .$$
(14)

For small values of n one obtains Bohr orbits for hydrogen atom like model. For n = 1 there is an upwards scaling of Bohr radius by $1/(1 - \epsilon^2)$. For large values of n the distances between sub-sequent radii begin to rapidly increase and at the limit $n \to 1/\epsilon$ the radius becomes infinite. Hence only $n < 1/\epsilon$ orbits are possible meaning that the system has necessarily a finite size for a given value of v_0 . Several values of v_0 are however suggested by the Bohr orbit model for the solar system.

3.2 Could one understand the different values of gravitational Planck constant for inner and outer planetary systems?

The model can be applied also to the solar system. Indeed, a cosmic string in the direction of rotation axis is predicted by the TGD inspired model for the final state of the star (or any astrophysical object [D3]). This string plays an important role in the TGD inspired model for gamma ray bursts from pulsars [D6].

In the simplest Bohr model for the solar system outer planets correspond to a smaller value of v_0 than inner planets ($v_0 = 2^{-11} \rightarrow v_0/5$). This is a grave objection against the model [D6].

3.2.1 The idea

One might hope that the inclusion of the gravitational force of cosmic string could explain the failure of the Bohr orbit model with single value of v_0 and give a physical justification for the modification of the value of v_0 . Indeed, only a finite number of Bohr orbits are possible in the presence of cosmic string and the radii for large $n \to \epsilon = v_1/v_0$ become infinite. For $\epsilon = v_1/v_0 = 1/5$ the radius of Earth's orbit with n = 5 would be infinite so that ϵ must be considerably smaller.

One can however consider the possibility that $1/5 > \epsilon > 1/10$ so that only inner planets would be allowed (first outer planet would correspond to n = 10 for v_0). If this were the case the phase transition reducing the value of v_0 to say $v_0/5$ would necessarily occur for the outer planets. Outer planetary system could be seen as a scaled up variant of the inner planetary system. $\epsilon \to \epsilon/5$ would scale up the upper bound for n by factor 5 and thus the upper bound for the radii of outer planets by factor 25. Note that if some fraction of the flux of cosmic string returns back in some length scale in the region between inner and outer planets ϵ is further reduced.

3.2.2 The failure of the idea in its simplest form

Unfortunately the proposed idea does not survive quantitative tests as such. The presence of an additional acceleration due to the cosmic string means that for circular orbits with a given radius the value of velocity is larger than that predicted by Newton's theory $(v^2 \rightarrow v^2 + v_1^2)$. This acceleration can be parameterized as

$$a = \epsilon^2 \times \frac{r_E}{r} \times \frac{v_0^2}{AU} = \epsilon^2 \times \left(\frac{r_E}{r}\right) \times .15 \ \frac{m}{s^2} \ . \tag{15}$$

In the region between Jupiter and Earth one certainly has $\epsilon < 30$ and this would give $a < 1.7 \times 10^{-4} (r_E/R) \text{ m/s}^2$.

This would mean the presence of an anomalous inwards radial acceleration v_1^2/ρ directed towards the rotation axis of the solar system. This acceleration is not probably related to the anomalous *constant* acceleration found for space-crafts [5, D6] and having the value $a = (8.74 \pm 1.33) \times 10^{-10}$ m/s². This bound is certainly satisfied for the $\epsilon < 2 \times 10^{-4}$.

The bound means that the Bohr orbit model is excellent for $n < 5 \times 10^3$ meaning size scale of order light year (the distance to nearest star is about 4 light years). Hence it would seem that the decomposition to inner and outer planetary systems cannot be due to the impossibly to have outer planets for

 $v_0 = 2^{-11}$. One must be however be very cautious since this process might have occurred during very early stage of the planetary evolution.

3.2.3 How could one modify the idea?

The proposed idea is too beautiful to be given up without fighting.

a) One could imagine that most of the cosmic string flux returns back in the region between inner and outer planets so that the outer planets would see only a very small cosmic string contribution to the gravitational force but somewhat larger solar mass. Very probably such a large value of the anomalous acceleration for inner planets would have been discovered as anomalously large velocities of the inner planets.

b) According to Masreliez [6] planetary radii seem to be shrinking with a velocity compensating exactly the cosmic expansion velocity. Bohr quantization allows to understand this effect [D6] and it has no connection with the cosmic string contribution to the gravitational force. The M^4 radial coordinate is the natural radial coordinate in the Bohr orbit model and the radii of Bohr orbits remain constant so that planetary system does not participate the cosmic expansion. This means that the radii measured in the Robertson-Walker coordinate $r = r_M/a$ appear to shrink.

This raises the possibility that cosmic expansion of the solar system has taken place in average sense involving discrete sequence of phase transitions reducing the value of ϵ . If this were the case, the decomposition to the inner and outer planetary systems might have taken place during the primordial stage. Inner and outer planetary systems could be also imagined to have originated from two separate mass shells emanating from the Sun and expanding via a discrete sequence of phase transitions reducing the value of the cosmic string tension. Part of the return flux and the flux on the rotation axes compensate each other. This could give rise to an emission of closed magnetic flux tubes.

3.3 Formation of rings like structures

One can consider an initial situation in which one has a continuous mass density rotating with a constant velocity around cosmic string defining the rotation axis of the planet. The situation is inherently unstable and a small perturbation forces the accumulation of both dark and visible matter to Bohr orbits and the upper bound for the value of n implies finite size of the system proportional to the central mass.

3.3.1 Rings of Saturn and Jupiter

The rings of Saturn and Jupiter [3, 4] could be seen a intermediate states in the process leading to the formation of satellites. Both planets indeed possess a large number of satellites [3, 4]. This would suggest that Saturn and Jupiter and outer planets in general are younger than the inner planets in accordance with the different values of v_0 . The orbital radii for lowest satellites correspond to $v_0 \rightarrow 16/15v_0$, and n = 5 for Saturn and $v_0 \rightarrow 2v_0$ and n = 5 for Jupiter from the requirement that the two lowest satellites correspond in a reasonable approximation to the two lowest Bohr orbits. The radii of satellites do not directly correspond to the radii for Bohr orbits. Also the formation of inner and outer satellite systems differing by a fractal scaling from each other can be considered. Same mechanism would be at work in all length scales and the recently observed dark matter ring associated with a galactic cluster could result by a similar mechanism [7].

The hierarchy of dark matters continues to elementary particle level and the differentiation by Bohr rules continues down to these levels. In particular, the formation of clumps of matter in Saturn rings [2] could be seen as a particular instance of this process.

The Z_n symmetry for the dark matter with very large n citePlanck suggests the possibility of more precise predictions. If n is a ruler-and-compass integer it has as factors only first powers of Fermat primes and a very large power of 2. The breaking of Z_n symmetry at the level of visible matter would naturally occur to subgroups $Z_m \subset Z_n$. Since m is a factor of n, the average number of matter clumps could tend to be a factor of n, and hence a ruler-and-compass integer. Also the hexagonal symmetry discovered near North Pole of Saturn [1] could have interpretation in terms of this symmetry breaking mechanism.

3.3.2 NASA Hubble Space Telescope Detects Ring of Dark Matter

The following announcement caught my attention during my morning webwalk.

NASA will hold a media teleconference at 1 p.m. EDT on May 15 to discuss the strongest evidence to date that dark matter exists. This evidence was found in a ghostly ring of dark matter in the cluster CL0024+17, discovered using NASA's Hubble Space Telescope. The ring is the first cluster to show a dark matter distribution that differs from the distribution of both the galaxies and the hot gas. The discovery will be featured in the May 15 issue of the Astrophysical Journal.

"Rings" puts bells ringing! Recall that in TGD Universe dark matter characterized by a gigantic value of constant [A9] making dark matter a macroscopic quantum phase in astrophysical length and time scales. Rotationally symmetric structures - such as rings- with an exact rotational symmetry Z_n , $n = GMm/v_0$ very large, of the "field body" of the system, is the basic prediction. In the model of planetary orbits the rings of dark matter around Bohr orbits force the visible matter at Bohr orbits. Rings, and also shell like structures, are expected in all length scales, even that for galaxy clusters and large voids.

Recall that the number theoretic hypothesis for the preferred values of Planck constants states that the gravitational Planck constant i/p_{ij}

$$\hbar = \frac{GMm}{v_0}$$

equals to a ruler-and-compass rational which is ratio $q = n_1/n_2$ of ruler-andcompass integers n_i expressible as a product of form $n = 2^k \prod F_s$, where all Fermat primes F_s are different. Only four of them are known and they are given by 3, 5, 17, 257, $2^{16} + 1$. $v_0 = 2^{-11}$ applies to inner planets and $v_0 = 2^{-11}/5$ to outer planets and the conditions from the quantization of hbar are satisfied.

