Chaos in Quantum Chromodynamics and the Hadron Spectrum

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Abstract

We present analytic evidence that the distribution of hadron masses follows from the universal transition to chaos in non-equilibrium field theory. It is shown that meson and baryon spectra obey a scaling hierarchy with critical exponents ordered in natural progression. Numerical predictions are found to be in close agreement with experimental data.

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1. Introduction

As an integral component of the Standard Model for particle physics, quantum chromodynamics (QCD) is a gauge field theory that successfully describes the coupling of quarks and gluons. Due to its rich dynamical content, QCD leads to many complex phenomena and it exhibits a number of remarkable features at both ends of the energy scale: asymptotic freedom, chiral symmetry breaking and color confinement [1, 2]. Strong interactions lead from free quarks and gluons in the high-energy limit (UV) to bound states forming mesons and baryons in the low-energy limit (IR). A unique manifestation of non-equilibrium QCD is the production of quark-gluon plasma (QGP) in collisions of heavy ions. QGP behaves like a strongly coupled liquid and unveiling its real-time dynamics outside lattice simulations remains a nontrivial task [3, 4].

Due to strong coupling at low energies, IR QCD is notoriously difficult to calculate with [3-5, 27-31]. It is for this reason that formulating analytic predictions directly from the equations of gauge field theory presents an ongoing challenge. Current understanding of QCD stems from several approximation tools such as weak-coupling perturbation methods (exclusively valid in UV), so-called “holographic” techniques inspired by the AdS/CFT correspondence, Euclidean lattice methods and effective formulations through phenomenological models [5]. The object of this work is to suggest that the hierarchy of hadron masses may be derived from a conceptually different baseline, that is, from the transition to chaos in non-equilibrium field theory. There are two basic premises that underlie our approach, namely:

1) a generic field theory can be modeled as a *statistical system* distributed on a space-time lattice [6].

2) *universal transition to chaos in nonlinear dynamics*, on the one hand, and *critical behavior of statistical systems*, on the other, share a common foundation [7].

Elaborating from this baseline, we find that the spectrum of hadron masses emerges from the universal scaling behavior of nonlinear maps near fully developed chaos. The paper is organized as follows: next section surveys the nonlinear behavior of field theory and its replica in non-equilibrium statistical physics; in section 3 we generalize the evolution of critical exponents and introduce the concept of flow in the space of universality classes. The link to observed fermion masses in the Standard Model as well as hadron masses in IR QCD forms the topic of section 4 and 5. Conclusions and open questions are elaborated upon in the last section.

We emphasize from the outset the introductory nature of this work (see also section 6). Its goal is strictly limited to exploring a new research avenue which, to the best of our knowledge, has received virtually no attention in previous publications. Given this rather modest goal, ideas discussed here are far from being the final word in understanding the puzzling physics of IR QCD. Independent studies are needed to reinforce or refute our preliminary findings.

2. Chaotic dynamics in non-equilibrium field theory

The starting point of our analysis is the framework developed in [8] which is applicable to generic fields that evolve in far-from-equilibrium conditions. Under some general assumptions, a quantum field theory *in steady contact* with its environment can be modeled as a distributed ensemble of coupled components, each representing a nonlinear dissipative system. In discrete time, the field dynamics may be formulated as

$$u_{n+1} = \hat{F}[u_n(x)] = \hat{g} f(u_n(x), \lambda) \tag{1}$$

Here, x is the spatial coordinate, n is the time index, $f(u,\lambda)$ stands for a generic nonlinear function, λ a control parameter and $\hat{g}$ a linear operator that defines the coupling. If x is continuous, $\hat{g}$ is given by the convolution

$$\hat{g} u(x-y) = \int g(y) u(x-y) dy \tag{2}$$

where the coupling constant g satisfies the set of constraints listed in [8]. Tuning λ near a set of critical values triggers transition to chaos in (1) whose universal signature is that it starts with a cascade of period-doubling bifurcations [8, 9, 26]. In the basin of attraction of fully developed chaos ($\lambda = \lambda_c$), the correlation length diverges according to

