

Hypothesis of dark matter and dark energy with negative mass

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From the observance of the HSS team in 1998, they gained the mass density of the negative($\Omega_M = -0.38(\pm 0.22)$), using field equations which do not have the cosmological constant. The quantity of the mass couldn't be negative value in they thought, the value is trashed away. We have to know that not the field equation has disposed the value, but our thought disposed that value. In the world of positive mass, ground state is a point that energy is low, but in case of negative mass, ground state is a point that energy is the highest. Accordingly, in the world of negative mass, energy level is filled from the highest to the lowest, and stable state means the highest energy state, so the catastrophe to energy level of minus infinity never happens even if negative mass spontaneously emits energy. Assuming that negative mass exists, Newton's Law of motion was derived in between negative and positive masses and also between negative and negative masses. As a method for proving the existence of negative mass, an explanation on the revolution velocity of the galaxy through negative mass has been presented. In this process, the existence of spherical mass distribution was given; furthermore, explanation was done using this, to show observation results where dark matter effect through negative mass is proportional to distance r . If Ω_M is -0.38, universe's age is 14.225 Gyr. It is in the range estimated by other observations. Universe's radius R is $96.76^{+12.13}_{-11.44} Gly = (85.32 \sim 108.89) Gly$. Assuming that negative mass and positive mass were born together at the beginning of universe, it satisfies the various problems that previous dark matter and dark energy possess, such as, centripetal force effects of galaxy and galaxy clusters from previous dark matters, mass effects that is proportional to the distance r , repulsive force needed for expansion, dark energy that has positive values, low interaction between dark matter when collision occurs between dark matter, deceleration expansion and acceleration expansion of universe, formation of void, inflation mechanism, fine tuning problem of mass density, collision of Bullet cluster, universe's age, universe's size, the reason of that dark energy seems to has a small and non-zero value. Also, we prove to the dark energy observation value($10^{-47} GeV^4$). As a result, the necessity of observation focusing on exact computation and detection of negative mass is stated.

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I. Introduction

The observation of Type Ia Supernovae tells us that the universe is expanding. [1] [2] In order to explain these accelerated expansions, the Λ CDM model is used to represent the energy which has a negative pressure to explain the expansion [3], and to explain the revolution curve within the Galaxy, the CDM is assumed. [4]

However, the mass density of the universe does not even closely explain the critical mass density(fine tuning problem), and does not provide a reason why the cosmological constant of the universe has a small and non-zero value. [5]

Therefore, as of this research, we suggest that if pair creations of negative and positive mass were existed at the beginning of universe, these dark energy, dark matter, fine tuning problem, Cosmological Constant problems [5] can be explained. Also, the current dark energy observed value was exactly(in 10^1 order) found through theoretical calculations.

The negative mass can possibly exist as a solution of relativistic the energy equation [6], Bondi in 1957 briefly mentioned about the negative mass, and tried the general relative theory to analyze [7], but yet have an observation. Also, looking at energy levels, there are no investigated cases because we believe that catastrophe of the minus infinite energy level has occurred.

It is necessary to watch carefully the fact that all sort of physical laws such as Law of energy conservation or Law of conservation of momentum do not deny negative mass [8]. All sorts of physical laws do not deny them, thus negative mass can exist!

If we consider the situation of antiparticle and antimatter [6], we can know they exist, although they are not observed in common situation. We have must explain the reason why they are not easily observed in common situation by their motion or characteristics, suggest explanation of existing phenomenon, and new prediction with their characteristics, and inspect it with observation.

With the assumption that negative mass exists, in the

paper, we tries to examine motion of negative mass and positive mass, and motion of negative mass and negative mass, and to suggest that negative mass can provide proper description of dark matter and dark energy.

Most of people have strong aversion of negative mass, but no law of physics says there is only positive mass in nature, and we think that we feel as if our thought of mass, namely mass that is the a synonym for the fact something exists denies negative value. However, negative charge and positive charge suggest that "something exists" does not mean positive value only. The great achievement of physics in theory of relativity and quantum mechanics request us to review the previous thought of physical quantities.

II. Property of negative mass

1.Extended Newton's motion law

When an object with mass of $+m_1$ is away from an object with mass of $+m_2$ by distance r , the force worked between two objects is described as following type.

$$\vec{F} = -G \frac{m_1 m_2}{r^2} \hat{r} \quad (1)$$

When an object with mass of $-m_1$ is away from an object with mass of $-m_2$ by distance r , what type does the force worked between two objects have?

Many people think "the force decides motion of object in gravity", but actually the acceleration decides the motion of object in gravity, and also plays a role of deciding the moving direction of object which is at the state of stop.

We can set up following dynamic equation to describe the motion of object. That is, the gravity created on mass m_1 by mass m_2 is expressed as follows:

$$\vec{F}_1 = m_1 \vec{a}_1 = -G \frac{m_1 m_2}{r^2} \hat{r} \quad (2)$$

$$m_1 \vec{a}_1 = -G \frac{m_1 m_2}{r^2} \hat{r} \quad (3)$$

$$\vec{a}_1 = -G \frac{m_2}{r^2} \hat{r} \quad (4)$$

As we can see in the equation above, the term of acceleration remains only because mass m_1 is erased from both terms. Now the equation of motion means the equation of acceleration, not the equation of force, the acceleration provides information of motion direction, and decides the direction of motion.

1-1)The law of motion of positive mass and positive mass

$$m_1 \vec{a}_1 = -G \frac{m_1 m_2}{r^2} \hat{r} \quad (5)$$



Figure 1: Positive mass $+m_1$ and positive mass $+m_2$ (initial velocity $=0$, $m_1 > 0$, $m_2 > 0$)

$$\vec{a}_1 = -G \frac{m_2}{r^2} \hat{r} \quad (6)$$

$$m_2 \vec{a}_2 = -G \frac{m_1 m_2}{r^2} \hat{r} \quad (7)$$

$$\vec{a}_2 = -G \frac{m_1}{r^2} \hat{r} \quad (8)$$

Positive mass and positive mass : The force worked between positive mass is attraction, and two objects move toward the center of mass. The force is attraction, thus their potential energy has negative value. The direction of acceleration is in the direction of $-\hat{r}$, so the distance between two objects are reduced gradually.

1-2)The law of motion of negative mass and positive mass



Figure 2: Negative mass $-m_1$ and positive mass $+m_2$ (initial velocity $=0$, $m_1 > 0$, $m_2 > 0$)

$$-m_1 \vec{a}_1 = -G \frac{(-m_1) m_2}{r^2} \hat{r} \quad (9)$$

$$\vec{a}_1 = -G \frac{m_2}{r^2} \hat{r} \quad (10)$$

$$+m_2 \vec{a}_2 = -G \frac{(-m_1) m_2}{r^2} \hat{r} \quad (11)$$

$$\vec{a}_2 = G \frac{m_1}{r^2} \hat{r} \quad (12)$$

Negative mass and positive mass : Negative mass is accelerated in the direction of positive mass, and positive mass is accelerated in the direction to be far away from negative mass.

The direction of acceleration a_1 worked on negative mass m_1 is $-\hat{r}$, so $-m_1$ moves in the direction of reducing distance r , and the direction of acceleration a_2 worked on positive mass $+m_2$ is $+\hat{r}$, so positive mass $+m_2$ is accelerated in the direction that distance r increases, namely the direction of being far away from negative mass.

If the absolute value of positive mass is bigger than that of negative mass, they will meet within finite time(attractive effect), and if the absolute value of positive mass is smaller than that of negative mass, the distance between them will be bigger, and they cannot meet(repulsive effect). The type of force is repulsion, so the potential energy has positive value.

1-3)The law of motion of negative mass and negative mass



Figure 3: Negative mass $-m_1$ and negative mass $-m_2$ (initial velocity $=0$, $m_1 > 0$, $m_2 > 0$)

$$-m_1 \vec{a}_1 = -G \frac{(-m_1)(-m_2)}{r^2} \hat{r} \quad (13)$$

$$\vec{a}_1 = +G \frac{m_2}{r^2} \hat{r} \quad (14)$$

$$-m_2 \vec{a}_2 = -G \frac{(-m_1)(-m_2)}{r^2} \hat{r} \quad (15)$$

$$\vec{a}_2 = +G \frac{m_1}{r^2} \hat{r} \quad (16)$$

Negative mass and negative mass: Both two objects are accelerated in the direction of $+\hat{r}$ which extends distance r , so as time passes, the distance between them is greater than initially given condition, and the force between them is attraction, but the effect is repulsive. The force is attraction($-Gm_1m_2/r^2$), thus the potential energy between them has negative value.

2. Negative mass cannot form the structure greater than atom

As examined the equation of motion for negative mass, it is marked in form of $F = -ma(m > 0)$, when attraction is applied together with nuclear force(when usually nuclear force is attraction, but has the form of repulsive core [6]), and assuming nuclear force has the form of $\vec{F} = -Q(r)\hat{r}$, $Q(r)$ is the positive function of distance r , thus nuclear force is in the form of attraction worked in the direction of $-\hat{r}$. Here, for the force worked on negative mass m ,

$$\vec{F} = -m\vec{a} = -Q(r)\hat{r} \quad (17)$$

$$\vec{a} = \frac{Q(r)}{m} \hat{r} \quad (18)$$

The term of acceleration is positive, so the effect of increasing distance r , namely repulsive effect appears. This

means that negative mass cannot form the structure like atom(massive nucleon, baryon, particle consists of multi elementary particle), because nuclear force has not binding negative mass when it is applied to negative mass. Also, gravity has not binding negative mass(repulsive)

Additionally, for the problem of mesons that mediates nuclear force or weak interaction, if there is no meson that delivers strong interaction or weak interaction, it is doubtful if strong interaction or weak interaction can be worked or not. For example, nucleon must have internal structure including meson or quark, but in case of negative mass, nuclear force is repulsive, so it cannot have the internal structure of nucleon from the beginning. That is, there is a great possibility that negative mass cannot include meson or quark which has negative mass in nucleus.

The fact that it cannot make nucleon means that it is impossible to form massive mass structure like a star in addition to atomic structure. This provides proper explanation of the fact that negative mass is not seen as it has visible massive mass structure. Also generally it satisfies the nonbaryonic [9] matters required for dark matter.

If negative mass was born at the beginning of universe, there is higher possibility that it exists until now as a certain basic state born at the beginning of universe, and that it does not have strong interaction like nuclear force, weak interaction, and electromagnetic interaction(neutral or has not internal structure(positive mass elementary particle has a $\pm 1/2, \pm 1/3, 2/3$ charge)). This point is keeping with current characteristics required for dark matter.