The obvious TGD inspired hypothesis is that the dark matter ring corresponds to Bohr orbit. If so, the radius of the ring is given by

$$r_n = n^2 r_0$$

where r_0 is Bohr radius and n is integer. The Bohr radius is given

$$r_0 = \frac{GM}{v_0^2}$$

where one has $1/v_0 = k \times 2^{11}$, k a small integer with preferred value k = 1. M is the total mass in the dense core region inside the ring. This would give a radius of about 2000 times Schwartschild radius for the lowest orbit.

This prediction can be confronted with the data [7].

a) From the "Summary and Conclusions" of the article the radius of the ring is about .4 Mpc, which makes in a good approximation r=1.2 Mly. The ring corresponds actually to a bump in the interval 60"-85" centered at 75" (figure 10 of [7] gives idea about the bump). The mass in the dense core within radius which is almost half of the ring radius is about $M = 1.5 \times 10^{14} \times M_{Sun}$. The mass estimate based on gravitational lensing gives

 $M = 1.8 \times 10^{14} \times M_{Sun}$. If the gravitational lensing involves dark mass not in the central core, the first value can be used as the estimate. The Bohr radius this system is therefore

$$r_0 = 1.5 \times 10^{14} \times r_0(Sun)$$
,

where I have assumed $v_0 = 2^{-11}$ as for the inner planets in the model for the solar system.

b) The Bohr orbit for our planetary system predicts correctly Mercury's orbital radius as n=3 Bohr orbit for $v_0 = 2^{-11}$ so that one has

$$r_0(Sun) = \frac{r_M}{9}$$

where r_M is Mercury's orbital radius. This gives

$$r_0 = 1.5 \times 10^{14} \times \frac{r_M}{9}$$

Mercury's orbital radius is in a good approximation $r_M = .4$ AU = .016 ly. This gives $r_0 = 11$ Mly to be compared with $r_0 = 1$ 1.2 Mly deduced from the observations. The result is 9 times too large.

c) If one replaces v_0 with $3v_0$ one obtains downwards scaling by a factor of 1/9, which gives $r_0 = 1.2$ Mly which can be found from the Summary and Conclusions of [7]. The general hypothesis indeed allows to scale v_0 by a factor 3.

d) If one considers instead of Bohr orbits genuine solutions of Schrödinger equation then only n > 1 structures can correspond to rings like structures. Minimal option would be n = 2 with v_0 replaced with $6v_0$.

The conclusion would be that the ring could correspond to the lowest possible Bohr orbit for $v_0 = 3 \times 2^{-11}$. I would have been really happy if the favored value of v_0 had appeared in the formula but the consistency with the ruler-and-compass hypothesis serves as a consolation. Skeptic can of course always argue that this is a pure accident. If so, it would be an addition to long series of accidents (planetary radii in solar system and radii of exoplanets). One can of course search rings at radii corresponding to n=2,3,... If these are found, I would say that the situation is settled.

3.4 A quantum model for the dark part of the central mass and rings

It is interesting to look for a simple quantum model for the dark part of the central mass and possibly also of rings. As a first approximation one can consider a cylindrically symmetric pan-cake of height L and radius R. Approximate spherical symmetry suggest L = 2R.

The governing conditions are

$$v^{2}(\rho) = G(dM/dl)(\rho) + v_{1}^{2} ,$$

$$v(\rho) = \frac{v_{0}}{n} .$$
(16)

Previous considerations suggest that the v_1^2 term from the cosmic string can be neglected. The general prediction is that the system has finite size and mass irrespective of the form of the distribution.

3.4.1 Four options

One can consider four kinds of mass distributions.

1) The scaling law $(dM/dl)(\rho) \propto K(\rho/\rho_0)^k$, $k \ge 0$, implies

$$\begin{aligned}
v(\rho) &= \sqrt{GK} (\rho/\rho_0)^{k/2} , \\
\omega(\rho) &= \sqrt{GK} (\rho/\rho_0)^{k/2-1} , \\
\rho(n) &= \rho_0 (v_0/\sqrt{GK})^{2/k} \times n^{-2/k} .
\end{aligned}$$
(17)

The radii decrease as $n^{-2/k}$ and largest radius is $\rho_0(v_0^2/GK)$. For constant mass density one obtains k = 2, rigid body rotation, and $\rho = \rho_0/n$ so that kind of reverted harmony of spheres would result. Quite generally, $v(\rho)$ is a non-decreasing function of ρ from the first condition. This reflects the 2-dimensionality of the situation.

2) If the mass distribution is logarithmic $M(\rho) = K \log^2(\rho/\rho_0)$ one has $v = \sqrt{GK} \log(\rho/\rho_0)$ and $\rho(n) = \rho_0 exp(k/n), \ k = v_0/\sqrt{GK}$. One obtains what might be regarded as a cylindrical shell $\rho/\rho_0 \in [1, e^k]$ and with density $dM/dl \propto 2\log(\rho)/\rho$. This kind of distribution could work in the case of planetary rings if the tidal effects of the central mass can be neglected.

3) p-Adic length scale hypothesis suggest the distribution $\rho(n) = 2^{-k}\rho_0$ for the radii of the "mass shells". This would give $v(\rho) = v_0/|log_2(\rho/\rho_0)|$ and

$$(dM/dl)(\rho) = \frac{v_0^2}{G|log_2(\rho/\rho_0)|^2} = \frac{M}{r_0|log_2(\rho/\rho_0)|^2}$$

Note that the most general form of p-adic length scale hypothesis allows $\rho(n) = 2^{-k/2}\rho_0$ This option defines the only working alternative for the dark

central mass. Note that this would explain Titius-Bode law [8] if planets have formed around dark matter shells or rings which have formed part of Sun during primordial stage.

4) The distribution of radii of form $\rho(n)/\rho_0 = x - n$ might serve as a model for planetary rings if the tidal effects of the central mass can be neglected. In this case one as

$$(dM/dl)(\rho) = \frac{M}{r_0(x - \frac{\rho}{\rho_0})^2}$$

The radius R must satisfy $R < x\rho_0$. The masses of the annuli must increase with ρ .

3.4.2 Only the p-adic variant works as a model for central mass

It is interesting to look what the three variants of the model would predict for the radius of Earth. If the pancake has height 2*R*, the relationship between radius and total mass can be expressed as $M = 2\pi (dM/dl)R^3$. Using $M_E = 3 \times 10^{-6} M_{Sun}$, and $r_0(Sun) \simeq R_M/9$, where $r_M = 5.8 \times 10^4$ Mm is the orbital radius of Mercury, one obtains by scaling $r_0 = GM_E/v_0^2 \simeq 20$ km for $v_0 = 2^{-11}$.

a) The options 1) and 2) fail. Constant density would give R = 140 km, which is about 2 per cent of the actual radius $R_E = 6.372797$ Mm and 10 percent about the radius 1.2 Mm of the inner core. The "inner inner core" of Earth happens to have radius of 300 km. For the logarithmic mass distribution one would obtain $R = r_0/2 \simeq 10$ km.

b) The option 3) inspired by the p-adic length scale hypothesis works and predicts $k^2 |log_2(R/\rho_0)|^2 = 2R/r_0$. $\rho_0 = 2R$ gives $k \simeq 25$. This alternative works also in the more general case since one can make the radius arbitrarily large by a proper choice of the integer k. The universal prediction would be that dark matter appears as shells corresponding to decreasing p-adic length scales coming as powers $p \simeq 2^k$. The situation would be very much analogous to that in atomic physics. The prediction conforms with the many-sheeted generalization of the model for the asymptotic state of the star for which the matter is concentrated on a thin cell [D3]. The model brings in mind also the large voids of size about 100 Mly.

c) The suspiciously small value of r_0 forces to ask whether the value of v_0 for Earth should be much smaller than $v_0 = 2^{-11}$. Also the radius of Moon's orbit would require $n \sim 138$ for this value to be compared with $n \geq 5$ for the moons of Saturn and Jupiter. If the age of Earth is much longer than that of outer planets, one would expect that more phase transitions

reducing v_0 forced by the cosmic expansion in average sense have taken place. $v_0 \rightarrow v_0/20$ would give $r_0 \simeq 8$ Mm to be compared with $R_E = 6.4$ Mm. Moon's orbit would correspond to n = 7 in a reasonable approximation. This choice of v_0 would allow k = 1.

The small value of v_0 might be understood from the fact that inner planets are older than outer ones so that the cosmic expansion in the average sense has forced larger number of phase transitions reducing the value of v_0 inducing a fractal scaling of the system. Ruler-and-compass hypothesis [D6] suggests preferred values of cosmic times for the occurrence of these transitions. Without this hypothesis the phase transitions could form almost continuum. For this option the failure of options 1) and 2) is even worse.

4 Quantum chaos in astrophysical length scales

The stimulus for writing this section came from the article "Quantum Chaos" by Martin Gurtzwiller [9]. Occasionally it can happen that even this kind of a masterpiece of scientific writing manages to stimulate only an intention to read it more carefully later. When you indeed read it again years later it can shatter you into a wild resonance. Just this occurred at this time.