$$\xi \sim \Delta (\lambda - \lambda_c)^{-\nu} \tag{3}$$

Here, Δ is a finite reference length and ν the critical exponent of the correlation length [10]. An infinite correlation length is physically equivalent to component fields having vanishing masses $(\mu = 0)$. Away from λ_c, masses flow with the control parameter as in

$$\mu \sim \xi^{-1} \sim \Delta^{-1} (\lambda - \lambda_c)^{\nu} \tag{4}$$

This picture is compliant with the postulated symmetries of equilibrium quantum theory (QFT) whose action functional involves only massless fields [7]. QFT in general (and gauge field theory in particular) approaches conformal behavior at the fixed point $\lambda = \lambda_c$ and may be regarded as the asymptotic manifestation of the flow embodied in (4).

It is known that the sequence of parameters leading to the emergence of period-doubling bifurcations satisfies the so-called Feigenbaum scaling [11, 26]

$$\lambda_n - \lambda_c = K_n \overline{\delta}^{-n} \tag{5}$$

where λ_n denotes the value of λ where a cycle of period 2^n first appears, K_n is a scaling factor and $\overline{\delta}$ a constant. We note that $\overline{\delta}$ is in general different from the standard Feigenbaum constant $\delta = 4.669...$ involved in the transition to chaos of unimodal maps. The scaling constant no longer depends on n for $n \gg 1$ and the asymptotic form of (5) is

$$\lambda_n - \lambda_c \propto \overline{\delta}^{-n} \tag{6}$$

Replacing (6) in (4) yields

$$\overline{\mu}_n \propto \overline{\delta}^{-n\nu} \tag{7a}$$

where $\overline{\mu}_n = \mu_n \Delta$. In section 4 we consider the case $n = 2^p$, $p \in \{N\}$ for which scaling (7a) becomes

$$\overline{\mu}_{2^p} \propto (\overline{\delta}^{-\nu})^{2^p} \tag{7b}$$

3. Generalized behavior of critical exponents

To an arbitrary scale transformation of the correlation length $\xi \to \xi/s$ with $s \neq 1$, Renormalization Group theory associates a corresponding flow in parameter space

$\lambda \to f(\lambda)$ [12]. Let $f(\lambda)$ represent an analytic function. Since λ_c is a fixed point of this flow, the following condition holds

$$\lambda_c = f(\lambda_c) \tag{8}$$

Using the language of iterated maps we write

$$\lambda_{m+1} = f(\lambda_m) = f(f(\lambda_{m-1}) = ... \tag{9}$$

in which m stands for the number of iteration steps. As it is known, critical exponent ν is intimately related to the behavior of the correlation length under the scale transformation $\xi \to \xi/s$. It is given by [12]

$$\nu = \frac{\log s}{\log f'(\lambda_c)} \quad \text{for} \quad \lambda \to \lambda_c \tag{10}$$

where

$$f'(\lambda_c) = \left.\frac{df}{d\lambda}\right|_{\lambda_c} \tag{11}$$

Relations (10) and (11) indicate that ν depends only on the slope of function f near λ_c and are insensitive to the details of the underlying field equations (1). For this reason, all systems that are characterized by the same ν are said to belong to the same *universality class*, regardless of their specific dynamics on the microscopic scale. As a result, the standard viewpoint is that $\nu = \nu_c$ is solely fixed by the properties and dimensionality of (1). The ansatz assumes, however, that there are no arbitrary perturbations acting on the system that arise from dimensional instability [13] or large deviations from equilibrium [14]. If this is no longer the case, ν is allowed to either drift away or towards the fixed point ν_c. We call this trajectory a flow in the *space of universality classes* ($\nu \to \nu_c$). Then,

$$\lambda_{m+1} = f(\lambda_m, \kappa) \tag{12}$$

represents a generic one-dimensional map that generates the flow $\nu \to \nu_c$ with κ being the corresponding control parameter. Let κ_N denote the value of κ leading to the birth of a cycle of period 2^N in (12). We are naturally led to assume that ν reaches its fixed value ν_c when κ_N lands on its fixed point κ_∞, that is, when $N \to \infty$. A different way to phrase this hypothesis is to state that, for sufficiently large N,

$$(\nu)_{N+1} - (\nu)_N \propto 1/N \tag{13}$$

4. Connection to experimental data

For the sake of clarity, let us summarize results obtained so far. If critical exponent ν is constrained to assume a fixed value ν_c, the ratio of two consecutive mass values derived from (7b) is given by

$$\boxed{\frac{\overline{\mu}_{2^p}}{\overline{\mu}_{2^{p+1}}} \propto (\overline{\delta}^{-\nu})^{2^p}} \tag{14a}$$