If negative mass and positive mass were born together at the beginning of universe, positive mass has attractive effect each other, so it forms star and galaxy structure now, but negative mass has repulsive effect each other and nuclear force cannot form nucleons by binding negative mass, so they cannot make massive mass structure like star or galaxy.

If dark matter is negative mass, non-observation of dark matter star and galaxy can be explained.

3.Negative mass is stable at the maximum point! [8]

Nature prefers stable state, and has the tendency to go to stable state. Additionally, this can be expressed in another way that nature prefers low energy state, and has the tendency to go to low energy state.

Such an idea is frequently used as a logic which denies the existence of negative mass. That is, if there is negative mass and negative energy level, negative mass spontaneously emits energy to be stable, and goes to energy

state of minus ∞ , so finally it is confronted by catastrophe. Is it right?

In case of positive mass, stable state means low energy state, therefore it is not necessary to divide which one nature prefers among two states(stable state and low energy state). By the way, does stable state mean low energy state also in case of negative mass?

We can get an answer, if we examine Harmonic oscillation.

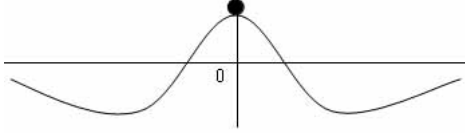


Figure 4: When there is negative mass in potential which has a point of maximum value and a point of minimum value.

We begin by considering the oscillatory motion of a particle that is constrained to move in one dimension. We assume that there exists a position of unstable equilibrium for the particle and we designate this point as the origin. Restoring force is in general some complicated function of the displacement and perhaps of the particle's velocity or even of some higher time derivative of the position coordinate.

We consider here only case in which the restoring force F is a function only of the displacement

$F(x)$ can be expanded in a Taylor series,

$$F(x) = F(0) + \frac{x}{1!}F'(0) + \frac{x^2}{2!}F''(0) + \frac{x^3}{3!}F'''(0) + \dots$$

$$+ \frac{x^n}{n!}F^{(n)}(0) + \dots \quad (19)$$

Since the origin is defined to be the equilibrium point, $F(0)$ must vanish, Then, if we confine our attention to displacements of the particle that are sufficiently small, we can neglect all terms involving x^2 and higher powers of x . We have, therefore, the approximate relation

$$F(x) = +kx \quad (20)$$

The force is always the opposite directed toward the unstable equilibrium position(the origin), the derivative $F'(0)$ is positive and therefore k is a positive constant.

$$-m\ddot{x} = +kx \quad (21)$$

$$\ddot{x} + \omega_0^2 x = 0 \quad (22)$$

$$\left(\omega_0^2 = \frac{k}{m}\right) \quad (23)$$

This form of differential equation is the same as that of particle which has positive mass. But we have to notice

that positive mass carries out harmonic oscillation on a point of minimum value, whereas negative mass carries out harmonic oscillation on a point of maximum value. Additionally, restoring force is $+kx$ at this time.

$$\vec{F} = -\nabla U \quad (24)$$

$$U = -\frac{1}{2}kx^2 \quad (25)$$

$$E_- = T + U = -\frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2 = -\frac{1}{2}m\omega_0^2 A^2 \quad (26)$$

In phase space

$$\frac{x^2}{\left(\frac{-2E_-}{k}\right)} + \frac{p^2}{(-2mE_-)} = 1 \quad (27)$$

This equation is ellipses equation, because total energy $E_- < 0$

As examined in the question of Harmonic oscillation, in case of positive mass, a point of minimum value which energy is the lowest is stable. However, in case of negative mass, stable equilibrium is a point of maximum value, not a point of minimum value. Therefore, negative mass is toward a point of maximum value to be stable, not a point of minimum value which energy is low.

In the world of positive mass, ground state is a point that energy is low, but in case of negative mass, ground state is a point that energy is the highest. Accordingly, in the world of negative mass, energy level is filled from the highest to the lowest, and stable state means the highest energy state, so the catastrophe to energy level of minus ∞ never happens even if negative mass spontaneously emits energy.

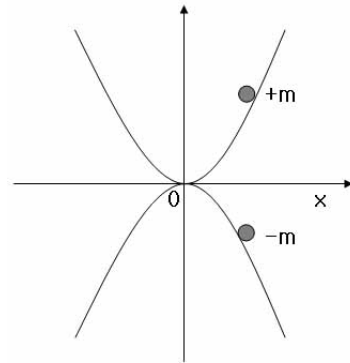


Figure 5: Harmonic oscillation of negative mass

In harmonic oscillation of negative mass

$$E_- = -\frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2 \quad (28)$$

$$H_+ \psi = E_+ \psi, H_- \psi = E_- \psi \quad (29)$$

$$[x, p_-] = [x, -p_+] = -[x, p_+] = -i\hbar \quad (30)$$

$$[x_i, p_{+j}] = +i\hbar\delta_{ij} \quad (31)$$

$$[x_i, p_{-j}] = -i\hbar\delta_{ij} \quad (32)$$

Hamiltonian H_- of negative mass (in harmonic oscillation)

$$H_- = -\frac{1}{2}m\dot{x}^2 - \frac{1}{2}mw^2x^2 \quad (33)$$

$$= -\hbar w \left(\sqrt{\frac{mw}{2\hbar}}x + i\frac{P_+}{\sqrt{2m\hbar w}} \right) \left(\sqrt{\frac{mw}{2\hbar}}x - i\frac{P_+}{\sqrt{2m\hbar w}} \right) \quad (34)$$

a (annihilation operator) and a^\dagger (creation operator) are defined

$$a = \sqrt{\frac{mw}{2\hbar}}x + i\frac{P_+}{\sqrt{2m\hbar w}} \quad (35)$$

$$a^\dagger = \sqrt{\frac{mw}{2\hbar}}x - i\frac{P_+}{\sqrt{2m\hbar w}} \quad (36)$$

$N(a^\dagger a)$ is

$$N = a^\dagger a = \frac{mw}{2\hbar} \left(x^2 + \frac{P_+^2}{m^2w^2} + \frac{i}{mw}[x, P_+] \right) \quad (37)$$

$$= \frac{1}{\hbar} \left[\frac{mwx^2}{2} + \frac{P_+^2}{2mw} \right] - \frac{1}{2} \quad (38)$$

$$= \frac{1}{\hbar} \left[-\frac{H_-}{w} \right] - \frac{1}{2} \quad (39)$$

$$H_- = -\hbar w \left(N + \frac{1}{2} \right) \quad (40)$$

Eigen value of number operator N

$$N|n\rangle = n|n\rangle \quad (41)$$

$$H_-|n\rangle = -\hbar w \left(N + \frac{1}{2} \right) |n\rangle \quad (42)$$

$$= -\hbar w \left(n + \frac{1}{2} \right) |n\rangle \quad (43)$$

$$= E_n^- |n\rangle \quad (44)$$

Therefore,

$$E_n^- = -\hbar w \left(n + \frac{1}{2} \right) \quad (n = 0, 1, 2, 3, \dots) \quad (45)$$

Ground state of negative mass is $n=0$ state. Eigen value is $E_0^- = -\frac{1}{2}\hbar w$, first-excite state $E_1^- = -\frac{3}{2}\hbar w$

4. The difference in mass when creating the pair of negative mass and positive mass

If positive mass and negative mass simultaneously repeat pair creation and pair annihilation microscopically at the vacuum state which satisfies energy conservation and momentum conservation, to materialize energy and momentum conservation, the size of two mass can be different when creating a pair of positive mass and negative mass because of the existence of potential energy, momentum conservation, etc.

If there is difference between $(-m_-c^2)$ and $(+M_+c^2)$ owing to momentum conservation and energy conservation at the state of repeating creation and annihilation of $(-m_-)$ and $(+M_+)$ in a vacuum in an instant (energy conservation can be broken for very short time because of uncertainty principle, so it can be created by such an energy as well), two mass are pair-created. If two mass are pair-annihilated, energy conservation is materialized on the whole, but the gravity by $(-m)$ and $(+M)$ can work on other objects in our universe for very short time before pair annihilation after pair creation

This study calculated dynamically the difference between two mass in accordance with energy and momentum conservation below, if negative mass and positive mass are pair-created in a vacuum.

If we consider the existence of difference in mass when pair creation negative mass and positive mass that energy conservation is materialized when pair creation and pair annihilation from $E_T = T + U + m_0c^2$

$$E_T = \frac{-m_-c^2}{\sqrt{1 - (\frac{v}{c})^2}} + \frac{M_+c^2}{\sqrt{1 - (\frac{V}{c})^2}} + \frac{(-GM_+(-m_-))}{r} \quad (46)$$

$$E_T = \frac{-m_-c^2}{\sqrt{1 - (\frac{v}{c})^2}} + \frac{M_+c^2}{\sqrt{1 - (\frac{V}{c})^2}} + \frac{GM_+m_-}{r} = 0 \quad (47)$$

$$P_T = \frac{-m_-v}{\sqrt{1 - (\frac{v}{c})^2}} + \frac{+M_+V}{\sqrt{1 - (\frac{V}{c})^2}} = 0 \quad (48)$$

(initial energy 0, initial momentum 0, $M_+ > 0, m_- > 0$.)

In equation (48),

$$\sqrt{1 - (\frac{v}{c})^2} = a \quad (49)$$

$$\sqrt{1 - (\frac{V}{c})^2} = b \quad (50)$$

are define, equation (48) is

$$\frac{m_-v}{a} = \frac{M_+V}{b} \quad (51)$$

$$m_- = M_+ \left(\frac{a}{b} \right) \left(\frac{V}{v} \right) \quad (52)$$

If equation (47) is substituted for equation (49), (50)

$$\frac{GM_+m_-}{r} = \frac{m_-c^2}{a} - \frac{M_+c^2}{b} \quad (53)$$

Substitution equation (52) in equation (53)

$$\frac{GM_+}{r} \times M_+\left(\frac{a}{b}\right)\left(\frac{V}{v}\right) = \frac{c^2}{a} \times M_+\left(\frac{a}{b}\right)\left(\frac{V}{v}\right) - \frac{M_+c^2}{b} \quad (54)$$

$$\frac{GM_+}{r} \times \left(\frac{a}{b}\right)\left(\frac{V}{v}\right) = \left(\frac{c^2}{b}\right)\left(\frac{V}{v}\right) - \frac{c^2}{b} \quad (55)$$

Both term $\left(\frac{b}{a}\right)\left(\frac{v}{V}\right)$ multiply,

$$\frac{GM_+}{r} = \frac{c^2}{a}\left(1 - \frac{v}{V}\right) \quad (56)$$

The left term is bigger than 0, thus the right term must be bigger than 0, accordingly,

$$V > v \quad (57)$$

If equation (57) is substituted for equation (52),

$$\left(\frac{V}{v}\right) = \frac{m_-b}{M_+a} > 1 \quad (58)$$

$$\frac{m_-}{M_+} > \frac{a}{b} = \frac{\sqrt{c^2 - v^2}}{\sqrt{c^2 - V^2}} \quad (59)$$

Considering $V > v$, which is the result from equation (57), in equation (59), numerator is bigger than denominator,

$$m_- > M_+ \quad (60)$$

So we can get the result. Equation (60) shows that, if negative mass and positive mass are pair-created, there is the difference between negative mass and positive mass when initial energy and momentum are 0 in the system that energy conservation and momentum conservation are materialized.

