

# From Ancient Africa

by Frank Dodd (Tony) Smith, Jr.

A little less than 15 billion years ago, our Universe emerged from the Void.

4 billion years ago, our Earth and Moon were orbiting our Sun.

2 billion years ago, bacteria built a nuclear fission reactor in Africa.

100,000 years ago, Humans were expanding from the African home-land to Eurasia and beyond.

12,000 years ago, Africans knew that the knowledge-patterns of 8 binary choices giving  $2^8 = 256 = 16 \times 16$  possibilities could act as an Oracle.

The 16x16 African Oracle patterns spread throughout the Earth, so that by the 13th century parts of them were found in:

Judaism as the 248 positive Commandments plus the 365 negative Commandments given to Moses during the 50 days from Egypt to Sinai;

India as the 240 parts of the first sukt of the Rig Veda;

Japan as the 128 possibilities of Shinto Futomani Divination;

China as the 64 possibilities of the I Ching;

Mediterranean Africa as the 16 possibilities of the Ilm al Raml.



In the early 13<sup>th</sup> century Ibn Arabi (born in Spain, died in Damascus) wrote the “Bezels of Wisdom” describing his Sufi Islamic World-View using terms consistent with the processes of Quantum Theory:

“mumkinat” = quantum possible Worlds of the Many-Worlds, or Bohmian beables;

“qada” = decoherence of quantum superposition of possibilities, or choice at an Event of a World of the Many-Worlds;

“qadar” = outcome of qada = the World or State that comes into existence as the outcome of an Event;

“al-khalq al-jadid” = the branching of the Worlds of the Many-Worlds that occurs at an Event.

According to the book *Sufism and Taoism*, by Toshihiko Izutsu (California 1983): “...Ibn Arabi says that ... the world in its entirety ... transforms itself kaleidoscopically from moment to moment ... 'new creation' (al-khalq al-jadid) ... ordinary people are not aware of the process ... If a man happens to obtain the true knowledge of qadar, the knowledge surely brings him a perfect peace of mind and an intolerable pain at the same time.

The unusual peace of mind arises from the consciousness that everything in the world occurs as it has been determined from eternity. ... Instead of struggling in vain for obtaining what is not in his capacity, he will be happy ... He must be tormented, on the other hand, by an intense pain at the sight of all the so-called 'injustices', 'evils', and 'sufferings' that reign rampant around him, being keenly conscious that it is not in his 'preparedness' to remove them from the world. ...".

Describing the kaleidoscopic process of qadar in specific physical detail requires, in addition to the processes of Ibn Arabi, a specific mathematical framework.

Near the end of the 13th century, Ramon Llull of Mallorca studied the 16 possibilities of the Ilm al Raml and realized that they had a Fundamental Organizational Principle that he summarized in a Wheel Diagram



with 16 vertices connected to each other by 120 lines. If the 16 vertices represented a 16-dimensional vector space, then the 120 lines connecting pairs of vectors represented 120-dimensional bivectors of rotations in that 16-dimensional vector space.

That total geometry is described by the Real 16-dimensional Clifford Algebra  $Cl(16)$ .

$Cl(16)$  not only describes rotations in vector space, but also spinors that describe left-handed and right-handed properties with respect to the space.

Half-spinors of  $Cl(16)$  have 128 dimensions.

When you combine the 120 bivectors of  $Cl(16)$  with the 128 half-spinors of  $Cl(16)$ , you get the 248-dimensional object called the exceptional Lie Algebra  $E_8$ .

In 8-dimensional space, 240 of the 248 generators of  $E_8$  form the 240 Root Vectors of  $E_8$ .

$Cl(16)$  factors into the tensor product of two copies of the Real 8-dimensional Clifford Algebra  $Cl(8)$ :

$$Cl(16) = Cl(8) \otimes Cl(8)$$

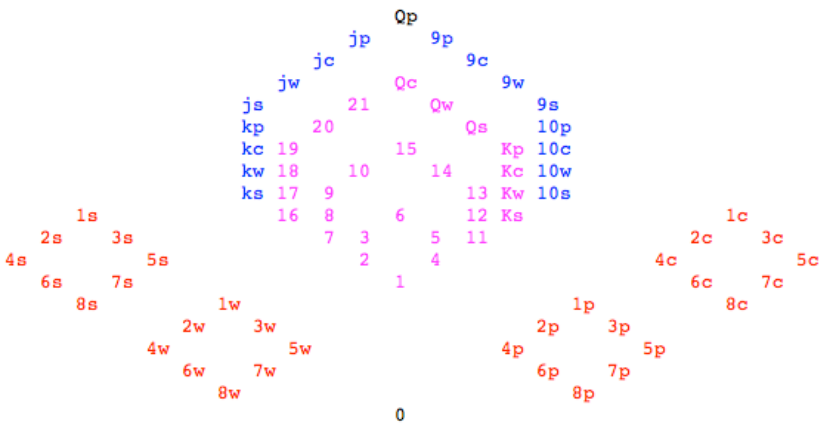
Since  $Cl(8)$ , based on an 8-dimensional vector space, has 256 dimensions, and  $256 = 16 \times 16$ ,

we have come full circle to the  $16 \times 16$  possibilities of the ancient African Oracle.

However, Ramon Llull was not as interested in making a physics model by combining his  $Cl(16)$  mathematical structure with Ibn Arabi's quantum processes as he was in using his structure to show the equivalence of Judaism, Christianity, and Islam, an effort that failed, perhaps because the political institutions of those



religions were in power struggles with each other and so were in fact hostile to reconciliation/unification. Ramon Lull's work failed to find institutional acceptance in Judaism or Islam, and Christian Dominican Catholicism suppressed his work, but his ideas were carried on in the underground world resulting in description of detailed substructures by Tarot



- The magenta 28 are the 28 Spin(8) adjoint bivectors of Cl(8).
- The 16 blue are the 8 vectors of Spin(8) and Cl(8) and their 8 dual/conjugates.
- The 32 red are the 16 spinors (8 +halfspinors and 8 -halfspinors) of Spin(8) and Cl(8) and their 16 dual/conjugates.
- The 2 black are diagonal degrees of freedom in 26-dim traceless J3(O) part of J3(O) Jordan algebra.

whose 78 cards represent the E6 subalgebra of the 248-dimensional Lie algebra E8 which is constructed from the Clifford algebra  $Cl(16) = Cl(8) \times Cl(8)$  where Cl(8) is the  $16 \times 16 = 256$ -dimensional algebra of IFA.

# Earlier Heaven Binary Sequence IFA

**The 16 Tetragrams in Earlier Heaven Binary Number sequence are:**

(o = 0 and oo = 1, ordered from top to bottom)

**Oludumare (original g-d) creates Earth, the 16 Odu, the Orishas, and humans:**

Odu	Orisha
0 Ogbe (light of creation) o      heaven Tao void o o o	- Orunmila (wisdom Tao) (next to Oluddomare)
1 Osa (creativity) oo o o o	- Oya (wind)
2 Otura (unity of everything) o oo o o	- Osain (forest spirits)
3 Owonrin (rain) oo oo o o	- Oshun (rivers) (He Xiangu)
4 Irete (fate) o o oo o	- Babalu Aye (healer) (TieGuai Li)
5 Ofun (taboo) oo o oo o	- Eshu Legba (trickster)

6  
Edi (womb) - Obatala (hermaphrodite father of humanity)  
o  
oo  
oo  
o

7  
Okanran (lightning) - Chango (thunder)  
oo  
oo  
oo  
o

8  
Ogunda (sword) - Ogun (iron) (Lu Dongbin)  
o  
o  
o  
oo

9  
Iwori (consciousness) - Eshu Legba (messenger)  
oo  
o  
o  
oo

10  
Ose (victim of abuse) - Osun (guardian angel)  
o - truth protects good and destroys evil  
oo  
o  
oo

11  
Oturopon (trap) - Ochosi (hunter)  
oo  
oo  
o  
oo

12  
Irosun (fire) - Aganyu (volcano)  
o  
o  
oo  
oo

13  
Ika (forest land) - Yemaya (ocean)  
oo  
o  
oo  
oo



correspond to the 256 elements of the Cl(8) Clifford Algebra with graded structure

$$1 + 8 + 28 + 56 + 70 + 56 + 28 + 8 + 1 = 256 = 16 \times 16$$

Grade:	0	1	2	3	4	5	6	7	8
000	00000000								
001		00000001							
002		00000010							
003			00000011						
004		00000100							
005			00000101						
006			00000110						
007				00000111					
008		00001000							
009			00001001						
010				00001010					
011				00001011					
012			00001100						
013				00001101					
014				00001110					
015					00001111				
016		00010000							
017			00010001						
018			00010010						
019				00010011					
020			00010100						
021				00010101					
022				00010110					
023					00010111				
024			00011000						
025				00011001					
026				00011010					
027					00011011				
028				00011100					
029					00011101				
030					00011110				
031						00011111			
032		00100000							
033			00100001						
034			00100010						
035				00100011					
036			00100100						
037				00100101					
038				00100110					
039					00100111				
040			00101000						
041				00101001					
042				00101010					
043					00101011				
044				00101100					
045					00101101				
046					00101110				

047				00101111		
048	00110000					
049		00110001				
050		00110010				
051			00110011			
052		00110100				
053			00110101			
054			00110110			
055				00110111		
056		00111000				
057			00111001			
058			00111010			
059				00111011		
060			00111100			
061				00111101		
062				00111110		
063					00111111	
064	01000000					
065		01000001				
066		01000010				
067			01000011			
068		01000100				
069			01000101			
070			01000110			
071				01000111		
072		01001000				
073			01001001			
074		01001010				
075				01001011		
076			01001100			
077				01001101		
078				01001110		
079					01001111	
080		01010000				
081			01010001			
082			01010010			
083				01010011		
084			01010100			
085				01010101		
086				01010110		
087					01010111	
088			01011000			
089				01011001		
090				01011010		
091					01011011	
092				01011100		
093					01011101	
094					01011110	
095						01011111
096		01100000				
097			01100001			
098			01100010			
099				01100011		
100			01100100			
101				01100101		
102				01100110		
103					01100111	
104			01101000			
105				01101001		

106			01101010		
107				01101011	
108			01101100		
109				01101101	
110				01101110	
111					01101111
112		01110000			
113			01110001		
114			01110010		
115				01110011	
116			01110100		
117				01110101	
118				01110110	
119					01110111
120			01111000		
121				01111001	
122				01111010	
123					01111011
124				01111100	
125					01111101
126					01111110
127					01111111
128	10000000				
129		10000001			
130		10000010			
131			10000011		
132		10000100			
133			10000101		
134			10000110		
135				10000111	
136		10001000			
137			10001001		
138			10001010		
139				10001011	
140			10001100		
141				10001101	
142				10001110	
143					10001111
144		10010000			
145			10010001		
146			10010010		
147				10010011	
148			10010100		
149				10010101	
150				10010110	
151					10010111
152			10011000		
153				10011001	
154				10011010	
155					10011011
156			10011100		
157				10011101	
158				10011110	
159					10011111
160		10100000			
161			10100001		
162			10100010		
163				10100011	
164			10100100		

165		10100101				
166		10100110				
167			10100111			
168	10101000					
169		10101001				
170		10101010				
171			10101011			
172		10101100				
173			10101101			
174			10101110			
175				10101111		
176	10110000					
177		10110001				
178		10110010				
179			10110011			
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183				10110111		
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185			10111001			
186			10111010			
187				10111011		
188			10111100			
189				10111101		
190				10111110		
191					10111111	
192	11000000					
193		11000001				
194		11000010				
195			11000011			
196		11000100				
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199				11000111		
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202			11001010			
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216			11011000			
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218				11011010		
219					11011011	
220				11011100		
221					11011101	
222					11011110	
223						11011111

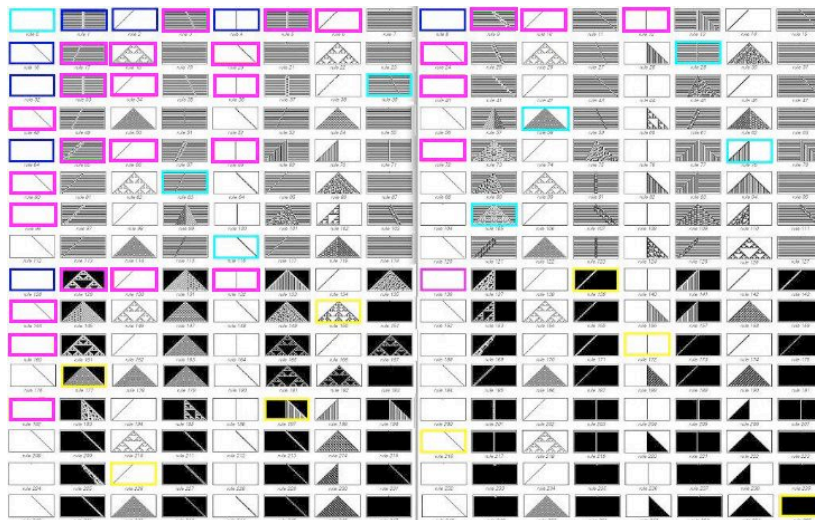


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224           11100000
225           11100001
226           11100010
227           11100011
228           11100100
229           11100101
230           11100110
231           11100111
232           11101000
233           11101001
234           11101010
235           11101011
236           11101100
237           11101101
238           11101110
239           11101111
240           11110000
241           11110001
242           11110010
243           11110011
244           11110100
245           11110101
246           11110110
247           11110111
248           11111000
249           11111001
250           11111010
251           11111011
252           11111100
253           11111101
254           11111110
255           11111111

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**and to the 256 1-dimensional Cellular Automata**



# IFA and Global Dynamics of Cellular Automata

## 88 Equivalence Classes ----- 256 Elementary Rules

Images and quoted text are from The Global Dynamics of Cellular Automata by Andrew Wuensche and Mike Lesser (Addison-Wesley 1992 and a scanned version in pdf format downloaded from the web) (unless otherwise specified).

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### 2.1 Cellular Automata

A cellular automaton (CA) is a discrete dynamical system which evolves by the iteration of a simple deterministic rule; as in any dynamical system, the system's variables change as a function of their current values.

An alternative approach is to view a CA as a parallel processing computer<sup>34</sup> where the data is considered to be the initial CA configuration.

Yet a third approach is that a CA is a "logical universe...with its own local physics."<sup>16</sup> Such a CA universe, in spite of its mathematically simple construction, seems to be capable of supporting complex emergent behaviour.

#### 2.1.1 CA Architecture

A CA is constructed as follows: Time is discrete and progresses in steps. A  $D$ -dimensional, potentially infinite space is partitioned into discrete "cells" (the CA array or lattice) according to a given geometry. Boundary conditions may be set to define a finite space. Each cell has one attribute (the cell's value) from a limited range of attributes, which may be labelled by an integer. The pattern of values across the whole array is the CA global state at a given time.

Any pattern may be set as an initial condition at time  $t_0$ . Each cell of the array simultaneously has its value updated to evolve a new global state at time  $t_1$ . The new value of any given cell (the target cell) at  $t_1$  is a function of the values and locations of a set of cells (the neighbourhood) at  $t_0$ , typically situated locally in relation to the target cell (see Fig. 2.1). The neighbourhood may be defined by a neighbourhood template or wiring diagram. The CA evolves through a succession of global states (its *trajectory*) by the iteration of this global updating procedure (the *transition function*). Provided that the transition function is constant and the system is closed to noise (the updating is error free), then the evolution of the CA from its initial global state is uniquely determined.

Two types of CA may be distinguished, both deterministic: The more general case may be described as having varying degrees of *disordered* architecture (non-local<sup>21</sup>), where the wiring diagram and/or function at each cell may differ, for example Walker's networks of Boolean functions<sup>27-32</sup> and Kauffman's random Boolean networks.<sup>14,15</sup>

CA with *ordered* architecture are a special case, where the wiring diagram and function are the same over the entire array. In addition, the ordered wiring may be confined to a *local neighbourhood*, an uninterrupted zone of cells typically centred on the target cell. CA of this type will be referred to as having *local architecture*, for example, the architecture of Wolfram's "elementary rules."<sup>33-40</sup> This paper is relevant to deterministic CA in general, however it deals mainly with the simplest possible local CA architecture.

Von Neumann first proposed CA to model self-reproduction.<sup>3,26</sup> His relatively complicated CA architectures were local and two-dimensional with 29 cell values. The tendency since then has been to find simpler architecture that could nonetheless support complex emergent behaviour. For example Conways' "game of life"<sup>2</sup> is a 2-D local

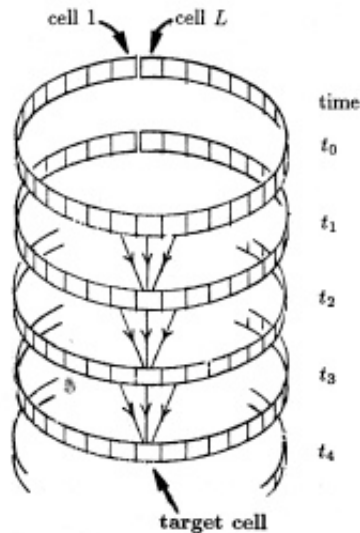


FIGURE 2.1 1-D, local binary CA with periodic boundary conditions, neighbourhood 3 (elementary rules), array length  $L$ .

CA with an orthogonal toroidal array, a 9-cell neighbourhood (the target cell and its eight nearest neighbours), and two cell values (binary value range).

### 2.1.2 Local 1-D Binary CA with Periodic Boundary Conditions

The simplest local CA architecture that we will investigate comprises a 1-D array of a small number of cells, a binary value range and a small local neighbourhood. The array is arranged in a circle; such a circular array is said to have *periodic boundary conditions*. Evolution of the CA may be represented as a sequence of global states on a cylinder, summed up by Fig. 2.1.

### 2.1.4 CA Dynamics

*State space* (also called *phase space*) is the set of all possible CA global states. In a finite CA, state space is finite; thus, any trajectory must eventually encounter a repeat of a global state that occurred at an earlier time. Because the system is deterministic, the trajectory will become trapped in this repeating sequence of states, a *cyclic attractor*, with a specific period of 1 or more.

States are either part of the attractor or belong to a *transient*, a sequence of states leading to the attractor. If transients exist, there must be states at their extremities (*garden-of-Eden states*), unreachable by evolution from any other state. The set of all possible transients leading to an attractor, plus the attractor itself, is the *basin of*

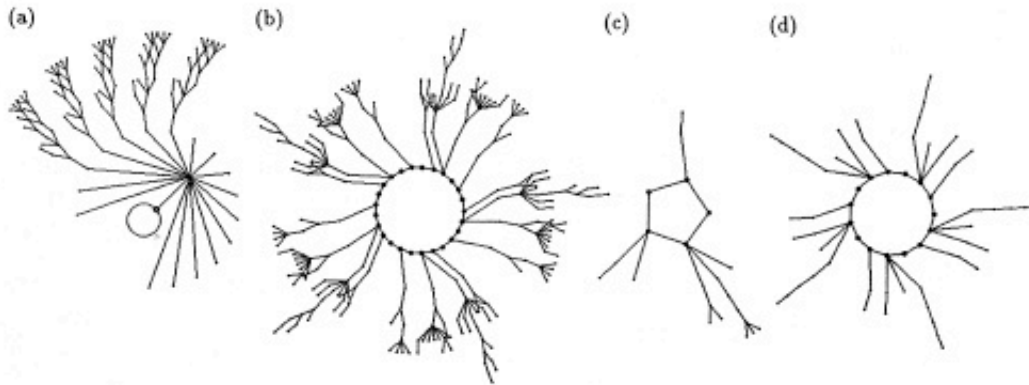


FIGURE 2.3 A basin of attraction field, local 3-neighbour rule 193,  $L = 10$ . The number of basins of each type is (a) 1, (b) 2, (c) 10, and (d) 2.

attraction of that attractor. State space is populated by one or more basins of attraction. These basins of attraction constitute the dynamical flow imposed on state space by the CA transition function.

A portrait of this global behaviour is the *basin of attraction field*, a discrete analogue of the familiar basin of attraction field found in the phase space of a continuous dynamical system, known as the system's *phase portrait*.

## 2.2 The Basin of Attraction Field

The *basin (of attraction) field* of a finite CA is the set of *basins of attraction* into which all possible states and trajectories will be organised by the cellular automaton transition function. The topology, or structure, of a single basin of attraction may be described by a diagram, the *state transition graph*. The set of graphs making up the field specifies the global behaviour of the system. Various other names have been used: state transition fragment,<sup>39</sup> contraction map,<sup>7</sup> topology of behaviour space,<sup>29</sup> and network of attraction.<sup>42</sup>

The notion of basin fields was proposed by Walker,<sup>27</sup> and examples<sup>[1]</sup> have been given by Martin et al.,<sup>22</sup> Pitsianis et al.,<sup>25</sup> Wolfram,<sup>39,40</sup> Feldberg and Rasmussen,<sup>5</sup> and by the authors<sup>42</sup> in an earlier edition of this atlas.

An example of a basin of attraction field is shown in Fig. 2.3 for the local, binary, 3-neighbour rule 193 (see chapter 3, section 3.3). The array length,  $L$ , equals 10, so that state space consists of  $2^{10} = 1024$  global states. The CA transition function connects these states into a set of basins, the basin (of attraction) field. In this case there are four different types of basins in the field, some of which occur more than once. The number of each type is indicated.

### 2.2.1 The State Transition Graph

A state transition graph links up all the states belonging to a single basin of attraction according to their specific evolutionary location; this will typically have a topology of *trees rooted on attractor cycles*.<sup>22</sup> Global states are represented by *nodes* which are linked by *directed arcs*. Each node will have zero or more incoming arcs from nodes at the previous time step (*pre-images*), but because the system is deterministic, exactly one outgoing arc

(“out degree”) to a single node (the *successor state*), at the next time step. Nodes with no incoming arcs represent garden-of-Eden states. The number of incoming arcs is referred to as the *degree of pre-imaging* (“in degree”).

For a given set of CA parameters, state space will, in a sense, crystallise into a set of one or more basins of attraction. The basin of attraction field is the set of state transition graphs representing all the basins in state space.

The make-up of a typical basin of attraction is illustrated by the state transition graph shown in Fig. 2.4 (it is part of the basin field shown in Fig. 2.6). In our graphic convention (see Appendix 2), the length of transition arcs decreases with distance away from the attractor, and the diameter of the graphic representation of the attractor asymptotically approaches an upper limit with increasing period, so that attractor cycles are drawn with approximately the same diameter irrespective of the number of nodes in the attractor. The forward direction of transitions is inward from garden-of-Eden states to the attractor, which is the only closed loop in the basin, and then clockwise around the attractor cycle.

Typically, the vast majority of nodes in a basin field, or a single basin of attraction, lie on *transient trees* outside the attractor cycle. A transient tree is the set of all paths from garden-of-Eden nodes leading to one node on the attractor cycle (an *attractor node*). A branch of the transient tree is termed a *transient branch*, and is the set of all paths from garden-of-Eden nodes leading to a state within a transient tree. A *transient* is one particular path from an arbitrary node in the transient tree leading to the attractor node. In all cases the attractor node itself is excluded from the definition.



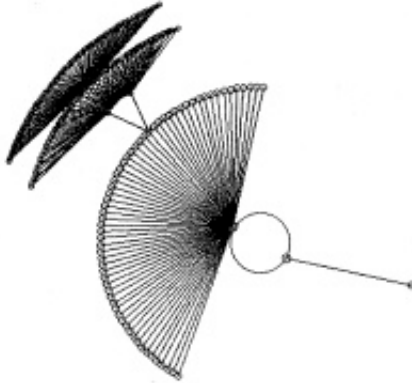


FIGURE 2.8 Example of a basin of the 3-neighbour rule 126,  $L = 31$ . The seed state is a single 0 followed by six copies of the string 01111.

### 2.2.6 The Atlas

The program has been used to produce an *atlas* of basin of attraction fields over a range of array lengths, presented in Appendix 2. The Atlas consists of two parts. Part 1 presents all 3-neighbour (elementary) rules. Part 2 presents all 5-neighbour totalistic codes. Selected data is also presented on each basin and basin field. There is an index to rules and codes at the beginning of each part.

A CA rule belongs to a set of up to four *equivalent* rules, the *equivalence class*, that differ only in that they have *negative* and/or *mirror image* space-time patterns, but which have identical basin field topology. Thus, only one rule representing each equivalence class is presented in the Atlas. Pairs of equivalence classes relate in that their rules have *complementary* rule tables, forming a *rule cluster*. The representative rules in the Atlas are presented according to their rule cluster, with complementary equivalence classes shown on facing pages (the relationships between rules are explained in detail in chapter 3, section 3.5)

The 3-neighbour rule clusters belong to one of three *symmetry categories*—*symmetric*, *semi-asymmetric*, and *fully asymmetric*—and are accordingly presented in three sections. The basin fields for the 88 *equivalence classes* of the 3-neighbour rules are shown for  $L = 1$  to 15.

### 2.2.7 Significance of Basin of Attraction Fields

The ability to represent basin of attraction fields may be significant in a number of areas such as CA theory, complex systems, dynamical systems, computational theory, artificial life, neural networks, and aspects of genetics.

Basin field topology represents a second order of complexity of CA behaviour, where the first order may be said to be the space-time patterns of particular trajectories. Easy access to a systematic presentation of basin fields, for a synoptic, qualitative, as well as explicit view of global behaviour, may provide insights into the dynamical theory of CA, and the structure of CA rule space. It may provide a useful analogue to the global behaviour of CA with more complex architecture, and dynamical systems in general.

The separate basins in a basin of attraction field classify state space. Attractors have been regarded as “memories,”<sup>13,14,15</sup> with implications for the mechanism underlying neural networks.

Basin fields may be of interest in genetics because mutations of the CA *rule table* by a small Hamming distance typically result in altered but related basin structures. Analogies have been made between a CA rule table and a DNA sequence.<sup>20</sup> Kauffman and others study CA with non-local architecture as models in biology and genetics.<sup>14,15</sup>

Attractors have been interpreted as “cell types” in ontogeny. Evolution is said to occur in an optimal “fitness landscape” by mutation and selection of the CA “genotype” resulting in adapted dynamics or “phenotype.”

A possible approach to basin of attraction fields is to see them as artificial morphologies, analogous to the “biomorphs” proposed by Dawkins.<sup>4</sup> Their explicit morphological form (including the global CA state at each node) is determined by the genetic code, the rule table; a mutation of the rule table results in a mutant morphology. In addition, the genetic code is implicit in the morphology because the rule table can be reconstructed from space-time patterns; this suggests possibilities for reproduction. Such a genotype-phenotype approach to CA may suggest applications in artificial life.

### 3.3 Rule Clusters

CA parameters may, finally, be restricted as follows: a binary value range ( $k = 2$ ) and local architecture, thus nearest-neighbour wiring, and periodic boundary conditions (a circular array). For a neighbourhood size  $n$ , the number of different neighbourhoods equals  $2^n$ , and the number of different rules equals  $2^{2^n}$ .

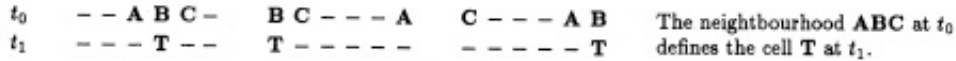
#### 3.3.1 Rule Numbering System, $n=3$ (Elementary Rules)

The  $k = 2, n = 3$  rules have the form,

$$P_i^{t+1} = f(P_{i-1}^t, P_i^t, P_{i+1}^t)$$

where  $P_i = 0$  or  $1$ ,  $i$  is the spatial position between  $1$  and  $L$ , and  $t$  is the time. For a circular array, length  $L$ ,  $P_1 = P_{L+1}$ .

The system may be represented as follows:



The  $n = 3$  rule table with  $2^3 = 8$  entries uniquely specifies each individual rule from a total of  $2^{2^3} = 256$ . Following Wolfram's convention<sup>33</sup> the rule table is ordered in descending values of the binary neighbourhood strings.

	111	110	101	100	011	010	001	000	neighbourhoods
Rule table..	$T_7$	$T_6$	$T_5$	$T_4$	$T_3$	$T_2$	$T_1$	$T_0$	outputs

If the rule table is regarded as a binary number, the rule number,  $R$ , is its decimal equivalent; thus, the rule table will range from 00000000 to 11111111, and  $R$  from 0 to 255.

#### 3.3.2 Complementary Transformation, $n=3$

Every rule,  $R$ , has a distinct complementary rule,  $R_c$ , where each entry in the rule table is inverted, so that for a given input line, the next line generated by  $R_c$  will be the negative of the next line generated by  $R$ .  $R$  and  $R_c$  may have equivalent behaviour (see collapsed clusters below).<sup>[3]</sup> In any case, their behaviour will be closely related. This will be reflected in the basin field structure. Deterministic structure and symmetry category (see below) will be common to  $R$  and  $R_c$ . In general, pre-imaging, cycle periods, and transient lengths will be related.

The  $n = 3$  rules can be listed in 128 complementary pairs.

$$\begin{array}{cccccccc} \text{if } R = & 0, & 1, & 2, & \dots, & 127 \\ & | & | & | & & | \\ R_c = & 255, & 254, & 253, & \dots, & 128 \end{array}$$

There are two types of symmetry that relate pairs of neighbourhoods:

Complementary neighbourhood pairs (0s changed to 1s and vice versa)	...	111, 000	$T_7, T_0$
		110, 001	$T_6, T_1$
		101, 010	$T_5, T_2$
		100, 011	$T_4, T_3$
Reflected neighbourhood pairs (mirror image)	...	110, 011	$T_6, T_3$
		100, 001	$T_4, T_1$

### 3.3.3 Negative Transformation, n=3

For any rule  $R$  and input line  $I$ , the CA will generate a space-time pattern  $P$ . There is a rule,  $R_n$ , that, given the negative input line,  $\bar{I}$  will generate the negative space-time pattern,  $\bar{P}$  (all cell values inverted).

Consider the rule table:

	111	110	101	100	011	010	001	000	neighbourhoods
Rule table..	$T_7$	$T_6$	$T_5$	$T_4$	$T_3$	$T_2$	$T_1$	$T_0$	outputs

To find  $R_n$ , suppose a *negative world* in which the rule table, including neighbourhoods, could be transformed by inverting all values into

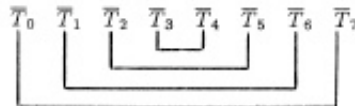
	000	001	010	011	100	101	110	111	neighbourhoods
Rule table..	$\bar{T}_7$	$\bar{T}_6$	$\bar{T}_5$	$\bar{T}_4$	$\bar{T}_3$	$\bar{T}_2$	$\bar{T}_1$	$\bar{T}_0$	outputs

The conventional order of neighbourhoods has been altered and must be restored, giving

	111	110	101	100	011	010	001	000	neighbourhoods
Rule table..	$\bar{T}_0$	$\bar{T}_1$	$\bar{T}_2$	$\bar{T}_3$	$\bar{T}_4$	$\bar{T}_5$	$\bar{T}_6$	$\bar{T}_7$	outputs

The transformed rule table is  $R_n$ . Thus the procedure to transform  $R$  to  $R_n$  is as follows in either order.

1. Take the complement of the rule table,  $R_c$ .
2. Swap the output of complementary neighbourhoods.



For example, rule 193-11000001 is transformed to rule 124-01111100.

### 3.3.4 Reflection Transformation, n=3

For any rule  $R$ , input line  $I$  and space-time pattern  $P$ , there is a rule  $R_r$  that, given the reflected input line,  $I_r$  will generate the reflected (mirror-image) space-time pattern  $P_r$ .