4.1 Brief summary about quantum chaos

The article discusses of Gurtzwiller the complex regime between quantal and classical behavior as it was understood at the time of writing (1992). As a non-specialist I have no idea about possible new discoveries since then.

The article introduces the division of classical systems into regular (R) and chaotic (P in honor of Poincare) ones. Besides this one has quantal systems (Q). There are three transition regions between these three realms.

a) R-P corresponds to transition to classical chaos and KAM theorem is a powerful tool allowing to organize the view about P in terms of surviving periodic orbits.

b) Quantum-classical transition region R-Q corresponds to high quantum number limit and is governed by Bohr's correspondence principle. Highly excited hydrogen atom - Rydberg atom - defines a canonical example of the situation.

c) Somewhat surprisingly, it has turned out that also P-Q region can be understood in terms of periodic classical orbits (nothing else is available!). P-Q region can be achieved experimentally if one puts Rydberg atom in a strong magnetic field. At the weak field limit quantum states are delocalized but in chaotic regime the wave functions become strongly concentrated along a periodic classical orbits.

At the level of dynamics the basic example about P-Q transition region discussed is the chaotic quantum scattering of electron in atomic lattice. Classical description does not work: a superposition of amplitudes for orbits, which consist of pieces which are fragments of a periodic orbit plus localization around atom is necessary.

The fractal wave function patterns associated with say hydrogen atom in strong magnetic field are extremely beautiful and far from chaotic. Even in the case of chaotic quantum scattering one has interference of quantum amplitudes for classical Bohr orbits and also now Fourier transform exhibits nice peaks corresponding to the periods of classical orbits. The term chaos seems to be an unfortunate choice referring to our limited cognitive capacities rather than the actual physical situation and the term quantum complexity would be more appropriate.

d) For a consciousness theorist the challenge is to try to formulate in a more precise manner this fact. Quantum measurement theory with a finite measurement resolution indeed provide the mathematics necessary for this purpose.

4.2 What does the transition to quantum chaos mean?

The transition to quantum chaos in the sense the article discusses it means that a system with a large number of virtually independent degrees of freedom (in very general sense) makes a transition to a phase in there is a strong interaction between these degrees of freedom. Perturbative phase becomes non-perturbative. This means emergence of correlations and reduction of the effective dimension of the system to a finite fractal dimension. When correlations become complete and the system becomes a genuine quantum system, the dimension of the system is genuinely reduced and again nonfractal. In this sense one has transition via complexity to new kind of order.

4.2.1 The level of stationary states

At the level of energy spectrum this means that the energy of system which correspond to sums of virtually independent energies and thus is essentially random number becomes non-random. As a consequence, energy levels tend to avoid each other, order and simplicity emerge but at the collective level. Spectrum of zeros of Zeta has been found to simulate the spectrum for a chaotic system with strong correlations between energy levels. Zeta functions indeed play a key role in the proposed description of quantum criticality associated with the phase transition changing the value of Planck constant.

4.2.2 The importance of classical periodic orbits in chaotic scattering

Poincare with his immense physical and mathematical intuition foresaw that periodic classical orbits should have a key role also in the description of chaos. The study of complex systems indeed demonstrates that this is the case although the mathematics and physics behind this was not fully understood around 1992 and is probably not so even now. The basic discovery coming from numerical simulations is that the Fourier transform of a chaotic orbits exhibits has peaks the frequencies which correspond to the periods of closed orbits. From my earlier encounters with quantum chaos I remember that there is quantization of periodic orbits so that their periods are proportional to log(p), p prime in suitable units. This suggests a connection of arithmetic quantum field theory and with p-adic length scale hypothesis.

The chaotic scattering of electron in atomic lattice is discussed as a concrete example. In the chaotic situation the notion of electron consists of periods spend around some atom continued by a motion along along some classical periodic orbit. This does not however mean loss of quantum coherence in the transitions between these periods: a purely classical model gives non-sensible results in this kind of situation. Only if one sums scattering amplitudes over all piecewise classical orbits (not all paths as one would do in path integral quantization) one obtains a working model.

4.2.3 In what sense complex systems can be called chaotic?

Speaking about quantum chaos instead of quantum complexity does not seem appropriate to me unless one makes clear that it refers to the limitations of human cognition rather than to physics. If one believes in quantum approach to consciousness, these limitations should reduce to finite resolution of quantum measurement not taken into account in standard quantum measurement theory.

In the framework of hyper-finite factors of type II_1 finite quantum measurement resolution is described in terms of inclusions $\mathcal{N} \subset \mathcal{M}$ of the factors and sub-factor \mathcal{N} defines what might be called \mathcal{N} -rays replacing complex rays of state space. The space \mathcal{M}/\mathcal{N} has a fractal dimension characterized by quantum phase and increases as quantum phase $q = exp(i\pi/n), n = 3, 4, ...,$ approaches unity which means improving measurement resolution since the size of the factor \mathcal{N} is reduced.

Fuzzy logic based on quantum qbits applies in the situation since the components of quantum spinor do not commute. At the limit $n \to \infty$ one obtains commutativity, ordinary logic, and maximal dimension. The smaller the *n* the stronger the correlations and the smaller the fractal dimension. In this case the measurement resolution makes the system effectively strongly correlated as *n* approaches its minimal value n = 3 for which fractal dimension equals to 1 and Boolean logic degenerates to single valued totalitarian logic.

Non-commutativity is the most elegant description for the reduction of dimensions and brings in reduced fractal dimensions smaller than the actual dimension. Again the reduction has interpretation as something totally different from chaos: system becomes a single coherent whole with strong but not complete correlation between different degrees of freedom. The interpretation would be that in the transition to non-chaotic quantal behavior correlation becomes complete and the dimension of system again integer valued but smaller. This would correspond to the cases n=6, n=4, and n=3 (D=3,2,1).

4.3 Quantum chaos in astrophysical scales?

4.3.1 Quantum criticality

a) TGD Universe is quantum critical. The most important implication of quantum criticality of TGD Universe is that it fixes the value of Kähler coupling strength, the only free parameter appearing in definition of the theory as the analog of critical temperature. The dark matter hierarchy characterized partially by the increasing values of Planck constant allows to characterize more precisely what quantum criticality might means. By quantum criticality space-time sheets are analogs of Bohr orbits. Since quantum criticality corresponds to P-Q region, the localization of wave functions around generalized Bohr orbits should occur quite generally in some scale.

b) Elementary particles are maximally quantum critical systems analogous to H_2O at tri-critical point and can be said to be in the intersection of imbedding spaces labelled by various values of Planck constants. Planck constant does not characterize the elementary particle proper. Rather, each field body of particle (em, weak, color, gravitational) is characterized by its own Planck constant and this Planck constant characterizes interactions. The generalization of the notion of the imbedding space allows to formulate this idea in precise manner and each sector of imbedding space is characterized by discrete symmetry groups Z_n acting in M^4 and CP_2 degrees of freedom. The transition from quantum to classical corresponds to a reduction of Z_n to subgroup Z_m , m a factor of n. Ruler-and-compass hypothesis implies very powerful predictions for the remnants of this symmetry at the level of visible matter. Note that the reduction of the symmetry in this chaos-to-order transition!

c) Dark matter hierarchy makes TGD Universe an ideal laboratory for studying P-Q transitions with chaos identified as quantum critical phase between two values of Planck constant with larger value of Planck constant defining the "quantum" phase and smaller value the "classical" phase. Dark matter is localized near Bohr orbits and is analogous to quantum states localized near the periodic classical orbits. Planetary Bohr orbitology provides a particularly interesting astrophysical application of quantum chaos.

d) The above described picture applies about chaotic quantum scattering applies quite generally in quantum TGD. Path integral is replaced with a functional integral over classical space-time evolutions and the failure of the complete classical non-determinism is analogous to the transition between classical orbits. Functional integral also reduces to perturbative functional integral around maxima of Kähler function.

4.3.2 Dark matter structures as generalization of periodic orbits

The matter with ordinary or smaller value of Planck constant can form bound states with these dark matter structures. The dark matter circles would be the counterparts for the periodic Bohr orbits dictating the behavior of the quantum chaotic system. Visible matter (and more generally, dark matter at the lower levels of hierarchy behaving quantally in shorter length and time scales) tends to stay around these periodic orbits and in the ideal case provides a perfect classical mimicry of quantum behavior. Dark matter structures would effectively serve as selectors of the closed orbits in the gravitational dynamics of visible matter.

As one approaches classicality the binding of the visible matter to dark matter gradually weakens. Mercury's orbit is not quite closed, planetary orbits become ellipses, comets have highly eccentric orbits or even nonclosed orbits. For non-closed quantum description in terms of binding to dark matter does not makes sense at all.