This difference in mass when negative mass and positive mass pair creation presents that, if negative mass and positive mass were born together at the beginning of universe, negative mass which is not offset from among them can exist in universe now, and also negative mass which is bigger than positive mass can exist. Accordingly, if negative mass is dark matter, it provides the description that total amount of dark matter is bigger than that of matter.

The existence of negative mass provides quantitative description of the existence of dark matter and dark energy, it can make it possible that energy value which is the origin of initial state of universe can be 0, and this has a very important meaning.

5. The universe expands even if the magnitude of negative mass is the same as that of positive mass

In previous particle physics, according to the case of the birth of antimatter, when pair-creating particles symmetrical to certain particle, their physical quantities are the same each other, according to the situation that has opposite specific value, when pair-creating negative mass and positive mass, the size of absolute value of negative mass is the same as that of positive mass, and negative mass and positive mass of same number were born, and this study tries to consider the cosmic expansion when the distance between them is similar.

5-1) Potential energy when there is one pair of negative mass and positive mass U :

$$U_1 = -\frac{G(m_+)(-m_-)}{r} = +\frac{G(m_+)(m_-)}{r} = \frac{Gm_+^2}{r} \quad (61)$$

(if, $m_+ = m_-$)

5-2) Potential energy when there is two pair of negative mass and positive mass U :

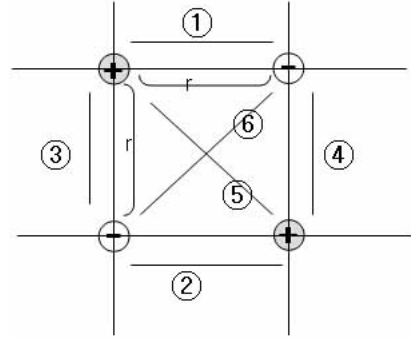


Figure 6: Arranges 1- when 2 pairs of negative and positive masses exist

$$U = \sum_{i=1}^6 U_i = \sum U_+ + \sum U_- \quad (62)$$

$$U = \sum U_+ + \sum U_- = (U_1 + U_2 + U_3 + U_4) + (U_5 + U_6) \quad (63)$$

$$= 4\frac{Gm_+m_-}{r} - \left(\frac{Gm_+m_+}{\sqrt{2}r} + \frac{Gm_-m_-}{\sqrt{2}r}\right) \quad (64)$$

$$= 4\frac{Gm_+^2}{r} + 2\left(-\frac{Gm_+^2}{\sqrt{2}r}\right) \quad (65)$$

$$= (4 - \sqrt{2})\left(\frac{Gm_+^2}{r}\right) \quad (66)$$

($m_+ = m_-$)

Potential energy has positive value, so there is repulsion, and the universe expands. We can know that the universe expands at the state that the size of negative mass is the same as that of positive mass, and the number was identically born

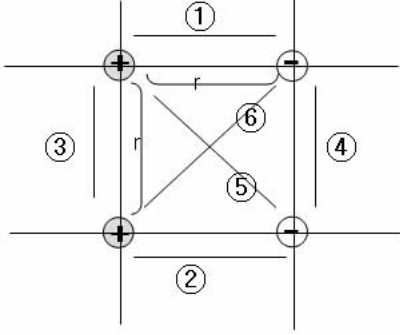


Figure 7: Arranges 2- when 2 pairs of negative and positive masses exist

$$U = \sum_{i=1}^6 U_i = \sum U_+ + \sum U_- \quad (67)$$

$$U = \sum U_+ + \sum U_- = (U_1 + U_2 + U_5 + U_6) + (U_3 + U_4) \quad (68)$$

$$= 2\left(\frac{Gm_+m_-}{r} + \frac{Gm_+m_-}{\sqrt{2}r}\right) - \left(\frac{Gm_+m_+}{r} + \frac{Gm_-m_-}{r}\right) \quad (69)$$

$$= 2\left(\frac{Gm_+^2}{\sqrt{2}r}\right) \quad (70)$$

$$= \sqrt{2}\left(\frac{Gm_+^2}{r}\right) \quad (71)$$

(if, $m_+ = m_-$)

Potential energy has positive value, so there is repulsion, and the universe expands.

We can see that potential energy keeps positive value at the state that the size of negative mass is the same as that of positive mass, and the number was identically born, although there is difference in the value of potential energy according to the distribution of each mass. Therefore, potential energy is positive, thus the force is positive, and the universe expands.

5-3) Potential energy when there are generally n pairs of negative mass and positive mass :

$$U_n = \sum_{i=1}^{n^2} U_{+i} + \sum_{j=1}^{n(n-1)} U_{-j} = nU_+ \quad (72)$$

(U_+ : positive potential energy, U_- : negative potential energy)

If n pairs of negative mass and positive mass are created, generally the number of term of positive potential energy is n^2 , the number of term of negative potential energy is $n(n-1)$, so the number of term of positive potential energy is greater than that of term of negative potential energy by n. Accordingly, even if the absolute value of negative mass is the same as that of positive mass at the beginning of universe, the universe has the value of positive potential energy, and expands.

III. Main assumption

: At the beginning of universe, negative mass and positive mass were born together in the energy and momentum conservation state.

If negative mass and positive mass were born together at the beginning of universe, positive mass has attractive effect each other as seen in Chapter II, so it forms star and galaxy structure now, and negative mass has repulsive effect each other, so it cannot form any structure, and may spread out uniformly in the whole area of universe as negative mass at the point that it was born.

Owing to the effect of negative mass and positive mass, negative mass was disappeared near massive positive mass structure (such as galaxy and galaxy cluster, etc.) after meeting positive mass, but negative mass which was born at the beginning of universe can still exist at the vacuum state out of general galaxy as much as the number of positive mass. This negative mass generates the effect of centripetal force which binding positive mass in galaxy or galaxy cluster.

IV. Explain for the problem of non-observation with the characteristics of negative mass

As examined with Newton dynamics of negative mass, attraction works between positive mass, and they have attractive effect, so they gradually clusters each other, and then make massive mass like planet or star. On the other hand, attraction works between negative mass, but they have repulsive effect, and nuclear force cannot form nucleons by binding negative mass, so they cannot make massive mass structure like planet or star. This explains that existing dark matter is not an interruption in forming galaxy while the universe is growing.

When matter and antimatter are pair creation and pair annihilation, antimatter also had basically positive mass, so gamma rays were emitted when pair annihilation, [6] and with this phenomenon, the existence of antimatter was proved, but pair annihilation of negative mass and positive mass is in the form of $(-E) + (+E) = 0$, thus

generally gamma rays are not emitted after pair annihilation. So it seems to be more difficult to detect.

Bubble box that detects a charged particle used in accelerator also is useless when negative mass is not charged, it cannot be valid means because negative mass cannot draw the trace itself with pair annihilation, and Thought wall of negative mass was too big. However, the essential reason is that energy values which can pair creation create negative mass and positive mass might be limited at the beginning of universe.

The reason why we have not found negative mass on the earth until now is that negative mass exists as the state of mass when it was born without forming massive mass structure which can be easily measured owing to basic characteristics of negative mass(repulsive effect) as described above, and that negative mass is not observed because negative mass which exists around massive positive mass such as earth or the solar system receives attractive effect from massive positive mass, and it was disappeared long time ago when forming galaxy, even if it existed at the beginning of universe.

There is no observation of negative mass until now, but basic characteristics of negative mass and aspect of dynamic motion with positive mass strangely provide the proper description of dark matter and dark energy in our universe.

The characteristics of negative mass uniformly satisfy the effect of centripetal force which existing dark matter in galaxy or galaxy cluster, the fact that it does not interrupt to form galaxy even though it has gravitational interaction, difficult element for the observation, very low interaction between dark matters when colliding with galaxy or galaxy clusters, repulsion energy required for cosmic expansion, repulsive effect on positive mass, etc.

On the other hand, negative mass also has the wall of thought that generally it is difficult to accept. However, we can say about the wall of thought, negative mass just collides with our fixed idea, and it does not collide with energy conservation or momentum conservation.

Some people deny negative mass because it cannot be accepted in the abstract, but it is necessary to refer to the fact that the quantum theory or the theory of relativity in 20th century is contradicted with existing fixed idea, provide quantitative explanation of dark matter and dark energy, and then to perform an experiment targeting on the observation of negative mass.