Consider the rule table:

	111	110	101	100	011	010	001	000	neighbourhoods
Rule table..	$T_7$	$T_6$	$T_5$	$T_4$	$T_3$	$T_2$	$T_1$	$T_0$	outputs

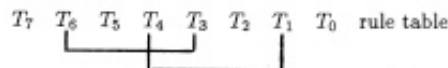
To find  $R_r$ , suppose a *mirror image world* in which the rule table, including neighbourhoods would be transformed by reflection into

	000	100	010	110	001	101	011	111	neighbourhoods
Rule table..	$T_0$	$T_1$	$T_2$	$T_3$	$T_4$	$T_5$	$T_6$	$T_7$	outputs

The conventional order of neighbourhoods has been altered and must be restored, giving

	111	110	101	100	011	010	001	000	neighbourhoods
Rule table..	$T_7$	$T_3$	$T_5$	$T_1$	$T_6$	$T_2$	$T_4$	$T_0$	outputs

The transformed rule table is  $R_r$ . Note that only the outputs of asymmetric neighbourhoods are altered. Thus to transform  $R$  to  $R_r$ , swap the outputs of the two pairs of asymmetric reflected neighbourhoods.



For example, rule 193-11000001 is transformed to rule 137-10001001.

### 3.3.5 Symmetry Categories

The reflection transformation allows rules to be placed into one of three symmetry categories:

1. *symmetric rules* ( $R = R_r$ ), if  $T_6 = T_3$  and  $T_4 = T_1$ .
2. *semi-asymmetric rules*, if either  $T_6 \neq T_3$  or  $T_4 \neq T_1$ , but not both.
3. *fully asymmetric rules*, if  $T_6 \neq T_3$  and  $T_4 \neq T_1$ .

Symmetric rules have space-time patterns whose structures appear to have no bias to move left or right; for semi-asymmetric rules, there is a clear bias towards either the left or right; and for fully asymmetric rules, there is an intersecting bias towards both left and right.



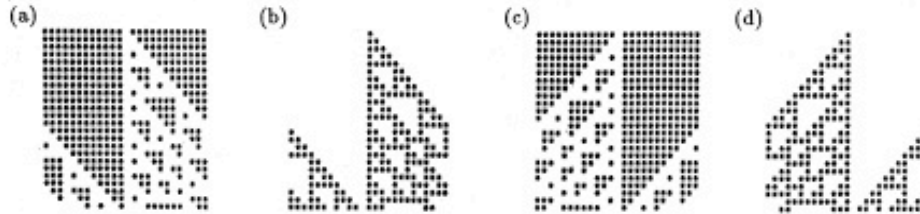


FIGURE 3.5 Equivalent space-time patterns, from equivalent rules. (a) rule  $R$  (193). (b) rule  $R_n$  (124). (c) rule  $R_r$  (137). (d) rule  $R_{nr}$  (110).

**3.3.6 Bilateral Symmetry of Symmetric Rules**

The space-time patterns of symmetric rules, given an input line with *bilateral symmetry* ( $bs$ ), must conserve bilateral symmetry, because the rule acts equivalently on both sides of the axis of symmetry.

As with rotation symmetry (see section 3.2), the degree of bilateral symmetry cannot decrease; in an attractor, bilateral symmetry must remain constant, because any increase would have to be balanced by a decrease as evolution returned around the attractor cycle, and a decrease cannot occur. Bilateral symmetry may increase only in a transient (see Fig. 3.4).

The bilateral axis may divide a circular array, length  $L$ , as follows: if  $L$  is even, the axis may divide the array into two reflected halves, for instance 0001 – 1000. If the maximum disclosure length is  $d_{max}$ , then  $d_{max} \leq 2^{L/2}$ . If  $L$  is odd, the axis may divide the array into two reflected halves bisecting one cell, for instance 0001000, then  $d_{max} \leq 2^{((L-1)/2)+1}$ . Alternatively, if  $L$  is even, there may be more than one axis of bilateral symmetry, for instance 00010001. Such an array has rotation symmetry as well as bilateral symmetry. The interaction between rotational and bilateral symmetry and consequences on behaviour is still unclear. All *totalistic rules* are symmetric (see 3.3.10).

**3.3.7 Equivalence Classes**

We have seen that a given rule,  $R$ , will have two equivalent rules,  $R_n$  and  $R_r$ .  $R$  will also have a third equivalent,  $R_{nr}$ , derived by performing the negative and reflection transformations successively, in either order. There will thus be a maximum of four rules grouped in an *equivalence class*; an example is shown in Fig. 3.5, based on rule 193.

The equivalent rules differ only in that they have negative and mirror-image space-time patterns; otherwise, CA behaviour is totally equivalent. There are 88 *equivalence classes* among the 256  $n = 3$  rules.<sup>28,42</sup> Basin field topology for the rules in each class will be identical, though the actual states will differ according to the transformations described.

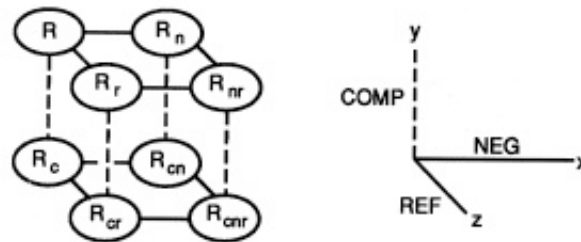


FIGURE 3.6 A rule cluster, two complementary sets of four equivalent rules.

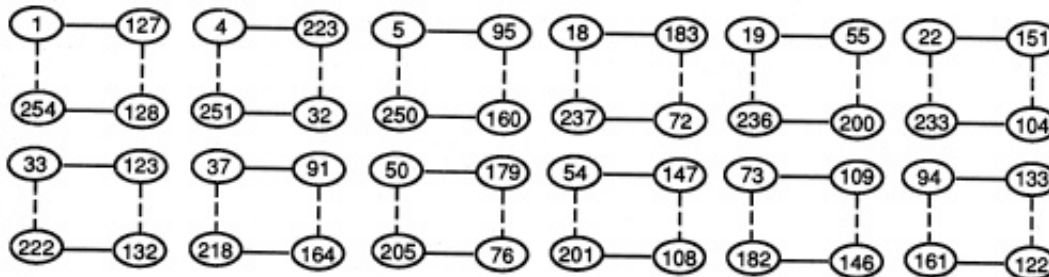
**3.3.8 Rule Clusters**

Every rule  $R$  has a distinct complement  $R_c$ . The 4-rule equivalence class relates to a complementary 4-rule equivalence class, resulting in an 8-rule cluster. We depict the cluster as a box with the rules at each corner (Fig. 3.6), with complementary links along the  $y$  axis (dashed line), negative links along the  $x$  axis, and reflection links along the  $z$  axis.

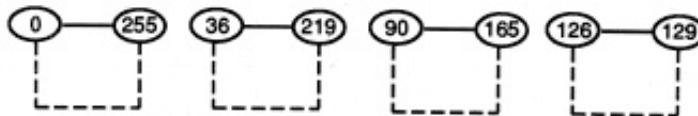
The rules in a rule cluster are described by two basin fields, one for the top and one for the bottom layer. The outcome of transformations may produce the same rule, resulting in more than one occurrence of that rule in the rule cluster. In this case the cluster is shown collapsed, so that each rule only occurs once. For the purpose of reference, the lowest rule number identifies the cluster, and is positioned in the top left-hand corner. The rule clusters for all  $n = 3$  (elementary rules) are set out below.



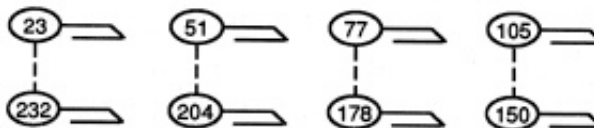
SYMMETRIC RULE CLUSTERS ( $T_6 = T_3$  and  $T_4 = T_1$ ). By definition,  $R = R_r$ , so the reflection links ( $x$  axis) will collapse.



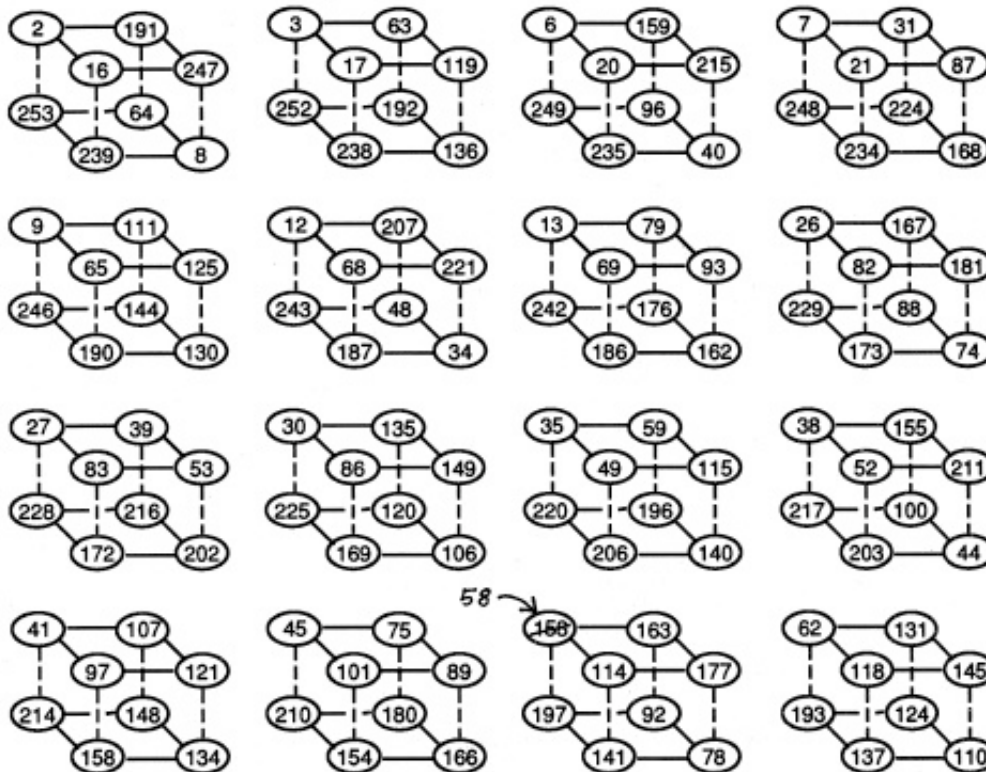
The cluster will collapse further, if for a given rule  $R$ ,  $R_c = R_n$ ,



and also if  $R = R_n$ ,



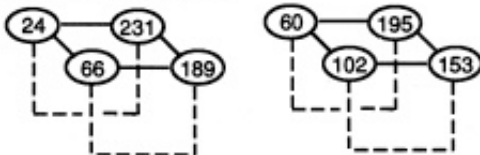
SEMI-ASYMMETRIC RULE CLUSTERS (either  $T_6 \neq T_3$  or  $T_4 \neq T_1$ ). There are no collapsed clusters among the semi-asymmetric rules.



FULLY ASYMMETRIC RULE CLUSTERS ( $T_5 \neq T_3$  and  $T_4 \neq T_1$ ).



The cluster will collapse if, for a given rule  $R$ ,  $R_c = R_n$ ,



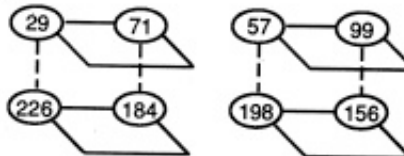
(continued)

Fully asymmetric rule clusters (continued):

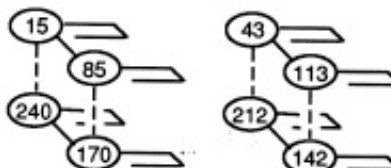
And if, for a given rule  $R$ ,  $R_c = R_{nr}$ ,



And if, for a given rule  $R$ ,  $R_n = R_r$ ,



And also if  $R = R_n$ ,



## 88 Equivalence Classes

To summarise, the 256 rules collapse into 88 equivalence classes<sup>20,28,42</sup> and 48 rule clusters in Table 3.1.

TABLE 3.1

	rules	equiv. classes	clusters
Symmetric rules	64	36	20
Semi-asymmetric rules	128	32	16
Fully asymmetric rules	64	20	12
<b>Total</b>	<b>256</b>	<b>88</b>	<b>48</b>

The 256 rules have been tabulated on a  $16 \times 16$  "rule-space matrix" (Appendix 4). Manipulations of the matrix simulate the clustering transformations.

## Rule-Space Matrix, 256 Rules

The 256  $n = 3$  rules (0 to 255) may be set out on a  $16 \times 16$  matrix. Rows  $i$  and columns  $j$  are numbered 0 to 15 as shown on the next page, and equivalently by the 4-bit binary numbers 0000 to 1111. Each entry in the matrix,  $a_{ij}$ , is a function of its position, and is assigned the decimal equivalent of the 8-bit binary number formed by the concatenation (denoted by the symbol +) of its 4-bit row and column binary expressions. If  $\text{bin}\$(x) =$  the 4-bit binary string equivalent of  $x$ , and  $\text{dec}(x\$\$) =$  the decimal equivalent of the binary string  $x\$\$,$  then

$$a_{ij} = \text{dec}(\text{bin}\$(i) + \text{bin}\$(j));$$

for example, to establish the entry  $a_{5,6}$

$$\begin{aligned} \text{bin}\$(5) &= "0101", \text{ and } \text{bin}\$(6) = "0110" \\ \text{bin}\$(5) + \text{bin}\$(6) &= "0101" + "0110" = "01010110", \text{ dec}("01010110") = 86 \end{aligned}$$

Conversely, given a rule number, 0 to 255, its position on the matrix is found by the separating its 8-bit binary expression into two equal parts. The left 4 bits denotes the row  $i$  and the right 4 bits the column  $j$ . The resulting matrix is set out on the next page.

	$j$															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
2	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47
3	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63
4	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79
5	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95
6	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111
7	112	113	114	115	116	117	118	119	120	121	122	123	124	125	126	127
8	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143
9	144	145	146	147	148	149	150	151	152	153	154	155	156	157	158	159
10	160	161	162	163	164	165	166	167	168	169	170	171	172	173	174	175
11	176	177	178	179	180	181	182	183	184	185	186	187	188	189	190	191
12	192	193	194	195	196	197	198	199	200	201	202	203	204	205	206	207
13	208	209	210	211	212	213	214	215	216	217	218	219	220	221	222	223
14	224	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239
15	240	241	242	243	244	245	246	247	248	249	250	251	252	253	254	255

The position of rules on the matrix depends on the rule numbering convention implicit in the sequence of the rule table entries. The conventional sequence as described in section 3.3 is as follows:

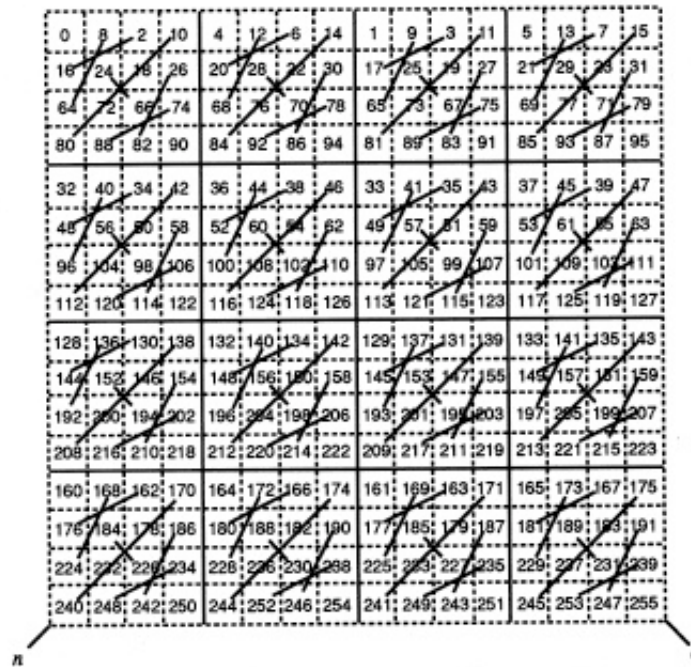
$$\text{Rule table.. } \begin{matrix} 111 & 110 & 101 & 100 & 011 & 010 & 001 & 000 & \text{neighbourhoods} \\ T_7 & T_6 & T_5 & T_4 & T_3 & T_2 & T_1 & T_0 & \text{outputs} \end{matrix}$$

Thus  $T_7T_6T_5T_4T_3T_2T_1T_0$  is the conventional binary expression of the rule. Other sequences of the rule table would be equally valid; indeed, there are  $8! = 13440$  permutations, and thus the same number of possible alternative numbering conventions.

If the equivalence relationships are indicated by drawing lines between equivalent rules on the matrix, a systematic pattern is apparent, however the clarity of this pattern varies for different numbering conventions. A limited search of alternatives has turned up a permutation that results in a pattern of exceptional clarity; this is the rearranged sequence:

$$\text{Rule table.. } \begin{matrix} 111 & 101 & 110 & 100 & 000 & 010 & 001 & 011 & \text{neighbourhoods} \\ T_7 & T_5 & T_6 & T_4 & T_0 & T_2 & T_1 & T_3 & \text{outputs} \end{matrix}$$

giving the alternative binary expression of the rule  $T_7T_5T_6T_4T_0T_2T_1T_3$ . In the matrix shown on the next page, rules are *positioned* according to the alternative numbering system, but are still numbered according to the conventional system. The diagonals, labeled  $c$  (complement) and  $n$  (negative), are indicated.



The matrix shows reflection equivalent rules linked by lines, and provides a graphic demonstration of rule categories and relationships as follows:

1. Simulates all the rule cluster transformations.
2. Distinguishes between symmetric, semi-asymmetric, and fully asymmetric rules.
3. Identifies special status rules resulting in collapsed clusters.
4. Accounts for the numbers of rules in equivalence classes and symmetry categories.

Rules that lie on the  $c$  diagonal have the property, for a given rule  $R$ ,  $R_c = R_n$ . Fully asymmetric rules whose reflection link is bisected by the  $c$  diagonal, have the property, for a given rule  $R$ ,  $R_c = R_{nr}$ . As stated earlier,  $R$  and  $R_n$  will be superimposed if the matrix is folded across the  $n$  diagonal; rules that lie on this diagonal (on the fold) have the property,  $R = R_n$ . Fully asymmetric rules whose reflection link is bisected by the  $n$  diagonal have the property that, for a given rule  $R$ ,  $R_n = R_r$ .

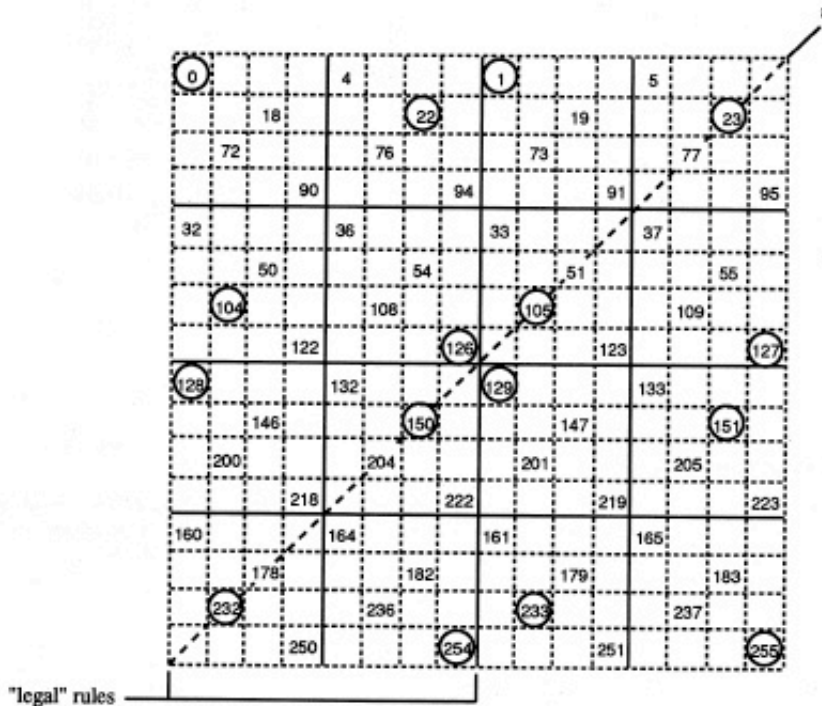
Semi-asymmetric rules are unrelated to the diagonals in the ways described above, and therefore have no collapsed clusters.



### A4.5 Symmetric Rules

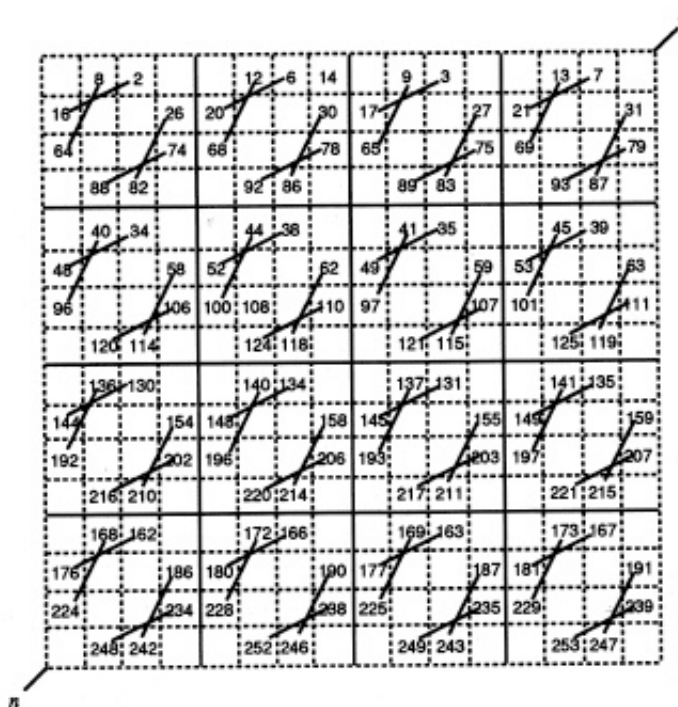
The equivalence classes among the symmetric rules (total 36) consist of the rules which are superimposed when the matrix is folded across the  $n$  diagonal.

The 32 rules that have sometimes been designated as "legal"<sup>33</sup> (both symmetric and even) are located in the left half of the matrix. The 16 totalistic rules among the  $n = 3$  rules are circled.



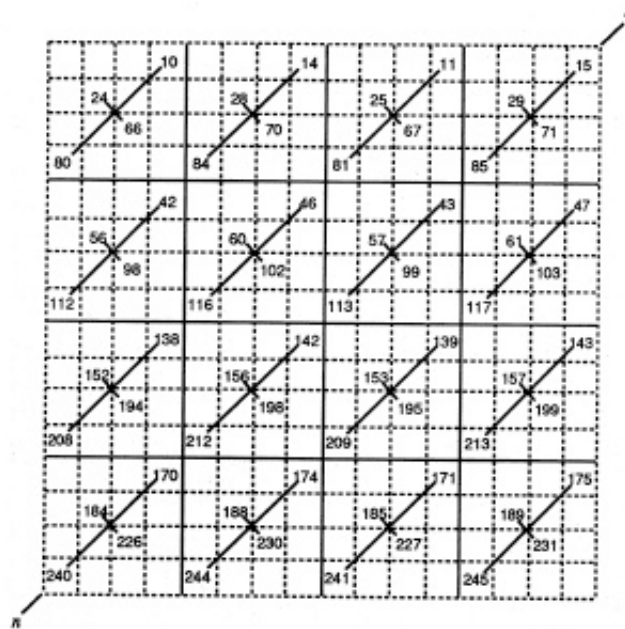
### A4.6 Semi-Asymmetric Rules

The equivalence classes among the fully asymmetric rules (total 32) consist of the linked rules which are superimposed when the matrix is folded across the  $n$  diagonal.



### A4.7 Fully Asymmetric Rules

The equivalence classes among the fully asymmetric rules (total 20) consist of the linked rules which are superimposed when the matrix is folded across the  $n$  diagonal.



## Wolfram CA Rule Classes

### 4.1 Basin Field Topology and Rule Space

In this section, some implications of the emerging basin field landscape on the current perception of the structure of rule space are examined. Wolfram has proposed that all CA rules belong to one of four universality classes.<sup>34,36,39</sup> These classes are essentially phenomenological,<sup>8</sup> based on the characteristic appearance of typical space-time patterns. Wolfram's classes, and their analogues in continuous dynamical systems, are said to exhibit the behaviour shown below.

<i>CA behaviour</i>	<i>dynamical systems analogue</i>
class 1 evolves to a fixed, homogeneous state .....	limit points
class 2 evolves to separated periodic regions .....	limit cycles
class 3 evolves to <i>chaotic</i> , aperiodic patterns .....	strange attractors
class 4 evolves to <i>complex</i> , localised structures .....	long transients, no analogue

It is accepted that many rules show "intermediate" behaviour,<sup>36</sup> and there is also scope for defining sub-classes,<sup>19,20</sup> but the discussion below uses a loose definition of the four classes listed. Of these classes, it is conjectured that the supposedly rare<sup>16</sup> *complex rules* (class 4) may in some cases be capable of supporting *universal computation*.<sup>17,34</sup> The space-time patterns of the complex rules contain interacting, static, and propagating structures, sometimes called *information structures*, similar to those illustrated in Figs. 2.2 and 4.1.

Langton proposed that class 4 rules are located at a *phase transition* in rule space. He suggested that the long transients typical of these rules have potential for information processing, and implications for understanding the origin and evolution of life.<sup>17</sup>

**4.1.1 The  $\lambda$  Parameter**

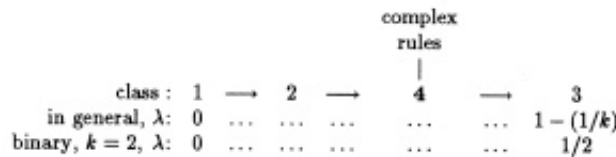
Rule space has been characterised by the  $\lambda$  parameter,<sup>16,17</sup> the proportion of non-zero entries in the rule table. If the value range is  $k$ , with possible values  $0, 1, 2, \dots, k - 1$ , then a particular value, say 0, is selected as the quiescent value. The  $\lambda$  parameter is the proportion of rule table entries other than 0.

Langton and others<sup>17,30,41</sup> have shown that the rule classes can be roughly selected by adjusting  $\lambda$  between 0 and  $1 - (1/k)$ . A rule table may be constructed by assigning one of the  $k$  values to each entry with equal probability, so that the density of all values, including the quiescent value, will be roughly equal, and  $\lambda \approx 1 - (1/k)$ . At this value of  $\lambda$ , space-time patterns are most likely to appear chaotic.

Rules may be selected according to other values of  $\lambda$ . This is typically done by assigning the quiescent value with a selected probability  $z$ , so that  $\lambda = 1 - z$ . The remaining values are then assigned with equal probability  $\lambda/(k - 1)$ .

For binary rules where  $k = 2$ ,  $\lambda$  is simply the density of 1s in the rule table. Maximum chaos in the appearance of space-time patterns is likely to occur at  $\lambda = 1/2$ , a rule table with an equal number of 0s and 1s.

As the  $\lambda$  parameter is varied from 0 to  $1/2$ , the various classes of behaviour are traversed. Complex, class 4 behaviour occurs at a phase transition between periodic, class 2 and chaotic, class 3 behaviour, reordering Wolfram's classes as follows<sup>16,17</sup>:



As  $\lambda$  increases from  $1/2$  to 1 the sequence is reversed, so that the complex rules occur in two limited regions on either side of  $\lambda = 1/2$ .

**4.1.2 Convergence of State Space**

It is suggested that the  $\lambda$  parameter is modulating the same aspect of CA global behaviour as the  $Z$  parameter (introduced in chapter 3, section 3.6)—the maximum pre-imaging (*mp*) as a function of array length. In general this will reflect the degree of pre-imaging typical of a rule. The degree of pre-imaging is apparent in the topology of a rule's basin of attraction field, and reflects the convergence of state space,<sup>14</sup> which may be equivalently measured as the density of garden-of-Eden nodes in state space (or in one basin).

Low pre-imaging implies low convergence and a low density of garden-of-Eden nodes; high pre-imaging implies high convergence and high density of garden-of-Eden nodes. The degree of pre-imaging and the density of garden-of-Eden nodes are available in the Atlas (or program).

**4.1.3 The  $Z$  Parameter and the  $\lambda$  Parameter**

The  $Z$  parameter is the probability that the next cell of a partial pre-image has a unique value (see chapter 3, section 3.6). For binary rules, when  $Z = 1$ , the quantity of 0s and 1s in the rule table is equal,  $\lambda = 1/2$ ; when  $Z = 0$ ,  $\lambda = 0$ . As  $Z$  varies between 0 and 1,  $\lambda$  varies between 0 and  $1/2$  or conversely between  $1$  and  $1/2$ .

The  $Z$  parameter may possibly be the mechanism underlying the operation of the  $\lambda$  parameter, because the probability of a rule table having a given value of  $Z$  depends on the proportions of 0s and 1s assigned at random to the rule table according to the setting of  $\lambda$ . Thus  $\lambda$  seems to be a measure of the probability of a particular value of  $Z$ . The  $Z$  parameter is concerned not only with the numbers of 0s and 1s in the rule table, but also their position and thus might be expected to modulate behaviour more closely.

For a more direct comparison between the values of  $\lambda$  and  $Z$  for a given binary rule table, the  $\lambda$  parameter may be modified as follows. The quiescent state is taken as the majority value in the rule table; the minority value is taken as the active state. The modified  $\lambda$  parameter, referred to as the  $\lambda$  ratio, is the ratio of the active values to  $1/2$  of the rule table (the potential maximum of active values). For example, in an  $n = 5$  rule table with 32 entries, say 10 entries in the rule table are 0, then the  $\lambda$  ratio equals  $10/16$ .

$$\begin{aligned} \text{If } \lambda \leq 1/2, \text{ then } \lambda \text{ ratio} &= 2\lambda. \\ \text{If } \lambda > 1/2, \text{ then } \lambda \text{ ratio} &= 2(1 - \lambda). \end{aligned}$$

The value of the  $\lambda$  ratio varies between 0 and 1 roughly in line with, but never smaller than, the  $Z$  parameter.  $Z \leq \lambda$  ratio, and  $Z = \lambda$  ratio = 1 only for limited pre-image rules.

As an example of the  $\lambda$  ratio and the  $Z$  parameter, consider the space-time patterns with complex structures that were illustrated in Figs. 2.2(a-c). The relevant rules have the following values for the  $\lambda$  ratio and  $Z$ :

- a. rule 3112581872  $\lambda$  ratio = 1,  $Z = .671875$
- b. rule 2334561936  $\lambda$  ratio = .8125,  $Z = .6875$
- c. rule 3583552890  $\lambda$  ratio = .875,  $Z = .75$

There are many examples of rules where the  $\lambda$  ratio equals 1 or close to 1, suggesting a chaotic space-time pattern, when in fact the pattern appears complex. In such cases the value of  $Z$  will typically be between .6 and .8.



	HIGH convergence and garden of Eden density		phase transition <i>mp</i> and ( <i>mc, ml, nb</i> ) in balance		LOW convergence and garden of Eden density
class	1	→ 2 →	4	→	3
<i>Z</i> :	0	... ..	...	...	1
$\lambda$ ratio :	0	... ..	...	...	1
$\lambda$ :	0	... ..	...	...	$1 - (1/k)$
<i>mp</i> :	diverges exponentially with <i>L</i>	... ..	...	...	fixed upper limit
( <i>mc, ml, nb</i> ) :	fixed lower limit	... ..	...	...	diverges exponentially with <i>L</i>

FIGURE 4.2 Relationship between the *Z* parameter (and  $\lambda$ ) and the change in basin of attraction field topology as *L* increases, reflecting convergence of state space.