The classical regular limit (R) would correspond to a decoupling between dark matter and visible matter. A motion along geodesic line is obtained but without Bohr quantization in gravitational sense since Bohr quantization using ordinary value of Planck constant implies negative energies for $GMm \geq 1$. The preferred extremal property of the space-time sheet could however still imply some quantization rules but these might apply in "vibrational" degrees of freedom.

4.3.3 Quantal chaos in gravitational scattering?

The chaotic motion of astrophysical object becomes the counterpart of quantum chaotic scattering. By Equivalence Principle the value of the mass of the object does not matter at all so that the motion of sufficiently light objects in solar system might be understandable only by assuming quantum chaos.

The orbit of a gravitationally unbound object such as comet could define the basic example. The rings of Saturn and Jupiter could represent interesting shorter length scale phenomena possible involving quantum scattering. One can imagine that the visible matter object spends some time around a given dark matter circle (binding to atom), makes a transition along a radial spoke to the next circle, and so on.

The prediction is that dark matter forms rings and cart-wheel like structures of astrophysical size. These could become visible in collisions of say galaxies when stars get so large energy as to become gravitationally unbound and in this quantum chaotic regime can flow along spokes to new Bohr orbits or to gravi-magnetic flux tubes orthogonal to the galactic plane. Hoag's object represents a beautiful example of a ring galaxy [11]. Remarkably, there is direct evidence for galactic cart-wheels (for pictures of them see [10]). There are also polar ring galaxies consisting of an ordinary galaxy plus ring approximately orthogonal to it and believed to form in galactic collisions [12]. The ring rotating with the ordinary galaxy can be identified in terms of gravi-magnetic flux tube orthogonal to the galactic plane: in this case Z_n symmetry would be completely broken.

5 Gravitational radiation and large value of gravitational Planck constant

The description of gravitational radiation provides a stringent test for the idea about dark matter hierarchy with arbitrary large values of Planck constants. In accordance with quantum classical correspondence, one can take the consistency with classical formulas as a constraint allowing to deduce information about how dark gravitons interact with ordinary matter. In the following standard facts about gravitational radiation are discussed first and

then TGD based view about the situation is sketched.

5.1 Standard view about gravitational radiation

5.1.1 Gravitational radiation and the sources of gravitational waves

Classically gravitational radiation corresponds to small deviations of the space-time metric from the empty Minkowski space metric [14]. Gravitational radiation is characterized by polarization, frequency, and the amplitude of the radiation. At quantum mechanical level one speaks about gravitons characterized by spin and light-like four-momentum.

The amplitude of the gravitational radiation is proportional to the quadrupole moment of the emitting system, which excludes systems possessing rotational axis of symmetry as classical radiators. Planetary systems produce gravitational radiation at the harmonics of the rotational frequency. The formula for the power of gravitational radiation from a planetary system given by

$$P = \frac{dE}{dt} = \frac{32}{\pi} \frac{G^2 M_1 M_2 (M_1 + M_2)}{R^5} \quad . \tag{18}$$

This formula can be taken as a convenient quantitative reference point.

Planetary systems are not very effective radiators. Because of their small radius and rotational asymmetry supernovas are much better candidates in this respect. Also binary stars and pairs of black holes are good candidates. In 1993, Russell Hulse and Joe Taylor were able to prove indirectly the existence of gravitational radiation. Hulse-Taylor binary consists of ordinary star and pulsar with the masses of stars around 1.4 solar masses. Their distance is only few solar radii. Note that the pulsars have small radius, typically of order 10 km. The distance between the stars can be deduced from the Doppler shift of the signals sent by the pulsar. The radiated power is about 10^{22} times that from Earth-Sun system basically due to the small value of R. Gravitational radiation induces the loss of total energy and a reduction of the distance between the stars and this can be measured.

5.1.2 How to detect gravitational radiation?

Concerning the detection of gravitational radiation the problems are posed by the extremely weak intensity and large distance reducing further this intensity. The amplitude of gravitational radiation is measured by the deviation of the metric from Minkowski metric, denote by h. Weber bar [14] provides one possible manner to detect gravitational radiation. It relies on a resonant amplification of gravitational waves at the resonance frequency of the bar. For a gravitational wave with an amplitude $h \sim 10^{-20}$ the distance between the ends of a bar with length of 1 m should oscillate with the amplitude of 10^{-20} meters so that extremely small effects are in question. For Hulse-Taylor binary the amplitude is about $h = 10^{-26}$ at Earth. By increasing the size of apparatus one can increase the amplitude of stretching.

Laser interferometers provide second possible method for detecting gravitational radiation. The masses are at distance varying from hundreds of meters to kilometers[14]. LIGO (the Laser Interferometer Gravitational Wave Observatory) consists of three devices: the first one is located with Livingston, Lousiana, and the other two at Hanford, Washington. The system consist of light storage arms with length of 2-4 km and in angle of 90 degrees. The vacuum tubes in storage arms carrying laser radiation have length of 4 km. One arm is stretched and one arm shortened and the interferometer is ideal for detecting this. The gravitational waves should create stretchings not longer that 10^{-17} meters which is of same order of magnitude as intermediate gauge boson Compton length. LIGO can detect a stretching which is even shorter than this. The detected amplitudes can be as small as $h \sim 5 \times 10^{-22}$.

5.2 Model for dark gravitons

In this subsection two models for dark gravitons are discussed. Spherical dark graviton (or briefly giant graviton) would be emitted in quantum transitions of say dark gravitational variant of hydrogen atom. Giant graviton is expected to de-cohere into topological light rays, which are the TGD counterparts of plane waves and are expected to be detectable by human built detectors.

5.2.1 Gravitons in TGD

Unlike the naive application of Mach's principle would suggest, gravitational radiation is possible in empty space in general relativity. In TGD framework it is not possible to speak about small oscillations of the metric of the empty Minkowski space imbedded canonically to $M^4 \times CP_2$ since Kähler action is non-vanishing only in fourth order in the small deformation and the deviation of the induced metric is quadratic in the deviation. Same applies to induced gauge fields. Even the induced Dirac spinors associated with the modified Dirac action fixed uniquely by super-symmetry allow only vacuum solutions in this kind of background. Mathematically this means that both the perturbative path integral approach and canonical quantization fail completely in TGD framework. This led to the vision about physics as Kähler geometry of "world of classical worlds" with quantum states of the universe identified as the modes of classical configuration space spinor fields.

The resolution of various conceptual problems is provided by the parton picture and the identification of elementary particles as light-like 3-surfaces associated with the wormhole throats. Gauge bosons correspond to pairs of wormholes and fermions to topologically condensed CP_2 type extremals having only single wormhole throat.

Gravitons are string like objects in a well defined sense. This follows from the mere spin 2 property and the fact that partonic 2-surfaces allow only free many-fermion states. This forces gauge bosons to be wormhole contacts whereas gravitons must be identified as pairs of wormhole contacts (bosons) or of fermions connected by flux tubes. The strong resemblance with string models encourages to believe that general relativity defines the low energy limit of the theory. Of course, if one accepts dark matter hierarchy and dynamical Planck constant, the notion of low energy limit itself becomes somewhat delicate.

5.2.2 Emission of dark gravitons

One must answer several non-trivial questions if one is to defend dark gravitational radiation.

Frequencies of dark gravitons turn out to correspond to orbital frequencies at large quantum number limit. However, if gravitational radiation is emitted as dark gravitons, they have enormous energies since the energy must correspond to the change of the energy of an astrophysical object jumping to a smaller Bohr orbit.

Hulse-Taylor binary system was used to demonstrate that the energy loss of the binary system equals to the classically predicted power of gravitational radiation. The power of gravitational radiation was deduced from the gradual reduction of the distance between the two stars. The obvious question is whether the consistency of the power emitted by Hulse-Taylor binary with the prediction of the classical theory kills the hypothesis about gigantic gravitational Planck constant. If one assumes that v_0 is of same order of magnitude as for planetary systems as the value of the orbital radius indeed suggests, the necessarily spherical dark graviton emitted in the transition would carry away a essentially astrophysical energy.

The only resolution of the problem is that dark graviton is spherical, or more generally correspond to partial wave with a definite value of angular momentum (in a sense to be specified), and decays gradually to gravitons with smaller values of Planck constant. As a matter fact, the measurement process should induce this kind of decay. The prediction is that energy is emitted in bunches and this should have testable experimental implications. The case of hydrogen atom inspires the question whether the lowest orbit is stable and does not emit gravitational radiation meaning that the binary ends up to the stable state rather than collapsing. Of course, the idealization as hydrogen atom type system might fail. The identification of dark gravitons as dark topological light rays (massless extremals, MEs) containing topologically condensed ordinary gravitons will be discussed later.

By quantum classical correspondence this process must have a space-time description and the natural proposal is that below the time scale associated with the emission process the space-time picture about the emission process looks like a continuous process, at least asymptotically when the space-time itself is replaced repeatedly with a new one. Thus the transition between orbitals at the level of space-time correlates must occur continuously below the time scale assigned to it classically. Quantum emission would quite generally mean in sub-quantum time scales continuous classical process at space-time level.