V. Method to prove the existence of negative mass

1.Description of cosmic decelerating expansion and accelerating expansion

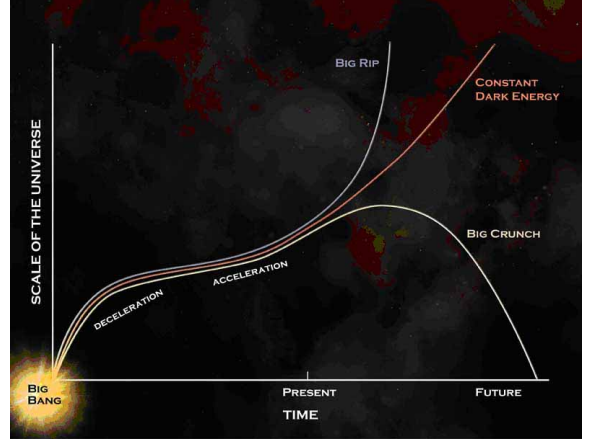


Figure 8: Decelerating expansion and accelerating expansion of the universe

Fig.8 shows current cosmic expansion. Currently expansion velocity of our universe was reduced owing to attraction of matter, but it has increased again for last 7 billion years. [10]

At the beginning of universe, most of negative energy was converted to negative mass owing to the characteristics of negative energy and positive energy pair-created, whereas some positive energy was exist as radiant energy(pair annihilation of matter and antimatter, radiation) and bond energy,

So, if we induce how many terms of potential energy survive when the number of negative mass is greater than that of positive mass,

$$U_+ : \text{positive potential } \left(+ \frac{GMm}{r} \right),$$

$$U_- : \text{negative potential } \left(- \frac{GMm}{r} \right), U_T : \text{Total potential}$$

Focus on the number of terms of remaining potential energy rather than the concrete value of potential energy,

*Potential energy between positive mass and positive mass has value: $U = \left(\frac{-Gm_+m_+}{r} \right) = 1U_-$

*Potential energy between negative mass and positive mass has + value: $U = \left(\frac{-G(-m_-)m_+}{r} \right) = 1U_+$

*Potential energy between negative mass and negative mass has value: $U = \left(\frac{-G(-m_-)(-m_-)}{r} \right) = 1U_-$

$$\text{(if, } U_+ = -U_- \text{)}$$

1-1) When the number of negative mass is 2, and the number of positive mass is 1

$$U_T = 2 \left(\frac{-G(-m_-)m_+}{r} \right) + \left(- \frac{G(-m_-)(-m_-)}{r} \right)$$

$$= 2U_+ + 1U_- = 1U_+ \quad (73)$$

1-2)When the number of negative mass is 3, and the number of positive mass is 1

$$U_T = 3U_+ + 3U_- = 0 \quad (74)$$

1-3)When the number of negative mass is 4, and the number of positive mass is 1

$$U_T = 4U_+ + 6U_- = -2U_+ \quad (75)$$

1-4)When the number of negative mass is 4, and the number of positive mass is 2

$$U_T = 8U_+ + 7U_- = 1U_+ \quad (76)$$

1-5)When the number of negative mass is n_- , and the number of positive mass is n_+ , total potential energy is given as follows.

$$U_T = (n_- \times n_+)U_+ + \left(\frac{n_-(n_- - 1)}{2}\right)U_- + \frac{n_+(n_+ - 1)}{2}U_- \quad (77)$$

Equation (77) is expressed as follows more strictly

$$U_T = \sum_{i,j}^{i=n_-,j=n_+} \left(\frac{Gm_{-i}m_{+j}}{r_{-+ij}}\right) + \sum_{i,j,i>j}^{i,j=n_-} \left(\frac{-Gm_{-i}m_{-j}}{r_{--ij}}\right) + \sum_{i,j,i>j}^{i,j=n_+} \left(\frac{-Gm_{+i}m_{+j}}{r_{++ij}}\right) \quad (78)$$

$$U_T = (n_- \times n_+) \left(\frac{Gm_{-}m_{+}}{\bar{r}_{-+}}\right) + \left(\frac{n_-(n_- - 1)}{2}\right) \left(\frac{-Gm_{-}m_{-}}{\bar{r}_{--}}\right) + \frac{n_+(n_+ - 1)}{2} \left(\frac{-Gm_{+}m_{+}}{\bar{r}_{++}}\right) \quad (79)$$

$$(U_+ = \left(\frac{Gm_{-}m_{+}}{\bar{r}_{-+}}\right), U_- = \left(\frac{-Gm_{-}m_{-}}{\bar{r}_{--}}\right), U_- = \left(\frac{-Gm_{+}m_{+}}{\bar{r}_{++}}\right))$$

\bar{r}_{-+} :The mean distance between negative mass and positive mass,

\bar{r}_{--} :The mean distance between negative mass and negative mass,

\bar{r}_{++} :The mean distance between positive mass and positive mass)

Calculate the value of $U_- = -U_+$, $n_- = 10, n_+ = 1 - 10$ with equation (77) when there is a difference in the number between negative mass and positive mass to examine changes,

$$\begin{aligned} (n_-, n_+) \\ (10, 1)U_T &= 10U_+ + 45U_- = -35U_+ \\ (10, 2)U_T &= 20U_+ + 46U_- = -26U_+ \\ (10, 3)U_T &= 30U_+ + 48U_- = -18U_+ \\ (10, 4)U_T &= 40U_+ + 51U_- = -11U_+ \\ (10, 5)U_T &= 50U_+ + 55U_- = -5U_+ \\ (10, 6)U_T &= 60U_+ + 60U_- = 0 \end{aligned}$$

$$\begin{aligned} (10, 7)U_T &= 70U_+ + 66U_- = 4U_+ \\ (10, 8)U_T &= 80U_+ + 73U_- = 7U_+ \\ (10, 9)U_T &= 90U_+ + 73U_- = 9U_+ \\ (10, 10)U_T &= 100U_+ + 90U_- = 10U_+ \end{aligned}$$

We can see the change in total potential energy in accordance with the difference in the number of negative mass and positive mass from 10 samples above.

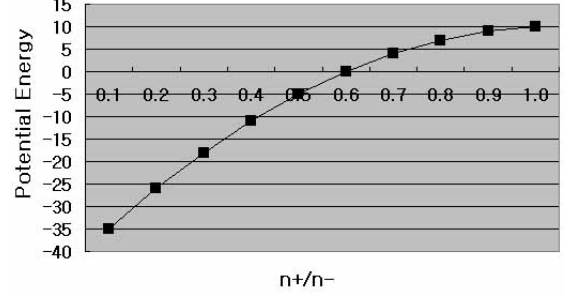


Figure 9: Potential energy from ratio of negative mass and positive mass

- The tendency of total potential energy in accordance with the number of negative mass and positive mass -

i)In Critical ratio of the number of negative mass to the number of positive mass, the total potential energy has 0. When the universe is flat.(ex. $(n_-, n_+) = (10, 6)$) Here, we note that total potential energy does not have 0 when general matter comes under 60% of dark matter. 60% is the proportion by assuming that all terms of potential energy are identical, and prescribing that the number of n_- is 10.

ii)If number of positive mass less than critical number, total potential energy has negative value. Cosmic decelerating expansion

iii)If number of positive mass over than critical number, total potential energy has positive value. Cosmic accelerating expansion

iv)The value of total potential energy increases as the number of positive mass approaches to the number of negative mass.

v)If the number of pair of negative mass and positive mass is n, n terms of positive potential remains as shown in equation (72)($U_T = nU_+$). Universe's last state.

$$\text{From eq.(72)} \quad (U_n = \sum_{i=1}^{n^2} U_{+i} + \sum_{j=1}^{n(n-1)} U_{-j} = nU_+)$$

The analysis above can describe that current universe expanded with decelerated speed approximately 7 billion years ago, whereas it expanded with accelerated speed in last 7 billion years, and most of negative energy was

converted to negative mass owing to the characteristics of negative energy and positive energy pair-created at the beginning of universe, whereas some positive energy was exist as radiant energy(pair annihilation of matter and antimatter, radiation) and bond energy, which does not have positive matter and mass.

At this time, it expanded with decelerated speed because total potential energy had negative value, then positive matter gradually increased as the universe was cooled down and currently it entered in accelerating expansion which total potential energy of universe has positive value after passing through the point that total potential energy of universe is zero.

2.Proof of that value of observed dark energy($10^{-47}GeV^4$)

In negative mass hypothesis, dark energy is corresponding to that positive potential term in total potential energy.

2-1)Proof from the definition equation of dark energy

In equation (79)

$$E_{de} = U_{de} = (n_- \times n_+) \left(\frac{Gm_- m_+}{\bar{r}_{-+}} \right) \quad (80)$$

If radius of the universe is 60Gyr, ordinary matter density is about proton $1ea/5m^3$. So, $m_+ = m_p$, $m_- = km_+ \simeq \left(\frac{23.3}{4.6}\right)m_+ = (5.06522)m_p$ (because that dark matter has about (23.3/4.6) times ordinary matter in WMAP) [11]

From equation (95)

$$\bar{r}_{-+} = (60\text{Gyr}/3.27273) = 1.73447 \times 10^{26}m$$

From analysis of V-5,

If $U_T \geq 0$, $n_- \approx n_+$, Therefore, Define, $n_- = n_+ = n$

$$V = \frac{4\pi R^3}{3} = \frac{4\pi \times (5.67648 \times 10^{26})^3}{3} = 7.66171 \times 10^{80}m^3$$

$$n = \frac{\rho V}{m_p} = \frac{(1m_p/5m^3)V}{m_p} = 1.53234 \times 10^{80}$$

(10^{80} is about total proton number of our universe).

$$U_{de} = (kn^2) \left(\frac{Gm_p^2}{\bar{r}_{-+}} \right) \quad (81)$$

$$U_{de} = (5.06522)n^2 \frac{(6.6726 \times 10^{-11})(2.79772 \times 10^{-54})}{1.73447 \times 10^{26}} J$$

$$U_{de} = (n^2) \times 5.45168 \times 10^{-90} J = 1.28009 \times 10^{71} J$$

$$1J = 1kg(m/s)^2 = 6.242 \times 10^{18} eV$$

$$U_{de} = 7.99031 \times 10^{89} eV \quad (82)$$

$$\rho_{de} = \frac{U_{de}}{V} = \frac{7.99031 \times 10^{89} eV}{7.66171 \times 10^{80} m^3} = \frac{1.04289 \times 10^{-6} GeV}{cm^3} \quad (83)$$

Planck Unit transformation($1cm = 0.5063 \times 10^{14} GeV^{-1}$)

$$\rho_{de} = \frac{1.04289 \times 10^{-6} GeV}{1.29784 \times 10^{41} GeV^{-3}} = 0.80355 \times 10^{-47} GeV^4$$

$$\rho_{de} = 0.80355 \times 10^{-47} GeV^4 \quad (84)$$

Observation value is $\rho_{obs} \approx 10^{-47} GeV^4$

If $R=90\text{Gyr}$, $\rho_{de} = 1.808 \times 10^{-47} GeV^4$ (refer to fig11).

$$\rho_{de} \approx \rho_{obs} \quad (85)$$

In Quantum Field Theory, the energy density of the vacuum is estimated as $10^{70} GeV^4$, which is about 10^{117} orders of magnitude large than the observation value $10^{-47} GeV^4$. [5]

Therefore, You can see that negative mass hypothesis how to close to the observation and the universe.

From equation (80), Origin of dark energy is particle not pressure or constant energy, because that n_- and n_+ are number of particle.

