Applications of Euclidian Snyder geometry
to the foundations of space-time physics

Dr. Andrew Beckwith
American Institute of Beam Energy Propulsion, life member (I will put in the address later!)

The following document is to answer if higher dimensions add value to answering fundamental cosmology questions. The results are mixed, **1st** with higher dimensions apparently helping in reconstructing and preserving the value of Plancks constant, and the fine structure constant from a prior to a present universe , while **2nd** failing to add anything different from four dimensional cosmological models to the question of what would cause an increase in the expansion rate of the universe, as of a billion years ago. Finally **3rd**,, higher dimensions may enable creation of a joint DM and DE model . A choice between LQG and brane world geometry is introduced by Snyder geometry, where Snyder geometry's minimum uncertainty length calculations Δx may help determine to what extent gravity is an emergent field that is partly or largely classical. Independent of the choice of LQG and branes (four dimensions versus higher dimensional cosmology models) is the following question: If gravity is largely classical, how much nonlinearity is involved? Gravitons and their structure as information carriers may help answer these questions. The main point of this document: DM and DE may be seen to be unified in terms of cosmological dynamics if the higher dimensional models of DM , as seen by KK towers of Gravitons are seen to be pertinent to increasing acceleration of the universe a billion years ago via a 4th dimensional small graviton mass term.added to the KK tower DM representation of Gravitons (a model of DM) In 4 dimensions, 11 dimensional DM structure 'imitates' DE in four dimensions.

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I. INTRODUCTION

Recently, a big bounce has been proposed[1] as an alternative to singularity conditions that Hawkings, Ellis [1], and others use. The **1st problem** is though that there as of yet appears to be no fundamental argument presented in either traditional Friedman metric GR or LQG for preservation of the same value for Plancks constant or the fine structure constant from prior universes (before ours) and the present universe. And if $\hbar$ and α (fine structure) constant were very different prior to our present universe, then cosmological evolution would likely be a highly random process. Abhay Ashtekar **[2]** , in the inaugural opening of the Penn State gravity center (2007) told the author that the universe preserves most of its " memory" in cosmological cycles, but the proof of this assertion does not show up in Rovelli's **[3]** reference on Quantum Gravity . This present document in part tries to ascertain what could be a way to preserve some of prior universe "memory" through the use of gravitons as information carriers. . I.e. as a way about the singularity theorems of classical GR [1], The author is fully aware of the Goswami' et al proof **[4]** of how cosmological singularities could also be resolved in four dimensions, but even with that proof, no evidence of a structure to preserve $\hbar$ and α (fine structure) constant appears to emerge from a prior universe to todays cosmos. Furthermore, as this paper shows, both higher dimensional cosmologies such as brane theory , and more classical 4 dimensional cosmologies can be employed to (if gravitons have a small rest mass) account for the rate of expansion of the universe increasing one billion years ago, without necessarily invoking DE. The 2nd problem is that , as demonstrated by Alves, et al **[5]** four dimensional geometries **also** can account for the same increase in acceleration of the cosmos, one billion years ago. This present paper presents a choice between LQG (a four dimensional cosmology) and brane worlds (involving more than four dimensions) , via the construction of Snyder geometry, in order to outline cosmology models as a way to find if additional dimensions can be justified in terms of preserving the cosmological parameters such as $\hbar$ and α (fine structure) constant. In particular, Batistini **[6]** uses Snyder geometry to find a common basis for a limiting approximation to determine either brane world or LQG conditions for cosmological evolution. The motivation for delineating a choice between either brane world (higher dimesions) and LQG (four dimensions) comes from Alves et al's **[5]** with regards to their following argument : **"**It is worth stressing that Gabadadze and Gruzinov **[7]** have analyzed the instabilities of the

[1] Papers on LCQ at the 12th Marcell Grossman Meeting in 2009 (http://www.icra.it/MG/mg12/en/)

background and ghosts produced by massive gravitons in 4D Minkowski space-time. They conclude that a natural way to account for a massive graviton on a(t) space is to invoke theories with extra dimensions. However, Visser's model **[8]** is truly continuous with general relativity (GR) in the limit of vanishing graviton mass. Together with Pauli-Fierz (PF) massive term, Visser's theory is the simplest attempt to incorporate mass for thegraviton in a ghost-free manner. Moreover, the van Dam-Veltman-Zakharov discontinuity **[9]** (vDVZ) present in the PF term can be circumvented in Visser's model by introducing a non-dynamical at-background metric **[10] "** While this paragraph above is excellent physics, it does not answer the question of if preserving the values of $\hbar$ and α (fine structure) constant between a prior to present universe is possible. The paragraph above indicates that higher dimensions do not add more to the problem of explaining what caused the universe to expand more quickly a billion years ago. The heart of **[6]** is a deformed Euclidian Snyder space, using $\hbar = c = 1$ units, obtaining $[x, p] = i \cdot \sqrt{1 - \alpha \cdot p^2} \Leftrightarrow \Delta x \Delta p \geq \frac{1}{2} \cdot \left| \left\langle \sqrt{1 - \alpha \cdot p^2} \right\rangle \right|$. The LQG condition implies $\alpha > 0$, while brane worlds have instead, $\alpha < 0$. As suggested in **[6],** it is possible to obtain a string-theory limit of $\Delta x \geq \left[(1/\Delta p) + l_s^2 \cdot \Delta p \right] \equiv (1/\Delta p) - \alpha \cdot \Delta p$. The Δx lower bound will be either lower or higher depending upon either using LQG condition $\alpha > 0$, or brane worlds condition. $\alpha < 0$ Finally, if higher dimensions of DM models, projected to 4 dimensions yield a KK type model of DM, with the lowest order mode of DM (4 dimensions) mimicking DE, then a new phenomenological model of partial unity between DM and DE is presented. Which would be better than the Chapygin Gas model , as stated by Roos [11]