If, on the other hand, if we assume that ν flows in the space of universality classes at a higher rate than the flow described by (6), combined use of (7a) and (13) yields:

$$\boxed{\frac{\overline{\mu}_{n,N}}{\overline{\mu}_{n,N+1}} \propto \overline{\delta}^{-(n/N)}} \tag{14b}$$

with $n \le m$ and $n \le N$.

These are our main results. It is instructive to note that (14a) recovers the mass ratios of quarks, leptons and gauge couplings if $\overline{\delta} = 3.9$ and $\nu = 1/2$ in four-dimensional space-time [14-16]. Proceeding along the same path, we now explore if (14b) may be linked to the observed spectrum of hadron masses. This is the topic of the next section.

5. Hadron spectrum

Despite being free and unbroken in UV, QCD is known to develop a non-vanishing energy scale in the infrared limit (Λ_{QCD}). The emergence of Λ_{QCD} is representative for field theories that are asymptotically free in the UV sector and is believed to be tied to the mechanism of mass generation in strong interactions [1, 2]. The current section is developed according to the following plan:

1) We interpret Λ_{QCD} as setting the natural resolution scale in the distribution of hadron masses. Meson and baryon spectra are partitioned in shells whereby the gap separating the shells is chosen to exceed Λ_{QCD}, that is, $\Delta \geq \Lambda_{QCD}$. Observed hadron masses are taken from the latest reports issued by the Particle Data Group [17].

2) The QCD scale is set to be equal to the confinement scale and computed using the so-called $\overline{MS}$ scheme $\Lambda_{QCD} = \Lambda_{\overline{MS}} = 130\ MeV$ [18].

3) Grouping of hadrons is captured in Tab. 1 and Tab. 2. The average mass for a given shell represents the arithmetic mean of that shell and is denoted by $\langle m^{(n)} \rangle$ and $\langle M^{(n)} \rangle$, respectively[1].

Results tabulated in Tabs 3, 4 and Figs 1, 2 show that:

a) the ratio of consecutive masses in the hadron spectrum also comply with (14b) for $\overline{\delta} = 3.9$ and,

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[1] Averaging within shells is not unique and may be performed according to a different scheme such as, for example, the use of weighted sums. Arithmetic averaging has been selected here for the sake of simplicity and clarity.

b) setting the lowest-lying value in (14b) ($n=1$), the group of numerical values for N that best fit observational data fall in the sequence $N = 1\cdot 1^1; 1\cdot 2^1; 2\cdot 2^1; 2\cdot 2^2; 1\cdot 3^1; 2\cdot 3^1$. In condensed form this series may be presented as

$$N = \begin{pmatrix} i\cdot 1^k \\ i\cdot 2^k \\ i\cdot 3^k \end{pmatrix} \quad \text{for } i,k = 1,2 \tag{15}$$

Shell definition	Makeup	$\langle m^{(n)} \rangle$
$m^{(1)}$	π^+, π^-, π^0	138
$m^{(2)}$	$K^+, K^-, K^0_{S,L}, \eta^0$	505
$m^{(3)}$	$\rho^+, \rho^-, \rho^0, \omega^0$	777
$m^{(4)}$	$\varphi, \eta^{0'}, K^{*+}, K^{*-}, K^{*0}, \overline{K}^{*0}$	925
$m^{(5)}$	$D^+, D^-, D^0, D_s^+, D_s^-$	1888
$m^{(6)}$	J/ψ	3097
$m^{(7)}$	B^+, B^-, B^0, B_s^0	5301
$m^{(8)}$	Υ	9460