Also, because that gravity is repulsive, it is strongly suggested that negative mass is exist. U_{de} is right. It means that U_T is also right. Thus, from the analysis of U_T , it means that analysis of the inflation, fine tuning problem, decelerating and accelerating expansion, future of our universe are right.

You will know the mean which magnitude of dark energy is proved. It is saying that cosmological constant has not existed and dark energy has not come from vacuum energy.

Definitely, it is against to the Λ CDM model.

For all that, why did the Λ CDM model show a similar result? We can find out if we look at the total potential energy section, but if we look at the total potential energy (78) equation, (79) equation, it is in a form of :

$U =$ (positive potential term) + (negative potential term) = $\Lambda +$ (ordinary gravitation potential)

Which the positive term played an independent potential role as Λ .

2-2)Proof from the gravitational self-energy

In case of, mass M and mass distribution is $0 \leq r \leq R$ and mass density is ρ

Gravitational self-energy of the universe [12]

$$U_S = -\frac{3}{5} \frac{GM^2}{R} \quad (86)$$

A coefficient 3/5 is constant for geometric shape of the universe.

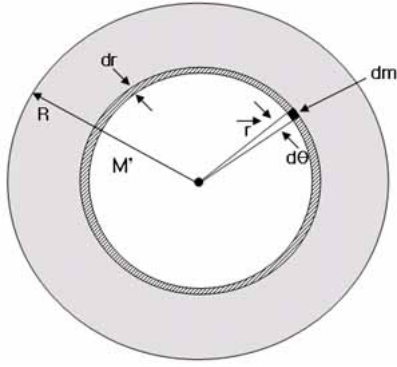


Figure 10: Gravitational self-energy

At this time, let's analyze to the relation between total potential energy and gravitational self-energy.

Equation (79) is total potential energy when the number of negative mass is n_- , and the number of positive mass is n_+ .

The other side, U_s is total potential energy when all particles are positive mass. Therefore, U_s is total potential energy when dark energy term has an opposite sign.

General gravitational potential defined,

$$U_{gp} = -\left(\frac{n_-(n_- - 1)}{2} \frac{Gm_- m_-}{r_{--}}\right) + \left(\frac{n_+(n_+ - 1)}{2} \frac{Gm_+ m_+}{r_{++}}\right) \quad (87)$$

Therefore,

$$U_T = U_{de} + U_{gp} \quad (88)$$

$$U_S = -U_{de} + U_{gp} \quad (89)$$

From WMAP [11]

$$a = \frac{|U_{gp}|}{U_{de}} = \left(\frac{23.9}{72.1}\right) = 0.33148 \quad (90)$$

$$U_{gp} = -aU_{de}, U_S = -(a+1)U_{de}$$

Finally,

$$U_{de} = -\frac{U_S}{(a+1)} = (0.75104) \left(\frac{3GM^2}{5R}\right) \quad (91)$$

i) In case of $n_- = n_+ = n$, $m_- = km_+$ ($k \geq 1$) Total mass $M = (n_- \times m_-) + (n_+ \times m_+) = (k+1)nm_p$

$$U_{de} = -\frac{U_S}{(a+1)} = \frac{3}{5(a+1)} \frac{G((k+1)nm_p)^2}{R} \quad (92)$$

$$U_{de} = \frac{3(k+1)^2 n^2 Gm_p^2}{5(a+1)R} = (0.45062)(k+1)^2 n^2 \frac{Gm_p^2}{R} \quad (93)$$

Equation (81) = Equation (93)

$$(kn^2) \left(\frac{Gm_p^2}{\bar{r}_{+-}}\right) = \frac{3(k+1)^2 n^2 Gm_p^2}{5(a+1)R}$$

$$\bar{r}_{+-} = \frac{5(a+1)k}{3(k+1)^2} R \quad (94)$$

If $k = \frac{(\text{dark_matter})}{(\text{ordinary_matter})} \approx \frac{23.3}{4.6} = (5.06522)$ (In WMAP)

$$\bar{r}_{+-} = \frac{R}{3.27273} \quad (95)$$

Used to the gravitational self-energy, dark energy value of today is explained. Therefore, it means that dark energy is gravitational potential. Also, because that gravitational potential is plus value, it is strongly suggested that negative mass is exist.

2-3) Relations between radius of universe and dark energy density

Radius (Gyr)	13.7	30	50	60
Proton no.	1.82E+78	1.92E+79	8.87E+79	1.53E+80
Dark Energy density (GeV ⁴)	4.19E-49	2.01E-48	5.58E-48	8.04E-48
Radius (Gyr)	90	100	120	150
Proton no.	5.17E+80	7.09E+80	1.23E+81	2.39E+81
Dark Energy density (GeV ⁴)	1.81E-47	2.23E-47	3.21E-47	5.02E-47

Figure 11: Relations between radius of universe and dark energy density

- * mass density of ordinary matter = $1\text{proton}/5m^3$
- * Proton mass = $1.67264 \times 10^{-27} \text{kg}$
- * $G = 6.6726 \times 10^{-11} \text{m}^3/\text{s}^2 \text{kg}$
- * $1\text{J} = 6.242 \times 10^{18} \text{eV}$
- * $n_- = n_+ = n, m_- = (23.3/4.6)m_+ = (5.06522)m_p$
- * $\bar{r}_{+-} = \frac{R}{3.27273}$
- * If $\bar{r}_{+-} = \frac{R}{2.17879}$ ($U_T \approx 0$), dark energy density has a 1/3 smaller than $\bar{r}_{+-} = \frac{R}{3.27273}$.

In a WMAP, observed value $\Lambda = 2.14 \pm 0.13 \times 10^{-47} \text{GeV}^4$

$$\rho_{de} = 2.09 \times 10^{-47} \left[\begin{smallmatrix} +0.557 \\ -0.465 \end{smallmatrix} \right] \text{GeV}^4$$

$$R_{UNI} = 96.76 \left[\begin{smallmatrix} +12.131 \\ -11.441 \end{smallmatrix} \right] \text{Gly} = 85.32 \sim 108.89 \text{Gly}$$

If $\bar{r}_{+-} = \frac{R}{2.17879}$, $R_{UNI} = 118.8 \left[\begin{smallmatrix} +14.91 \\ -14.0 \end{smallmatrix} \right] \text{Gly} = 104.8 \sim 133.7 \text{Gly}$

From Neil J. Cornish, the universe' radius is at least 24Gpc(78Gly). [13]

3.Dark matter and dark energy come from one origin

As considered the problem of the gravity at the point placed on arbitrary radius R of spherical object like the

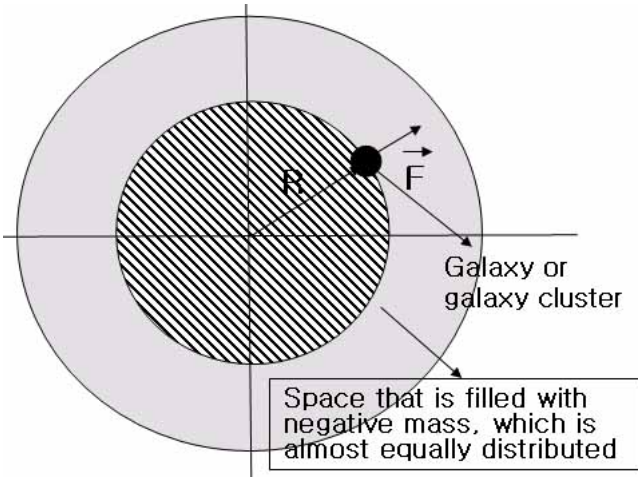


Figure 12: Expansion of the universe by negative mass distribution. The force, which is applied to the galaxy and galaxy cluster and located within radius R from the center of a 3-dimensional space, when assuming equal distribution of negative mass from repulsive effects between negative masses across space is valid.

earth that mass is uniformly distributed [14], the gravity by mass distribution out of radius R is offset each other, and there is only the gravity effect by mass within radius R.

If negative mass was born at the beginning of universe together with positive mass, it should be uniformly distributed in universe by repulsion between negative mass. Of course, there can be difference in negative mass density owing to the local distribution of massive positive mass.

For the gravity effect that almost uniformly distributed negative mass has on positive mass(galaxy or galaxy cluster) located in radius R from the center of universe(3-dimensional center), negative mass within radius R only has the gravity effect on positive mass located within radius R, and at this time, positive mass is accelerated in the direction of begin far away from negative mass.

The direction that positive mass becomes farther from negative mass is in keeping with the direction of cosmic expansion, and also negative mass continuously works the force, so finally positive mass is gradually accelerated(expansion velocity becomes faster). As examined in Chapter II, in the description of the force of positive mass and negative mass, the force is repulsion(the direction of +R), and integrated potential energy is positive value. Therefore, the assumption of the existence of negative mass can suggest the explanation of repulsive effect required for cosmic expansion, and the accompanying positive dark energy.

The hypothesis of dark matter and dark energy with negative mass insists that dark matter is not different from dark energy each other, dark matter is the effect of centripetal force by negative mass out of galaxy, and dark energy is that positive potential term in total potential energy, and total potential energy has positive value at now. So universe has accelerating expansion.

Accordingly, if we calculate the distribution and mean density of negative mass that describes rotation velocity within galaxy, and explain accelerating expansion of current universe with the size of potential energy that has positive value by this negative mass, we can prove the hypothesis that negative mass is the origin of dark matter and dark energy.

3-1) Negative mass has observed, but it was trashed away

In 1998 year, observation by HSS(The High-z Supernova Search team) [1] team and SCP team(Supernova Cosmology Project team) [2], I have seen a calculation that if the cosmological constant is 0(It means that they got a negative mass density from inspected field equation during 70years.),

$$\Omega_M = -0.38(\pm 0.22) [1]$$

However, HSS team and SCP team have asserted that cosmological constant must be revival. Because Ω_M must not be negative value in they thought.

They did not know that negative mass is stable at the maximum point!

In case of negative mass, ground state is a point that energy is the highest. Accordingly, in the world of negative mass, energy level is filled from the highest to the lowest, and stable state means the highest energy state, so the catastrophe to energy level of minus infinity never happens even if negative mass spontaneously emits energy.(Refer to II-3)

We must to know that not the equation has disposed the value, but our thought disposed the value.

In hypothesis with negative mass,

$$E_T = \text{positive mass}(4.6) + \text{negative mass}(y) + \Lambda(0) \\ (\text{negative mass} \geq \text{positive mass, cosmological constant } \Lambda = 0)$$

$$E_T = \text{positive mass}(x) + \text{negative mass}(y) + U_T$$

We assume over 50% of the dark matter as negative mass.