TGD based quantum model for living system suggests that the transition occurs in a fractal manner proceeding from long to short dark time scales. First a quantum jump in the longest time scale occurs and induces the replacement of the entire space-time with a new one differing dramatically from the previous one. This quantum jump is followed by quantum jumps in shorter time scales. At each step space-time sheet characterizing the system is replaced by a new one and eventually by a space-time surface which describes the process as more or less continuous one. The final spacetime could be regarded as symbolic description of the process as a classical continuous process.

The time interval for the occurrence of the transition at space-time level should correspond to a dark p-adic time scale and in the case of Hulse-Taylor binary be of same order as the lifetime of the period during which the system ends up to a stable state. In the Hulse-Taylor case the emission would correspond to small values of n, most naturally $n = 2 \rightarrow n = 1$ transition so that the frequency of the gravitational radiation would not correspond to the orbital frequency. This might some day be used as a test for the theory. The time duration T for the transition can be estimated from $T = \Delta E/P$, where P is the classical formula for the emission power.

5.2.3 Model for the giant graviton

Detector, giant graviton, source, and topological light ray will be denoted simply by D, G, and S, and ME in the following. Consider first the model for the giant graviton.

a) Orbital plane defines the natural quantization axis of angular momentun. Giant graviton and all dark gravitons corresponds to n_a -fold coverings of CP_2 by M^4 points, which means that one has a quantum state for which fermionic part remains invariant under the transformations $\phi \rightarrow \phi + 2\pi/n_a$. This means in particular that the ordinary gravitons associated with the giant graviton have same spin so that the giant graviton can be regarded as Bose-Einstein condensate in spin degrees of freedom. Only the orbital part of state depends on angle variables and corresponds to a partial wave with a small value of L.

b) The total angular momentum of the giant graviton must correspond to the change of angular momentum in the quantum transition between initial and final orbit. Orbital angular momentum in the direction of quantization axis should be a small multiple of dark Planck constant associated with the system formed by giant graviton and source. These states correspond to Bose-Einstein condensates of ordinary gravitons in eigen state of orbital angular with ordinary Planck constant. Unless S-wave is in question the intensity pattern of the gravitational radiation depends on the direction in a characteristic non-classical manner. The coherence of dark graviton regarded as Bose-Einstein condensate of ordinary gravitons is what distinguishes the situation in TGD framework from that in GRT.

c) If all elementary particles with gravitons included are maximally quantum critical systems, giant graviton should contain $r(G, S) = n_a/n_b$ ordinary gravitons. This number is not an integer for $n_b > 1$. A possible interpretation is that in this case gravitons possess fractional spin corresponding to the fact that rotation by 2π gives a point in the n_b -fold covering of M^4 point by CP_2 points. In any case, this gives an estimate for the number of ordinary gravitons and the radiated energy per solid angle. This estimate follows also from the energy conservation for the transition. The requirement that average power equals to the prediction of GRT allows to estimate the geometric duration associated with the transition. The condition $\hbar\omega = E_f - E_i$ is consistent with the identification of \hbar for the pair of systems formed by giant-graviton and emitting system.

5.2.4 Dark graviton as topological light ray

Second kind of dark graviton is analog for plane wave with a finite transversal cross section. TGD indeed predicts what I have called topological light rays, or massless extremals (MEs) as a very general class of solutions to field equations [D1].

MEs are typically cylindrical structures carrying induced gauge fields and gravitational field without dissipation and dispersion and without weakening with the distance. These properties are ideal for targeted long distance communications which inspires the hypothesis that they play a key role in living matter [J4, K4] and make possible a completely new kind of communications over astrophysical distances. Large values of Planck constant allow to resolve the problem posed by the fact that for long distances the energies of these quanta would be below the thermal energy of the receiving system.

Giant gravitons are expected to decay to this kind of dark gravitons having smaller value of Planck constant via de-decoherence and that it is these gravitons which are detected. Quantitative estimates indeed support this expectation.

At the space-time level dark gravitons at the lower levels of hierarchy would naturally correspond to n_a -Riemann sheeted ($r = GmE/v_0 = n_a/n_b$ for m >> E) variants of topological light rays ("massless extremals", MEs), which define a very general family of solutions to field equations of TGD [D1]. n_a -sheetedness is with respect to CP_2 and means that every point of CP_2 is covered by $n_a M^4$ points related by a rotation by a multiple of $2\pi/n_a$ around the propagation direction assignable with ME. n_b -sheetedness with respect to M^4 is possible but does not play a significant role in the following considerations. Using the same loose language as in the case of giant graviton, one can say that $r = n_a/n_b$ copies of same graviton have suffered a topological condensation to this kind of ME. A more precise statement would be n_a gravitons with fractional unit \hbar_0/n_a for spin.

5.3 Detection of gravitational radiation

One should also understand how the description of the gravitational radiation at the space-time level relates to the picture provided by general relativity to see whether the existing measurement scenarios really measure the gravitational radiation as they appear in TGD. There are more or less obvious questions to be answered (or perhaps obvious after a considerable work).

What is the value of dark gravitational constant which must be assigned

to the measuring system and gravitational radiation from a given source? Is the detection of primary giant graviton possible by human means or is it possible to detect only dark gravitons produced in the sequential decoherence of giant graviton? Do dark gravitons enhance the possibility to detect gravitational radiation as one might expect? What are the limitations on detection due to energy conservation in de-coherence process?

5.3.1 TGD counterpart for the classical description of detection process

The oscillations of the distance between the two masses defines a simplified picture about the receival of gravitational radiation. Now ME would correspond to n_a -"Riemann-sheeted" (with respect to CP_2)graviton with each sheet oscillating with the same frequency. Classical interaction would suggest that the measuring system topologically condenses at the topological light ray so that the distance between the test masses measured along the topological light ray in the direction transverse to the direction of propagation starts to oscillate.

Obviously the classical behavior is essentially the same as as predicted by general relativity at each "Riemann sheet". If all elementary particles are maximally quantum critical systems and therefore also gravitons, then gravitons can be absorbed at each step of the process, and the number of absorbed gravitons and energy is r-fold.

5.3.2 Sequential de-coherence

Suppose that the detecting system has some mass m and suppose that the gravitational interaction is mediated by the gravitational field body connecting the two systems.

The Planck constant must characterize the system formed by dark graviton and measuring system. In the case that E is comparable to m or larger, the expression for $r = n_a/n_b = \hbar/\hbar_0$ must replaced with the relativistically invariant formula in which m and E are replaced with the energies in center of mass system. This gives

$$r = \frac{GmE}{v_0(1+\beta)\sqrt{1-\beta}}$$
, $\beta = z(-1+\sqrt{1+\frac{2}{x}})$, $x = \frac{E}{2m}$. (19)

Assuming $m >> E_0$ this gives in a good approximation $r = Gm_1E_0/v_0 = G^2m_1mM/v_0^2$. Note that in the interaction of identical masses ordinary \hbar

is possible for $m \leq \sqrt{v_0} M_{Pl}$. For $v_0 = 2^{-11}$ the critical mass corresponds roughly to the mass of water blob of radius 1 mm.

One can interpret the formula by saying that de-coherence splits from the incoming dark graviton dark piece having energy $E_1 = (Gm_1E_0/v_0)\omega$, which makes a fraction $E_1/E_0 = (Gm_1/v_0)\omega$ from the energy of the graviton. At the *n*:th step of the process the system would split from the dark graviton of previous step the fraction

$$\frac{E_n}{E_0} = (\frac{G\omega^n}{v_0})^n \prod_i (m_i) \;\; .$$

from the total emitted energy E_0 . De-coherence process would proceed in steps such that the typical masses of the measuring system decrease gradually as the process goes downwards in length and time scale hierarchy. This splitting process should lead at large distances to the situation in which the original spherical dark graviton has split to ordinary gravitons with angular distribution being same as predicted by GRT.

The splitting process should stop when the condition $r \leq 1$ is satisfied and the topological light ray carrying gravitons becomes 1-sheeted covering of M^4 . For $E \ll m$ this gives $GmE \leq v_0$ so that $m \gg E$ implies $E \ll M_{Pl}$. For $E \gg m$ this gives $GE^{3/2}m^{1/2} \leq 2v_0$ or

$$\frac{E}{m} \le \left(\frac{2v_0}{Gm^2}\right)^{2/3} \ . \tag{20}$$

5.3.3 Information theoretic aspects

The value of $r = \hbar/\hbar_0$ depends on the mass of the detecting system and the energy of graviton which in turn depends on the de-coherence history in corresponding manner. Therefore the total energy absorbed from the pulse codes via the value of r information about the masses appearing in the de-coherence process. For a process involving only single step the value of the source mass can be deduced from this data. This could some day provide totally new means of deducing information about the masses of distant objects: something totally new from the point of view of classical and string theories of gravitational radiation. This kind of information theoretic bonus gives a further good reason to take the notion of quantized Planck constant seriously.