Tab 1: Structure of meson mass shells

Shell definition	Makeup	$\langle M^{(n)} \rangle$
$M^{(1)}$	p, n	939
$M^{(2)}$	$\Lambda^0, \Sigma^+, \Sigma^-, \Sigma^0, \Delta^{++}, \Delta^+, \Delta^-, \Delta^0$	1203
$M^{(3)}$	$\Xi^0, \Xi^-, \Xi^{*0}, \Xi^{*-}, \Sigma^{*+}, \Sigma^{*-}, \Sigma^{*0}$	1408
$M^{(4)}$	Ω^-	1672
$M^{(5)}$	Λ_c^+	2286

Tab 2: Structure of baryon mass shells

Mass shell ratio	Exponent	Predicted scaling behavior	Relative error (%)
$\langle m^{(1)} \rangle / \langle m^{(2)} \rangle$	1	$\overline{\delta}^{\,-1}$	6.5
$\langle m^{(2)} \rangle / \langle m^{(3)} \rangle$	$1/3$	$\overline{\delta}^{\,-1/3}$	2.25
$\langle m^{(3)} \rangle / \langle m^{(4)} \rangle$	$1/8$	$\overline{\delta}^{\,-1/8}$	0.40

$\langle m^{(4)}\rangle / \langle m^{(5)}\rangle$	$\frac{1}{2}$	$\bar{\delta}^{-\frac{1}{2}}$	3.20
$\langle m^{(5)}\rangle / \langle m^{(6)}\rangle$	$\frac{1}{3}$	$\bar{\delta}^{-\frac{1}{3}}$	4.10
$\langle m^{(6)}\rangle / \langle m^{(7)}\rangle$	$\frac{1}{3}$	$\bar{\delta}^{-\frac{1}{3}}$	8.65
$\langle m^{(7)}\rangle / \langle m^{(8)}\rangle$	$\frac{1}{2}$	$\bar{\delta}^{-\frac{1}{2}}$	9.75

Tab 3: Predicted versus actual mass ratios for mesons

Mass shell ratio	Exponent	Predicted scaling behavior	Relative error (%)
$\langle M^{(1)}\rangle / \langle M^{(2)}\rangle$	$\frac{1}{6}$	$\bar{\delta}^{-\frac{1}{6}}$	2.07
$\langle M^{(2)}\rangle / \langle M^{(3)}\rangle$	$\frac{1}{8}$	$\bar{\delta}^{-\frac{1}{8}}$	1.28
$\langle M^{(3)}\rangle / \langle M^{(4)}\rangle$	$\frac{1}{8}$	$\bar{\delta}^{-\frac{1}{8}}$	0.16
$\langle M^{(4)}\rangle / \langle M^{(5)}\rangle$	$\frac{1}{4}$	$\bar{\delta}^{-\frac{1}{4}}$	2.78