**4.1.5 Examples of Typical Basin Topology in Relation to Rule Class**

To illustrate the relationships in Fig. 4.2, an example of a typical basin of attraction for *n* = 3 rules characteristic of each rule class is given in Figs. 4.3–4.6, including the following data: *mp*, maximum pre-imaging; *g-density*, density of garden-of-Eden nodes; *period*, of attractor cycle; and *ml*, maximum transient length.

In these examples, rotation equivalent transients are suppressed for the sake of clarity. General examples of the complete basin field over a range of array length *L* are presented in the Atlas.

**CLASS 1.** Rules where *Z* = .25 will have a large proportion of ambiguous permutations in the rule table, so *mp* will diverge exponentially with *L*; *mc*, *ml*, and *nb* will either remain fixed or relate arithmetically to *L*. Most states are locked into pre-image fans flowing into point attractors, or two-state attractors, especially the uniform states (all 0s or 1s). The few states outside the influence of these attractors will only have enough scope to form basins with short cycles and transients. (See Fig. 4.3.)

**CLASS 2.** Rules where *Z* = .5 may have many basins separated from the uniform attractor. *mp* will diverge exponentially with *L*, but to a lesser extent than for class 1. *mc*, *ml*, and *nb* will relate arithmetically to *L*. Again many states are locked up in pre-image fans leaving enough scope for only short cycles or transients. (See Fig. 4.4.)

**CLASS 4.** The complex, class 4 rules occur typically at *Z* = .75. *mp* and some combination of *mc*, *ml*, and *nb* will be finely balanced and will diverge by some intermediate function with *L*. Controlled pre-imaging allows enough scope for moderately long cycles and transients. (See Figure 4.5.)

**CLASS 3.** The chaotic, class 3 rules are limited pre-image rules where *Z* = 1. *mp* is fixed irrespective of *L*. Some combination of *mc*, *ml*, and *nb* diverge exponentially with increasing *L*. For instance, clusters 30 and 45, where *mp* = 3, have the greatest values of *mc* and *ml* among the *n* = 3 rules. (See Fig. 4.6.)

In general, the space-time patterns of the *n* = 3 and *n* = 5 rules have been found to correlate with the *Z* parameter; however, as with the  $\lambda$  parameter, there are exceptions. It has been noted that the transition to chaotic behaviour may occur at different  $\lambda$  values, although there is a well-defined distribution around a mean value.<sup>17,19</sup> Chaotic space-time patterns may also occur at low values of *Z*. For example, the *n* = 3 rules 18 and 126 have a value of *Z* = .5. Although the typical basin topology of these rules correlates with *Z*, their space-time patterns appear chaotic, but are confined to only a subset of possible configurations of neighbourhoods. This is clearly seen when colors are assigned to cells in space-time patterns according to the neighbourhood which determined the cell's value, implemented in the program Space1 following a suggestion by Warrell.<sup>43</sup> The process whereby active neighbourhoods are eliminated requires further investigation.





FIGURE 4.3 Class 1, rule 251.  $L = 12$ , seed 111111111111.  $mp = 853$ ,  $g\text{-density} = .32$ , period = 1,  $ml = 6$ .  $mp$  diverges exponentially with  $L$ ;  $Z = .25$ ; ( $\lambda$  ratio = .25). Equivalent transients above level 1 are suppressed.

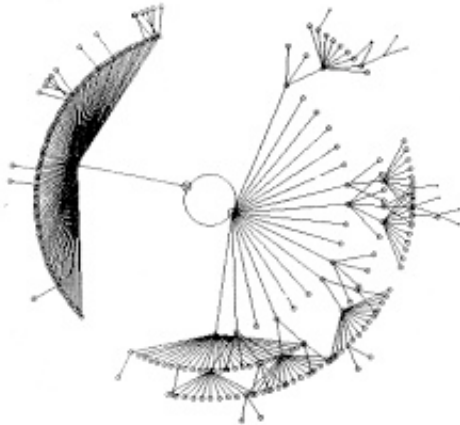


FIGURE 4.4 Class 2, rule 33.  $L = 16$ , seed 0111111111100000.  $mp = 97$ ,  $g\text{-density} = .84$ , period = 2,  $ml = 5$ .  $mp$  diverges exponentially with  $L$ ;  $Z = .5$ ; ( $\lambda$  ratio = .5).

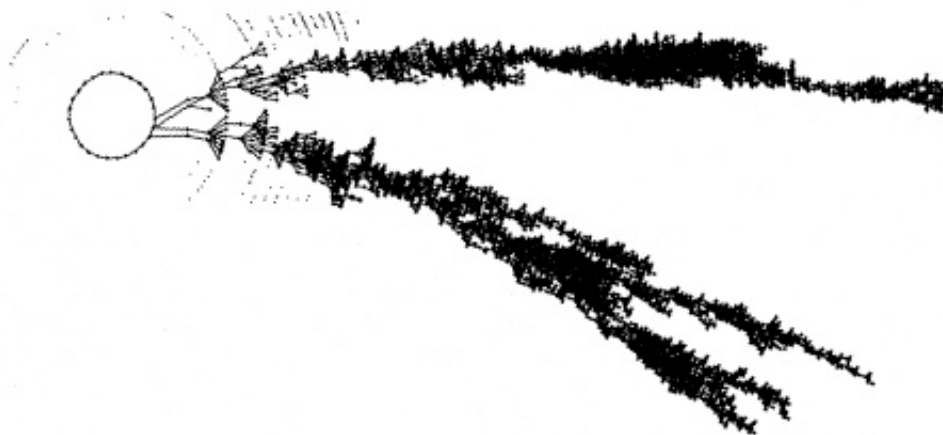


FIGURE 4.5 Class 4, rule 193.  $L = 18$ , seed 011010001110000010.  $mp = 70$ ,  $g\text{-density} = .61$ , period = 27,  $ml = 120$ .  $mp$ ,  $mc$ , and  $ml$  diverge with  $L$ ;  $Z = .75$ ; ( $\lambda$  ratio = .75). Equivalent transients are suppressed.

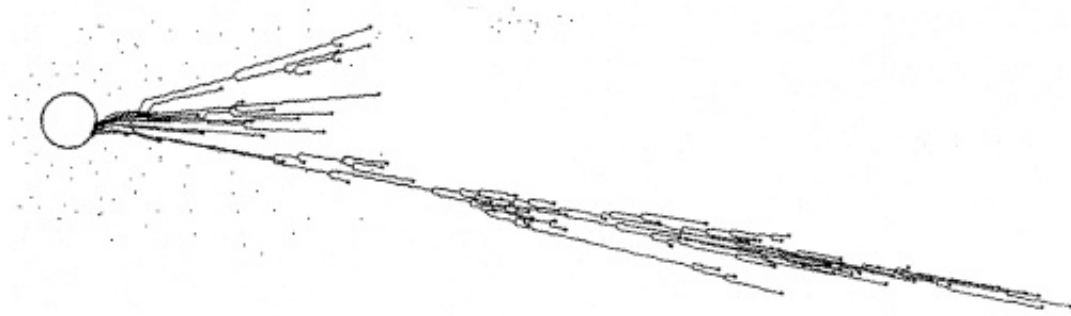


FIGURE 4.6 Class 3, rule 30.  $L = 15$ , seed 110110111000000.  $mp = 2$ ,  $g$ -density = .04, period = 1455,  $ml = 321$ .  $mp$  and  $ml$  diverge exponentially with  $L$ ;  $Z = 1$ ; ( $\lambda$  ratio = 1). Equivalent transients and pre-image nodes are suppressed; angle between pre-image arcs is increased.

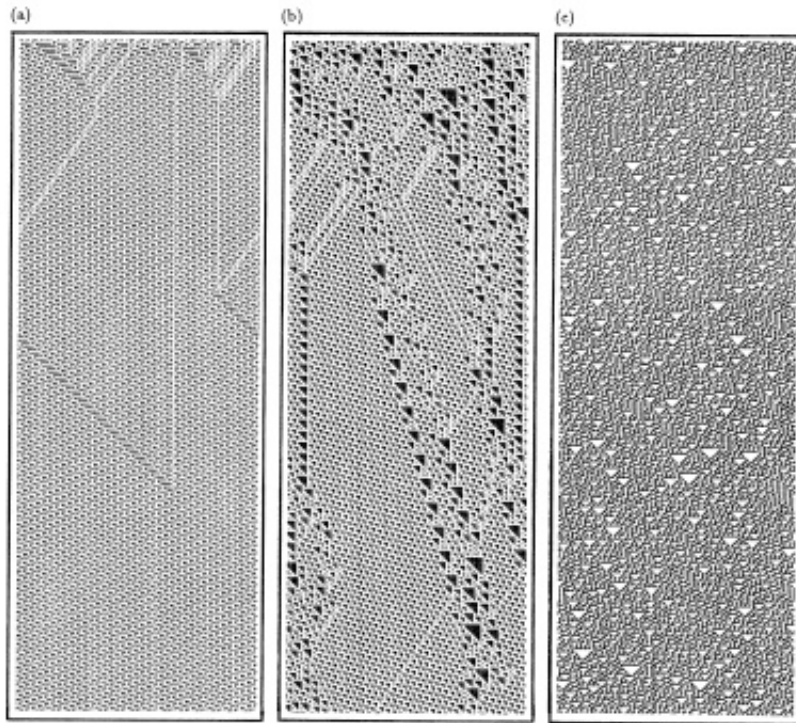


FIGURE 4.7 Space-time patterns for  $n = 3$  rules. Array size is 150, 420 time steps from a random initial state. Rule numbers and  $Z$  parameter are: (a) Class 2: rule 25,  $Z = .5$ ; (b) Class 4: rule 193,  $Z = .75$ ; (c) Class 3: rule 30,  $Z = 1$ .

## Parameters for 48 $n=3$ Clusters

TABLE 3.3  $n = 3$  rules

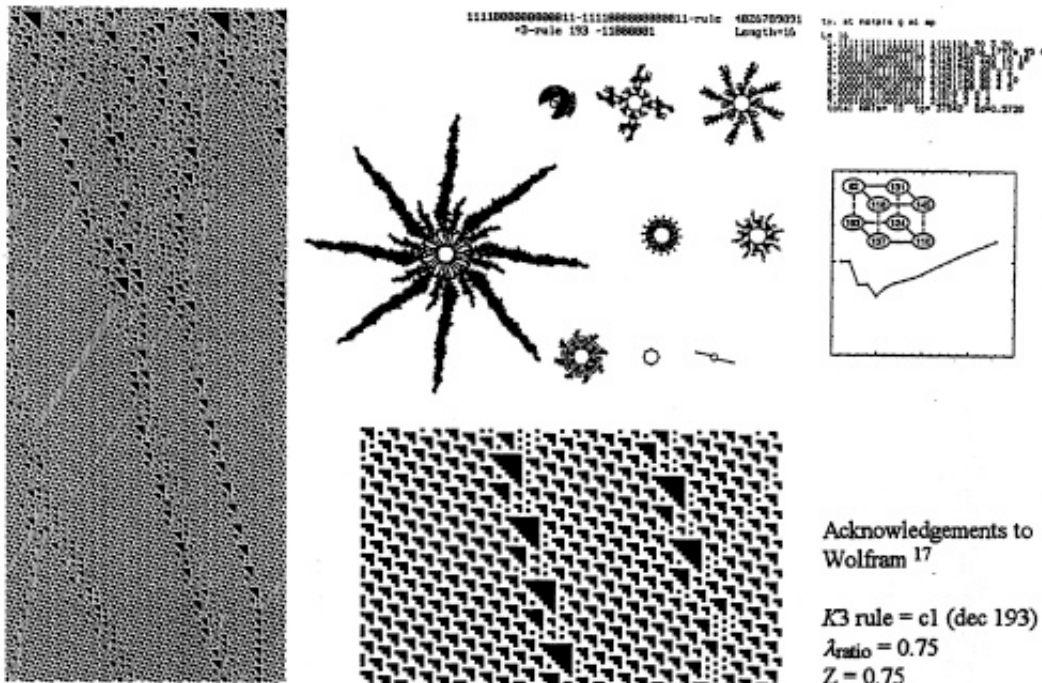
	rule number	$\lambda$ parameter	$\lambda$ ratio	$Z$ parameter
symmetric rules	0	0/8	0	0
	1	1/8	0.25	0.25
	4	1/8	0.25	0.25
	5	2/8	0.5	0.5
	18	2/8	0.5	0.5
	19	3/8	0.75	0.625
	22	3/8	0.75	0.75
	23	4/8	1	0.5
	33	2/8	0.5	0.5
	36	2/8	0.5	0.5
	37	3/8	0.75	0.75
	50	3/8	0.75	0.625
	51	4/8	1	1
	54	4/8	1	0.75
	73	3/8	0.75	0.75
	77	4/8	1	0.5
90	4/8	1	1	
94	5/8	0.75	0.75	
105	4/8	1	1	
126	6/8	0.5	0.5	
semi-asymmetric rules	2	1/8	0.25	0.25
	3	2/8	0.5	0.5
	6	2/8	0.5	0.5
	7	3/8	0.75	0.75
	9	2/8	0.5	0.5
	12	2/8	0.5	0.5
	13	3/8	0.75	0.75
	26	3/8	0.75	0.75
	27	4/8	1	0.75
	30	4/8	1	1
	35	3/8	0.75	0.625
	38	3/8	0.75	0.75
	41	3/8	0.75	0.75
	45	4/8	1	1
	58	4/8	1	0.75
62	5/8	0.75	0.75	
fully asymmetric rules	10	2/8	0.5	0.5
	11	3/8	0.75	0.75
	14	3/8	0.75	0.75
	15	4/8	1	1
	24	2/8	0.5	0.5
	25	3/8	0.75	0.75
	28	3/8	0.75	0.75
	29	4/8	1	0.5
	43	4/8	1	0.5
	46	4/8	1	0.5
	57	4/8	1	0.75
	60	4/8	1	1

### In his 1996-97 D.Phil. Thesis at the University of Sussex, Andrew Wuensche said:

"... Glider Dynamics ... corresponds to Wolfram's class 4 behaviour ... , to notions of emergent computation on the edge of chaos, and to a phase transition between order and chaos ... Gliders are analagous to autocatalytic sets of polymers in the sense of Kauffman ... members of such sets have a survival advantage in occupying space, and the set acquires its own identity as an observed object at a higher level. Gliders are also discrete examples of Prigogine's far-from-equilibrium dissipative structures ... in CA the process of formation, persistence and interaction of gliders and other dissipative structures can be traced at the lowest level of the system's basic components and their local interactions which are completely defined. This ability to see two levels of behaviour simultaneously, the underlying and emergent, may lead to insights into the mechanics of self-organization. ... Among the 256 binary 3-neighbor rules, the "elementary rules" ... , an

exhaustive search reveals two sets of glider rules, rule 54 and 110, and their equivalents. ...

Appendix 2.4 [re rule 193] ...



...[The]... spectrum of CA behaviour ... rang[es] progressively through ordered, complex and chaotic dynamics, corresponding to Wolfram's ... classes

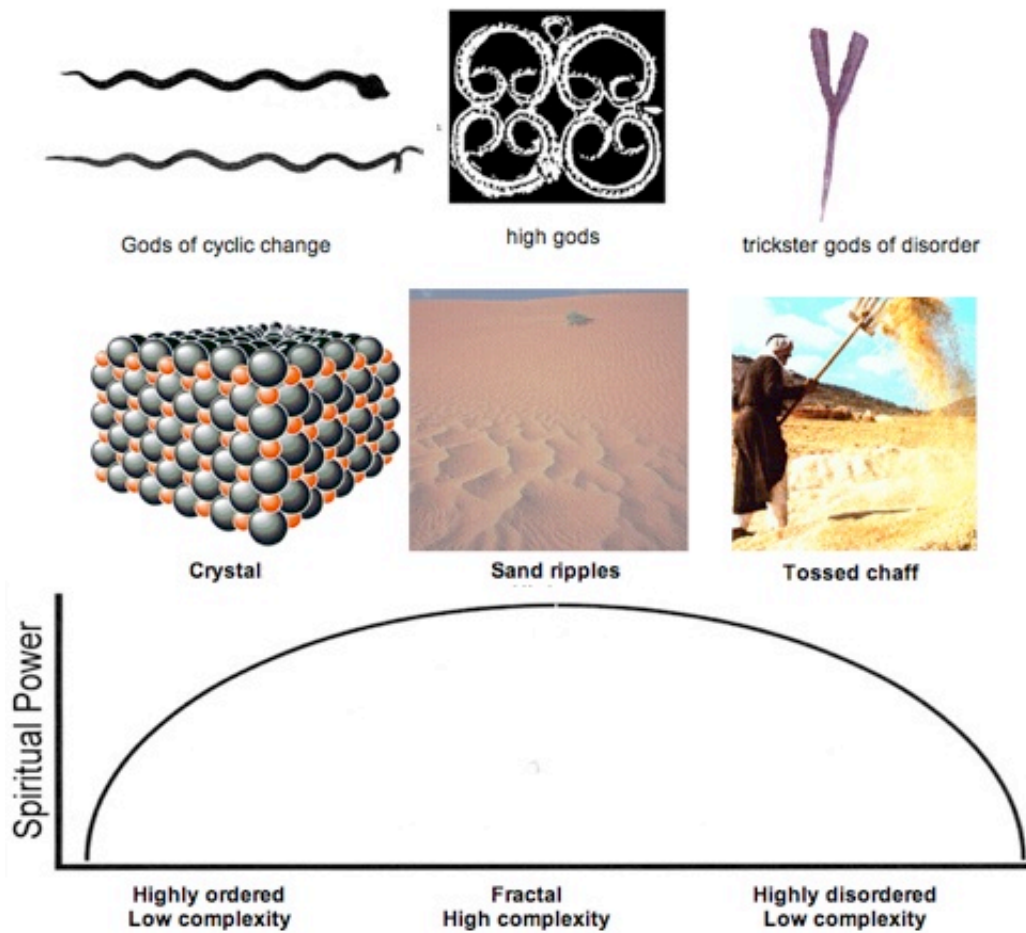
**(1 and 2) [ordered] - (4) [complex, including 54 and 193] - (3) [chaotic]**

...".

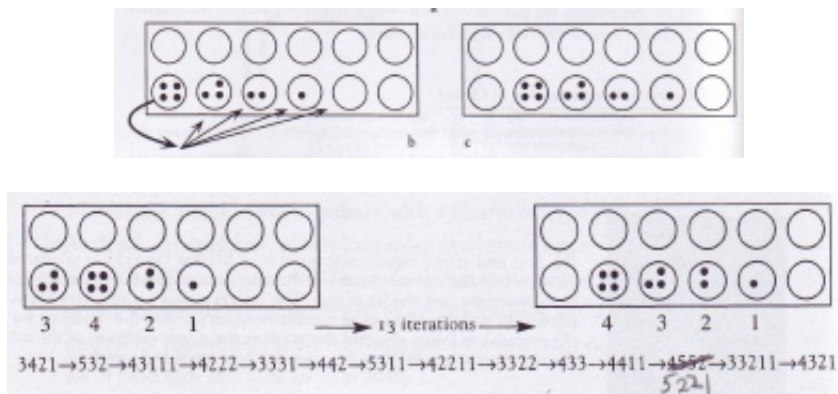
**Ron Eglash** (in his book "African Fractals" (Rutgers 1999) and on his web site at [www.csdt.rpi.edu](http://www.csdt.rpi.edu)) says:

"...





... the owari marching-group system can be used as a one-diemnsional cellular automaton ...



... transients of many different lengths can be produced. ... the constant pattern is called a "point attractor", and the transients would be said to lie in the "basin of attraction".

The marching group rule can also produce periodic behavior (a "limit cycle" or "periodic attractor" ...). Here is a period-3 system using only four conters:

$$211 \rightarrow 22 \rightarrow 31 \rightarrow 211$$

Total number of counters	Behavior (after transients)
1	Marching
2	Period 2
3	Marching
4	Period 3
5	Period 3
6	Marching
7	Period 4
8	Period 4
9	Period 4
10	Marching
11	Period 5
12	Period 5
13	Period 5
14	Period 5
15	Marching

... The numbers which lead to marching groups - 1 , 3 , 6 , 10 , 15 ... - ...[are]... the triangular numbers ...

[ the triangular numbers correspond to the dimension of the grade-2 bivectors in Clifford Algebras -

- for the case of the  $2^8 = 256$  Elementary CA Rules, there are 28 grade-2 CA Rules ]

... One-dimensional versions can ... be used as a kind of parallel computer. Consider, for example, a rule that in each iteration the number of counters in a cup is replaced by the sum of itself and its left neighbor.

Starting with one:

$$0100000 \rightarrow 0110000 \rightarrow 0121000 \rightarrow 0133100 \rightarrow 0146410$$

This fourth iteration gives the us the binomial coefficients for expansion of  $(a+b)^4$  ,

$$\text{which equals to } a^4 + 4 a^3 b + 6 a^2 b^2 + 4 a b^3 + b^4 .$$

[ Such a rule reproduces at each step succeeding rows of the Yang Hui triangle. ]

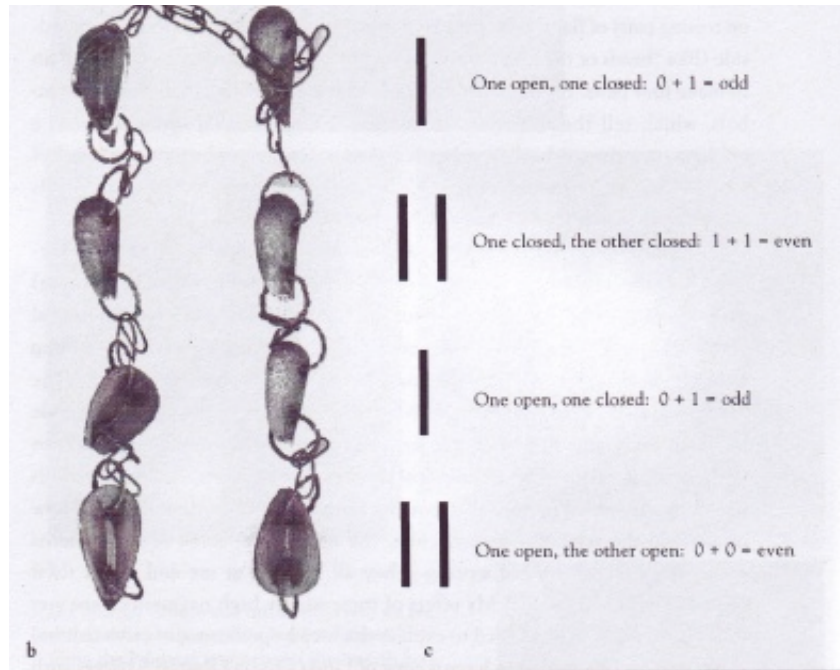


FIGURE 7.6

**Binary codes in divination**

(a) This Nigerian priest is telling the future by Ifa divination, in which pairs of flat shells or seeds split in two are tossed with each landing open-side or closed-side. They are connected by a doubled chain to make four pairs, giving a total of 16 divination symbols. In this version of Ifa (used in the Abigba region of Nigeria) they use two doubled chains and consider the cast more accurate if there is a correlation between the two sets. (b) Here we see a chain using split seeds. Each half lands either "closed" (meaning we see the rounded outside) or "open" (meaning we see the interior). By using open to represent 0 (double lines), and closed to represent 1 (single line), we can see how the divination symbol is obtained. (c) The divination chain is interpreted as pairs summing to odd (one stroke) or even (two strokes).  
 (a, photo by E. M. McClelland, courtesy Royal Anthropological Institute.)

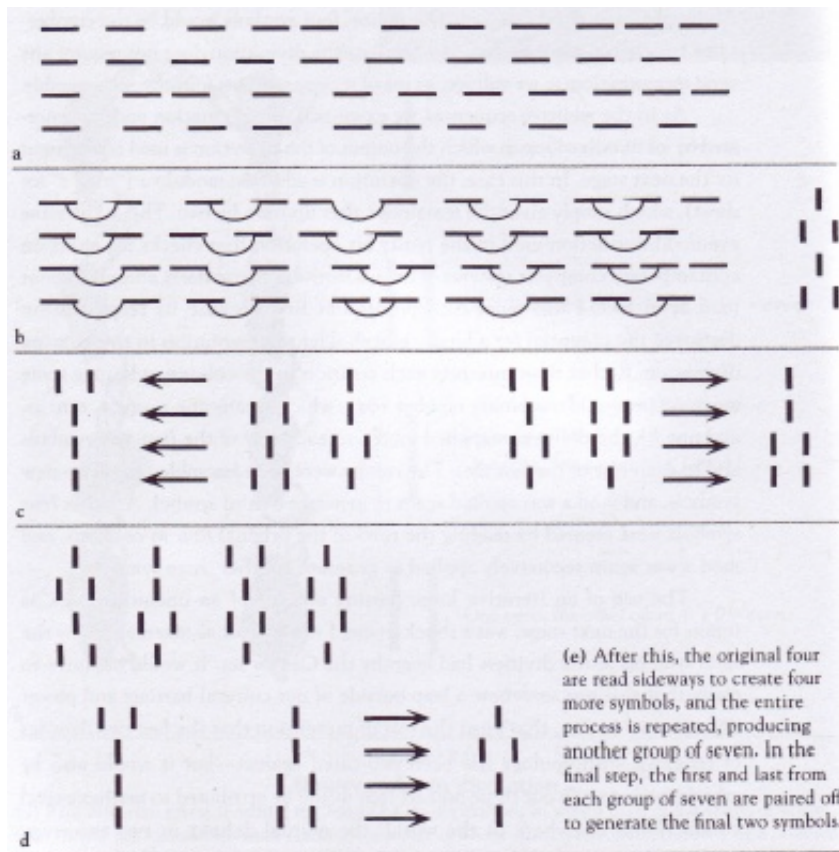


FIGURE 7-7  
**Bamana sand divination**  
 (a) Four sets of random dashes are drawn. (b) Each of the dashes is paired, and the odd/even results are recorded. (c) The process is repeated four times, resulting in four symbols. Each row of the first two symbols and the last two symbols are paired off to generate two new symbols. (d) The two newly generated symbols, now placed below the original four, are again paired off to generate a seventh symbol.

... If we think of the two-strokes as zero and single stroke as one, the Bamana divination system is almost identical to the process of pseudorandom number generation used by digital circuits called "shift registers". Here the circuit takes mod 2 of the last two bits in the register and places the result in the first position. The other bits are shifted to the right, with the last discarded. ...

- 1111
- 0111
- 0011
- 0001
- 1000
- 0100
- 0010
- 1001
- 1100
- 0110
- 1011
- 0101
- 1010
- 1101
- 1110

... This four-bit register will only produce 15 binary words before the cycle starts over, but the period of the



cycle increases with more bits ... For the entire 16 bits ... that begin the Bamana divination, 65,535 binary words can be produced before repeating the cycle. ...

Skinner ... "Terrestrial Astrology: Divination by Geomancy". London: Routledge and Kegan Paul, 1980 ... provides a well-documented history of the diffusion evidence ... for ... Arabic, European, West African, and East African ... "geomancy" ...divination technique. ...

[ Such diffusion seems also to have extended



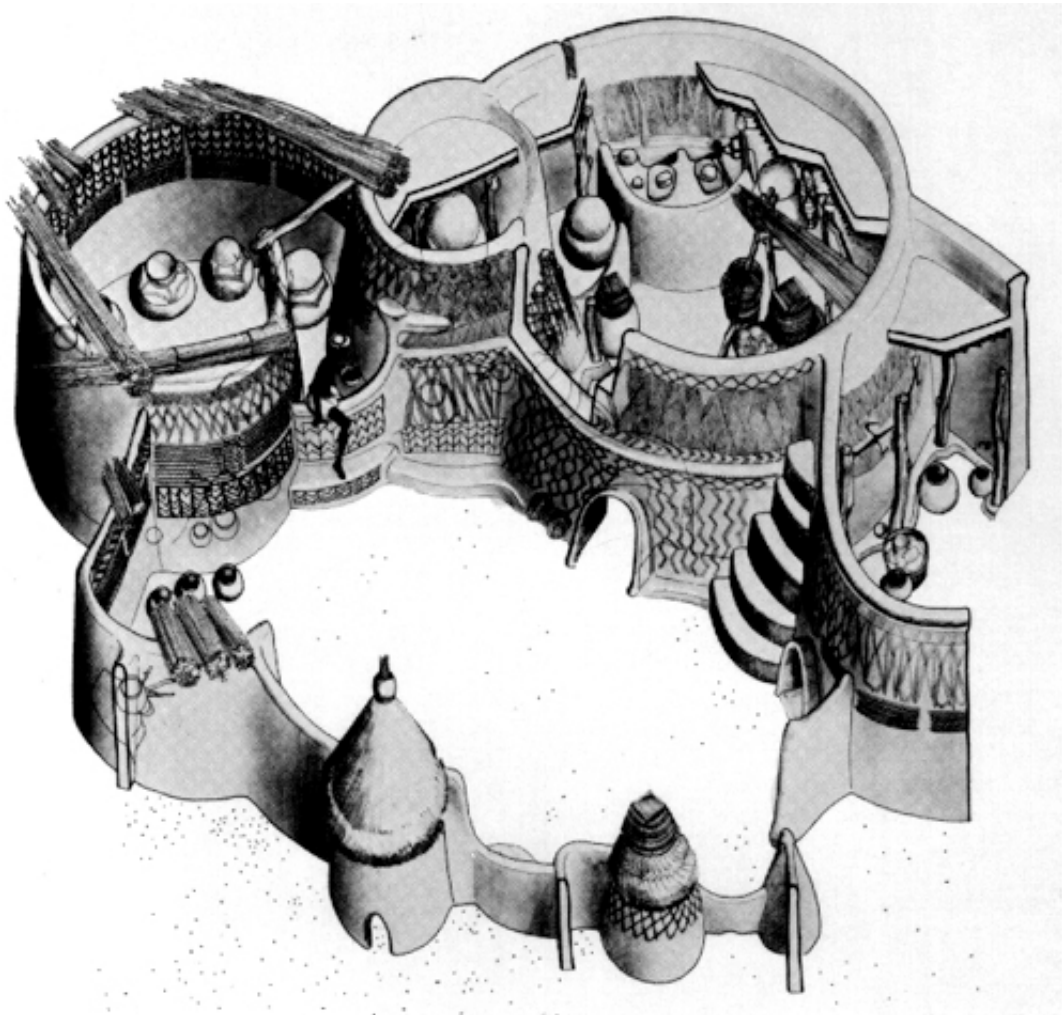
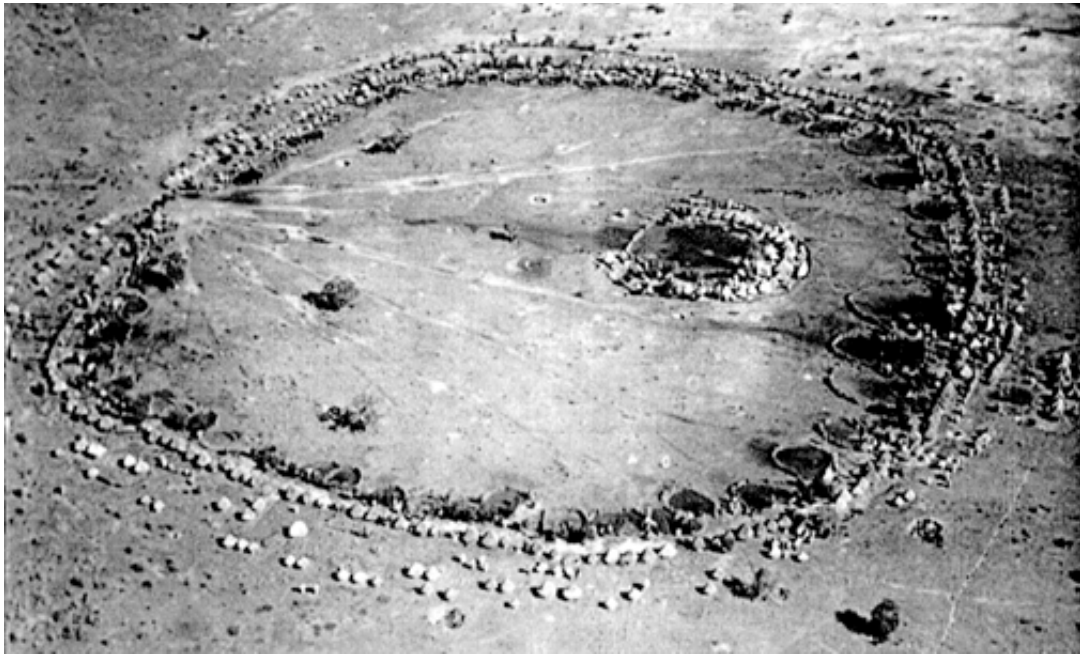
from Africa to India, China, and Japan. ]

... The implications of this trajectory - from sub-Saharan Africa to North Africa to Europe - are quite significant for the history of mathematics.