3-2) Age of universe

If cosmological constant $\Lambda=0$ and mass density of universe is negative value $\Omega_M = -0.38$, to get the age of the universe.

From Friedmann equation,

$$t_0 = \frac{1}{H_0} \int_0^1 \frac{xdx}{\sqrt{\Omega_M x + \Omega_R + \Omega_\Lambda x^4 + \Omega_K x^2}} \quad (96)$$

$$\Omega_M = -0.38(\pm 0.22), \Omega_R h^2 \approx 4.16 \times 10^{-5}, \Omega_\Lambda = 0$$

Use to $\Omega_K = 1 - \Omega_\Lambda - \Omega_M = 1.38$, $H_0 = 100h$ km/s/Mpc, $h=0.701$

define to $\Omega_K = a, \Omega_M = b, \Omega_R = c$,

$$\frac{1}{H_0} \int_0^1 \frac{xdx}{\sqrt{\Omega_K x^2 + \Omega_M x + \Omega_R}} = \frac{1}{H_0} \int_0^1 \frac{xdx}{\sqrt{ax^2 + bx + c}} \quad (97)$$

$$t_0 = \frac{1}{H_0} \left[\frac{\sqrt{ax^2 + bx + c}}{a} - \frac{b}{2a} \int_0^1 \frac{dx}{\sqrt{ax^2 + bx + c}} \right] \quad (98)$$

$$t_0 = \frac{1}{H_0} \left[\frac{\sqrt{ax^2 + bx + c}}{a} \right]_0^1 - \frac{1}{H_0} \left[\frac{b}{2a} \frac{1}{\sqrt{a}} \ln(2\sqrt{a}\sqrt{ax^2 + bx + c} + 2ax + b) \right]_0^1 \quad (99)$$

$$t_0 = 14.225 \text{Gyr} \quad (100)$$

This value is approach to the $t=13.73$ Gyr calculated in a WMAP.(+3.6% large) considering the very large changes($\Lambda=0$ and $\Omega_M = -0.38$; Λ is a coefficient of x^4), it seems to increase existence probability of negative mass.

Additionally, if $\Omega_M = \Omega_{visible}(4.6) + \Omega_{dark}(-23.3) = -18.7$, we get the $t_0=14.927$ Gyr

Universe' age by other method

Radioactive decay method : 15.2 ± 2.1 Gyr [15],

HR diagram(Hertzsprung-Russel diagram): 13.2 ± 2 Gyr [16],

The arithmetic mean of two methods is 14.2 ± 2.05

So, it(14.225 Gyr from negative mass density) is in the range estimated by other observations.

4. Calculating centripetal force in galaxy from negative mass distributed out of galaxy

We observed not positive mass but gravitation effect. That is, universe has not said that dark matter is positive mass.

This study mentioned the effect of centripetal force described previously, and the effect of centripetal force of negative mass with the description of dark matter. The researcher suggests that it is possible to prove the existence of negative mass with the comparison between observed value and revolution velocity of star according to the distance in galaxy which is calculated from negative mass distributed out of galaxy.

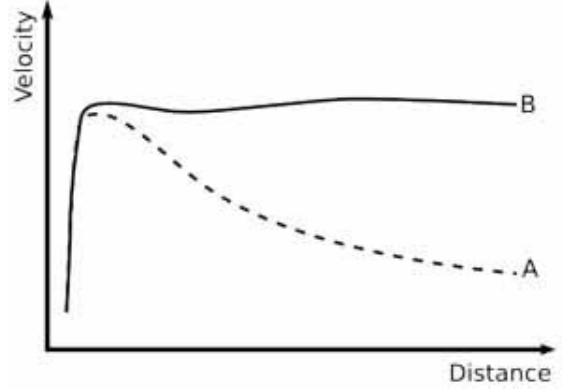


Figure 13: Revolution velocity of galaxy. The revolution velocity through the total quantity of matter that can be observed is A, while the actual observed value is B.

When negative mass is distributed uniformly, if we examine the area of galaxy diameter working the gravity on the basis of mass m for the analysis, there is no negative mass in the space of spiral space on the left, so negative mass can exist as much as the size of black and white on the right. negative mass on the right works the force on positive mass m on the origin to the left(positive mass moves after receiving the force that becomes farther from negative mass), and this direction will be in keeping with the central direction of gravity of real galaxy.

$$V(r)^2 = GM(r)/r \quad (101)$$

$$M(r) = M(r)_{visible} + M(r)_{dark} \quad (102)$$

From the calculation of centripetal force in Newton dynamics, the important matters to calculate gravity effect of negative mass are the size and model of empty space of negative mass by real galaxy on the left, in case of spiral galaxy or oval galaxy, it may be close to oval if we consider the fact that most of mass is placed on galaxy core, the rotation of galaxy, and the gravitational radius of galaxy core, thus we regard empty space of negative mass as spherical or oval, and perform calculation to compare with observed value.

$$\frac{mv^2}{r_+} = \frac{GM_+(r_+)m}{r_+^2} + \frac{GM_-(r_-)m}{r_-^2} \quad (103)$$

$$(M_+(r_+) > 0, M_-(r_-) > 0)$$

Adding the term of centripetal force by negative mass to the term of centripetal force by positive mass as described above, we compare revolution velocity of star and the density of negative mass, then expand this to apply to other galaxies, and we can inspect if the interpretation of dark matter distributed out of galaxy is proper or not.

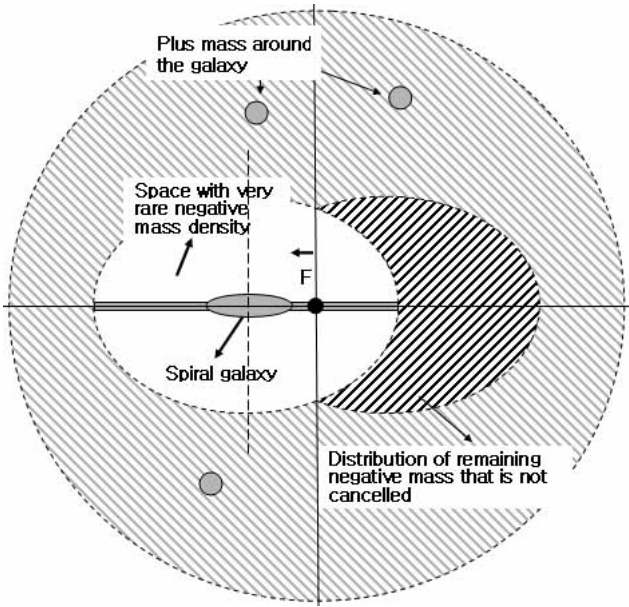


Figure 14: Negative mass effect on arms of galaxy. The actual observed spiral galaxy is on the left. Since repulsive effects occur between negative masses in Newtons dynamics, negative masses will be distributed all over space because it cannot form large mass structures like stars. Negative mass within the galaxy is cancelled out by attraction from large positive mass during the galaxy formation process. Furthermore, the space, other than the galaxy, will maintain the distribution state of negative mass.

The distribution of negative mass that remains without being offset in Fig. 15 is crescent-shaped, so it seems to be difficult to calculate, but if we assume that the white empty space on the left is full with negative mass and positive mass at the same density, here negative mass is uniformly distributed over the whole area, so the effect of negative mass on mass m is 0. Remaining positive mass is distributed over the white area on the left at the density of negative mass, and the gravity that uniformly distributed positive mass works on positive mass m place on radius r is worked only by the distribution of mass within radius r .

Therefore, the effect of negative mass that remains out of galaxy without being offset can make it approximate to the gravity generated by the distribution of positive mass within the radius R in galaxy

$$M_- = \rho_-(r)V = \rho_-(r)\left(\frac{4\pi r^3}{3}\right) \quad (104)$$

In equation(103)

$$\frac{mv^2}{r_+} = \frac{GM_+(r_+)m}{r_+^2} + \frac{GM_-(r_+)m}{r_+^2} \quad (105)$$

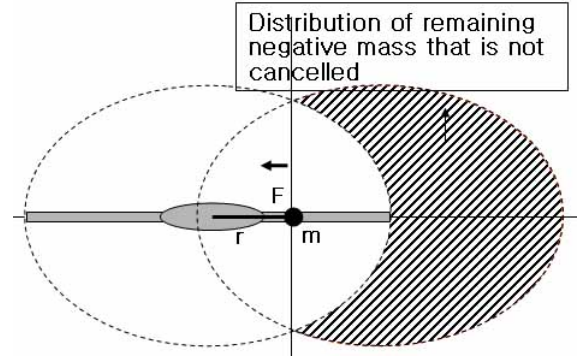


Figure 15: Effective negative mass. The negative mass distribution, where gravity is applied without cancellation of the oblique field located on the right side.

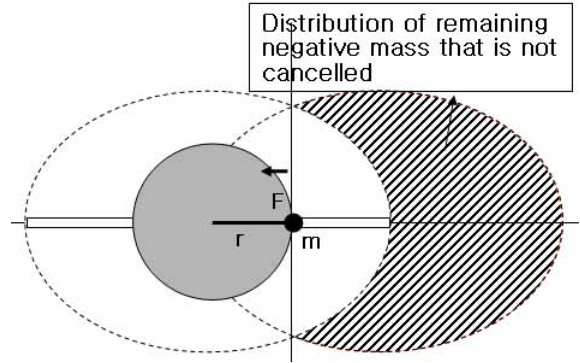


Figure 16: Equal to the degree of gravity that is applied on the equally distributed positive mass within the radius r of the left side. The gravitational effect from negative mass, which functions at mass m is equal to the gravitational effect from positive mass within radius r .

$$v = \sqrt{\frac{G(M_+ + M_-)}{r_+}} = \sqrt{\frac{G(M_+ + \rho_-(r_+)V)}{r_+}} \quad (106)$$

$$v = \sqrt{\frac{GM_+}{r_+} + \rho_-(r_+)\left(\frac{4\pi r_+^2}{3}\right)} \quad (107)$$

The mass density of collisionless isothermal sphere is given as $\rho(r) = \frac{\sigma^2}{2\pi G r^2}$ [17] in galaxy dynamics. σ^2 is velocity dispersion.

$$M_{dark} = M_{nega} = \frac{2\sigma^2}{3G}r \quad (108)$$

Equation (107) is applied to arbitrary spherical or oval galaxy, and in case of normal shaped spiral galaxy, the distance which spherical approximation can be applied from the center of galaxy will be effective.