Tab 4: Predicted versus actual mass ratios for baryons

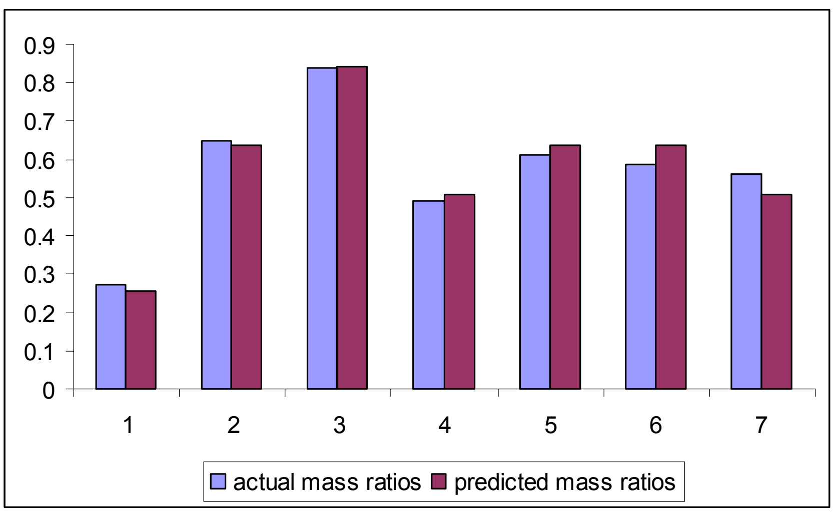

Fig 1: Actual versus predicted mass ratios for mesons

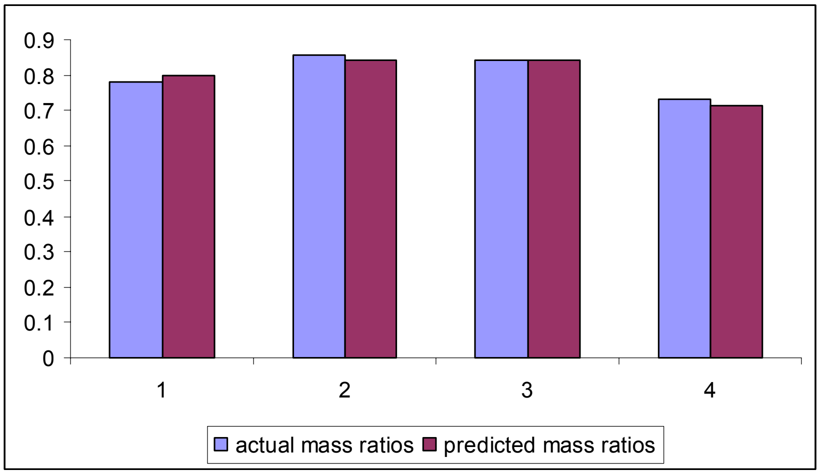

Fig 2: Actual versus predicted mass ratios for baryons

6. Conclusions and open questions

Many authors have stressed the fact that understanding the physics of IR QCD remains an outstanding challenge [3, 27-31]. The analytic computation of hadron masses at the level of experimental data precision is hampered by major technical obstacles related to color confinement and chiral symmetry breaking. For instance, rigorous lattice simulations suffer from artifacts that prevent reliable results in the hadronization region [28]. The Schwinger-Dyson formalism contains an infinite tower of equations which require truncations that are not gauge-independent and implicitly affect outcomes [29]. Models based on analytical confinement have led to some satisfactory results but are far from being confirmed as a realistic picture of QCD in the low-energy limit [30, 31].

Technical difficulties associated with the physics of IR QCD have prompted us to take an alternate route. To avoid a direct plunge into the intricate dynamics of gauge field theory, we have decided to start by exploring a straightforward yet sufficiently general model. The model consists of a distributed ensemble of coupled fields, each representing a nonlinear dissipative system. Using this baseline, we found that universal transition to chaos in non-equilibrium dynamics (as discussed for example in [8]) suggests a relatively straightforward explanation of hadron masses. It is instructive to note that the mechanism of mass generation discussed here is consistent with the conceptual framework of [19-20] and [25]. Needles to say, much remains to be done for a satisfactory clarification of quark and gluon physics in the low-energy sector. As pointed out in section 1, our arguments are introductory in nature and inevitably lack the depth expected from a rigorous and comprehensive analysis. Among the many issues in need for further clarification we mention the following:

a) how is our model related to individual hadron masses and not to their shell averages?

b) can our model explain the angular momentum and parity of hadron states?

c) can our model be expanded to include the glueball spectrum and the spectrum of light flavored mesons?

d) how do various nonlinear effects induced by (1) contribute to the observed hadron properties?

e) is transition to chaos applied to a full quantum context compatible with our analysis?

Interestingly enough, the contents of (15) seem to match the so-called Sharkovskii's ordering of periodic orbits in unimodal maps [11]. It is also surprising that the pattern of masses does not appear to be directly related to the quark content of hadrons but rather to the universal behavior of correlation length near criticality. One is led to suspect that there might be a deep connection between *non-equilibrium critical behavior in coupled stochastic systems* and the physics of hadronization [32-34]. Of particular interest is to investigate the link between the critical exponent $\beta_k = \left(1/2\right)^k$ of [32, 33] and (13). The results of this analysis will be reported elsewhere.

To conclude, although these findings are encouraging first steps, concurrent work is needed to confirm, expand or falsify our approach. We hope that similar techniques inspired by transition to chaos and non-equilibrium dynamics will soon play an increasingly important role in understanding the physics of IR QCD [21-24].

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