... a historical path for base-2 calculation ... begins with African divination, runs through the geomancy of European alchemists, and is finally transformed into binary calculation, where it is now applied in every digital circuit ...

... Following the introduction of geomancy to Europe by Hugo of Santalla in twelfth-century Spain ... European geomancers ... Ramon Lull ... and others ... persistently replaced the deterministic aspects of the system with chance. By mounting the 16 figures on a wheel and spinning it, they maintained their society's exclusion of any connections between determinism and unpredictability. The Africans, on the other hand, seem to have emphasized such connections ...[with]... a "trickster" god, one who is both deterministic and unpredictable. ...

The fractal settlement patterns



of Africa stand in sharp contrast to the Cartesian grids of Euro-American settlements. ... Euro-American cultures are organized by ... "top-down" organization. Precolonial African cultures included ... societies that

are organized "bottom-up" rather than "top-down". ... African architecture tends to be fractal because that is a prominent design theme in African culture ... most of the indigenous African societies were neither utterly anarchic, nor frozen in static order; rather they utilized an adaptive flexibility ... African traditions of decentralized decision making could ... be combined with new information technologies, creating new forms that combine democratic rule with collective information sharing ... what is needed is not ... "small is beautiful", but rather a self-organized approach to changes in the relations between scale and the socioenvironmental systems - not just appropriate technology, but appropriate scaling. ...

we are trapped between the periodic stasis of the preservationists' limit cycle,

and the white noise of the profiteering positive feedback loop.

... both are lacking in flexible interactions with memory;

the ... preservationists' ... limit cycle being too tied to it, and the ... profiteering ... white noise being too free from it. ...".

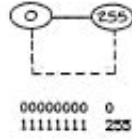
**During the decade since Ron Eglash published his book in 1999, the USA/UK global financial system is collapsing due to grossly excessive profiteering.**

**Perhaps "African traditions of decentralized decision making" could produce a successor global financial system that will better serve the needs of human civilization.**

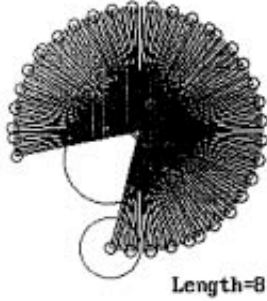
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## **Atlas of Basin of Attraction Fields for 88 $n=3$ Equivalence Classes**

00000000000000000000-00000000000000000000-rule  
=3-rule 0 -00000000

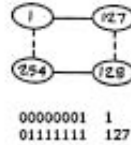
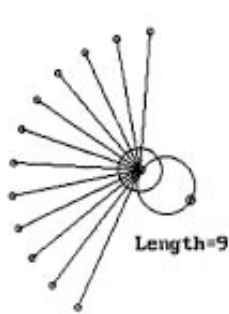


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```



3-rule 0 =C  
ty. at noip  
L= 1  
1.0 1(1)2  
total NATs  
L= 2  
1.00 1(1)4  
total NATs  
L= 3  
1.000 1(1)8  
total NATs  
L= 4  
1.0000 1(1)16  
total NATs  
L= 5  
1.00000 1(1)32  
total NATs  
L= 6  
1.000000 1(1)64  
total NATs  
L= 7  
1.0000000 1(1)128  
total NATs  
L= 8  
1.00000000 1(1)256  
total NATs

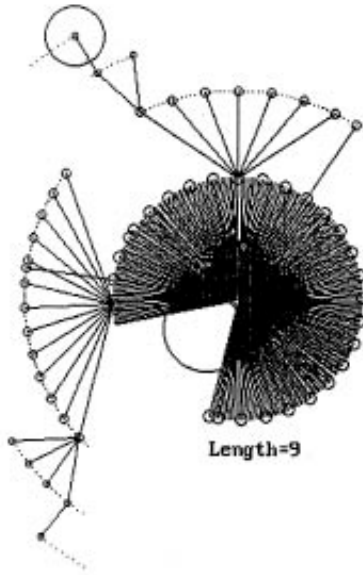
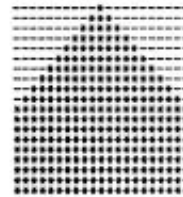
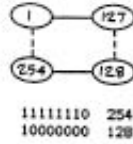
Z = .25 0000000000000000000011-0000000000000000000011-rule  
=3-rule 1 -00000001



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```

3-rule 1 =0000  
ty. at noip's  
L = 1

1111111111111100-1111111111111100-rule  
=3-rule 254 -11111110



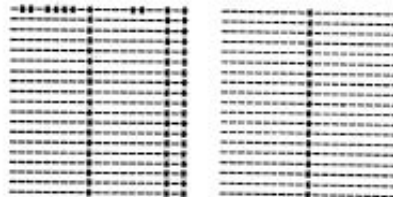
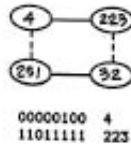
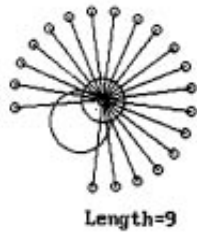
3-rule 254 =1  
ty. at no(p)s

```

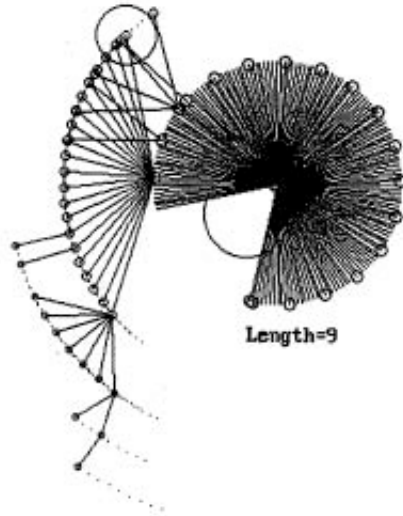
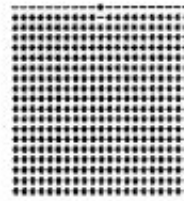
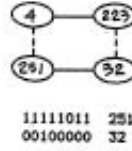
L=1
1.0 1111 0
Total NATs =
L=2
1.00 1111 0
Total NATs = 2
L=3
1.000 1111 0
Total NATs = 4
L=4
1.0000 1111 0
Total NATs = 8
L=5
1.00000 1111 0
Total NATs = 16
L=6
1.000000 1111 0
Total NATs = 32
L=7
1.0000000 1111 0
Total NATs = 64
L=8
1.00000000 1111 0
Total NATs = 128
L=9
1.000000000 1111 0
Total NATs = 256

```

0000000000110000-0000000000110000-rule 3145776  
=3-rule 4 -00000100 Length=1 -13

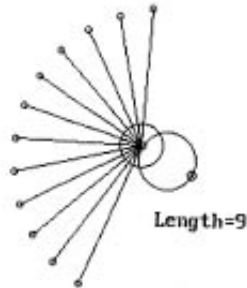
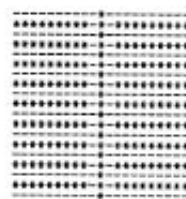
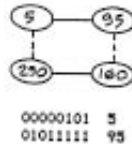


.25 1111111111001111-1111111111001111-rule  
 =3-rule 251 -1111011

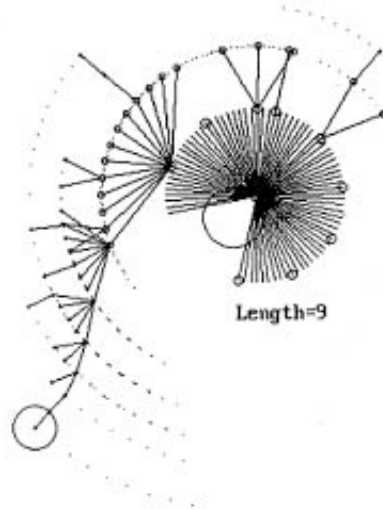
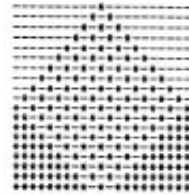
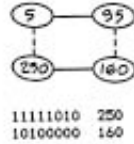


3-rule 251 :  
 ty. at noip:  
 L=1 1(1)2 :  
 total NATs  
 L=2 1112 :  
 total NATs  
 L=3 11112 :  
 total NATs  
 L=4 111112 :  
 total NATs  
 L=5 1111112 :  
 total NATs  
 L=6 11111112 :  
 total NATs  
 L=7 111111112 :  
 total NATs  
 L=8 1111111112 :  
 total NATs  
 L=9 11111111112 :  
 total NATs  
 L=10 111111111112 :  
 total NATs  
 L=11 1111111111112 :  
 total NATs

0000000000110011-0000000000110011-rule  
 =3-rule 5 -00000101



111111111001100-111111111001100-rule  
=3-rule 250 -1111010



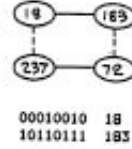
```

3-rule 250 =111101
ty. at no(pis q ml
L= 1
1. 111110000
Total NATs = 2
L= 2
1. 111110000
2. 111110000
Total NATs = 3
L= 3
1. 111110000
2. 111110000
3. 111110000
Total NATs = 4
L= 4
1. 111110000
2. 111110000
3. 111110000
4. 111110000
Total NATs = 5
L= 5
1. 111110000
2. 111110000
3. 111110000
4. 111110000
5. 111110000
Total NATs = 6
L= 6
1. 111110000
2. 111110000
3. 111110000
4. 111110000
5. 111110000
6. 111110000
Total NATs = 7
L= 7
1. 111110000
2. 111110000
3. 111110000
4. 111110000
5. 111110000
6. 111110000
7. 111110000
Total NATs = 8
L= 8
1. 111110000
2. 111110000
3. 111110000
4. 111110000
5. 111110000
6. 111110000
7. 111110000
8. 111110000
Total NATs = 9
L= 9
1. 111110000
2. 111110000
3. 111110000
4. 111110000
5. 111110000
6. 111110000
7. 111110000
8. 111110000
9. 111110000
Total NATs = 10

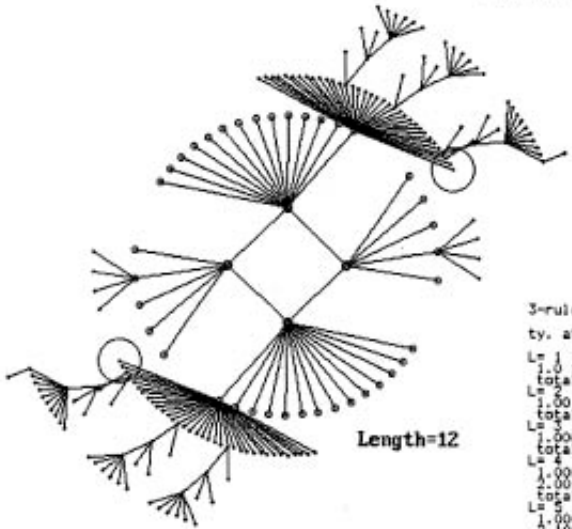
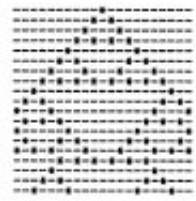
```



0000001100001100-0000001100001100-rule  
=3-rule 18 -00010010

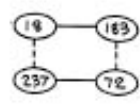


00010010 18  
10110111 183

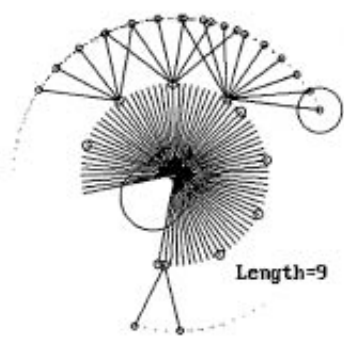
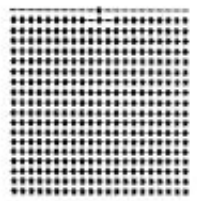


3-rule 18 =000100  
 ty. at no(p)s g a:  
 L= 1  
 1:0 1(1)2 1 1 2  
 total NATs = 1  
 L= 2  
 1:00 1(1)4 3 1 4  
 total NATs = 1  
 L= 3  
 1:000 1(1)8 4 2 1  
 total NATs = 1  
 L= 4  
 1:0000 1(1)12 9 1  
 total NATs = 3  
 L= 5  
 1:00000 1(1)16 12 1  
 total NATs = 6  
 L= A

111110011110011-111110011110011-rule  
=3-rule 237 -11101101



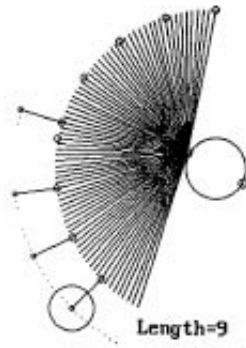
11101101 237  
01001000 72



3-rule 237 =11  
 ty. at no(p)s g a:  
 L= 1  
 1:1 1(1)2 1 1  
 total NATs = 1  
 L= 2  
 1:11 1(1)4 3 1  
 total NATs = 1  
 L= 3  
 1:111 1(1)8 4 2  
 total NATs = 1  
 L= 4  
 1:1111 1(1)12 9 1  
 total NATs = 3  
 L= 5  
 1:11111 1(1)16 12 1  
 total NATs = 6  
 L= 6  
 1:111111 1(1)20 15 1  
 total NATs = 11  
 L= 7  
 1:1111111 1(1)24 18 1  
 total NATs = 16  
 L= 8  
 1:11111111 1(1)28 21 1  
 total NATs = 21  
 L= 9  
 1:111111111 1(1)32 24 1  
 total NATs = 26  
 L= A

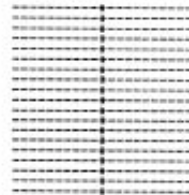
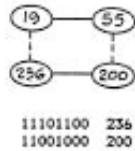
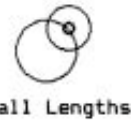


0000001100001111-0000001100001111-rule  
=3-rule 19 -00010011



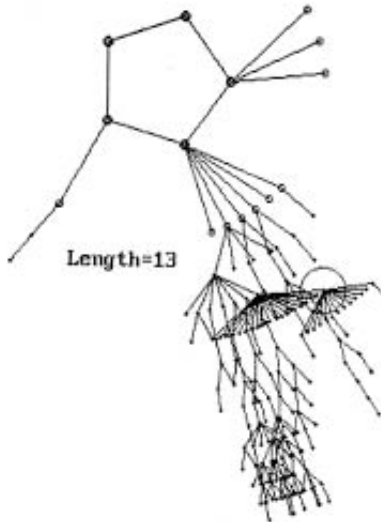
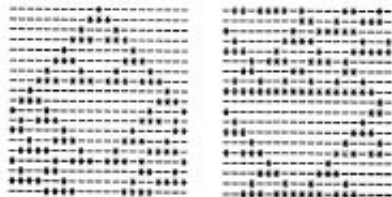
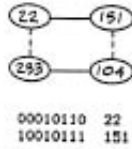
3-rule 19 =  
ty. at nosp:  
L= 1  
1.0 1(2)2 1  
total NHTs

111110011110000-111110011110000-rule  
=3-rule 236 -11101100



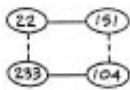
=3-rule 22 -00010110

Length=1 -15

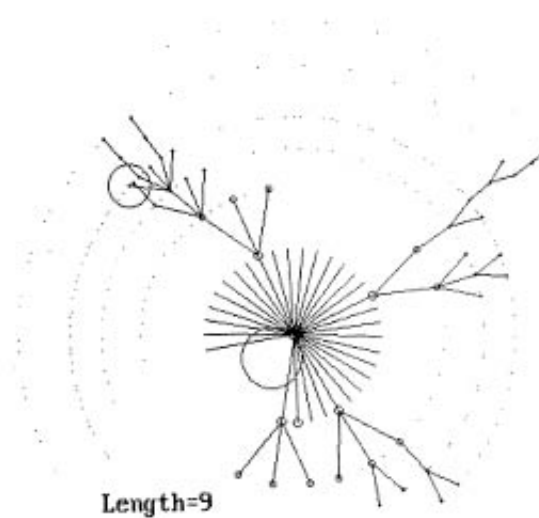
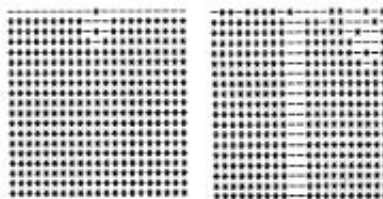


=3-rule 233 -11101001

Length=1 -15



11101001 233  
01101000 104

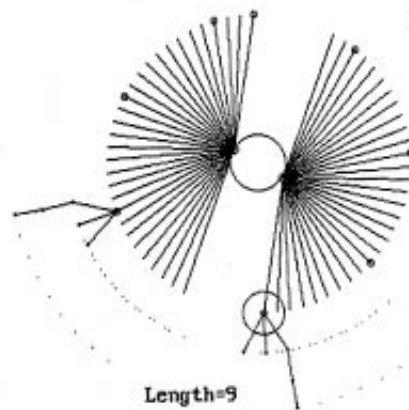
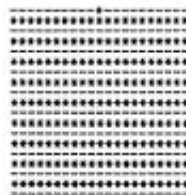


Length=9

0000001100111111-0000001100111111-rule  
=3-rule 23 -00010111



00010111 23

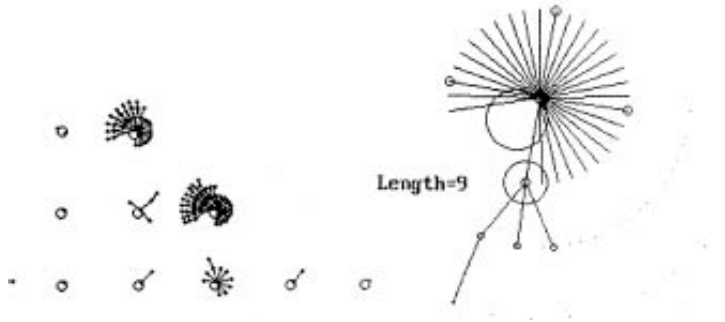
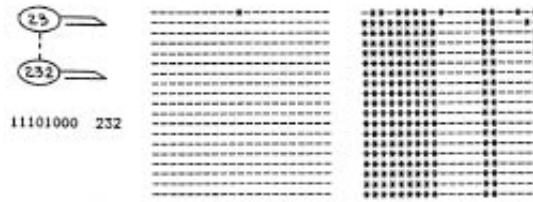


Length=9

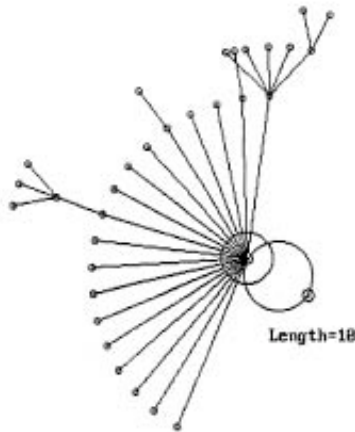
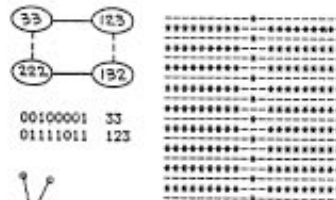
3-rule 23 =0  
ty. at no(p):  
L= 1  
1 0 1(2)2 0  
Total NRTs :  
L= 2  
2 00 1(2)2 1  
2 01 2(1)1 1  
Total NRTs :  
L= 3  
1 000 1(2)8  
Total NRTs :  
L= 4  
4 0000 1(2)16

=3-rule 232 -11101000

Length=1 -14

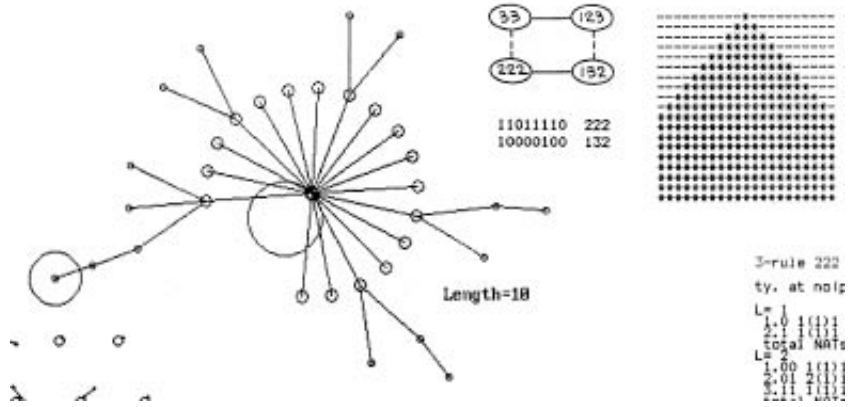


0000110000000011-0000110000000011-rule  
 =3-rule 33 -00100001

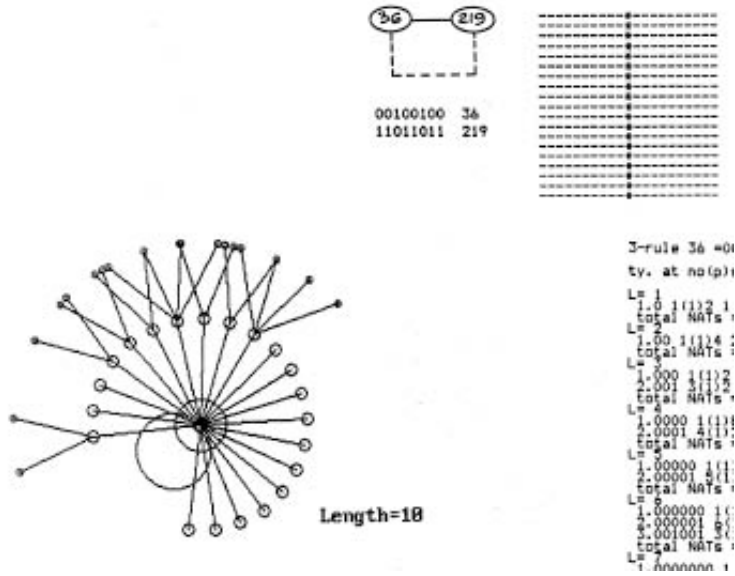


3-rule 33 =01  
 ty. at noip11  
 L= 1  
 1.0 1(2)2 0  
 Total NATs =  
 L= 2  
 1.00 1(2)2 1  
 2.01 1(2)2 1  
 Total NATs =  
 L= 3  
 1.000 1(2)18  
 Total NATs =  
 L= 4

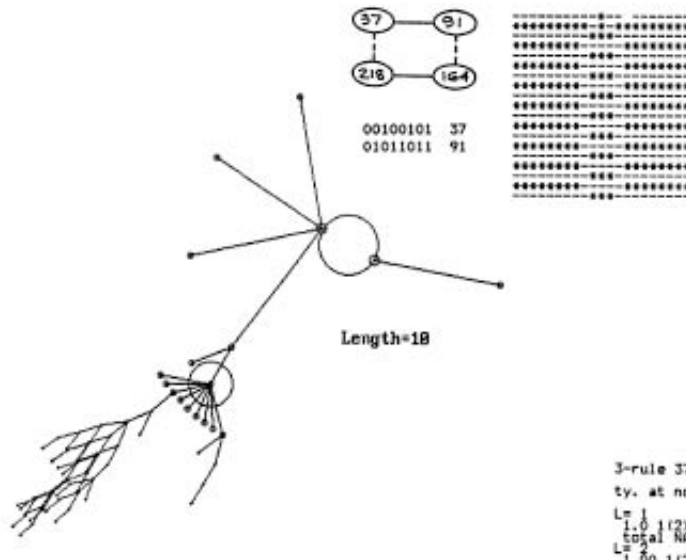
$Z = .5$  **1111001111111100-1111001111111100-rule**  
**=3-rule 222 -11011110**



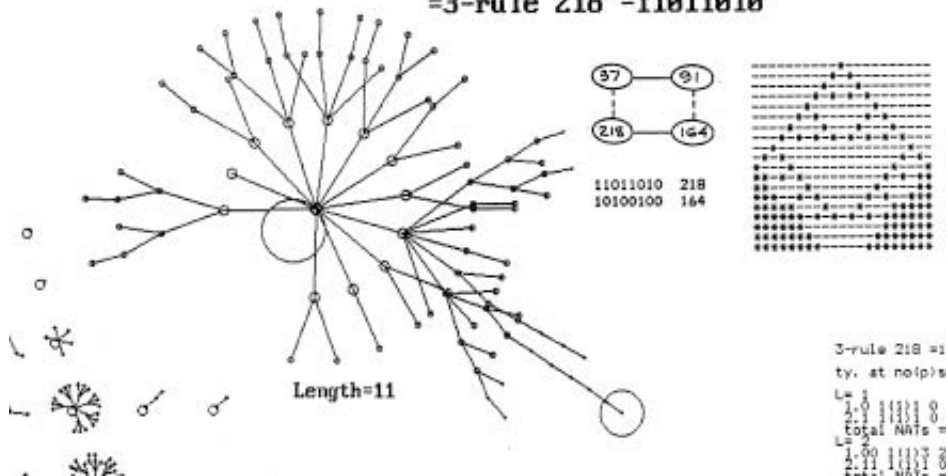
$Z = .5$  **0000110000110000-0000110000110000-rule**  
**=3-rule 36 -00100100**



75 0000110000110011-0000110000110011-rule  
=3-rule 37 -00100101

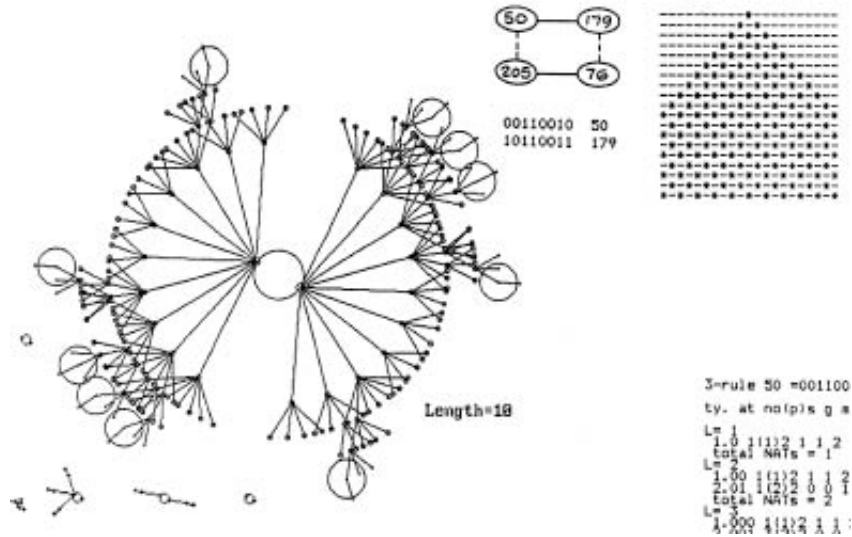


$\lambda$  ratio = .75  $Z$  = .75 1111001111001100-1111001111001100-rule  
=3-rule 218 -11011010

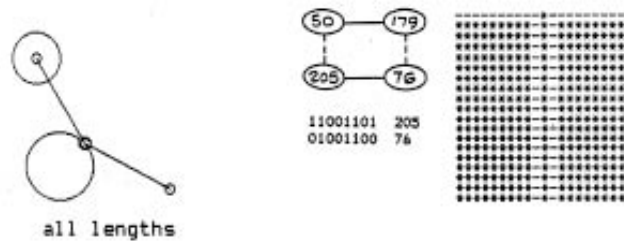




.75  $Z = .625$  **0000111100001100-0000111100001100-rule**  
**=3-rule 50 -00110010**



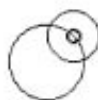
**1111000011110011-1111000011110011-rule**  
**=3-rule 205 -11001101**



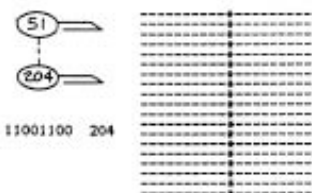
$Z = 1$  **0000111100001111-0000111100001111-rule**  
**=3-rule 51 -00110011**



1111000011110000-1111000011110000-rule  
=3-rule 204 -11001100

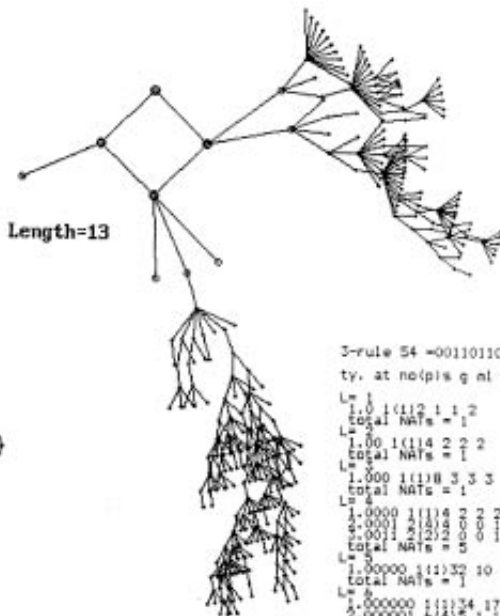
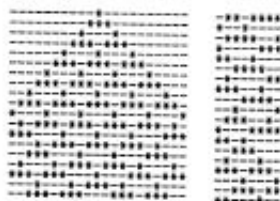
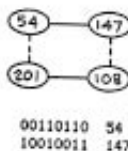


all lengths



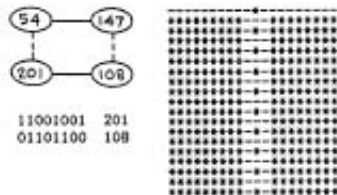
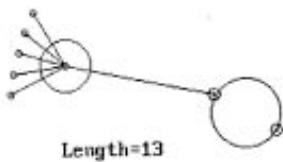
=3-rule 54 -00110110

Length



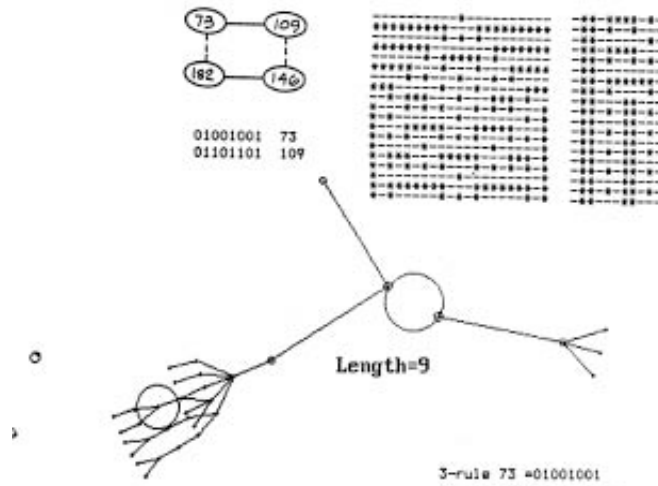
3-rule 54 =00110110  
ty. at no(pis q ml np  
L=1 1(112 1 1 2  
total NATs = 1  
L=2 1.00 1(114 2 2 2  
total NATs = 1  
L=3 1.000 1(118 3 3 3  
total NATs = 1  
L=4 1.0000 1(114 4 4 4  
total NATs = 1  
L=5 1.00000 1(1132 10 6 5  
total NATs = 1  
L=6 1.000000 1(1134 17 3 5  
total NATs = 7  
L=7