If one makes the stronger assumption that the values of r correspond to ruler-and-compass rationals expressible as ratios of the number theoretically preferred values of integers expressible as $n = 2^k \prod_s F_s$, where F_s correspond to different Fermat primes (only four is known), very strong constraints on the masses of the systems participating in the de-coherence sequence result. Analogous conditions appear also in the Bohr orbit model for the planetary masses and the resulting predictions were found to be true with few per cent. One cannot therefore exclude the fascinating possibility that the decoherence process might in a very clever manner code information about masses of systems involved with its steps.

5.3.4 The time interval during which the interaction with dark graviton takes place?

If the duration of the bunch is T = E/P, where P is the classically predicted radiation power in the detector and T the detection period, the average power during bunch is identical to that predicted by GRT. Also T would be proportional to r, and therefore code information about the masses appearing in the sequential de-coherence process.

An alternative, and more attractive possibility, is that T is same always and correspond to r = 1. The intuitive justification is that absorption occurs simultaneously for all r "Riemann sheets". This would multiply the power by a factor r and dramatically improve the possibilities to detect gravitational radiation. The measurement philosophy based on standard theory would however reject these kind of events occurring with 1/r time smaller frequency as being due to the noise (shot noise, seismic noise, and other noise from environment). This might relate to the failure to detect gravitational radiation.

5.4 Quantitative model

In this subsection a rough quantitative model for the de-coherence of giant (spherical) graviton to topological light rays (MEs) is discussed and the situation is discussed quantitatively for hydrogen atom type model of radiating system.

5.4.1 Leakage of the giant graviton to sectors of imbedding space with smaller value of Planck constant

Consider first the model for the leakage of giant graviton to the sectors of H with smaller Planck constant.

a) Giant graviton leaks to sectors of H with a smaller value of Planck constant via quantum critical points common to the original and final sector

of H. If ordinary gravitons are quantum critical they can be regarded as leakage points.

b) It is natural to assume that the resulting dark graviton corresponds to a radial topological light ray (ME). The discrete group Z_{n_a} acts naturally as rotations around the direction of propagation for ME. The Planck constant associated with ME-G system should by the general criterion be given by the general formula already described.

c) Energy should be conserved in the leakage process. The secondary dark graviton receives the fraction $\Delta\Omega/4\pi = S/4\pi r^2$ of the energy of giant graviton, where S(ME) is the transversal area of ME, and r the radial distance from the source, of the energy of the giant graviton. Energy conservation gives

$$\frac{S(ME)}{4\pi r^2}\hbar(G,S)\omega = \hbar(ME,G)\omega .$$
(21)

or

$$\frac{S(ME)}{4\pi r^2} = \frac{\hbar(ME,G)}{\hbar(G,S)} \simeq \frac{E(ME)}{M(S)} .$$
(22)

The larger the distance is, the larger the area of ME. This means a restriction to the measurement efficiency at large distances for realistic detector sizes since the number of gravitons must be proportional to the ratio S(D)/S(ME) of the areas of detector and ME.

5.4.2 The direct detection of giant graviton is not possible for long distances

Primary detection would correspond to a direct flow of energy from the giant graviton to detector. Assume that the source is modellable using large \hbar variant of the Bohr orbit model for hydrogen atom. Denote by $r = n_a/n_b$ the rationals defining Planck constant as $\hbar = r\hbar_0$.

For G-S system one has

$$r(G,S) = \frac{GME}{v_0} = GMmv_0 \times \frac{k}{n^3} .$$
(23)

where k is a numerical constant of order unity and m refers to the mass of planet. For Hulse-Taylor binary $m \sim M$ holds true.

For D-G system one has

$$r(D,G) = \frac{GM(D)E}{v_0} = GM(D)mv_0 \times \frac{k}{n^3}$$
 (24)

The ratio of these rationals (in general) is of order M(D)/M.

Suppose first that the detector has a disk like shape. This gives for the total number n(D) of ordinary gravitons going to the detector the estimate

$$n(D) = \left(\frac{d}{r}\right)^2 \times n_a(G,S) = \left(\frac{d}{r}\right)^2 \times GMmv_0 \times n_b(G,S) \times \frac{k}{n^3} \quad .$$
 (25)

If the actual area of detector is smaller than d^2 by a factor x one has

$$n(D) \to xn(D)$$
.

n(D) cannot be smaller than the number of ordinary gravitons estimated using the Planck constant associated with the detector: $n(D) \ge n_a(D,G) =$ $r(D,G)n_b(D,G)$. This gives the condition

$$\frac{d}{r} \geq \sqrt{\frac{M(D)}{M(S)}} \times \sqrt{\frac{n_b(D,G)}{n_b(G,S)}} \times (\frac{k}{xn^3})^{1/2} .$$
(26)

Suppose for simplicity that $n_b(D,G)/n_b(G,S) = 1$ and $M(D) = 10^3$ kg and $M(S) = 10^{30}$ kg and r = 200 MPc $\sim 10^9$ ly, which is a typical distance for binaries. For x = 1, k = 1, n = 1 this gives roughly $d \ge 10^{-4}$ ly $\sim 10^{11}$ m, which is roughly the size of solar system. From energy conservation condition the entire solar system would be the natural detector in this case. Huge values of $n_b(G,S)$ and larger reduction of $n_b(G,S)$ would be required to improve the situation. Therefore direct detection of giant graviton by human made detectors is excluded.

5.4.3 Secondary detection

The previous argument leaves only the secondary detection into consideration. Assume that ME results in the primary de-coherence of a giant graviton. Also longer de-coherence sequences are possible and one can deduce analogous conditions for these.

Energy conservation gives

$$\frac{S(D)}{S(ME)} \times r(ME,G) = r(D,ME) .$$
(27)

Using the expression for S(ME) from Eq. 22, this gives an expression for S(ME) for a given detector area:

$$S(ME) = \frac{r(ME,G)}{r(D,ME)} \times S(D) \simeq \frac{E(G)}{M(D)} \times S(D) .$$
⁽²⁸⁾

From $S(ME) = \frac{E(ME)}{M(S)} 4\pi r^2$ one obtains

$$r = \sqrt{\frac{E(G)M(S)}{E(ME)M(D))}} \times \sqrt{S(D)}$$
(29)

for the distance at which ME is created. The distances of binaries studied in LIGO are of order $D = 10^{24}$ m. Using $E(G) \sim Mv_0^2$ and assuming $M = 10^{30}$ kg and S(D) = 1 m² (just for definiteness), one obtains $r \sim 10^{25}(kg/E(ME))$ m. If ME is generated at distance $r \sim D$ and if one has $S(ME) \sim 10^6 m^2$ (from the size scale for LIGO) one obtains from the equation for S(ME) the estimate $E(ME) \sim 10^{-25}$ kg $\sim 10^{-8}$ Joule.

5.4.4 Some quantitative estimates for gravitational quantum transitions in planetary system

To get a concrete grasp about the situation it is useful to study the energies of dark gravitons in the case of planetary system assuming Bohr model.

The expressions for the energies of dark gravitons can be deduced from those of hydrogen atom using the replacements $Ze^2 \rightarrow 4\pi GMm$, $\hbar \rightarrow GMm/v_0$. The energies are given by

$$E_n = \frac{1}{n^2} E_1 ,$$

$$E_1 = (Z\alpha)^2 \frac{m}{4} = (\frac{Ze^2}{4\pi\hbar})^2 \times \frac{m}{4} \to \frac{m}{4} v_0^2 .$$
 (30)

 E_1 defines the energy scale. Note that v_0 defines a characteristic velocity if one writes this expression in terms of classical kinetic energy using virial theorem T = -V/2 for the circular orbits. This gives $E_n = T_n = mv_n^2/2 = mv_0^2/4n^2$ giving

$$v_n = \frac{v_0}{\sqrt{2}n}$$

Orbital velocities are quantized as sub-harmonics of the universal velocity $v_0/sqrt2 = 2^{-23/2}$ and the scaling of v_0 by 1/n scales does not lead out from the set of allowed velocities.

Bohr radius scales as

$$r_0 = \frac{\hbar}{Z\alpha m} \to \frac{GM}{v_0^2}$$
 (31)

For $v_0 = 2^{11}$ this gives $r_0 = 2^{22}GM \simeq 4 \times 10^6 GM$. In the case of Sun this is below the value of solar radius but not too much.

The frequency $\omega(n, n-k)$ of the dark graviton emitted in $n \to n-k$ transition and orbital rotation frequency ω_n are given by

$$\omega(n, n-k) = \frac{v_0^3}{GM} \times \left(\frac{1}{n^2} - \frac{1}{(n-k)^2}\right) \simeq k\omega_n \quad .$$

$$\omega_n = \frac{v_0^3}{GMn^3} \quad . \tag{32}$$

The emitted frequencies at the large n limit are harmonics of the orbital rotation frequency so that quantum classical correspondence holds true. For low values of n the emitted frequencies differ from harmonics of orbital frequency.