So if we consider that the distribution of negative mass out of galaxy comes under isothermal, the effect of mass in galaxy by the distribution of negative mass out of

galaxy is proportional to r as shown in equation (108), and this is in keeping with observed dark matter.

Positive mass dark matter distributed in galaxy does not form the spherical distribution, and it is broken owing to the gravity of galaxy core, gravitational action with stars, and rotation of galaxy, if its shape is not very intentionally assumed. Furthermore, there are various heat sources in galaxy according to position, so it cannot be regarded as collisionless Isothermal.

However, the effect of centripetal force that negative mass works from the exterior of galaxy can keep the spherical distribution of mass, and make the mass effect that increases linearly in accordance with distance r as seen above because it receives less influence from elements such as the gravity of galaxy core, or rotation of galaxy. Also ideal state of collisionless can be achieved by negative mass, because negative mass has repulsive effect each other.

At the solar system, non-observation of dark matter is explained. In negative mass hypothesis, dark matter exists at the exterior of galaxy, so dark matter is not observed at the solar system.

It described that the quantity of negative mass decided at the beginning of universe can explain the effect of centripetal force in galaxy(V-4), repulsion effect on cosmic expansion(V-3), decelerating expansion at the first half, and accelerating expansion(V-1) of universe at the second half(7 billion years ago) at the same time.

Therefore, it is necessary to try to calculate and observe negative mass more strictly, laying aside the abstract aversion of negative mass.

5. Fine tuning problem and inflation mechanism

Pair creation of negative mass and positive mass seems to provide the proper explanation of homogeneity, local non-homogeneity, and flatness of our universe without separate assumption with basic characteristics of Newton dynamics.

With reference to the flatness of universe, the calculation of cosmic critical density with previous positive mass leads to the problem that present density should be close to critical density, the cosmic density at the beginning should be close to critical density, and initial condition should be set delicately. [5]

However, according to the hypothesis of negative mass, potential energy of universe is not single potential when there is only positive mass(potential that has both + and

-), and the density of universe close to critical density was from basic mechanism of pair creation, namely 1:1 correspondence of negative mass to positive mass, and form of potential energy.

If we try to explain the start and natural end of inflation with negative mass for inflation mechanism, and phase transition which the gravity is separated was generated at the beginning of universe when vacuum state was $E_T = (E_-) + (E_+) = 0$, the separation of gravity means the existence of mass, $E_- = m_-c^2$, $E_+ = m_+c^2$ so we can consider that both negative energy and positive energy were converted to mass.

5-1) Bigbang

As seen in equation (72) and V-1 above, total potential energy has the biggest positive value when the number of positive mass is similar or equal to that of negative mass, if all energy was converted to mass when the gravity was birthed or separated, it can explain that there was very big positive potential value at that time, and it can explain the reason of the start of Big Bang and inflation.

$$E_T = 0 = (+E) + (-E) = (\sum +m_+c^2) + (\sum -m_-c^2) + (\sum U) = 0 = E_T$$

positive energy and negative energy simultaneously repeat pair creation and pair annihilation microscopically at the vacuum state.

A moment, when the gravity was birthed or separated, energy was changed to mass.

At this time, total gravitational potential energy has the biggest positive value($U_T = nU_+$). So universe is started and expanded.

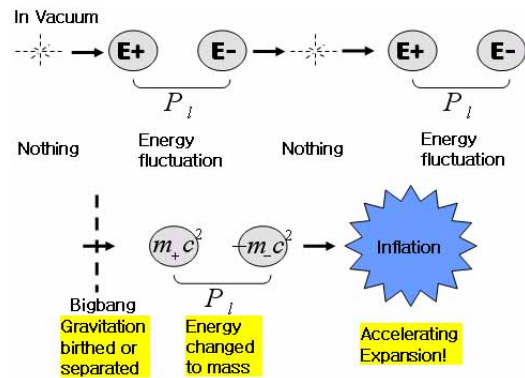


Figure 17: Mechanism of Bigbang and Inflation

Existence can be started from nothing. Also, equation means that our universe can be starting at zero energy by physically. And it's means that our universe must not be starting at specially energy value.

5-2) Inflation mechanism

- Inflation start -

If we consider that inflation started at the point of time that there were large-scale pair creations of negative mass and positive mass after Big Bang, not at the point of time that the gravity was separated, the number of positive mass and negative mass might be born together at this time, so here also the term of positive potential energy by equation (72) nU_+ (if pair creations are 10^{80} , $U_T = 10^{80}U_+$ survives), and this can provide the start power of inflation.

- Inflation finish -

If positive mass is converted to radiant energy (pair annihilation of matter and antimatter, radiation) or energy when strong interaction (Some mass changed to the bond energy. Therefore, positive mass defect has come.), weak interaction, and electromagnetic force are separated after the start of inflation, the number of positive mass falls down below critical ratio which total potential energy is 0 in (V-1.), so at this time, inflation also is naturally finished.

5-3) Fine tuning problem

For fine tuning problem of mass density and cosmological constant,

Universe mass density is same critical mass density, that is correspond with total potential energy=0

Define, $x = n_-$, $y = n_+$, if

$$U_{-+} = -U_{--} = -U_{++} = U \quad (109)$$

I can't seek exactly solution for total potential energy equation. So I consider that these three potential can be the same U, and I will seek to the whole tendency.

Maybe, consider of uniformly distribution, it is near to stable state when three value of potential energy are almost in same level.

Equation(77) is

$$U_T = \frac{2xy - x^2 + x - y^2 + y}{2} U \quad (110)$$

If $x=y=n$,

$$U_{\max} = nU \quad (111)$$

If, $x \rightarrow \infty, y \rightarrow \infty; n_h(t) = \frac{y}{x} = \frac{n_+}{n_-}$

$$U_T = -\frac{x^2}{2} \left[\left\{ 1 + \left(\frac{y}{x}\right)^2 - \left\{ \frac{2y}{x} + \frac{1}{x} + \frac{y}{x^2} \right\} \right\} U \right] \quad (112)$$

$$U_T \simeq -\frac{x^2}{2} \left[\left\{ 1 + \left(\frac{y}{x}\right)^2 - \left\{ \frac{2y}{x} + 0 + 0 \right\} \right\} U \right] \quad (113)$$

$$U_T = -\frac{x^2}{2} \left(1 - \frac{y}{x} \right)^2 U \quad (114)$$

$$U_T = -\frac{n_-^2}{2} (1 - n_h(t))^2 U \quad (115)$$

$y \neq n$, at most the section, Total potential energy is proportion square term of negative mass number (similar positive mass number)

Divide U_{\max} by equation(115),

Ratio of potential size

$$\frac{U_{\max}}{|U_T|} = \frac{n_-}{\frac{n_-^2}{2} (1 - n_h(t))^2} = \frac{2}{n_- (1 - n_h(t))^2} \quad (116)$$

- Cosmological constant problem -

n_- is total number of negative mass. If n_- is 10^{80} , we can know how the present potential values are smaller than the absolute value of minimum potential values of universe, how this present potential value is close to zero.

It is means that present dark energy (cosmological constant) has a small positive value. [5] Because that dark energy is gravitational potential energy (eq.(80), eq.(91)) of universe and then total potential energy has positive value at now.

Other process, total potential energy term by $2n$ general particles, has a $\frac{2n(2n-1)}{2}U$. The other side, U_{\max} (positive maximum potential energy by negative mass and positive mass) has a nU .

So, if $n = 10^{80}$

$$\frac{U_{\max}}{U_{GP}} = \frac{nU}{n(2n-1)U} = \frac{1}{(2n-1)} \approx \frac{1}{2 \times 10^{80}}$$

U_{\max} is that the upper limit of total potential energy.

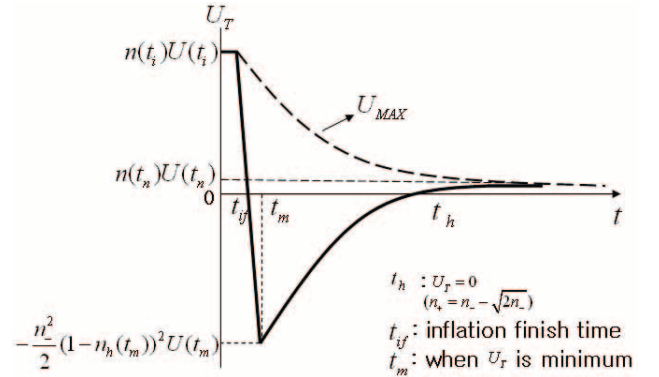


Figure 18: Changes in universe's potential energy over time

- Fine tuning problem of mass density -

About the fine tuning of mass density in the early universe, even though the mass density of present universe is equally same with the value of critical mass density, when negative mass and positive mass coexist, it doesn't mean

that the density of early universe must be very close with the value of critical density.

And also because negative and positive mass's rate is going close to 1, that present universe is almost at the same with critical density values.

The mass density of universe close to critical mass density was from basic mechanism of pair creation, namely 1:1 correspondence of negative mass to positive mass, and form of potential energy.

In fig.18, negative and positive mass created n pairs, and in this period that potential energy values in $U_{\max} = nU_+$. That cause accelerates expansion of universe(Bigbang or Inflation).

Strong interaction, weak interaction and electromagnetic force are separate(some mass change to the bond energy) and also antimatter and matter generate pair annihilation in during inflation. So most of positive matter became radiation. Then total potential energy becomes negative value(result of V-1, fig.9). Therefore we call this time t_{if} (inflation finish time)

As the universe is getting cooler, the radiation is getting lower and that radiation changed the matter with positive mass. And after values of positive mass became the critical ratio of negative mass, that potential energy becomes zero. We call this time by t_h .

At this time, positive mass number is

$$n_+ = n_- - \sqrt{2n_-} \quad (117)$$

According to hypothesis with negative mass, The last state of universe is when positive mass and negative mass almost the same and potential energy has the value of $U_T = n(t)U(t)$ at this time.

- Phase transition problem of dark energy -

This value is strong power that can generate inflation in early universe. But roll of this value is very smaller than inflation period in the growth universe(mean distance r is very bigger than initial mean distance r_0) and because of our universe have positive potential energy, so it's doing expansion now.

$$\begin{aligned} \text{If } r_0 &= l_p = 1.61624 \times 10^{-35} m, \\ r_{now} &= ct_0/2 \sim ct_0 = (6.48065 \sim 12.9613) \times 10^{25} m \\ U_0 &= (4.0097 \sim 8.0194) \times 10^{60} U_{now}, \text{ also } \rho_0 \gg \rho_{now} \end{aligned}$$

It is very important. Because that U_{de} explain dark energy that very big positive value(Inflation Energy) in early universe and very small positive value(Cosmological Constant) at now.

$$\lim_{t \rightarrow \infty} U_{\max} \approx 0 \quad (118)$$

Considering of pair annihilation, potential energy of universe can under the zero again.