Z = .75 1111000011000011-1111000011000011-rule  
=3-rule 201 -11001001

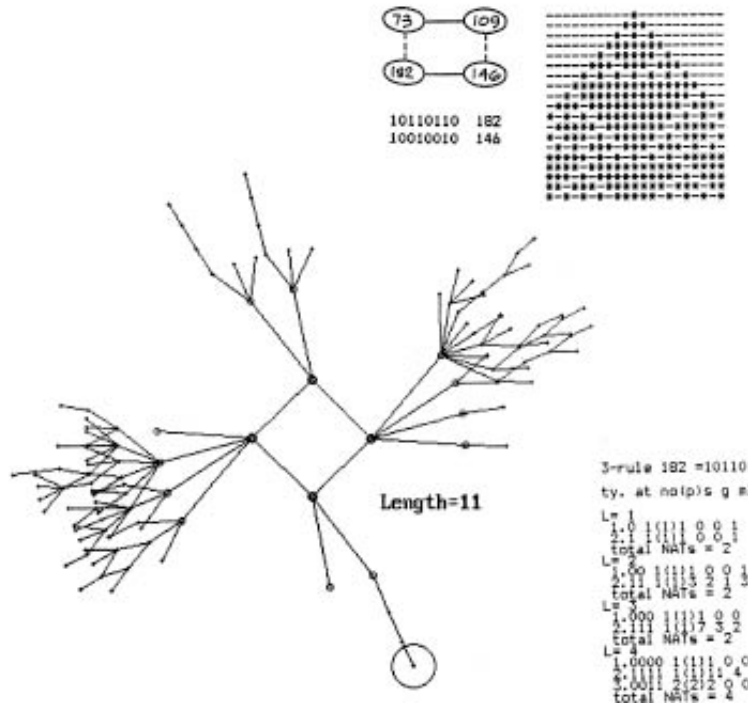


=3-rule 73 -01001001

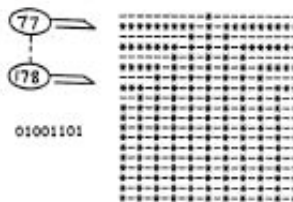
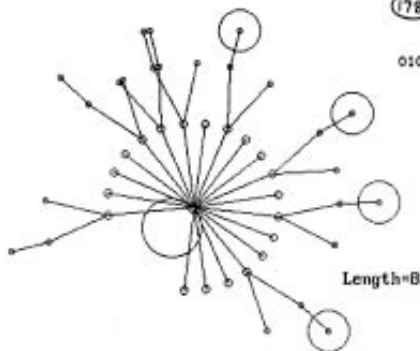
Length:



Z = .75 1100111100111100-1100111100111100-rule  
 =3-rule 182 -10110110

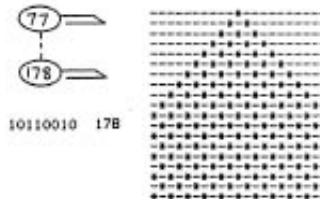
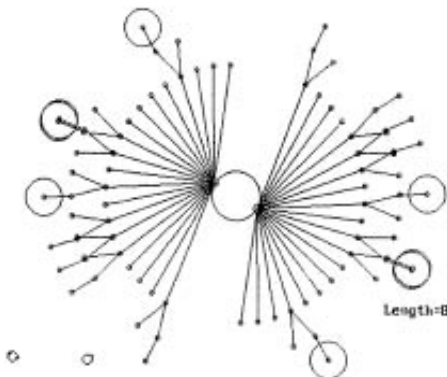


0011000011110011-0011000011110011-rule {  
=3-rule 77 -01001101 I



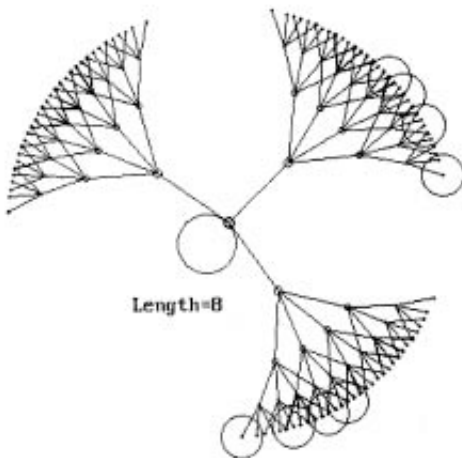
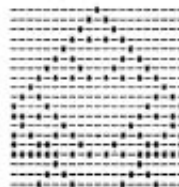
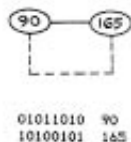
3-rule 77 =0  
ty. at noip)  
L= 1 1(2)2 0  
Equal NATs  
L= 2 1(2)2  
1-01 1(2)2  
Equal NATs  
L= 3 1(2)2  
1-001 1(2)2  
1-011 1(2)2  
Total NATs  
L= 4 1(2)2

Z = .5 1100111100001100-1100111100001100-rule  
=3-rule 178 -10110010



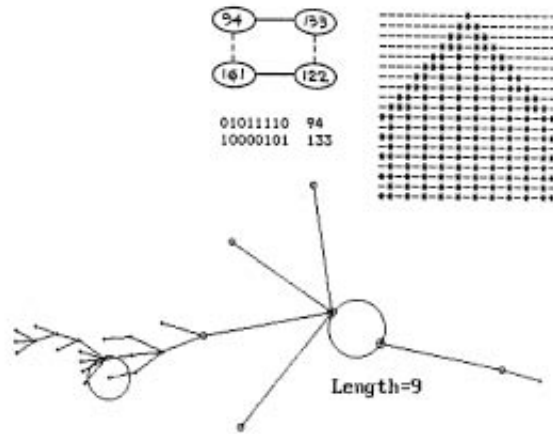
3-rule 178  
ty. at noip)  
L= 1 1(2)2 0  
Equal NATs  
L= 2 1(2)2  
1-01 1(2)2  
Equal NATs  
L= 3 1(2)2  
1-001 1(2)2  
1-011 1(2)2  
Total NATs  
L= 4 1(2)2

0011001111001100-0011001111001100-rule  
=3-rule 90 -01011010



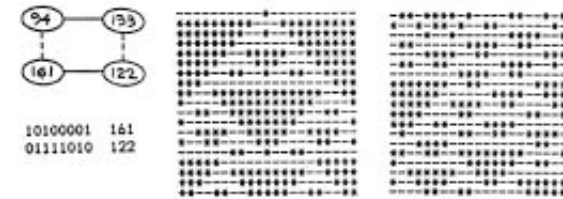
3-rule 90  
ty. at no  
L= 1 1(2)2 0  
Equal NATs  
L= 2 1(2)2  
1-00 1(2)2  
Equal NATs  
L= 3 1(2)2  
1-000 1(2)2  
1-001 1(2)2  
1-010 1(2)2  
1-011 1(2)2  
Total NATs  
L= 4 1(2)2  
1-0000 1(2)2  
1-0001 1(2)2  
1-0010 1(2)2  
1-0011 1(2)2  
1-0100 1(2)2  
1-0101 1(2)2  
1-0110 1(2)2  
1-0111 1(2)2  
Total NATs  
L= 5 1(2)2  
1-00000 1(2)2  
1-00001 1(2)2  
1-00010 1(2)2  
1-00011 1(2)2  
1-00100 1(2)2  
1-00101 1(2)2  
1-00110 1(2)2  
1-00111 1(2)2  
1-01000 1(2)2  
1-01001 1(2)2  
1-01010 1(2)2  
1-01011 1(2)2  
1-01100 1(2)2  
1-01101 1(2)2  
1-01110 1(2)2  
1-01111 1(2)2  
Total NATs  
L= 6 1(2)2  
1-000000 1(2)2  
1-000001 1(2)2  
1-000010 1(2)2  
1-000011 1(2)2  
1-000100 1(2)2  
1-000101 1(2)2  
1-000110 1(2)2  
1-000111 1(2)2  
1-001000 1(2)2  
1-001001 1(2)2  
1-001010 1(2)2  
1-001011 1(2)2  
1-001100 1(2)2  
1-001101 1(2)2  
1-001110 1(2)2  
1-001111 1(2)2  
1-010000 1(2)2  
1-010001 1(2)2  
1-010010 1(2)2  
1-010011 1(2)2  
1-010100 1(2)2  
1-010101 1(2)2  
1-010110 1(2)2  
1-010111 1(2)2  
1-011000 1(2)2  
1-011001 1(2)2  
1-011010 1(2)2  
1-011011 1(2)2  
1-011100 1(2)2  
1-011101 1(2)2  
1-011110 1(2)2  
1-011111 1(2)2  
Total NATs  
L= 7 1(2)2  
1-0000000 1(2)2  
1-0000001 1(2)2  
1-0000010 1(2)2  
1-0000011 1(2)2  
1-0000100 1(2)2  
1-0000101 1(2)2  
1-0000110 1(2)2  
1-0000111 1(2)2  
1-0001000 1(2)2  
1-0001001 1(2)2  
1-0001010 1(2)2  
1-0001011 1(2)2  
1-0001100 1(2)2  
1-0001101 1(2)2  
1-0001110 1(2)2  
1-0001111 1(2)2  
1-0010000 1(2)2  
1-0010001 1(2)2  
1-0010010 1(2)2  
1-0010011 1(2)2  
1-0010100 1(2)2  
1-0010101 1(2)2  
1-0010110 1(2)2  
1-0010111 1(2)2  
1-0011000 1(2)2  
1-0011001 1(2)2  
1-0011010 1(2)2  
1-0011011 1(2)2  
1-0011100 1(2)2  
1-0011101 1(2)2  
1-0011110 1(2)2  
1-0011111 1(2)2  
1-0100000 1(2)2  
1-0100001 1(2)2  
1-0100010 1(2)2  
1-0100011 1(2)2  
1-0100100 1(2)2  
1-0100101 1(2)2  
1-0100110 1(2)2  
1-0100111 1(2)2  
1-0101000 1(2)2  
1-0101001 1(2)2  
1-0101010 1(2)2  
1-0101011 1(2)2  
1-0101100 1(2)2  
1-0101101 1(2)2  
1-0101110 1(2)2  
1-0101111 1(2)2  
1-0110000 1(2)2  
1-0110001 1(2)2  
1-0110010 1(2)2  
1-0110011 1(2)2  
1-0110100 1(2)2  
1-0110101 1(2)2  
1-0110110 1(2)2  
1-0110111 1(2)2  
1-0111000 1(2)2  
1-0111001 1(2)2  
1-0111010 1(2)2  
1-0111011 1(2)2  
1-0111100 1(2)2  
1-0111101 1(2)2  
1-0111110 1(2)2  
1-0111111 1(2)2  
Total NATs  
L= 8 1(2)2  
1-00000000 1(2)2  
1-00000001 1(2)2  
1-00000010 1(2)2  
1-00000011 1(2)2  
1-00000100 1(2)2  
1-00000101 1(2)2  
1-00000110 1(2)2  
1-00000111 1(2)2  
1-00001000 1(2)2  
1-00001001 1(2)2  
1-00001010 1(2)2  
1-00001011 1(2)2  
1-00001100 1(2)2  
1-00001101 1(2)2  
1-00001110 1(2)2  
1-00001111 1(2)2  
1-00010000 1(2)2  
1-00010001 1(2)2  
1-00010010 1(2)2  
1-00010011 1(2)2  
1-00010100 1(2)2  
1-00010101 1(2)2  
1-00010110 1(2)2  
1-00010111 1(2)2  
1-00011000 1(2)2  
1-00011001 1(2)2  
1-00011010 1(2)2  
1-00011011 1(2)2  
1-00011100 1(2)2  
1-00011101 1(2)2  
1-00011110 1(2)2  
1-00011111 1(2)2  
1-00100000 1(2)2  
1-00100001 1(2)2  
1-00100010 1(2)2  
1-00100011 1(2)2  
1-00100100 1(2)2  
1-00100101 1(2)2  
1-00100110 1(2)2  
1-00100111 1(2)2  
1-00101000 1(2)2  
1-00101001 1(2)2  
1-00101010 1(2)2  
1-00101011 1(2)2  
1-00101100 1(2)2  
1-00101101 1(2)2  
1-00101110 1(2)2  
1-00101111 1(2)2  
1-00110000 1(2)2  
1-00110001 1(2)2  
1-00110010 1(2)2  
1-00110011 1(2)2  
1-00110100 1(2)2  
1-00110101 1(2)2  
1-00110110 1(2)2  
1-00110111 1(2)2  
1-00111000 1(2)2  
1-00111001 1(2)2  
1-00111010 1(2)2  
1-00111011 1(2)2  
1-00111100 1(2)2  
1-00111101 1(2)2  
1-00111110 1(2)2  
1-00111111 1(2)2  
1-01000000 1(2)2  
1-01000001 1(2)2  
1-01000010 1(2)2  
1-01000011 1(2)2  
1-01000100 1(2)2  
1-01000101 1(2)2  
1-01000110 1(2)2  
1-01000111 1(2)2  
1-01001000 1(2)2  
1-01001001 1(2)2  
1-01001010 1(2)2  
1-01001011 1(2)2  
1-01001100 1(2)2  
1-01001101 1(2)2  
1-01001110 1(2)2  
1-01001111 1(2)2  
1-01010000 1(2)2  
1-01010001 1(2)2  
1-01010010 1(2)2  
1-01010011 1(2)2  
1-01010100 1(2)2  
1-01010101 1(2)2  
1-01010110 1(2)2  
1-01010111 1(2)2  
1-01011000 1(2)2  
1-01011001 1(2)2  
1-01011010 1(2)2  
1-01011011 1(2)2  
1-01011100 1(2)2  
1-01011101 1(2)2  
1-01011110 1(2)2  
1-01011111 1(2)2  
1-01100000 1(2)2  
1-01100001 1(2)2  
1-01100010 1(2)2  
1-01100011 1(2)2  
1-01100100 1(2)2  
1-01100101 1(2)2  
1-01100110 1(2)2  
1-01100111 1(2)2  
1-01101000 1(2)2  
1-01101001 1(2)2  
1-01101010 1(2)2  
1-01101011 1(2)2  
1-01101100 1(2)2  
1-01101101 1(2)2  
1-01101110 1(2)2  
1-01101111 1(2)2  
1-01110000 1(2)2  
1-01110001 1(2)2  
1-01110010 1(2)2  
1-01110011 1(2)2  
1-01110100 1(2)2  
1-01110101 1(2)2  
1-01110110 1(2)2  
1-01110111 1(2)2  
1-01111000 1(2)2  
1-01111001 1(2)2  
1-01111010 1(2)2  
1-01111011 1(2)2  
1-01111100 1(2)2  
1-01111101 1(2)2  
1-01111110 1(2)2  
1-01111111 1(2)2  
Total NATs

.100111111100-001100111111100-rule  
=3-rule 94 -01011110



=3-rule 161 -10100001

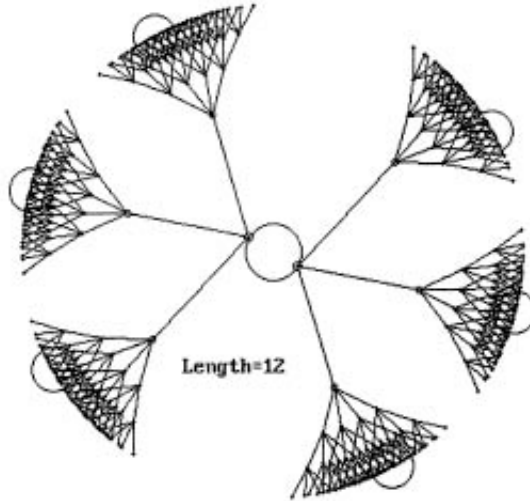
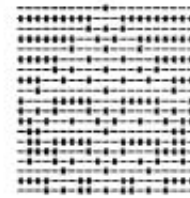
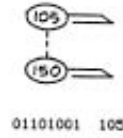
Length=1 -15





Z = 1

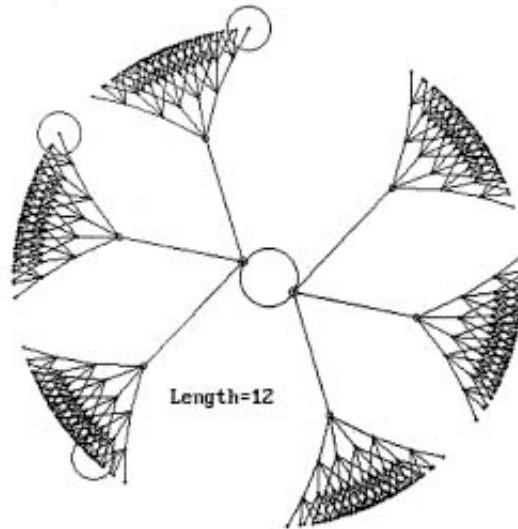
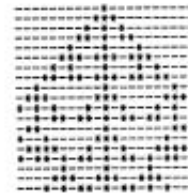
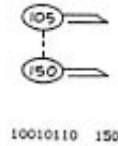
0011110011000011-0011110011000011-rule  
=3-rule 105 -01101001



3-rule 105  
ty. at no  
L= 1  
1.0 1(2)  
total NA  
L= 2  
1.00 1(2)  
2.01 1(2)  
total NA  
L= 3  
1.000 1(2)  
total NA  
L= 4  
1.0000 1(2)  
2.0001 2(1)  
3.0011 4(1)  
4.0111 1(1)  
5.0111 2(1)  
total NA  
L= 5  
1.00000 1(2)  
2.00001 2(1)  
total NA

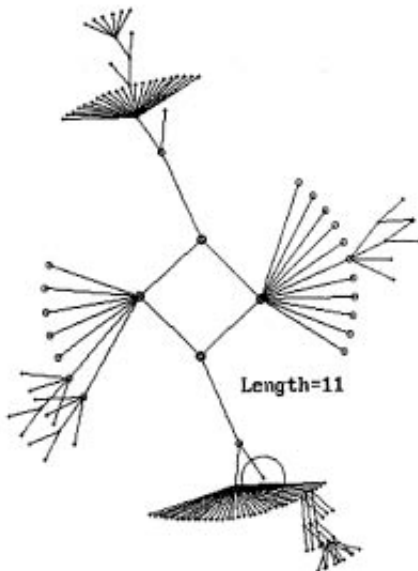
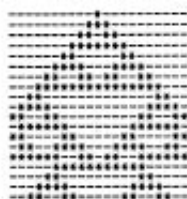
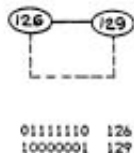
Z = 1

1100001100111100-1100001100111100-rule  
=3-rule 150 -10010110



3-rule 150  
ty. at no  
L= 1  
1.0 1(1)  
total NA  
L= 2  
1.00 1(1)  
2.01 1(1)  
total NA  
L= 3  
1.000 1(1)  
2.011 1(1)  
total NA  
L= 4  
1.0000 1(1)  
2.0001 2(1)  
3.0011 2(1)  
4.0111 1(1)  
total NA  
L= 5  
1.00000 1(1)  
2.00001 2(1)  
total NA

001111111111100-001111111111100-rule  
=3-rule 126 -01111110



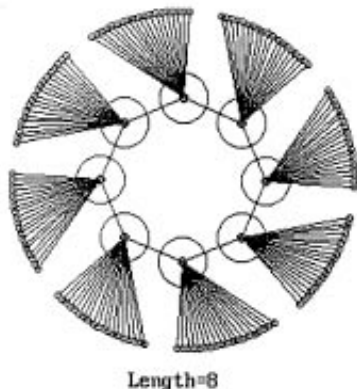
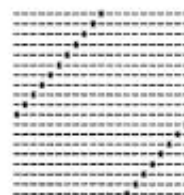
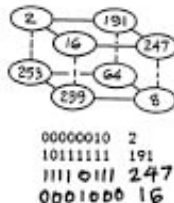
3-rule 126 =01111  
ty. at no(p)s q n

```

L= 1
1.0 11112 1 1 2
total NATs = 1
L= 2
1.00 11114 2 2 2
total NATs = 1
L= 3
1.000 11118 4 2
total NATs = 1
L= 4
1.0000 11116 4 2
total NATs = 1
L= 5
1.00000 11114 4 2
total NATs = 1
L= 6
1.000000 11112 4 2
total NATs = 1
L= 7
1.0000000 11110 4 2
total NATs = 1
L= 8
1.00000000 11108 4 2
total NATs = 1
L= 9
1.000000000 11106 4 2
total NATs = 1
L= 10
1.0000000000 11104 4 2
total NATs = 1
L= 11
1.00000000000 11102 4 2
total NATs = 1
L= 12
1.000000000000 11100 4 2
total NATs = 1

```

.25 0000000000001100-0000000000001100-rule  
=3-rule 2 -00000010



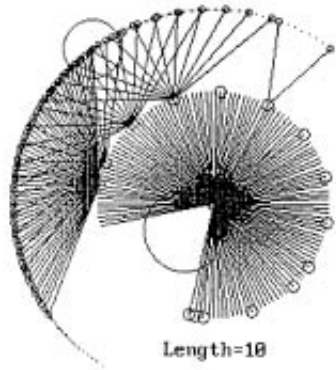
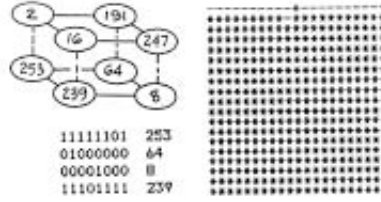
3-rule 2 =000000  
ty. at no(p)s q n

```

L= 1
1.0 11112 1 1 2
total NATs = 1
L= 2
1.00 11114 2 2 2
total NATs = 1
L= 3
1.000 11118 4 2
total NATs = 2
L= 4
1.0000 11116 4 2
total NATs = 2
L= 5
1.00000 11114 4 2
total NATs = 2
L= 6
1.000000 11112 4 2
total NATs = 2
L= 7
1.0000000 11110 4 2
total NATs = 2
L= 8
1.00000000 11108 4 2
total NATs = 2
L= 9
1.000000000 11106 4 2
total NATs = 2
L= 10
1.0000000000 11104 4 2
total NATs = 2
L= 11
1.00000000000 11102 4 2
total NATs = 2
L= 12
1.000000000000 11100 4 2
total NATs = 2

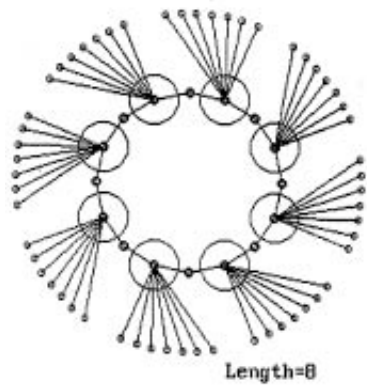
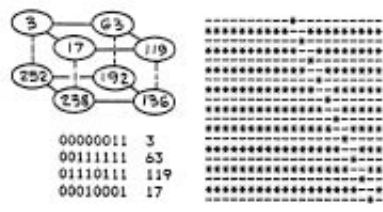
```

.25 1111111111110011-1111111111110011-rule  
 =3-rule 253 -1111101



3-rule 253 =1111101  
 ty. at no(p)ic q al sp  
 $L_n = 1 \quad 1(1)2 \quad 1 \quad 1 \quad 2$   
 total NATs = 1  
 $L_n = 1 \quad 1(1)4 \quad 3 \quad 1 \quad 4$   
 total NATs = 1

Z = .5 0000000000001111-0000000000001111-rule  
 =3-rule 3 -00000011



3-rule 3 =0  
 ty. at no(p)  
 $L_n = 1 \quad 1(2)2$   
 total NATs = 1  
 $L_n = 1 \quad 1(2)4$   
 total NATs = 1  
 $L_n = 1 \quad 1(2)8$   
 total NATs = 1  
 $L_n = 1 \quad 1(2)16$   
 total NATs = 1  
 $L_n = 1 \quad 1(2)32$   
 total NATs = 1

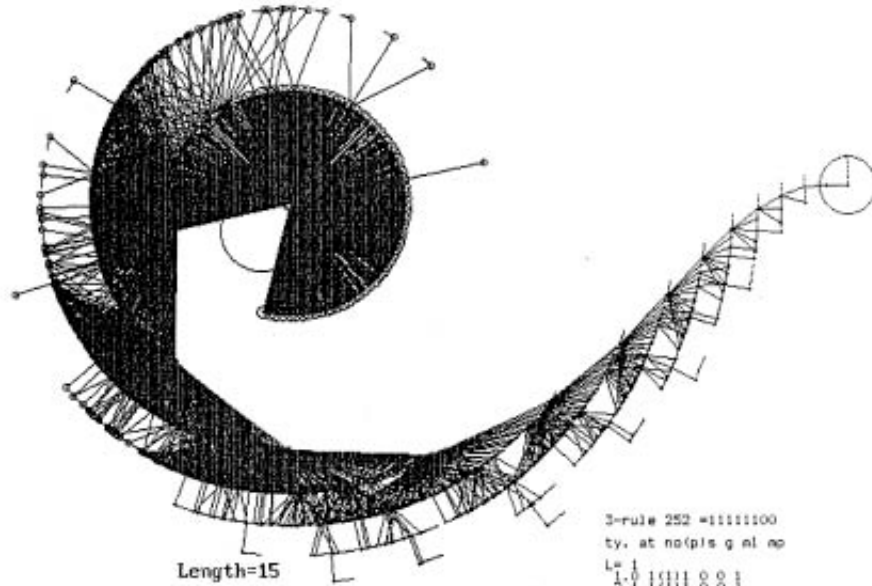
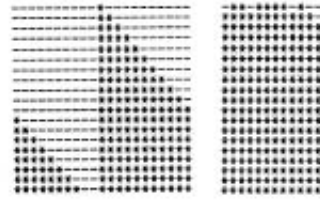
Z = .5

1111111111110000-1111111111110000-rule  
=3-rule 252 -11111100

4293984  
Length=

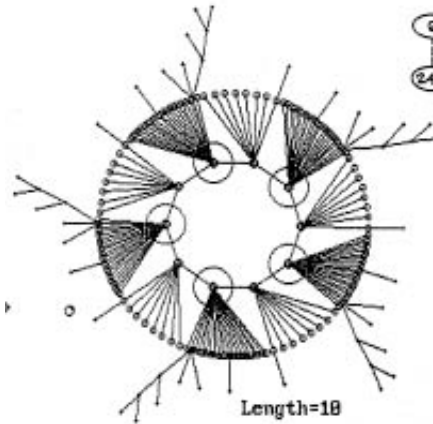


11111100	252
11000000	192
10001000	136
11101110	238

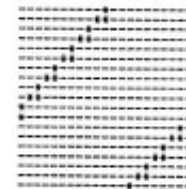


= .5 Z = .5

0000000000111100-0000000000111100-rule  
=3-rule 6 -00000110

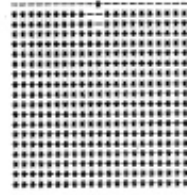


00000110	6
10011111	159
11010111	215
00010100	20

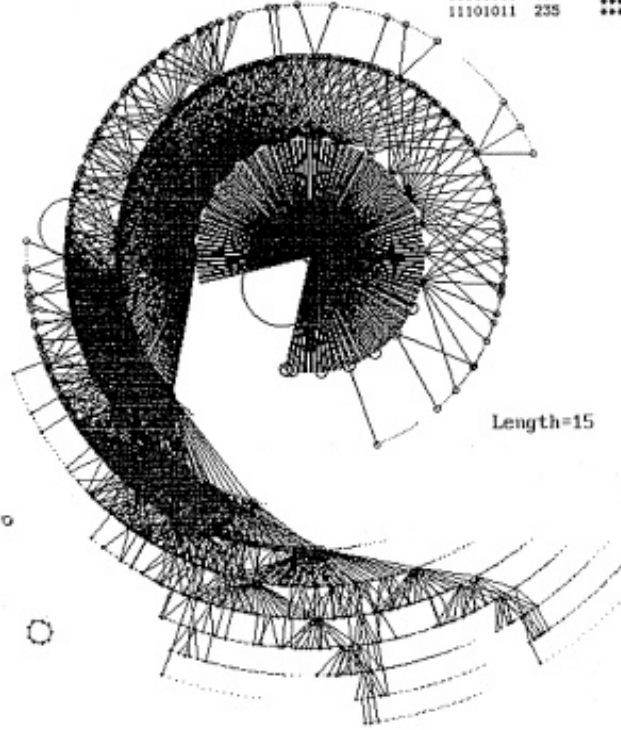


3-rule 6 =000  
ty. at no(p)is  
L= 1  
1:0 1(112 1  
total NATs =  
L= 2  
1:00 1(112 1  
2:01 1(111 0  
total NATs =  
L= 3

.5 Z = .5 1111111111000011-1111111111000011-rule  
=3-rule 249 -11111001



11111001 249  
01100000 96  
00101000 40  
11101011 235



Length=15

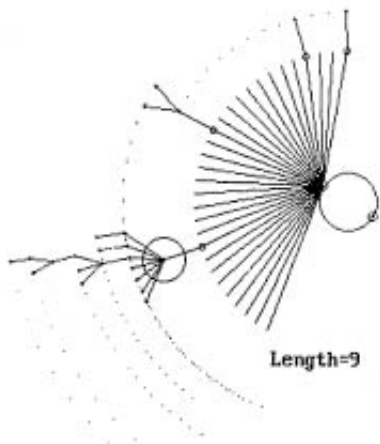
3-rule 249 =11111  
ty. at noip's g e  
L=1 1 1 1 2 1 1 2  
total NATs = 1  
L=2 1 1 1 2 1 1 2  
total NATs = 2  
L=3 1 1 1 1 1 1 1 4 1  
total NATs = 2  
L=4 .....