The energy emitted in $n \rightarrow n - k$ transition would be

$$E(n, n-k) = mv_0^2 \times \left(\frac{1}{n^2} - \frac{1}{(n-k)^2}\right) , \qquad (33)$$

and obviously enormous. Single giant (spherical) dark graviton would be emitted in the transition and should decay to gravitons with smaller values of \hbar . Bunch like character of the detected radiation might serve as the signature of the process. The bunch like character of liberated dark gravitational energy means coherence and might play role in the coherent locomotion of living matter. For a pair of systems of masses m = 1 kg this would mean $Gm^2/v_0 \sim 10^{20}$ meaning that exchanged dark graviton corresponds to a bunch containing about 10^{20} ordinary gravitons. The energies of graviton bunches would correspond to the differences of the gravitational energies between initial and final configurations which in principle would allow to deduce the Bohr orbits between which the transition took place. Hence dark gravitons could make possible the analog of spectroscopy in astrophysical length scales.

5.5 Generalization to gauge interactions

The situation is expected to be essentially the same for gauge interactions. The first guess is that one has $r = Q_1 Q_2 g^2 / v_0$, were g is the coupling constant of appropriate gauge interaction. v_0 need not be same as in the gravitational case. The value of $Q_1 Q_2 g^2$ for which perturbation theory fails defines a plausible estimate for v_0 . The naive guess would be $v_0 \sim 1$. In the case of gravitation this interpretation would mean that perturbative approach fails for $GM_1M_2 = v_0$. For r > 1 Planck constant is quantized with rational values with ruler-and-compass rationals as favored values. For gauge interactions r would have rather small values. The above criterion applies to the field body connecting two gauge charged systems. One can generalize this picture to self interactions assignable to the "personal" field body of the system. In this case the condition would read as $\frac{Q^2g^2}{v_0} > 1$.

5.5.1 Some applications

One can imagine several applications.

a) A possible application would be to electromagnetic interactions in heavy ion collisions.

b) Gamma ray bursts might be one example of dark photons with very large value of Planck constant. The MEs carrying gravitons could carry also gamma rays and this would amplify the value of Planck constant form them too.

c) Atomic nuclei are good candidates for systems for which electromagnetic field body is dark. The value of \hbar would be $r = Z^2 e^2/v_0$, with $v_0 \sim 1$. Electromagnetic field body could become dark already for Z > 3 or even for Z = 3. This suggest a connection with nuclear string model [F9] in which $A \leq 4$ nuclei (with Z < 3) form the basic building bricks of the heavier nuclei identified as nuclear strings formed from these structures which themselves are strings of nucleons.

d) Color confinement for light quarks might involve dark gluonic field bodies.

e) Dark photons with large value of \hbar could transmit large energies through long distances and their phase conjugate variants could make possi-

ble a new kind of transfer mechanism [K6] essential in TGD based quantum model of metabolism and having also possible technological applications. Various kinds of sharp pules [15] suggest themselves as a manner to produce dark bosons in laboratory. Interestingly, after having given us alternating electricity, Tesla spent the rest of his professional life by experimenting with effects generated by electric pulses. Tesla claimed that he had discovered a new kind of invisible radiation, scalar wave pulses, which could make possible wireless communications and energy transfer in the scale of globe (for a possible but not the only TGD based explanation [G3]). This notion of course did not conform with Maxwell's theory, which had just gained general acceptance so that Tesla's fate was to spend his last years as a crackpot. Great experimentalists seem to be able to see what is there rather than what theoreticians tell them they should see. They are often also visionaries too much ahead of their time.

5.5.2 In what sense dark matter is dark

The notion of dark matter as something which has only gravitational interactions brings in mind the concept of ether and is very probably only an approximate characterization of the situation. As I have been gradually developing the notion of dark matter as a hierarchy of phases of matter with an increasing value of Planck constant, the naivete of this characterization has indeed become obvious.

If the proposed view is correct, dark matter is dark only in the sense that the process of receiving the dark bosons (say gravitons) mediating the interactions with other levels of dark matter hierarchy, in particular ordinary matter, differs so dramatically from that predicted by the theory with a single value of Planck constant that the detected dark quanta are unavoidably identified as noise. Dark matter is there and interacts with ordinary matter and living matter in general and our own EEG in particular provide the most dramatic examples about this interaction. Hence we could consider the dropping of "dark matter" from the glossary altogether and replacing the attribute "dark" with the spectrum of Planck constants characterizing the particles (dark matter) and their field bodies (dark energy).

6 Appendix: Orbital radii of exoplanets as a test for the theory

Orbital radii of exoplanets serve as a test for the theory. Hundreds of them are already known and in [13] tables listing basic data for for 136 exoplanets can be found. Tables provide also references and links to sources giving data about stars, in particular star mass M using solar mass M_S as a unit. Hence one can test the formula for the orbital radii given by the expression

$$\frac{r}{r_E} = \frac{n^2}{5^2} \frac{M}{M_S} X ,
X = \left(\frac{n_1}{n_2}\right)^2 ,
n_i = 2^{k_i} \times \prod_{s_i} F_{s_i} , F_{s_i} \in \{3, 5, 17, 257, 2^{16} + 1\} .$$
(34)

Here a given Fermat prime F_{s_i} can appear only once.

It turns out that the simplest option assuming X = 1 fails badly for some planets: the resulting deviations of order 20 per cent typically but in the worst cases the predicted radius is by factor of ~ .5 too small. The values of X used in the fit correspond to $X \in \{(2/3)^2, (3/4)^2, (4/5)^2, (5/6)^2, (15/17)^2, (15/16)^2, (16/17)^2\} \simeq \{.44, .56, .64, .69, .78, .88, .89\}$ and their inverses. The tables summarizing the resulting fit using both X = 1 and value giving optimal fit are given below. The deviations are typically few per cent and one must also take into account the fact that the masses of stars are deduced theoretically using the spectral data from star models. I am not able to form an opinion about the real error bars related to the masses.

In the tables R denotes the value of minor semiaxis of the planetary orbit using AU as a unit and M the mass of star using solar mass M_S as a unit. n is the value of the principal quantum number and R_1 the radius assuming $X = (r/s)^2 = 1$ and R_2 the value for the best choice of X as ratio of "ruler and compass integers". The data about radii of planets are from tables at http://exoplanets.org/almanacframe.html and star masses from the references contained by the tables.

Star Name	R	М	n	R1	R1/R	r	s	R2/R
HD73256	0.037	1.05	1	0.042	1.14	16	15	1.00
HD83443	0.040	0.79	1	0.032	0.79	15	17	1.01
HD46375	0.040	1.00	1	0.040	1.00	1	1	1.00
HD179949	0.040	1.24	1	0.050	1.24	17	15	0.97
HD187123b	0.040	1.06	1	0.042	1.06	1	1	1.06
HD120136	0.050	1.30	1	0.052	1.04	1	1	1.04
HD330075	0.046	0.70	1	0.028	0.61	4	5	0.95
BD-103166	0.050	1.10	1	0.044	0.88	15	16	1
HD209458	0.050	1.05	1	0.042	0.84	16	17	0.95
HD76700	0.050	1.00	1	0.040	0.8	15	17	1.03
HD217014	0.050	1.06	1	0.042	0.85	15	16	0.96
HD9826b	0.059	1.30	1	0.052	0.88	15	16	1.00
HD49674	0.060	1.00	1	0.040	0.67	5	6	0.96
HD68988	0.070	1.20	1	0.048	0.69	5	6	0.99
HD168746	0.065	0.88	1	0.035	0.54	3	4	0.96
HD217107	0.070	0.98	1	0.039	0.56	3	4	1
HD162020	0.074	0.75	1	0.030	0.41	2	3	0.91
HD130322	0.088	0.79	1	0.032	0.36	3	5	1
HD108147	0.102	1.27	1	0.051	0.50	3	4	0.89
HD38529b	0.129	1.39	1	0.056	0.43	2	3	0.97
HD75732b	0.115	0.95	1	0.038	0.33	3	5	0.92
HD195019	0.140	1.02	2	0.163	1.17	16	15	1.02
HD6434	0.150	0.79	2	0.126	0.84	15	16	0.96
HD192263	0.150	0.79	2	0.126	0.84	15	16	0.96
GJ876c	0.130	0.32	3	0.115	0.89	15	16	1.01
HD37124b	0.181	0.91	2	0.146	0.80	15	17	1.03
HD143761	0.220	0.95	2	0.152	0.69	5	6	0.99
HD75732c	0.240	0.95	2	0.152	0.63	4	5	0.99
HD74156b	0.280	1.27	2	0.203	0.73	5	6	1.05
HD168443b	0.295	1.01	2	0.162	0.55	3	4	0.97
GJ876b	0.210	0.32	4	0.205	0.98	1	1	0.98
HD3651	0.284	0.79	3	0.284	1.00	1	1	1
HD121504	0.320	1.18	2	0.189	0.59	3	4	1.05
HD178911	0.326	0.87	3	0.313	0.96	1	1	0.96
HD16141	0.350	1.00	3	0.360	1.03	1	1	1.03
HD114762	0.350	0.82	3	0.295	0.84	15	16	0.96
HD80606	0.469	1.10	3	0.396	0.84	15	16	0.96
HD117176	0.480	1.10	3	0.396	0.83	15	16	0.94
HD216770	0.460	0.90	3	0.324	0.70	5	6	1.01