6. Other methods

6-1) Description of formation of void and galaxy

If there were positive mass and negative mass at the beginning of universe, and there was minute cave caused by pair annihilation of positive mass and negative mass, it could grow to the present scale through cosmic expansion. It is difficult to explain present large-scale Void only with the uniform distribution of positive mass.

Also, as considered cosmic expansion velocity in the process of forming galaxy, it is difficult to form galaxy structure with uniformly distribute positive mass. On the other hand, from the uniform distribution of negative mass and positive mass, we can explain the formation of galaxy owing to the situation that naturally breaks uniform distribution of matters such as attraction effect between positive mass, repulsion effect between negative mass, attractive effect between massive positive mass and negative mass, and Void caused by pair annihilation.

Making simulation model of the initial state of universe under the assumption of uniform distribution of positive mass and negative mass, we can prove the formation of current Void and galaxy structure and on the basis of this, we can prove the possibility of the existence of negative mass.

6-2) Result of observing WAMP

From the results of observing WAMP at the beginning of universe, the existence of temperature deviation [11] has an influence on cosmic background radiation owing to non-uniformity of matter caused by the existence of negative mass, or the gravity that negative mass works on time and space. Accordingly, if we try to interpret the result of observing WMAP with negative mass, we can find the basis of negative mass.

6-3) The effect of gravitational lensing

Generally, it is possible to examine the existence of dark matter with the existence of additional mass with the effect of gravitational lensing. [18] If negative mass is dark matter, so if we try to examine the effect of gravitational lensing, previously the gravity between positive mass is attraction, so it has the shape of convex lens to collect within the form, whereas the force between negative mass and positive mass is repulsion, so a set of massive negative mass can make the effect that distorts observation target in the form of concave lens.

If existing measuring instrument recognizes the effect of gravitational lens with distorted aspect of galaxy or galaxy cluster, the effect of concave gravitational lensing owing to the existence of negative mass also plays a role of distorting galaxy or galaxy cluster, so it can be the basis of invisible dark matter. Additionally, in the distribution of dark matter suggested with previously measured effect of gravitational lens, dark matter may be created purely by positive mass, but also it can be created by the distribution of dark matter by negative mass.

But in the hypothesis of dark matter with negative mass in this study, negative mass is distributed out of galaxy, not within galaxy. Therefore, the effect of concave gravitational lensing by negative mass out of galaxy will be presented to an observer on the earth in the same form as the effect of convex gravitational lensing which galaxy works.

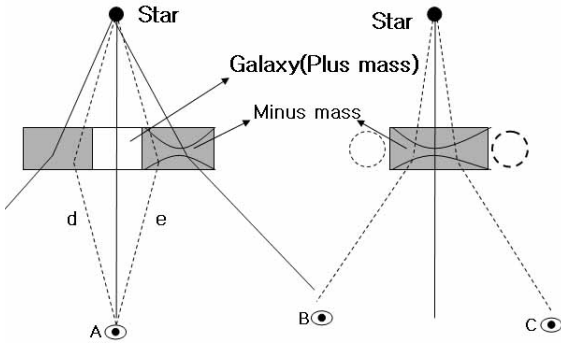


Figure 19: Concave Gravitational lensing effect. The left side is the Gravitational lensing effect that occurs when the galaxy is surrounded by negative mass and the right side is Gravitational lensing effect that occurs when positive mass exists alone

No the effect of concave gravitational lensing by negative mass has been observed. However, we have never thought about negative mass itself, so we might not have thought about the concave gravitational lensing as well, and it might be difficult to classify the effect of concave gravitational lensing and the effect of convex gravitational lensing.

For the situation presented on the left when observing the light on the earth which passed through galaxy surrounded with negative mass, this study suggested that there is invisible dark matter in galaxy, and owing to this gravity it can be happened owing to the effect of convex lens, it means, if there is any matter on the left and right side of galaxy that plays a role of concave lens, it is difficult to classify convex lens and its form.

According to the explanation of the effect of concave gravitational lensing from the viewpoint of gravity, not

geometrical optics, there is the effect of centripetal force by the distribution of negative mass out of galaxy with the effect of centripetal force in galaxy as described previously, so there is real mass of galaxy plus the gravity when filling the whole galaxy with positive mass that has the same density as that of negative mass in galaxy. Therefore, if the light from the exterior passes inside of galaxy or near galaxy, real gravity of galaxy plus gravity of dark matter(by negative mass) is added, and finally it will present much greater effect of gravitational lensing.

If we consider the situation that we can observe purely the effect of concave gravitational lensing, that situation is just the same as the situation when there is single concave lens on the right in Figure above.

First, an observer on the earth must observe it on Position B and C after moving by considerably great distance as compared with the size of galaxy, but the earth has never changed the position of observation from the target for distant observation.

second, real observation on the earth is the single B or C situation in Figure on the right, and if an observer on the earth performed observation previously on C, generally an observer of C would describe the effect of gravitational lensing is generated because there is invisible dark matter on the right of black and white area(circle drawn with dotted line on the right).

As observed the effect of concave gravitational lensing, this can be the evidence for the existence of negative mass.

6-4) We must observe the exterior of galaxy, not its interior to observe dark matter(negative mass)

Most of observation equipments focus on the interior including galaxy core in exploring dark matter, but according to the hypothesis of negative mass, dark matter made up with negative mass exists out of galaxy, so we must observe the exterior of galaxy to find the evidence of dark matter.

6-5) Simulation of the aspect of colliding with Bullet Cluster

As shown in the recent observation of dark matter, dark matter seems not to interact with each other when colliding with galaxy clusters [18], and it can be predicted with essential characteristics of negative mass.

Uniformly distributed negative mass receives attractive effect from massive positive mass, so dark matter which has negative mass is clustered around galaxy because of attraction of galaxy. If the type of force worked

between negative mass when colliding with galaxy clusters is repulsive, and negative mass is distributed almost uniformly without forming massive mass structure, the shape of dark matter is not distorted or transformed even if dark matter(negative mass) and dark matter(negative mass) pass through similar area, that is, they will seem not to interact each other. Also there is repulsive effect between dark matters that are made up with negative mass, thus there will be almost no direct collision as well.

Repulsive effect that is the nature between negative mass as described above provides the proper description that existing dark matters do not interact each other, although they interact gravitationally with positive mass.

As the factor that breaks the uniform distribution of negative mass,

First, negative mass receives the attractive effect from massive positive mass, thus for the distribution of negative mass near massive positive mass such as galaxy or galaxy cluster, the density of negative mass is higher as it is closer to galaxy or galaxy cluster, and is lower as it is farther.

Second, If positive mass(like galaxy cluster) that has strong gravity or interstellar cloud that has positive mass pass through existing area that negative mass is distributed, negative mass can be disappeared when meeting positive mass or it can be drawn owing to attractive effect of massive positive mass at this moment, so there can be the area that negative mass, namely, dark matter is not uniformly distributed

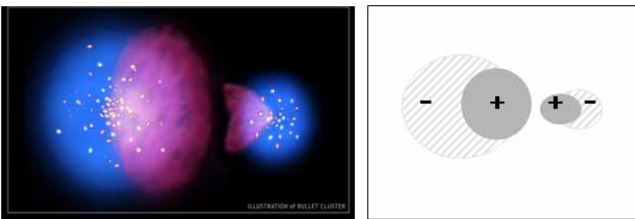


Figure 20: Collision of Bullet Cluster. We can see that general matters(red color) are close to each other, and dark matters(blue color) are on the far side. [18]

- i) Positive mass, positive mass : attractive
- ii) Negative mass(dark matter), negative mass(dark matter) : repulsive
- iii) Massive positive mass, negative mass : attractive
- iv) Result : At least, from 3 characteristics above, we can predict that positive mass and positive mass are arranged on the close side, and negative mass(dark matter)

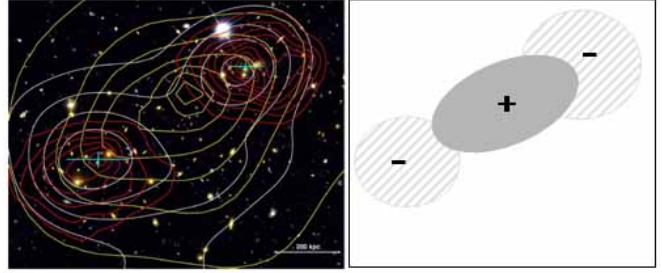


Figure 21: MACS J0025.4-1222 Cluster. We can see that general matters(yellow color) are close to each other(center), and dark matters(red color) are on the far side. [19]

and negative mass(dark matter) are arranged on the far side.

Can we explain the phenomenon that dark matter is arranged on the far side and visible positive matter is arranged on the close side in Fig.20 with other dark matters?

Negative mass shows the result matched with the phenomena, and it means that it is necessary to perform more strict simulation with negative mass.

6-6) Investigation of negative mass in accelerator

There is the possibility that negative mass is born in accelerator, so it is necessary to consider negative mass with opening the possibility of negative mass while performing an experiment with accelerator.

VI. Conclusion

If negative energy and positive energy were born together at the time of Big Bang, then negative energy may create negative mass, and positive energy may exist in the universe at the state of positive energy such as positive mass, electromagnetic wave, etc. Newton's law of motion can explain that positive mass formed massive positive mass structure such as planet or galaxy, negative mass was disappeared when meeting positive mass near massive positive mass structure at the period of galaxy formation, but negative mass born at the beginning of universe still exists out of galaxy, this negative mass can perform a role of centripetal force that binding galaxy in galaxy cluster, or stars in individual galaxy, and repulsion and dark energy accelerate positive mass in the direction of cosmic expansion.

The existence of negative mass seems to satisfy evenly all characteristics of existing dark matter and dark energy such as the effect of centripetal force in galaxy or galaxy cluster, the mass effect proportional to distance r ,

repulsion required for cosmic expansion, dark energy that has positive value, element only for gravity gravitational interaction, low interaction between dark matters when cluster of galaxies collision, decelerating expansion at the first half of universe and current accelerating expansion, formation of void and galaxy, fine tuning problem, and difficulty in observation.

So it is necessary to perform more experiments focusing more strict theoretical calculation and observation of negative mass.

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