00000000111111-00000000011111-rule  
=3-rule 7 -00000111



00000111 7  
 00011111 31  
 01010111 87  
 00010101 21

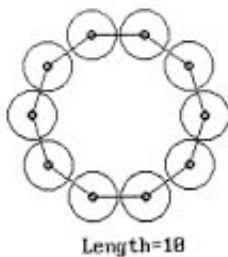
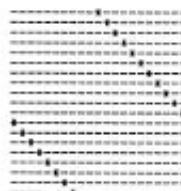


3-rule 7 =00000111  
 ty. at noip)s g al  
 L= 1  
 1.0 1(2)2 0 0 1  
 total NArts = 1  
 L= 2  
 1.00 1(2)7 0 0 1  
 1.01 2(1)7 0 0 1  
 total NArts = 3  
 L= 3  
 1.000 1(2)8 3 2 4  
 total NArts = 1  
 L= 4  
 1.0000 1(2)14 4 0 1  
 1.0001 1(1)14 0 0 1  
 total NArts = 3 0  
 L= 5  
 1.00000 1(2)27 1(1)15 0 1  
 1.00001 1(1)15 0 0 1  
 total NArts = 2  
 L= 6  
 1.000000 1(2)44 1  
 1.000001 1(1)44 1  
 1.010101 2(1)11 0  
 total NArts = 4  
 L= 7  
 1.0000000 1(2)79  
 1.0000001 1(1)79 21  
 1.0000111 1(1)28  
 total NArts = 3  
 L= 8  
 1.00000000 1(2)151  
 1.00000001 1(1)151  
 1.00000111 1(1)60  
 1.01010101 2(1)11  
 total NArts = 5  
 L= 9

111100000-111111111000000-rule  
=3-rule 248 -1111000



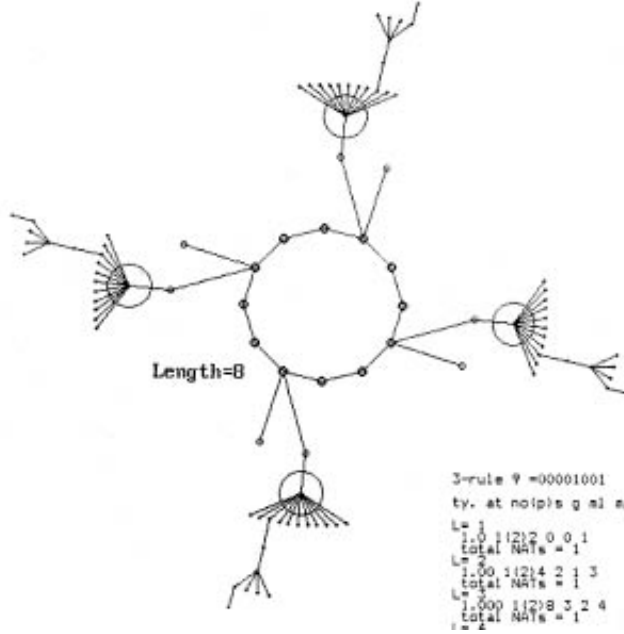
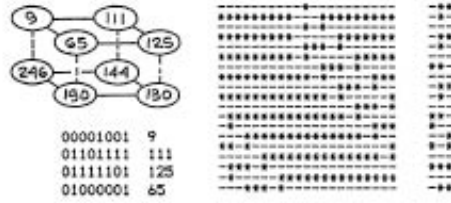
11111000 248  
 11100000 224  
 10101000 168  
 11101010 234



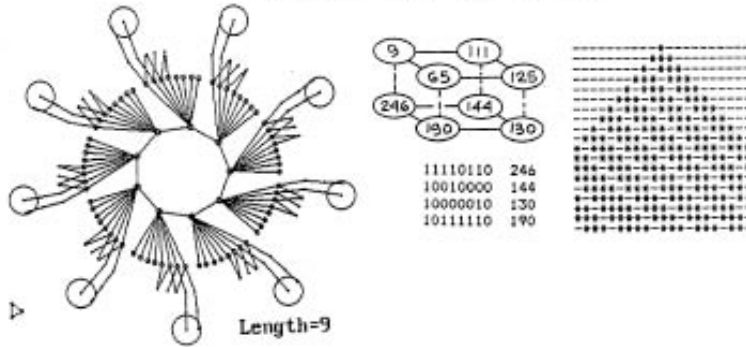
3-rule 248 :  
 ty. at noip:  
 L= 1  
 1.0 11111 1  
 total NArts  
 L= 2  
 1.00 11111  
 1.01 1(2)2  
 1.11 11111  
 total NArts  
 L= 3  
 1.000 1(1)1  
 1.001 1(1)1  
 1.011 1(1)1  
 total NArts  
 L= 4  
 1.0000 1(1)1  
 1.0001 1(1)4  
 1.0101 1(1)2  
 1.0101 1(1)2  
 total NArts  
 L= 5  
 1.00000 1(1)1  
 1.00001 1(1)1  
 1.00011 1(1)1  
 1.00011 1(1)1  
 1.010101 1(1)1  
 total NArts

=3-rule 9 -00001001

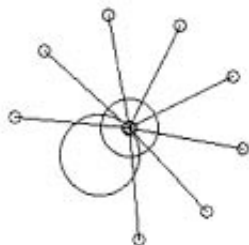
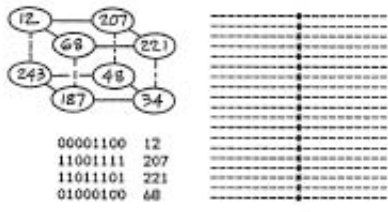
Le



.5 Z = .5 111111100111100-111111100111100-rule  
 =3-rule 246 -11110110



000000011110000-0000000011110000-rule  
 =3-rule 12 -00001100

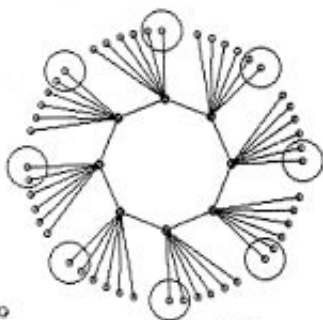


all lengths

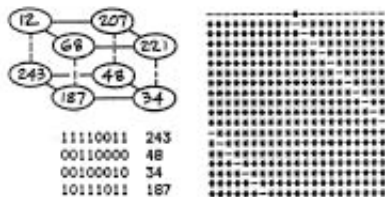
```

3-rule 1
ty. at r
L= 1
1.0 101
total 1
L= 2
1.01 11
1.01 11
total 2
L= 3
1.000 1
1.001 1
total 3
L= 4
1.0000
1.0001
1.0010
1.0011
total 4
L= 5
1.00000
1.00001
1.00010
1.00011
1.00100
1.00101
total 5
L= 6
1.000000
1.000001
1.000010
1.000011
1.000100
1.000101
total 6
L= 7
1.0000000
1.0000001
1.0000010
1.0000011
1.0000100
1.0000101
1.0000110
1.0000111
total 7
L= 8
1.00000000
1.00000001
1.00000010
1.00000011
1.00000100
1.00000101
1.00000110
1.00000111
total 8
    
```

Z = .5 1111111100001111-1111111100001111-rule  
 =3-rule 243 -11110011



Length=8

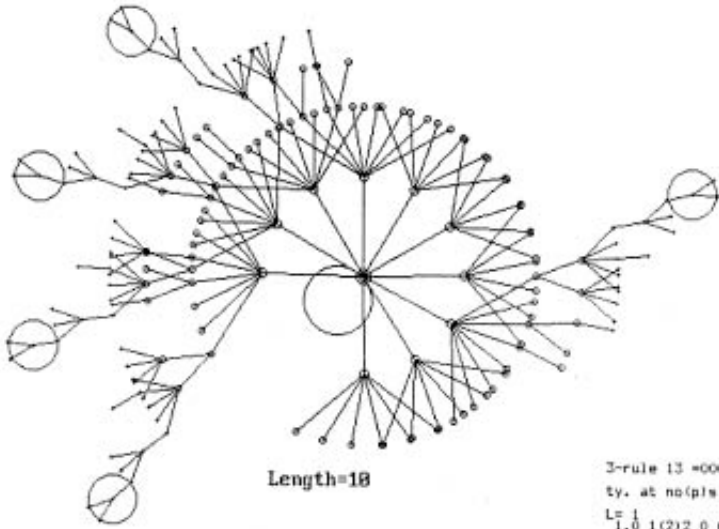
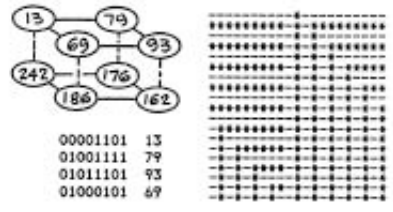


```

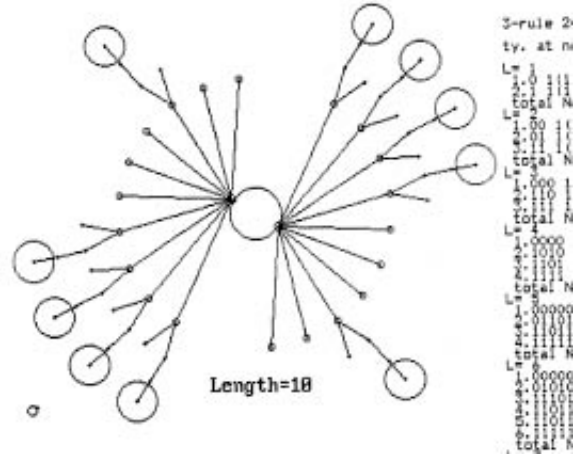
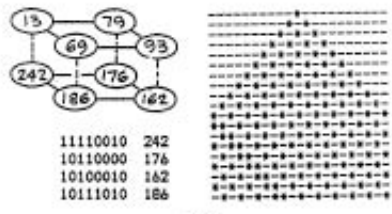
3-rule 243
ty. at n01
L= 1
1.1 11112
total NAT
L= 2
1.11 11111
    
```

15

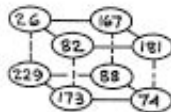
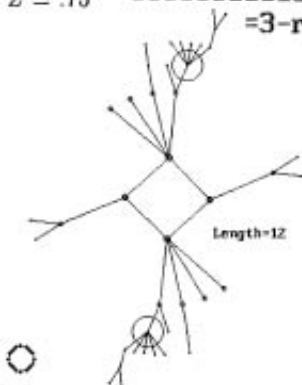
0000000011110011-0000000011110011-rule  
 =3-rule 13 -00001101



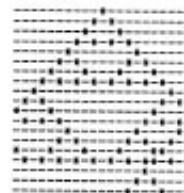
111100001100-111111100001100-rule  
 =3-rule 242 -11110010



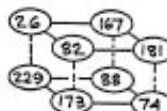
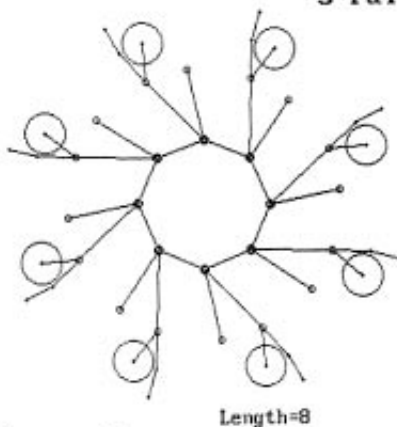
Z = .75 **0000001111001100-0000001111001100-rule**  
**=3-rule 26 -00011010**



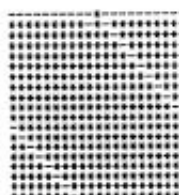
00011010 26  
 10100111 167  
 10110101 181  
 01010010 82



Z = .75 **111110000110011-111110000110011-rule**  
**=3-rule 229 -11100101**

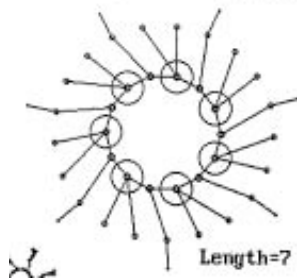


11100101 229  
 01011000 88  
 01001010 74  
 10101101 173

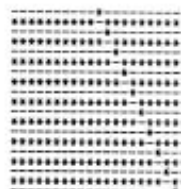


3-rule 229  
 ty. at no  
 L=1 1 1 1 1  
 total NA  
 L=2 1 1 1 1 1  
 total NA  
 L=3 1 1 1 1 1 1  
 total NA

5 **0000001111001111-0000001111001111-rule**  
**=3-rule 27 -00011011**



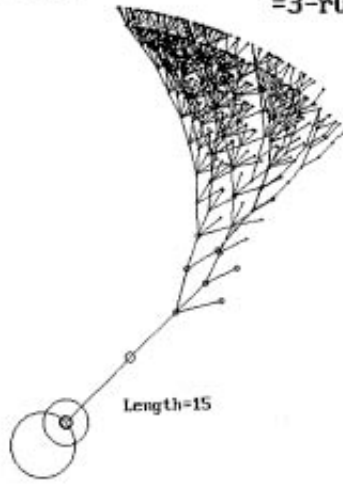
00011011 27  
 00100111 39  
 00110101 53  
 01010011 83



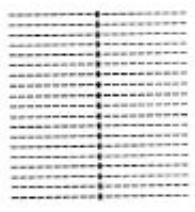
3-rule 27  
 ty. at no



$Z = .75$  111110000110000-111110000110000-rule  
 =3-rule 228 -11100100



11100100 228  
 11011000 216  
 11001010 202  
 10101100 172



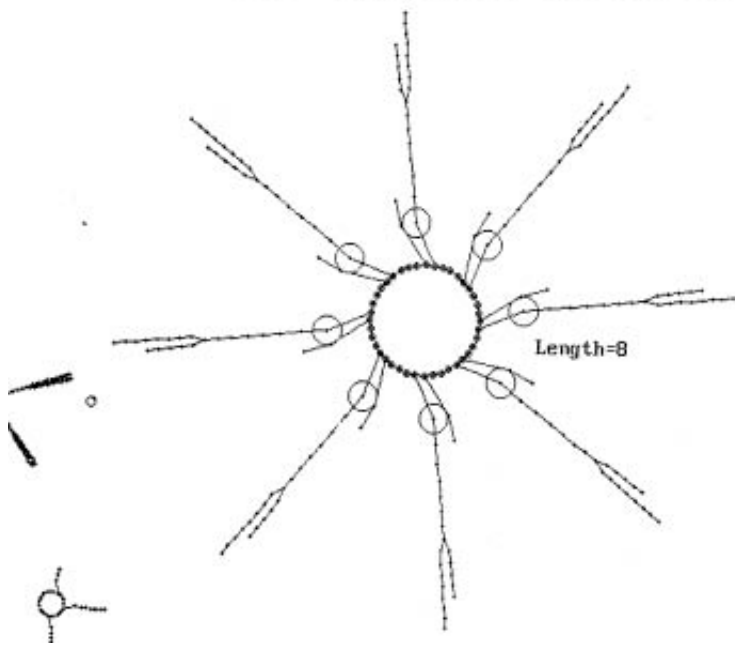
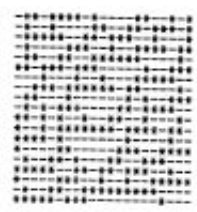
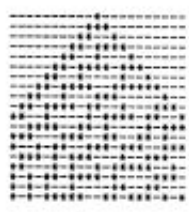
3-rule  
 tv. at  
 L= 1  
 1,0 |  
 2,1 |  
 total |  
 L= 2  
 1,00 |  
 2,11 |  
 total |

=3-rule 30 -00011110

Length=1 -14

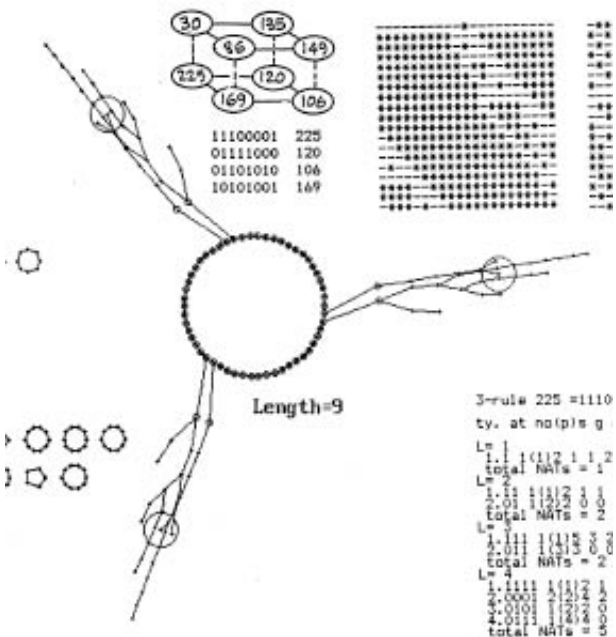


00011110 30  
 10000111 135  
 10010101 149  
 01010110 86

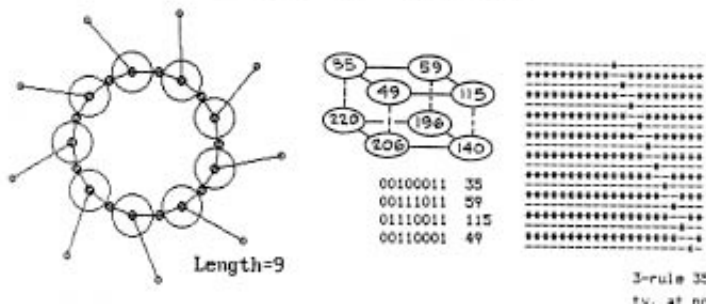


=3-rule 225 -11100001

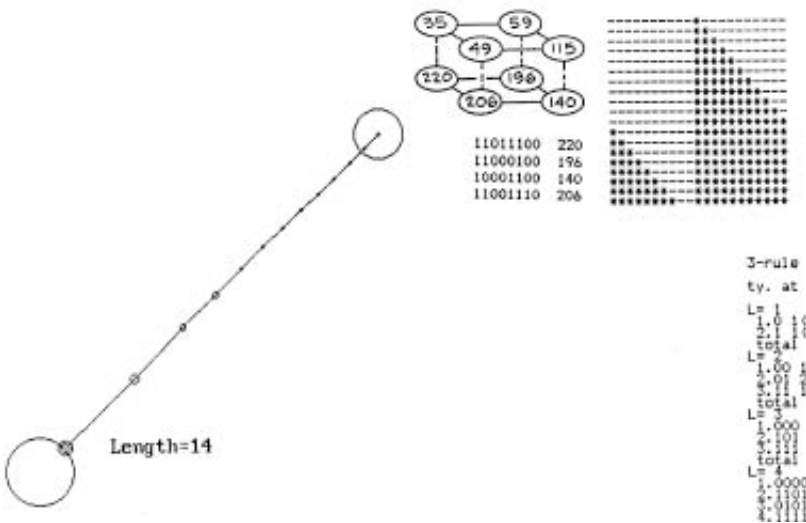
Len



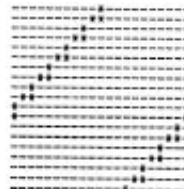
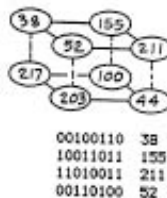
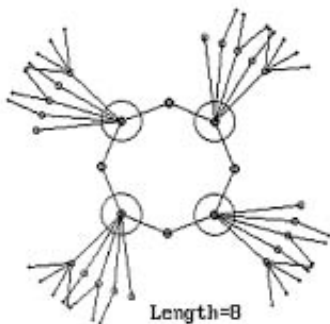
625 **0000110000001111-0000110000001111-rule**  
**=3-rule 35 -00100011**



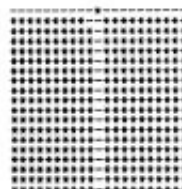
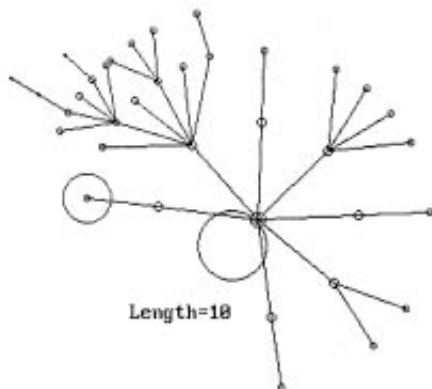
Z = .625 **1111001111110000-1111001111110000-rule**  
**=3-rule 220 -11011100**



Z = .75 0000110000111100-0000110000111100-rule  
=3-rule 38 -00100110

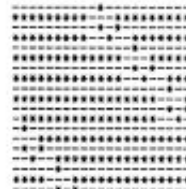
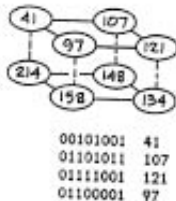
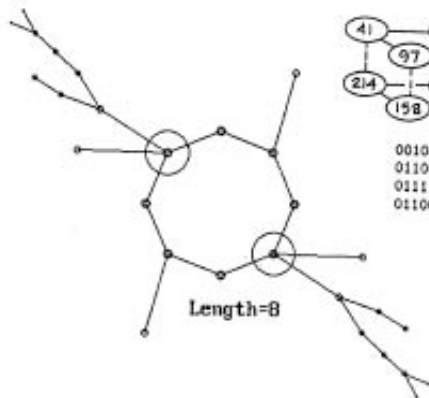


Z = .75 1111001111000011-1111001111000011-rule  
=3-rule 217 -11011001



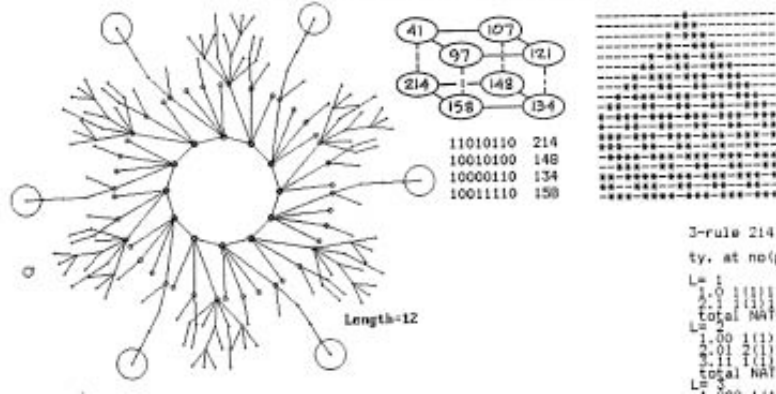
3-rule 217 :  
ty. at no(p  
L= 1  
1 1 1 (112  
Total NRTS  
L= 2  
1 1 1 1 (114  
Total NRTS  
L= 3  
1 1 1 1 1 (116  
Total NRTS  
L= 4  
1 1 1 1 1 1 (118  
Total NRTS  
L= 5  
1 1 1 1 1 1 1 (120  
Total NRTS  
L= 6  
1 1 1 1 1 1 1 1 (122  
Total NRTS  
L= 7  
1 1 1 1 1 1 1 1 1 (124  
Total NRTS  
L= 8  
1 1 1 1 1 1 1 1 1 1 (126  
Total NRTS

Z = .75 0000110011000011-0000110011000011-rule  
=3-rule 41 -00101001

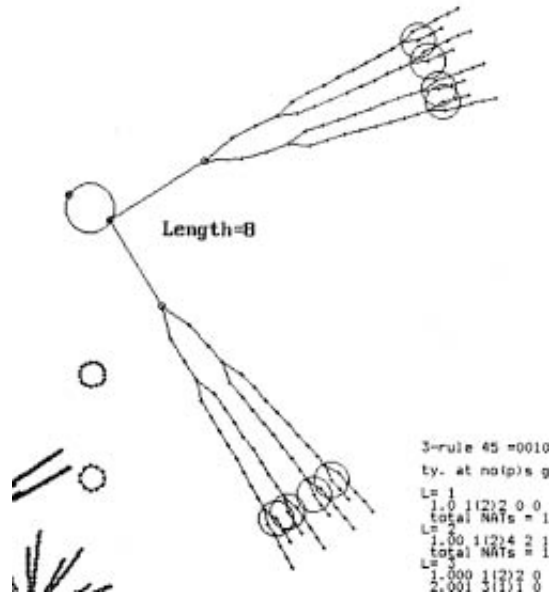
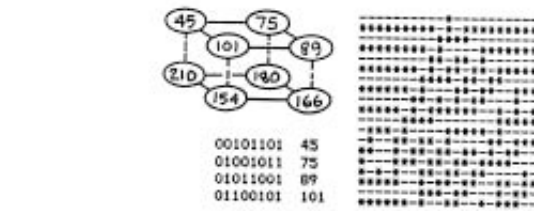


3-rule 41 =00101001  
ty. at no(p)s g ml  
= 1

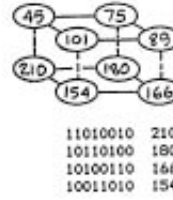
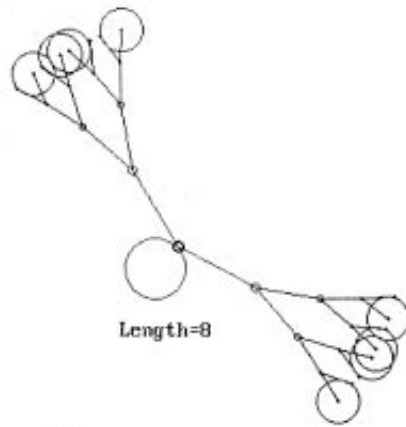
$Z = .75$  **1111001100111100-1111001100111100-rule**  
**=3-rule 214 -11010110**



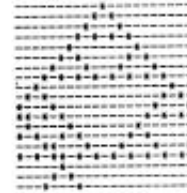
**111110011-0000110011110011-rule**  
**=3-rule 45 -00101101**



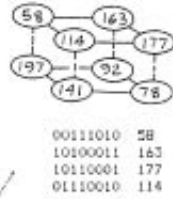
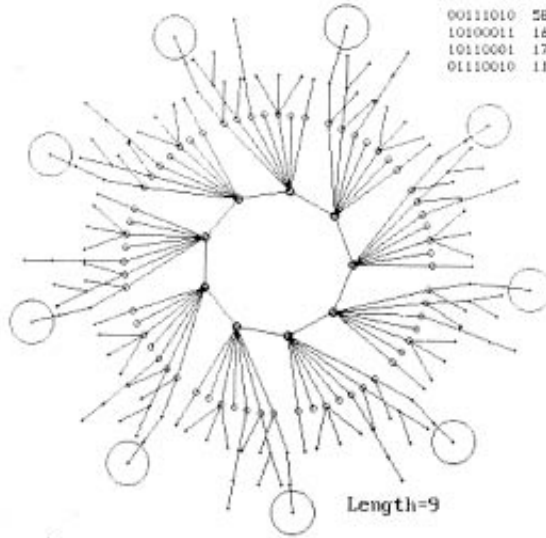
1  $Z = 1$       1111001100001100-1111001100001100-rule  
 =3-rule 210 -11010010



11010010 210  
 10110100 180  
 10100110 166  
 10011010 154



$Z = .75$       0000111111001100-0000111111001100-rule  
 =3-rule 58 -00111010



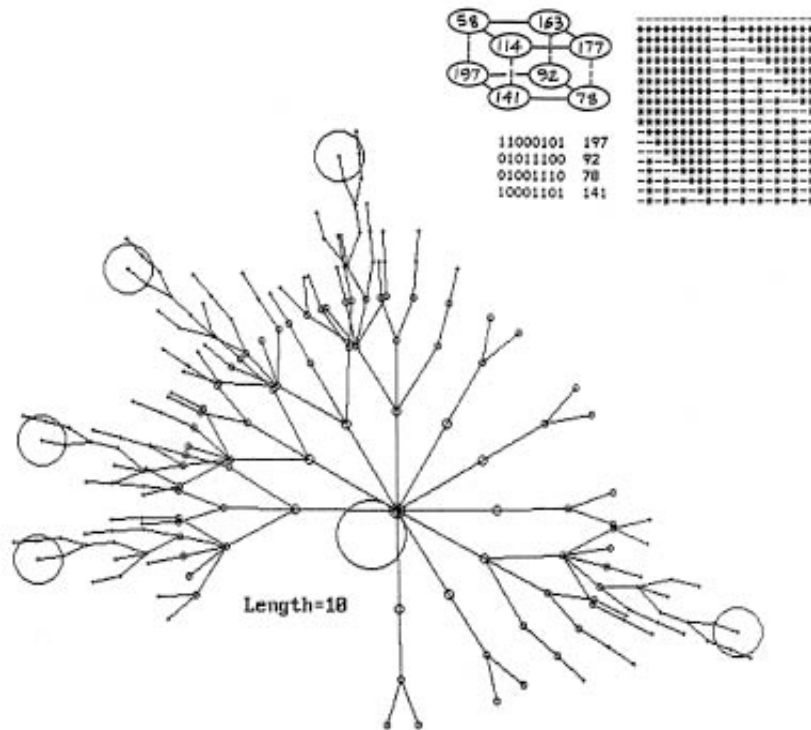
00111010 58  
 10100011 163  
 10110001 177  
 01110010 114



3-rule 58 =00  
 ty. at n0ip>e  
 L= 1  
 2.0 11112 1  
 Total NNTs =  
 L= 2  
 1.00 11112 1  
 2.01 11112 0  
 Total NNTs =  
 L= 3  
 1.000 11112 1  
 2.110 11112 0  
 Total NNTs =  
 L= 4  
 1.0000 11112  
 2.0010 11112  
 Total NNTs =  
 L= 5  
 1.00000 11111  
 2.01100 11111  
 3.00111 11111  
 Total NNTs =  
 L= 6  
 1.000000 11111  
 2.010101 11111  
 3.101101 11111

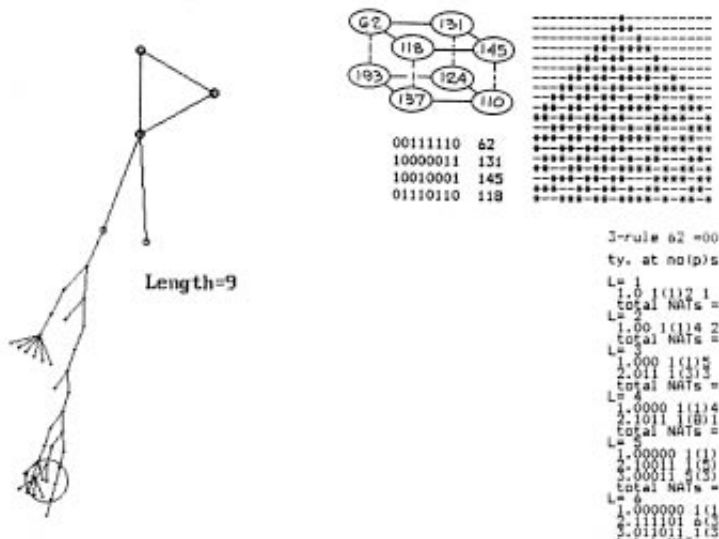


$Z = .75$  1111000000110011-1111000000110011-rule  
 =3-rule 197 -11000101



3-rule 197 =111

5 0000111111111100-0000111111111100-rule  
 =3-rule 62 -00111110

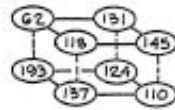


3-rule 62 =00  
ty. at noip/s

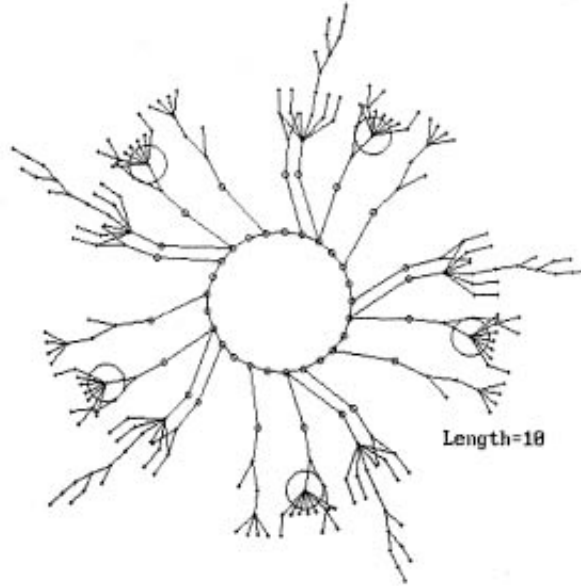
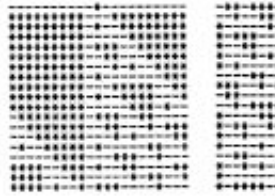
L=1  
 1.0 1(1)2 1  
 total NATs =  
 L=2  
 1.00 1(1)4 2  
 total NATs =  
 L=3  
 1.000 1(1)8  
 2.011 1(3)3  
 total NATs =  
 L=4  
 1.0000 1(1)4  
 2.1011 1(8)1  
 total NATs =  
 L=5  
 1.00000 1(1)1  
 2.10011 4(3)1  
 3.00011 4(3)1  
 total NATs =  
 L=6  
 1.000000 1(1)1  
 2.111101 8(3)1  
 3.011011 1(3)1