Star Name	R	М	n	R1	R1/R	r	s	R2/R
HD52265	0.49	1.13	3	0.41	0.83	15	16	0.94
HD73526	0.65	1.02	4	0.65	1	1	1	1.00
HD82943c	0.73	1.05	4	0.67	0.92	16	17	1.04
HD8574	0.77	1.17	4	0.75	0.97	1	1	0.97
HD169830	0.82	1.40	4	0.90	1.09	17	16	0.97
HD9826c	0.83	1.30	4	0.83	1.00	1	1	1.00
HD202206	0.83	1.15	4	0.74	0.89	15	16	1.01
HD89744	0.89	1.40	4	0.9	1.01	1	1	1.01
HD134987	0.81	1.05	4	0.67	0.83	15	16	0.94
HD12661b	0.82	1.07	4	0.68	0.84	15	16	0.95
HD150706	0.82	0.98	5	0.98	1.20	16	15	1.05
HD40979	0.81	1.08	4	0.69	0.85	15	16	0.97
HD92788	0.95	1.06	5	1.06	1.12	16	15	0.98
HD142	0.97	1.10	5	1.1	1.13	16	15	1.00
HD28185	1.03	0.99	5	0.99	0.96	1	1	0.96
HD142415	1.07	1.03	5	1.03	0.96	1	1	0.96
HD108874b	1.06	1.00	5	1.00	0.94	1	1	0.94
HD4203	1.09	1.06	5	1.06	0.97	1	1	0.97
HD177830	1.14	1.17	5	1.17	1.03	1	1	1.03
HD128311b	1.02	0.80	6	1.15	1.13	1	1	1.13
HD27442	1.18	1.20	5	1.20	1.02	1	1	1.02
HD210277	1.12	0.99	5	0.99	0.88	15	16	1.01
HD82943b	1.16	1.05	5	1.05	0.91	15	16	1.03
HD20367	1.25	1.17	5	1.17	0.94	1	1	0.94
HD114783	1.19	0.92	6	1.32	1.11	1	1	1.11
HD137759	1.28	1.05	5	1.05	0.82	15	17	1.05
HD19994	1.42	1.34	5	1.34	0.94	1	1	0.94
HD147513	1.26	1.11	5	1.11	0.88	15	16	1.00
HD222582	1.35	1.00	6	1.44	1.07	1	1	1.07
HD65216	1.31	0.92	6	1.32	1.01	1	1	1.01
HD141937	1.52	1.10	6	1.58	1.04	1	1	1.04
HD41004A	1.31	0.70	7	1.37	1.05	1	1	1.05
HD160691b	1.87	1.08	7	2.12	1.13	16	15	0.99
a	-			.	. . /-			D.C. /F
Star Name	R	М	n	R1	R1/R	r	S	R2/R
HD52265	0.49	1.13	3	0.41	0.83	15	16	0.94
HD73526	0.65	1.02	4	0.65	1	1	1	1.00

HD82943c	0.73	1.05	4	0.67	0.92	16	17	1.04
HD8574	0.77	1.17	4	0.75	0.97	1	1	0.97
HD169830	0.82	1.40	4	0.90	1.09	17	16	0.97
HD9826c	0.83	1.30	4	0.83	1.00	1	1	1.00
HD202206	0.83	1.15	4	0.74	0.89	15	16	1.01
HD89744	0.89	1.40	4	0.9	1.01	1	1	1.01
HD134987	0.81	1.05	4	0.67	0.83	15	16	0.94
HD12661b	0.82	1.07	4	0.68	0.84	15	16	0.95
HD150706	0.82	0.98	5	0.98	1.20	16	15	1.05
HD40979	0.81	1.08	4	0.69	0.85	15	16	0.97
HD92788	0.95	1.06	5	1.06	1.12	16	15	0.98
HD142	0.97	1.10	5	1.1	1.13	16	15	1.00
HD28185	1.03	0.99	5	0.99	0.96	1	1	0.96
HD142415	1.07	1.03	5	1.03	0.96	1	1	0.96
HD108874b	1.06	1.00	5	1.00	0.94	1	1	0.94
HD4203	1.09	1.06	5	1.06	0.97	1	1	0.97
HD177830	1.14	1.17	5	1.17	1.03	1	1	1.03
HD128311b	1.02	0.80	6	1.15	1.13	1	1	1.13
HD27442	1.18	1.20	5	1.20	1.02	1	1	1.02
HD210277	1.12	0.99	5	0.99	0.88	15	16	1.01
HD82943b	1.16	1.05	5	1.05	0.91	15	16	1.03
HD20367	1.25	1.17	5	1.17	0.94	1	1	0.94
HD114783	1.19	0.92	6	1.32	1.11	1	1	1.11
HD137759	1.28	1.05	5	1.05	0.82	15	17	1.05
HD19994	1.42	1.34	5	1.34	0.94	1	1	0.94
HD147513	1.26	1.11	5	1.11	0.88	15	16	1.00
HD222582	1.35	1.00	6	1.44	1.07	1	1	1.07
HD65216	1.31	0.92	6	1.32	1.01	1	1	1.01
HD141937	1.52	1.10	6	1.58	1.04	1	1	1.04
HD41004A	1.31	0.70	7	1.37	1.05	1	1	1.05
HD160691b	1.87	1.08	7	2.12	1.13	16	15	0.99

Star Name	R	М	n	R1	R1/R	r	s	R2/R
HD23079	1.65	1.10	6	1.58	0.96	1	1	0.96
HD186427	1.67	1.01	6	1.45	0.87	15	16	0.99
HD4208	1.67	0.93	7	1.82	1.09	16	15	0.96
HD114386	1.62	0.68	8	1.74	1.07	17	16	0.95
HD213240	2.03	1.22	6	1.76	0.87	15	16	0.98
HD10647	2.10	1.07	7	2.10	1.00	1	1	1
HD10697	2.13	1.10	7	2.16	1.01	1	1	1.01
HD95128b	2.09	1.03	7	2.02	0.97	1	1	0.97
HD190228	2.00	0.83	8	2.12	1.06	1	1	1.06
HD114729	2.08	0.93	7	1.82	0.88	15	16	1
HD111232	1.97	0.78	8	2.00	1.01	1	1	1.01
HD2039	2.19	0.98	7	1.92	0.88	15	16	1
HD136118	2.40	1.24	7	2.43	1.01	1	1	1.01
HD50554	2.32	1.07	7	2.09	0.9	15	16	1.02
HD9826d	2.53	1.30	7	2.55	1.01	1	1	1.01
HD196050	2.43	1.10	7	2.16	0.89	15	16	1.01
HD216437	2.43	1.07	8	2.74	1.13	17	15	0.88
HD216435	2.70	1.25	7	2.45	0.91	1	1	0.91
HD169830c	2.75	1.40	7	2.74	1	1	1	1
HD106252	2.54	0.96	8	2.46	0.97	1	1	0.97
HD12661c	2.60	1.07	8	2.74	1.05	1	1	1.05
HD23596	2.86	1.30	7	2.55	0.89	15	16	1.01
HD168443c	2.87	1.01	8	2.59	0.9	15	16	1.03
HD145675	2.85	1.00	8	2.56	0.9	15	16	1.02
HD11964b	3.10	1.10	8	2.82	0.91	16	17	1.03
HD39091	3.29	1.10	9	3.56	1.08	17	16	0.96
HD38529c	3.71	1.39	8	3.56	0.96	1	1	0.96
HD70642	3.30	1.00	9	3.24	0.98	1	1	0.98
HD33636	3.56	0.99	9	3.21	0.9	15	16	1.03
HD95128c	3.73	1.03	10	4.12	1.1	16	15	0.97
HD190360	3.65	0.96	10	3.84	1.05	1	1	1.05
HD74156c	3.82	1.27	9	4.11	1.08	1	1	1.08
HD22049	3.54	0.80	11	3.87	1.09	16	15	0.96
HD30177	3.86	0.95	10	3.80	0.98	1	1	0.98
HD89307	4.15	0.95	10	3.80	0.92	1	1	0.92
HD72659	4.50	0.95	11	4.60	1.02	1	1	1.02
HD75732d	5.90	0.95	13	6.42	1.09	16	15	0.96

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