=3-rule 193 -11000001

Length

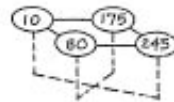


11000001 193  
 01111100 124  
 01101110 110  
 10001001 137

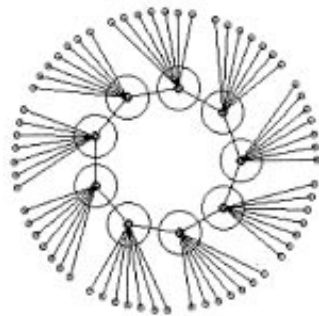
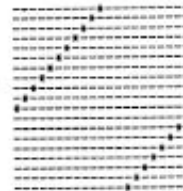


Length=18

.5 0000000011001100-0000000011001100-rule  
 =3-rule 10 -00001010



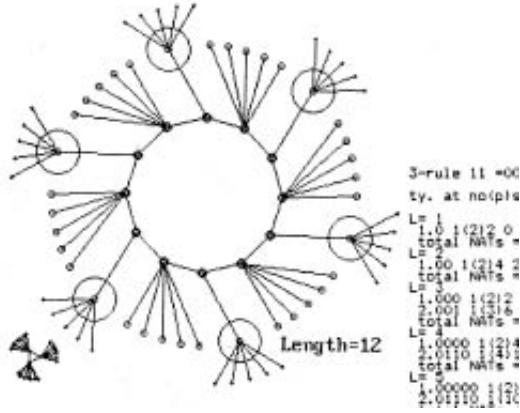
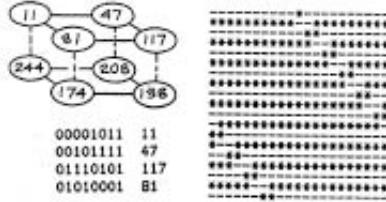
00001010 10  
 10101111 175  
 11110101 245  
 01010000 80



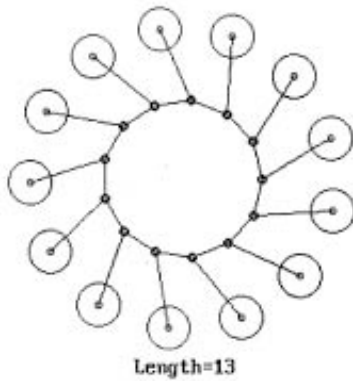
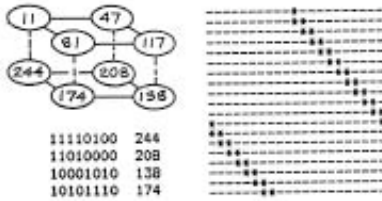
Length=9

3-rule 10 =  
 ty. at n0ip  
 L= 1  
 1.0 1(1)2  
 Total NATs  
 L= 2  
 1.00 1(1)4  
 Total NATs  
 L= 3  
 1.000 1(1)8  
 2.000 1(1)3  
 Total NATs  
 L= 4  
 1.0000 1(1)16  
 2.0001 1(1)2  
 3.0011 1(1)4  
 Total NATs  
 L= 5  
 1.00000 1(1)32

011001111-0000000011001111-rule  
 =3-rule 11 -00001011

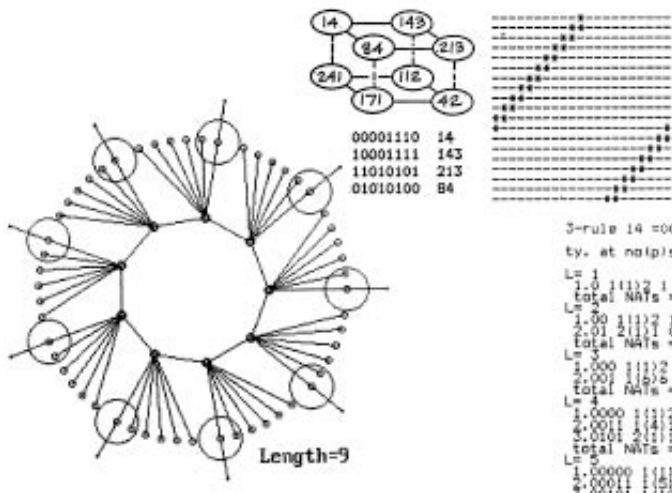


Z = .75 111111100110000-111111100110000-rule  
 =3-rule 244 -11110100

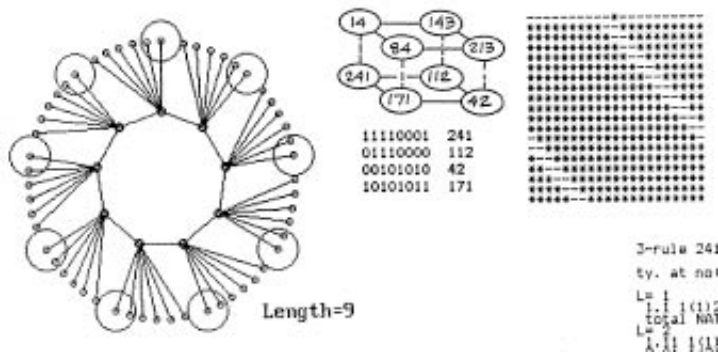


3-rule  
 ty. at  
 L=1  
 1.0  
 total  
 L=2  
 1.00  
 total  
 L=3  
 1.000  
 total  
 L=4  
 1.0000  
 total

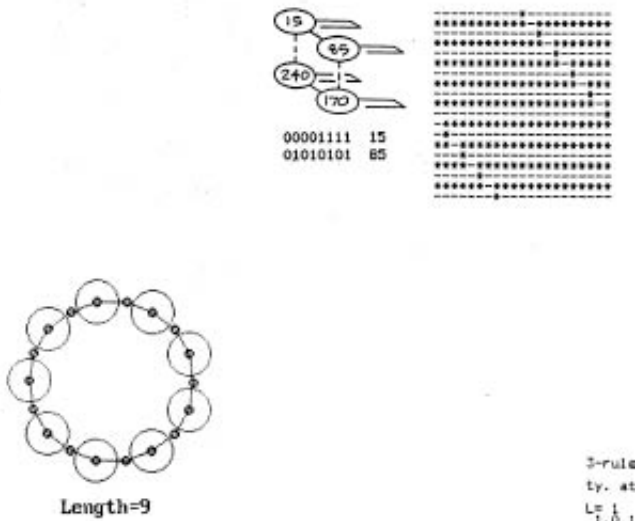
0000000011111100-0000000011111100-rule  
=3-rule 14 -00001110



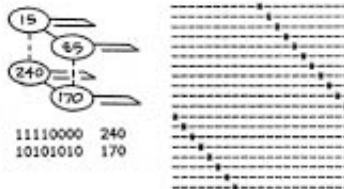
5 1111111100000011-1111111100000011-rule  
=3-rule 241 -11110001



0000000011111111-0000000011111111-rule  
=3-rule 15 -00001111

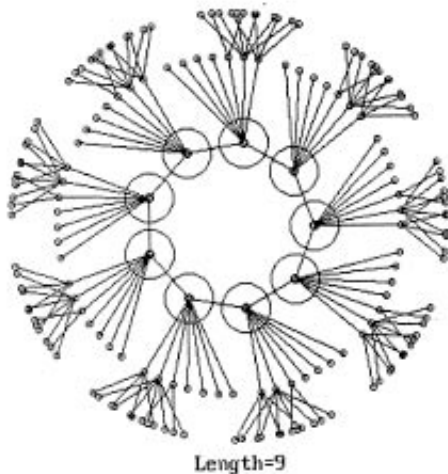
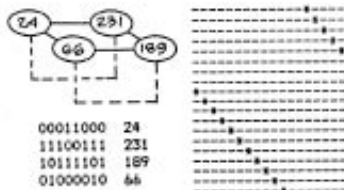


1111111100000000-1111111100000000-rule  
=3-rule 248 -11110000



3-ru  
ty.  
L=1  
L=2  
L=3  
L=4  
L=5  
L=6  
L=7  
L=8  
L=9

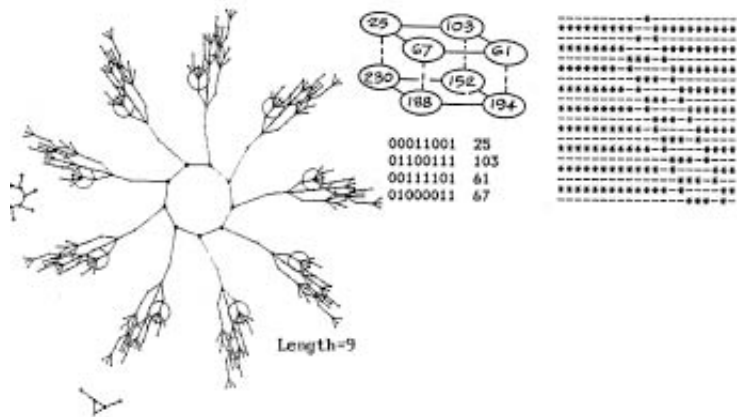
0000001111000000-0000001111000000-rule  
=3-rule 24 -00011000



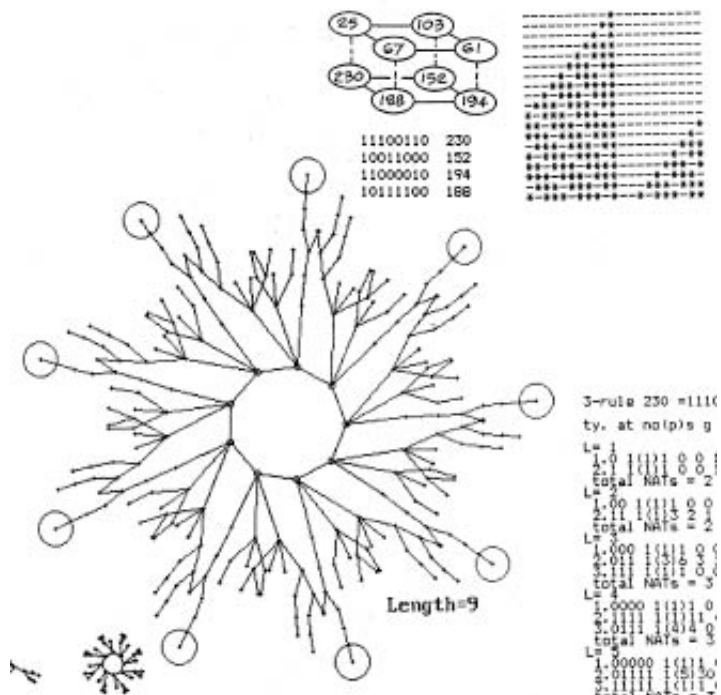
3-rule 24 =000  
ty. at no(p)s  
L=1  
1.0 1(1)2 1 1  
total NATs =  
L=2  
1.00 1(1)4 3  
total NATs =  
L=3  
1.000 1(1)8 3  
total NATs =  
L=4  
1.0000 1(1)16 3  
total NATs =  
L=5  
1.00000 1(1)32 3  
total NATs =  
L=6  
1.000000 1(1)64 3  
total NATs =  
L=7  
1.0000000 1(1)128 3  
total NATs =  
L=8  
1.00000000 1(1)256 3  
total NATs =  
L=9  
1.000000000 1(1)512 3  
total NATs =



= .75 0000001111000011-0000001111000011-rule  
=3-rule 25 -00011001

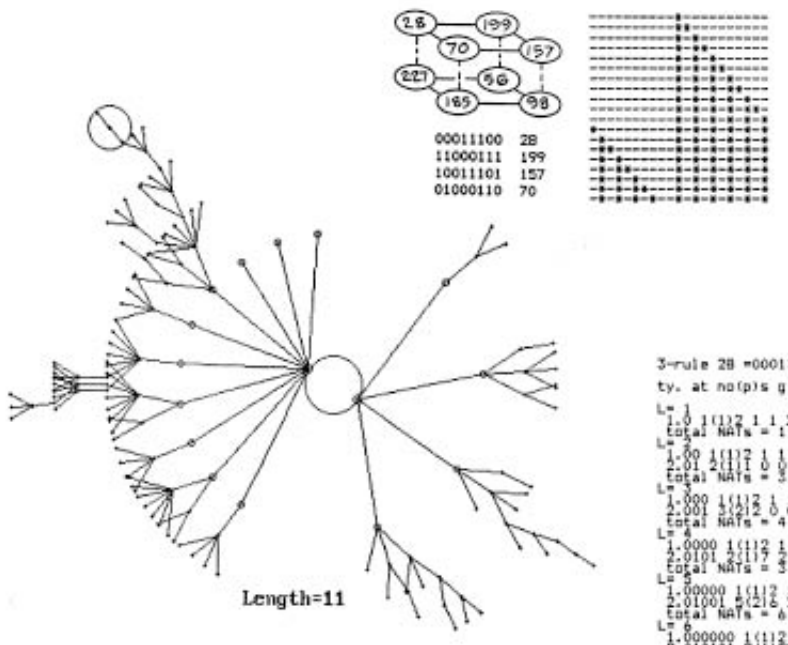


111110000111100-111110000111100-rule  
=3-rule 230 -11100110

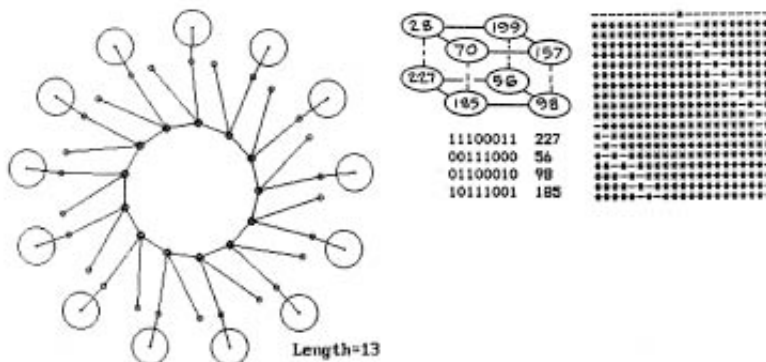


3-rule 230 =1110  
 ty. at noip's g  
 Lw 1  
 1-0 11111111 00 1  
 total NATs = 00 2  
 Lw 20  
 1-11 11111111 110 0  
 total NATs = 110 3  
 Lw 100  
 1-0111 11111111 010 0  
 total NATs = 010 3  
 Lw 1000  
 1-0000 11111111 1 0  
 total NATs = 1 3  
 Lw 8  
 1-00000 11111111  
 total NATs = 1 7

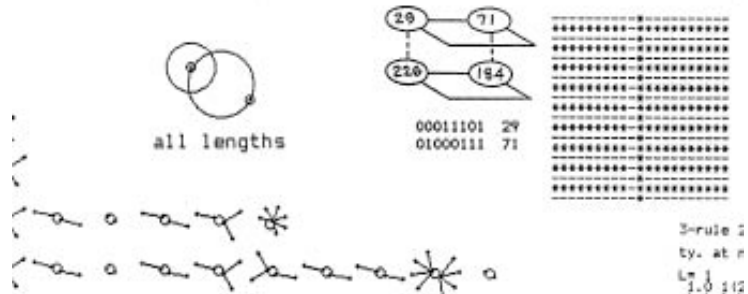
= .75 **000000111110000-000000111110000-rule**  
**=3-rule 28 -00011100**



Z = .75 **111110000001111-111110000001111-rule**  
**=3-rule 227 -11100011**



**000000111110011-000000111110011-rule** {  
**=3-rule 29 -00011101** I



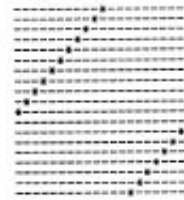
**111110000001100-111110000001100-rule**  
**=3-rule 226 -11100010**



Length=9

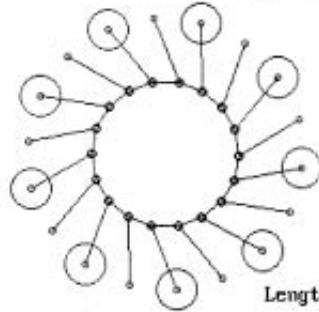


11100010 226  
 10111000 194



3-rule 2  
 ty. at n  
 $L_n = 1011$

**0000110011001111-0000110011001111-rule**  
**=3-rule 43 -00101011**



Length=9

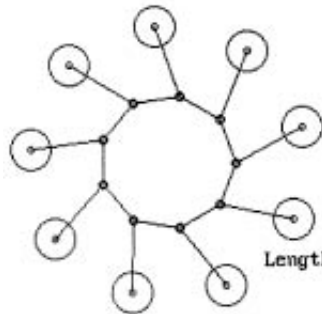


00101011 43  
 01110001 113



3-rule 43  
 ty. at n  
 $L_n = 1011$

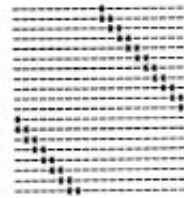
**1111001100110000-1111001100110000-rule**  
**=3-rule 212 -11010100**



Length=9

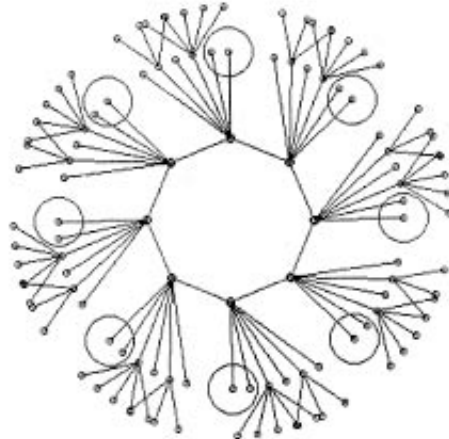
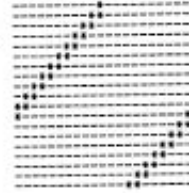
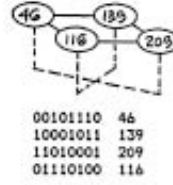


11010100 212  
 10001110 142



3-rule 212  
 ty. at n  
 $L_n = 1011$

= .5      0000110011111100-0000110011111100-rule  
 =3-rule 46 -00101110



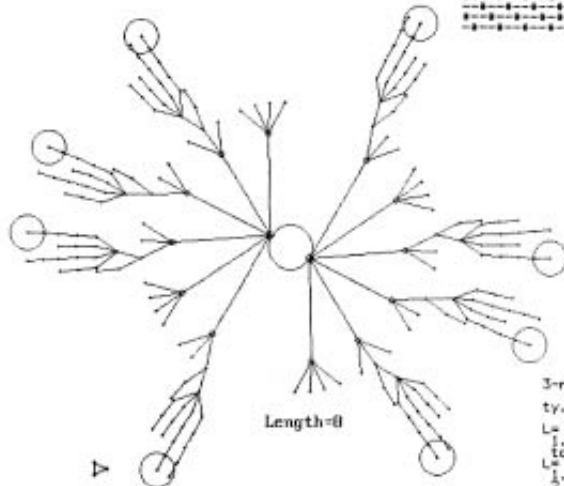
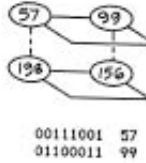
Length=8

```

3-rule 46 =001011
ty. at no(p)s g 4
L=1
1.0 1112 1 1 2
total NATs = 1
L=2
1.00 1114 2 2 2
total NATs = 2
L=3
1.000 1112 4 1
total NATs = 2
L=4
1.0000 1112 2 2
total NATs = 2
L=5
1.00000 1112 1
1.00001 1113 0
total NATs = 2
L=6
1.000000 1114 1
1.000001 1114 0
1.011011 1114 2
total NATs = 3
L=7
1.0000000 1114 2
1.0000001 1114 1
1.0000010 1114 1
1.0001001 1114 1
total NATs = 5
L=8
1.00000000 1111
1.00000001 1111
1.0000100 1111
1.0011001 1111
total NATs = 4
L=9
1.000000000 1111
1.000000001 1111
1.00001001 1111
1.00110011 1111
total NATs = 5
L=10
1.0000000000 1111
1.0000000001 1111
1.000010011 1111
1.001100111 1111
total NATs = 5
L=10
1.0000000000 1111
1.0000000001 1111
1.000010011 1111
1.001100111 1111
total NATs = 5

```

0000111111000011-0000111111000011-rule ;  
 =3-rule 57 -00111001 ]



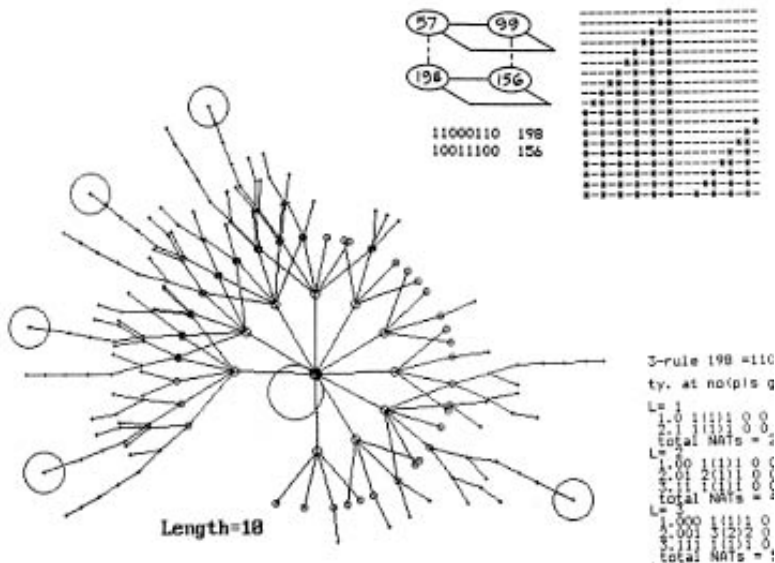
Length=8

```

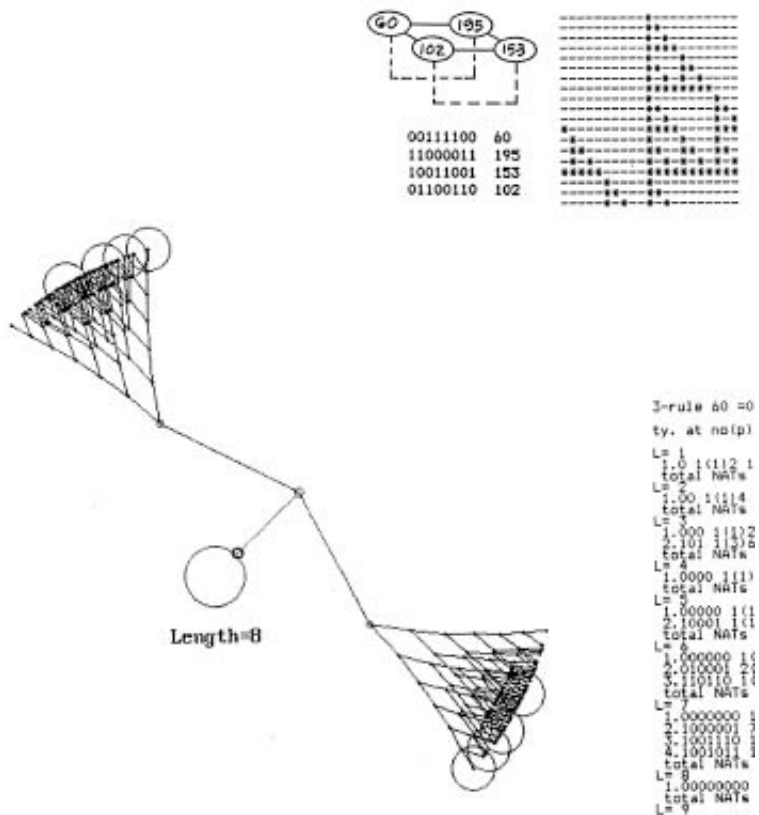
3-rule 57 =00:
ty. at no(p)s
L=1
1.0 112 0 1
total NATs = 1
L=2
1.00 112 0 0
total NATs = 1
L=3
1.000 112 0 0
total NATs = 1

```

.75 **1111000000111100-1111000000111100-rule**  
**=3-rule 198 -1100110**



= 1 **0000111111110000-0000111111110000-rule**  
**=3-rule 60 -00111100**



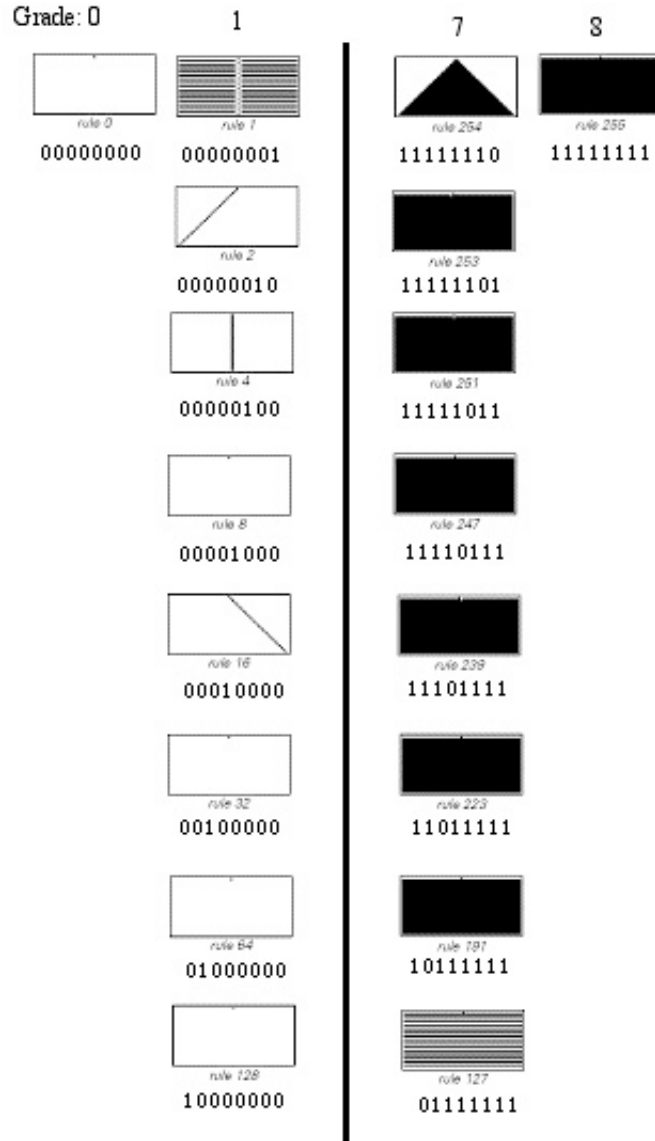
## 256 Elementary Rule Patterns



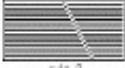



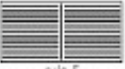

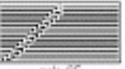
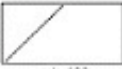

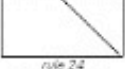
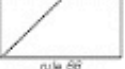
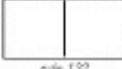
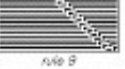
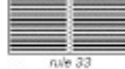








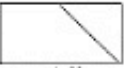

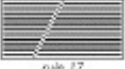



# with Graded Structure

## 1 8 28 56 70 56 28 8 1





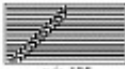



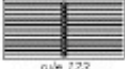



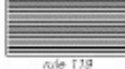


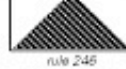
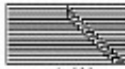



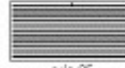



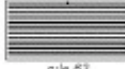



( images from "A New Kind of Science" by Stephen Wolfram (Wolfram 2002) )



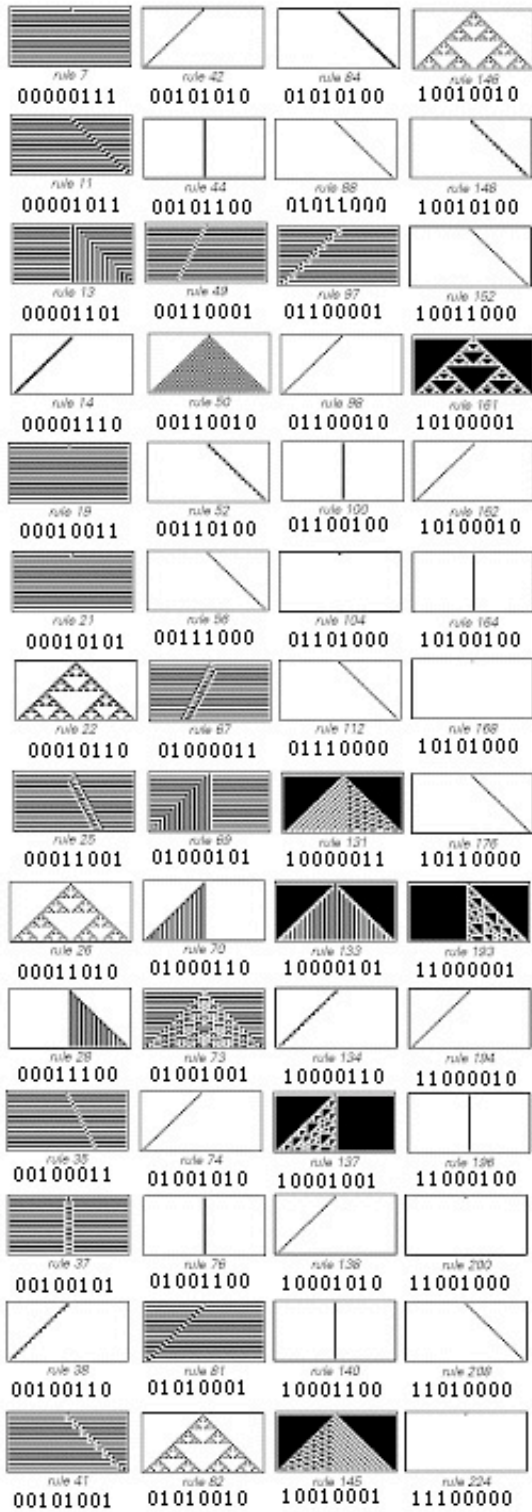
Grade: 2

 rule 3 00000011	 rule 18 00010010	 rule 48 00110000	 rule 129 10000001
 rule 5 00000101	 rule 20 00010100	 rule 65 01000001	 rule 130 10000010
 rule 6 00000110	 rule 24 00011000	 rule 66 01000010	 rule 132 10000100
 rule 8 00001001	 rule 33 00100001	 rule 68 01000100	 rule 136 10001000
 rule 10 00001010	 rule 34 00100010	 rule 72 01001000	 rule 144 10010000
 rule 12 00001100	 rule 36 00100100	 rule 60 01010000	 rule 160 10100000
 rule 17 00010001	 rule 40 00101000	 rule 96 01100000	 rule 192 11000000

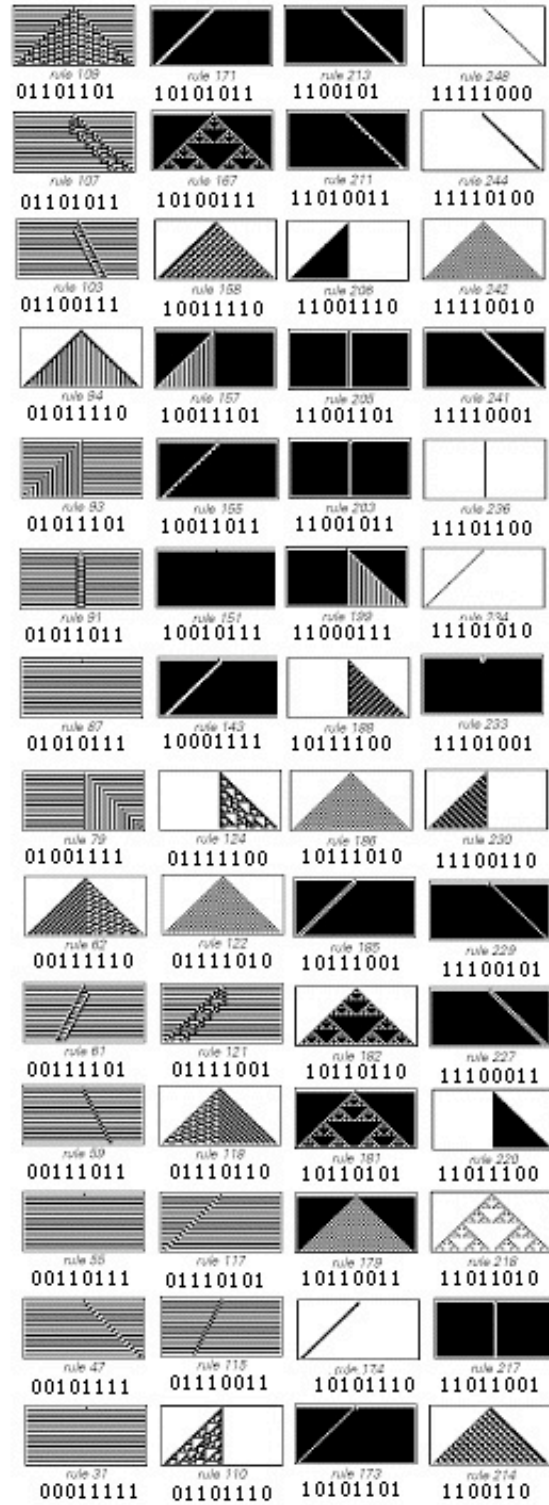
6

 rule 126 01111110	 rule 207 11001111	 rule 237 11101101	 rule 252 11111100
 rule 128 01111101	 rule 180 10111110	 rule 235 11101011	 rule 250 11111010
 rule 129 01111011	 rule 189 10111101	 rule 237 11100111	 rule 249 11111001
 rule 119 01110111	 rule 187 10111011	 rule 222 11011110	 rule 246 11110110
 rule 117 01101111	 rule 183 10110111	 rule 221 11011101	 rule 245 11110101
 rule 95 01011111	 rule 175 10101111	 rule 219 11011011	 rule 243 11110011
 rule 63 00111111	 rule 159 10011111	 rule 215 11010111	 rule 238 11101110

Grade: 3

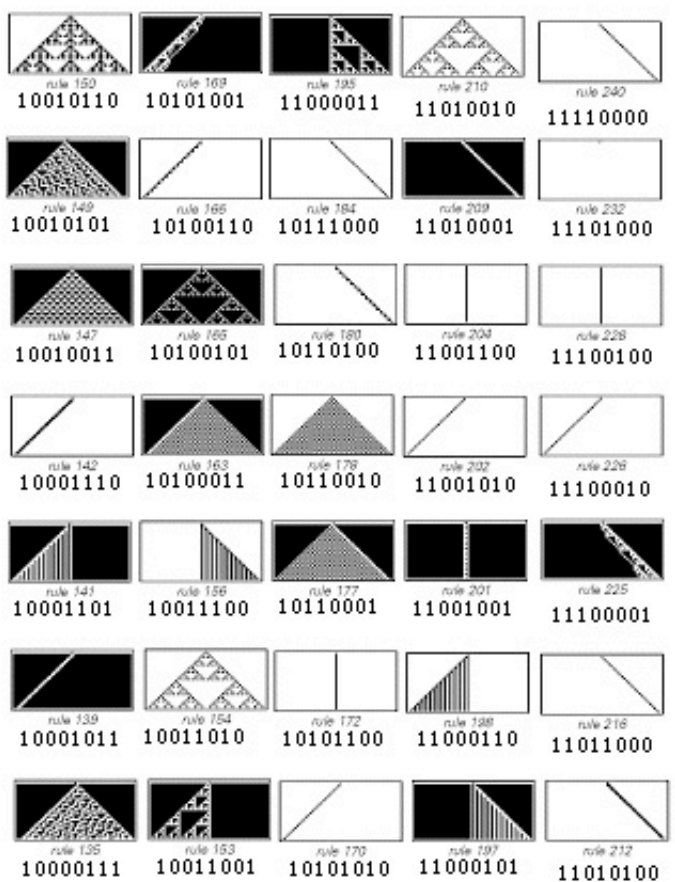
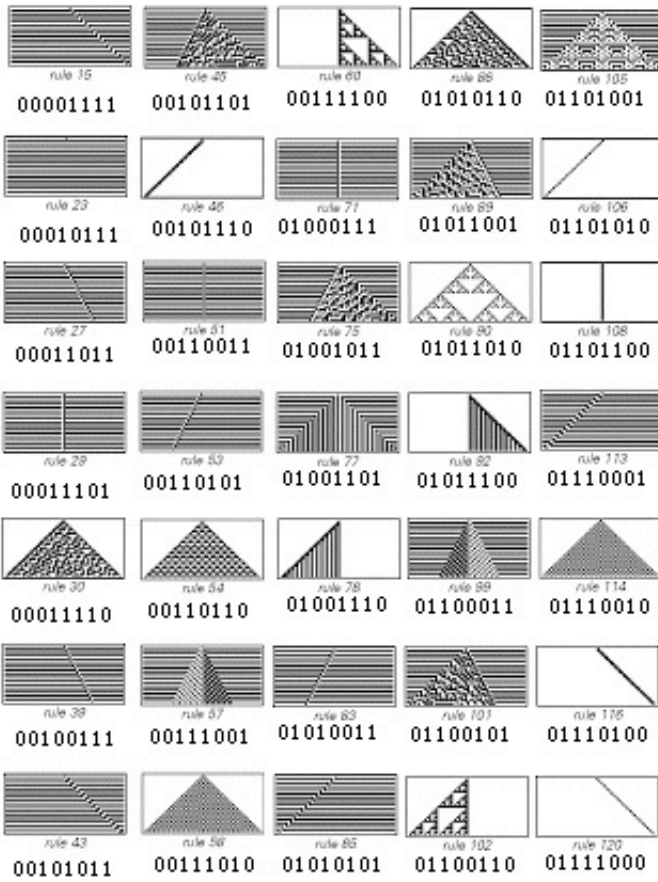


5



Grade:

4



the 16 terms in the Cl(8) primitive idempotent

$$f = (1/2)(1 + e_{1248}) (1/2)(1 + e_{2358}) (1/2)(1 + e_{3468}) (1/2)(1 + e_{4578}) =$$

$$= (1/16)(1 + e_{1248} + e_{2358} + e_{3468} + e_{4578} + e_{5618} + e_{6728} + e_{7138} - e_{3567} - e_{4671} - e_{5712} - e_{6123} - e_{7234} - e_{1345} - e_{2456} + e_J)$$

correspond to 16 of the 256 Cellular Automata



rule 255

- + e<sub>12345678</sub> 11111111



rule 226      rule 172      rule 216

- + e<sub>6728</sub> + e<sub>3468</sub> + e<sub>4578</sub> to 11100010    10101100    11011000



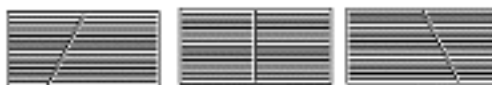
rule 150      rule 139      rule 177      rule 197

- + e<sub>2358</sub> + e<sub>1248</sub> + e<sub>5618</sub> + e<sub>7138</sub> 10010110    10001011    10110001    11000101



rule 0

- + 1 to 00000000



rule 83      rule 29      rule 38

- - e<sub>5712</sub> - e<sub>1345</sub> - e<sub>6123</sub> 01010011    00011101    00100111



rule 105      rule 78      rule 56      rule 176

- - e<sub>4671</sub> - e<sub>7234</sub> - e<sub>2456</sub> - e<sub>3567</sub> 01101001    01001110    00111010    01110100

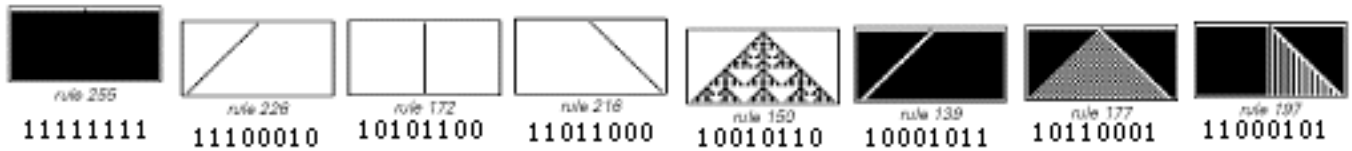


Note the  $Cl(0,8) = Cl(1,7)$  triality correspondences among:

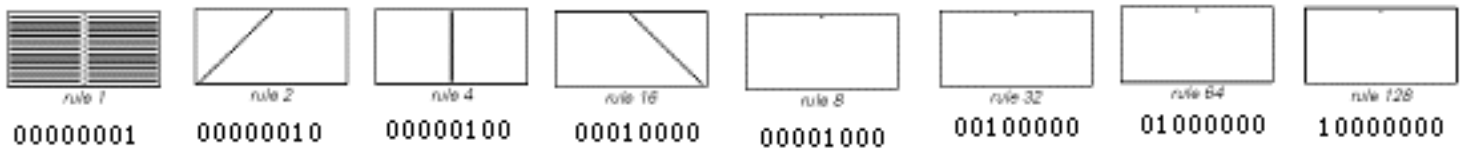
- the 8 [+half-spinors](#)



- the 8 [-half-spinors](#)



- the 8 [vectors](#)



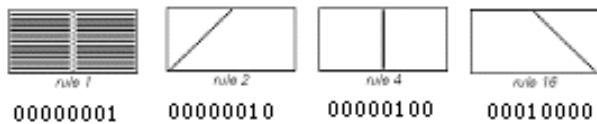
Note that:

the grade-0 scalars



[are related to the Spinors and Primitive Idempotents of Cl\(0,8\);](#)

[the grade-1 vectors 1, 2, 4, 16](#) (the subset sequence  $2^0 = 1, 2^1 = 2, 2^2 = 4, 2^4 = 16$  related to [Fermat primes](#))



correspond to [the 4 dimensions of physical spacetime;](#)

- 1 gives a succession of bands, the procession of time;
- 2 gives a slope to the left, one of three space dimensions;
- 4 gives a vertical slope, a second of three space dimensions;
- 16 gives a slope to the right, the third of three space dimensions;

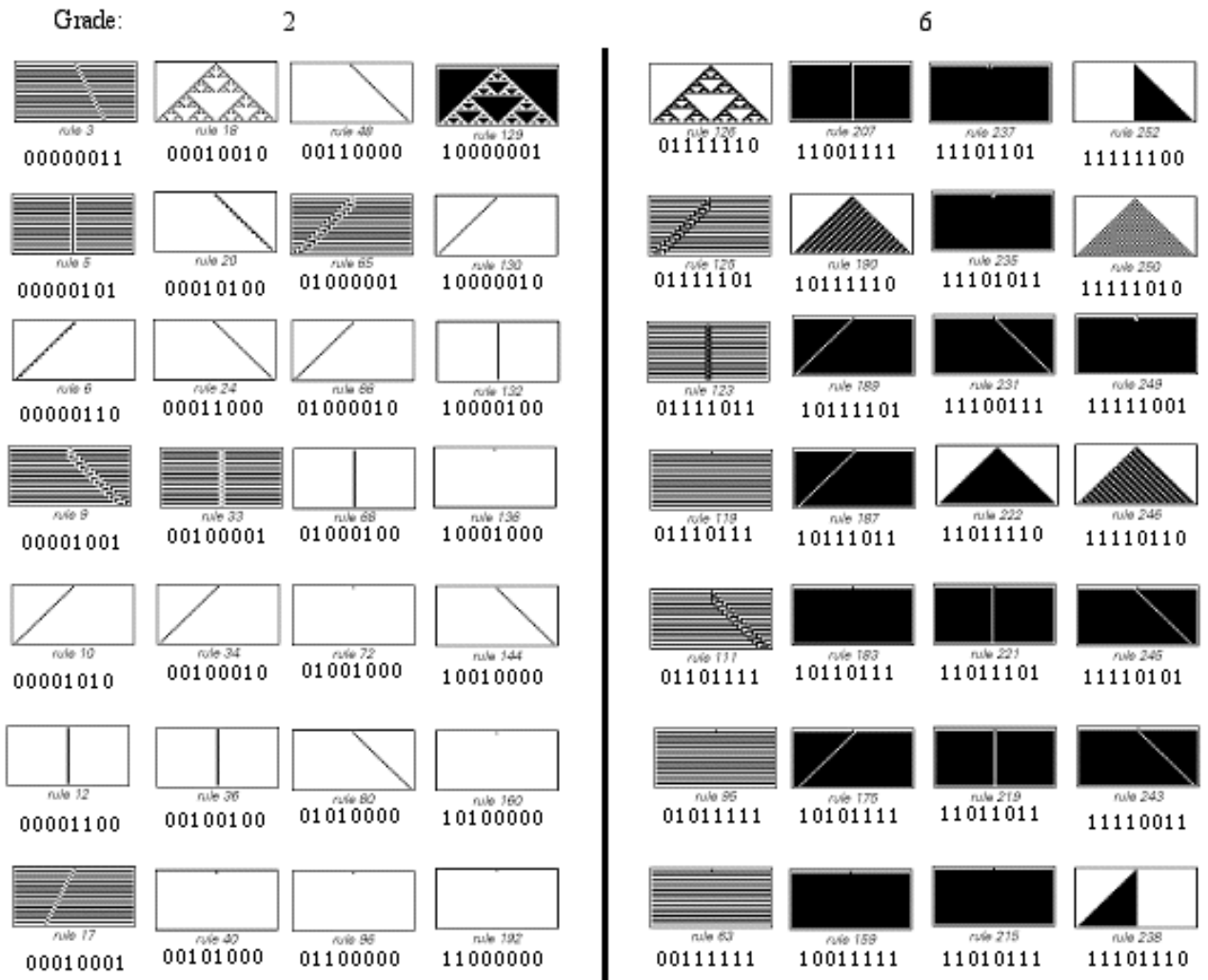
[the grade-1 vectors 8, 32, 64, 128](#) (all giving all white)



correspond to [the 4 dimensions of internal symmetry space;](#)

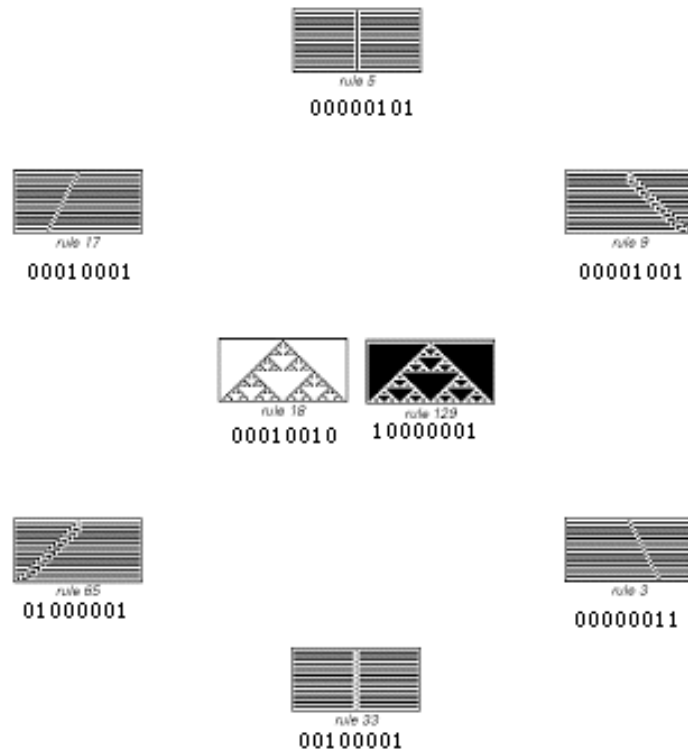
- rule 18 = 00010010 is the first rule to include both 16 = 00010000 with right slope and 2 = 00000010 with left slope and is the first rule with triangular self-similar fractal structure;
- rule 30 = 00011110 is the first rule to include 16, 8, 4, and 2 and is in the self-dual grade-4 and is the first rule with triangular chaotic behavior.

Here are all 28 rules for each of grades 2 and 6.



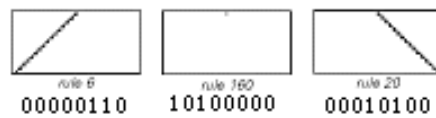
all 28 grade-2 bivectors correspond to the 28 generators of the Spin(8) [Lie algebra](#);

8 of the grade-2 bivectors,



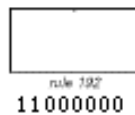
after [dimensional reduction to 4-dimensional physical spacetime](#), correspond to [the 8 generators of color force SU\(3\)](#), whose [root vector diagram](#) is illustrated above;

3 of the grade-2 bivectors,



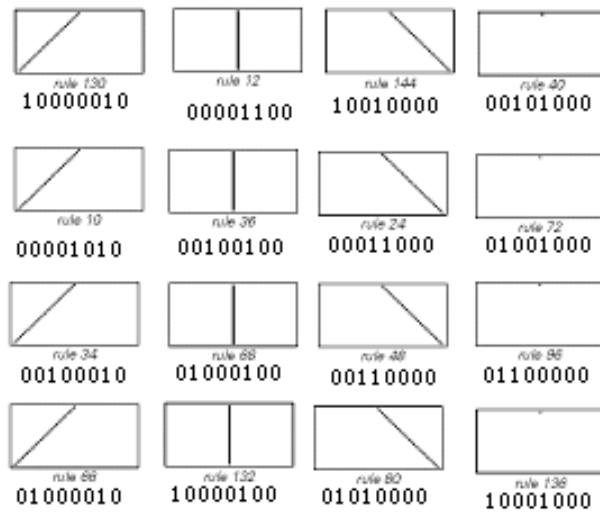
after [dimensional reduction to 4-dimensional physical spacetime](#), correspond to [the 3 generators of weak force SU\(2\)](#);

1 of the grade-2 bivectors,



after [dimensional reduction to 4-dimensional physical spacetime](#), correspond to [the 1 generator of electromagnetic U\(1\)](#);

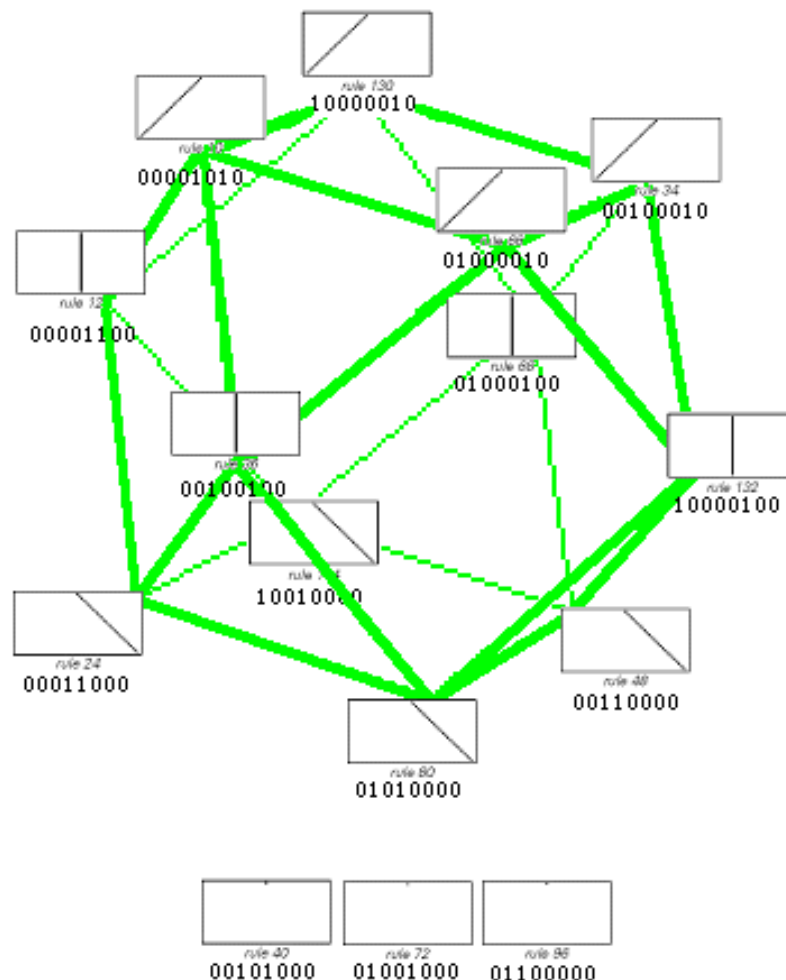
16 of the grade-2 bivectors,



after [dimensional reduction to 4-dimensional physical spacetime](#), correspond to [the 16 generators of Gravity/Higgs/phase U\(2,2\)](#). One of them



corresponds to the propagator phase U(1) while the other 15 correspond to the [Conformal](#) Group  $SU(2,2) = Spin(2,4)$  [whose root vector diagram](#)



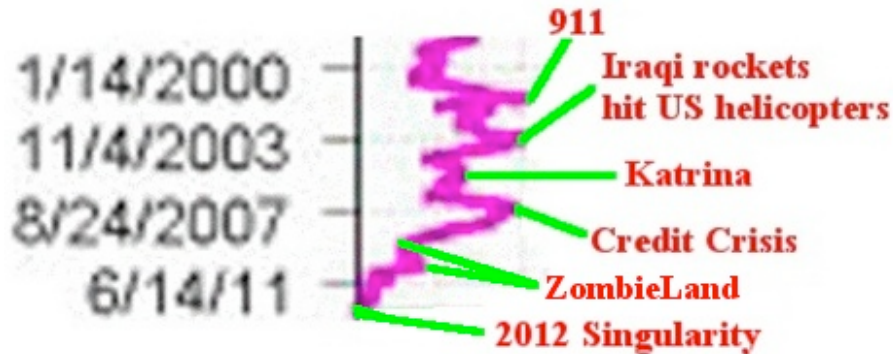
is a 12-vertex cuboctahedron (the other 3 bivectors corresponding to the 3 generators of the Cartan Subalgebra).



# Can the African IFA tell us about Past and Future History ?

A few thousand years ago in Shang China, King Wen took the 64-element I Ching subset of IFA and modified its binary number Earlier Heaven sequence to describe the History of that Time and Place. In the 20th century, Terence McKenna used a similar technique to describe History of the Earth through 2012. His Predictions for 2000 to 2012 seem to be quite relevant to the State of Our Earth as of now (2009):

## Terence McKenna died in April 2000, but the Predictions of his



## I Ching Resonance TimeWave of History live on to 2012.

Now ( April 2009 ) we know that his 2000 to 2009 TimeWave History peaks:

- [9/11](#)
- [Iraqi War escalation \(Rocket Attacks on USA helicopters\)](#)
- [Hurricane Katrina](#)
- [the Credit Crisis of the USA/UK Global Financial System](#)

all did happen coincident with TimeWave Peaks.

## **What does the TimeWave tell us about next 3 years, 2009 to 2012 ?**

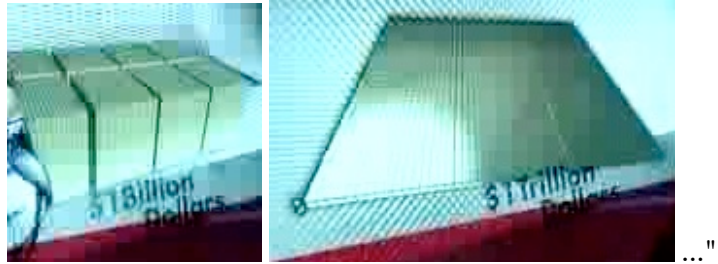
From now (April 2009) into 2010 is a nearly flat (or seemingly slightly rising) plateau that I call **ZombieLand**. In ZombieLand, Trillions of USA Dollars are fed to keep insolvent (but politically influential) financial institutions in an undead state - that is, not a living useful part of the Global Economy, but not officially recognized as being dead. To understand ZombieLand, you need to understand what is a **Trillion USA Dollars**. Glenn Beck (on his TV show) made a very good effort to show what is a Trillion USA Dollars. He said (I am paraphrasing): :

"... Here are \$100 USA Dollars and \$1 Million USA Dollars (It would fit in a suitcase):



Here are \$1 Billion USA Dollars (It is about the size of a car) and **\$1 Trillion USA Dollars (It is about the**

size of a Thousand Cars in a big parking lot.)



Another way to visualize the size of \$1 Trillion USA Dollars is to realize that the population of the USA (around 300 Million people) is about 100 Million Families.

**\$1 Billion is about \$10 for every Family in the USA.**

**\$1 Trillion is about \$10,000 for every Family in the USA.**

Another thing to keep in mind is that the amount of Bailouts so far is on the order of a few Trillion USA Dollars, but **the total amount of possibly-worthless derivatives is on the order of \$500 Trillion USA Dollars.** That would be  $500 \times 10,000 = \$5 \text{ Billion}$  for every Family in the USA.

**Since the population of the Earth is around 6 Billion, that would be almost \$100,000 for every Person on Earth.**

According to a 15 March 2009 huffingtonpost.com article by Arianna Huffington:

"... The battle lines over **how to deal with the banking crisis** have been drawn. **On the one side are those who know what needs to be done. On the other are those who know what needs to be done -- but won't admit it.** Because it is against their self-interest. Unlike the conflict over the stimulus package, this is not an ideological fight. **This is a battle between the status quo and the future, between the interests of the financial/lobbying establishment and the public interest. What needs to be done is hard but straightforward.** As Martin Wolf of the Financial Times sums it up: "**Admit reality, restructure banks and, above all, slay zombie institutions at once.**" This tough love for bankers is being promoted by everyone from Nouriel Roubini, Paul Krugman, and Ann Pettifor to Niall Ferguson, the Wall Street Journal, and Milton Friedman's old partner, Anna Schwartz ... "They should not be recapitalizing firms that should be shut down," says Schwartz. "Firms that made wrong decisions should fail."... **Tim Geithner ... is on the wrong side of the issue, more worried about the banking industry than the American people.** Like Hank Paulson before him, Geithner appears more concerned about saving particular banks than saving the banking system. ... As Ann Pettifor puts it on HuffPost: "**Much of Wall Street is effectively insolvent. It's not that these banks lack cash or capital -- it's just that they're never going to meet all their financial liabilities -- i.e. repay their debts. Ever.**" ...".

According to a 23 March 2009 democracynow.org interview of Paul Krugman:

"... **"The Zombie Ideas Have Won" - Paul Krugman on \$1 Trillion Geithner Plan to Buy Toxic Bank Assets ... Treasury Secretary Timothy Geithner is preparing to unveil a plan ... to purchase as much as \$1 trillion in troubled mortgages and other assets from banks. ... The Obama administration has described the plan as a public-private partnership, but most of the actual money will be put up by the government.** ... Paul Krugman: A zombie

idea is ... a bad idea, but it just keeps on coming back. ... to ... have taxpayers go in and buy ... these toxic assets ... 85 percent of the money is going to be a loan from the government, which is a non-recourse loan ... this is not Geithner. Ultimately, **the buck stops in the Oval Office.** The question is, why is President Obama going with the soft side, the hope over analysis, on this stuff? ... **the view still, apparently, dominant ... in this administration is that there's nothing really fundamentally wrong with the system. ... those people who we thought were so smart ... really are smart, and we want to keep them on the job. ... the Obama administration is still partying like it's 2006. ...**".

According to a 22 April 2009 [economictimes.indiatimes.com](http://economictimes.indiatimes.com) article by Swaminathan S. Anklesaria Alyar:

**"... the Obama administration is prolonging the recession by avoiding surgery to remove dead wood from its financial sector.** Some call this cowardice. Others, such as former IMF chief economist Simon Johnson, writing in The Atlantic, say **Wall Street has captured the White House.** ... Johnson says **the US now resembles Russia, where business oligarchs and government officials protect each others' financial interests, at the expense of the economy.** ... This ... highlights the priority given by the Obama administration to save the titans of Wall Street rather than end the recession quickly. ... **Technically, the financial sector is comprehensively bust.** It needs to recognise the losses, writing off trillions. ... **The market solution would be to force insolvent banks into bankruptcy, with shareholders and creditors taking a huge hit** ... Many titans of Wall Street will disappear ... **the Obama administration refuses to contemplate this obvious solution. ... Wall Street has captured the White House, so nothing will be done to imperil the politico-financial network that rules the US.** Robert Rubin and Hank Paulson, treasury secretaries of Clinton and Bush, were both from Goldman Sachs. Larry Summers, the current treasury secretary, earned millions as a hedge fund consultant. **In a market economy, well-managed companies should be rewarded with profits, while mismanaged companies should go bust. This basic rule has been suspended almost entirely for the titans of Wall Street. ... Accounting norms have been tweaked to permit zombie banks to pretend they are alive and solvent...."**

**If Terence McKenna's TimeWave is accurate, then for the next year or so we will live in a ZombieLand of accounting fiction and Empty Words of Hope, only to see a sharp collapse of the USA/UK Global Financial System during 2010.**

## **What will the 2012 Singularity Be Like ?**

On the bad side, maybe the USA/UK leaders will be so unhappy at the prospect of losing their Global Hegemony that they might have a Temper Tantrum and Kick Over the Table of the Game of Life and have a Big War.

**A more optimistic possibility is that a new Global Hegemony might form consistent with a [Confucian Mandate of Heaven](#).**

The Confucian I Ching (used by Terence McKenna in constructing the TimeWave) seems to be derived from Ancient African Mathematical Divination. Ron Eglash, in his book "African Fractals" (Rutgers 1999), said: "... fractal settlement patterns of Africa stand in sharp contrast to the Cartesian grids of Euro-American settlements. ... Euro-American cultures are organized by ... "top-down" organization. Precolonial African cultures included ... societies that are organized "bottom-up" rather than "top-down". ... most of the indigenous African societies were neither utterly anarchic, nor frozen in static order; rather they utilized an

adaptive flexibility ... African traditions of decentralized decision making could ... be combined with new information technologies, creating new forms that combine democratic rule with collective information sharing ...".

## **How does such a New Collective Democracy fit with the most likely successor entity to the USA/UK Global Hegemon - China ?**

**Although some regard China as a top-down dictatorship, the true situation is quite different, as is exemplified by the Chinese Computer Hacker Community.** According to a 23 April 2009 popsci.com article by Mara Hvistendahl: "... In the past two years, Chinese hackers have intercepted critical NASA files, breached the computer system in a sensitive Commerce Department bureau, and launched assaults on the Save Darfur Coalition, pro-Tibet groups and CNN. And those are just the attacks that have been publicly acknowledged. Were these initiated by the Chinese government? Who is doing this? ... **It's hundreds of thousands of everyday civilians.** ... This ... Hacker ... culture thrives on a viral, Internet-driven nationalism. ... **China's Internet patriots, who call themselves "red hackers," may not be acting on direct behalf of their government, but the effect is much the same.** ... The Red Hacker Alliance, often described in the Western press as a monolithic group, is in fact a loose association allowing disparate cells to coordinate their efforts. ... **the largest unifying characteristic is nationalism.** In a 2005 Hong Kong Sunday Morning Post article, a man identified as "the Godfather of hackers" explains, "Unlike our Western [hacker] counterparts, most of whom are individualists or anarchists, Chinese hackers tend to get more involved with politics because most of them are young, passionate, and patriotic." Nationalism is hip, and hackers -- who spearhead nationalist campaigns with just a laptop and an Internet connection -- are figures to revere. Henderson ... emphasizes that **the relationship between citizen and state is fluid in China**, and that the Chinese government tends not to prosecute hackers unless they attack within China. To Henderson, that lack of supervision is tacit approval, and it constitutes a de facto partnership between civilian hackers and the Chinese government. Jack Linchuan Qiu, a communications professor at the Chinese University of Hong Kong ... agrees. "**Chinese hackerism** is not the American 'hacktivism' that wants social change," he says. "**It's actually very close to the state. The Chinese distinction between the private and public domains is very small.**" ...".

**Perhaps the Chinese Government/People System, is in fact in accord with "African traditions of decentralized decision making",**

and so might give us a Better World than the USA/UK system that is now collapsing under the weight of its own greedy profiteering.