A. What role do gravitons play in distinguishing between $\alpha < 0$ and $\alpha > 0$ in terms of Snyder geometry ?

The author finds no difference in either the $\alpha < 0$ and $\alpha > 0$ situations as far as determining if a speed up of the universe's rate of expansion occurred a billion years ago. Alves [5] has shown that an accelerating expansion at the present time is closely related to the value of the graviton mass, while not using brane world dark flow, C or contributions from higher dimensions. If $m_{graviton} \cong 205 \times 10^{-65}$ grams in four dimensions yields few difference in behavior in the LQG and Brane world situations in a speed up of the rate of acceleration of the cosmos one billion years ago, then what worth are higher dimensions to cosmology ? The next paragraph presents how higher dimensions may preserve the values of $\hbar$ and α (fine structure) constant from prior to present universes. To do that , the author tries to establish the graviton as a carrier of information to be placed into both initial values of $\hbar$ and α at the big bang. Modeling how much information to be carried by an individual graviton can be achieved by measuring the graviton . Normalized energy density of gravitational waves, as given by Maggiore **[12]** is

$$\Omega_{gw} \equiv \frac{\rho_{gw}}{\rho_c} \equiv \int_{\nu=0}^{\nu=\infty} d(\log \nu) \cdot \Omega_{gw}(\nu) \Rightarrow h_0^2 \Omega_{gw}(\nu) \cong 3.6 \cdot \left[\frac{n_\nu}{10^{37}} \right] \cdot \left(\frac{\nu}{1kHz} \right)^4 \tag{1.1}$$

Where n_ν is a frequency-based count of gravitons per unit cell of phase space. In terms of early universe nucleation, the choice of n_ν may also depend upon interaction of gravitons with neutrinos **[13]**. The supposition is that eventually, (1.1) could be actually modified with a change of

$$n_\nu \propto n_\nu [graviton] + n_\nu [neutrinos] \tag{1.2}$$

Eq. (1.2) is part of a weighted average of neutrino-graviton coupled frequency $\langle \nu \rangle$, so that for detectors,

$$h_0^2 \Omega_{gw}(\nu) \cong \frac{3.6}{2} \cdot \left[\frac{n_\nu [graviton] + n_\nu [neutrino]}{10^{37}} \right] \cdot \left(\frac{\langle \nu \rangle}{1kHz} \right)^4 \tag{1.3}$$

Having confirmed a linkage between Gravitons and GW energy density the next step will be to confirm if possible that higher dimensional cosmology models do not invalidate or alter the Alves {5} results

II. Proof that gravitons with a small mass in four dimensions , $m_{graviton} \cong 205 \times 10^{-65}$ grams ,contribute to a brane world speed up of the acceleration of the universe

As suggested by Beckwith [14] , gravitons may contribute to the re-acceleration of the universe one billion years ago, when both LQG and brane world models showed increased acceleration of the rate of expansion**.** Is there direct proof of the above assertion? Yes, if the value of q(z) as appearing in Fig 1 would be almost the same in LQG and brane world geometry**.** . The results for Beckwith [14] show the same speed up of acceleration which Alves [5] obtained for cosmological expansion one billion years ago, but with higher dimensions. Generally, as specified by **[5, 11, 14]**, the dimensionless decelerating parameter, Eq. (1.4), models the rate of expansion of the universe, q, as a function of the scale factor $a \sim 1/1 + redshift$. When q becomes negative, the rate of acceleration of the universe is actually increasing, rather than slowing down.[5] , [11],[14]

$$q = -\frac{\ddot{a}a}{\dot{a}^2} \tag{1.4}$$

Eq. (1.16) follows what Beckwith [14] did (to duplicate **[5]** for a brane world) in order to plot an analysis of the deceleration , q(z), with z set = X in the calculation for Eq. (1.4). Eq. (1.16) is the input for for Figure 1 of page 6. The derivation of Eq. (1.16) as explained by **Appendix A** assumes setting a small mass for the graviton, for a brane world treatment of the Friedman equation, with the density of a brane world given in Eq (1.5), as used by Alves, et al. [6] As, in **Appendix A**

$$\rho \equiv \rho_0 \cdot \left(\frac{a_0}{a}\right)^3 - \left[\frac{m_g c^6}{8\pi G \hbar^2}\right] \cdot \left(\frac{a^4}{14} + \frac{2a^2}{5} - \frac{1}{2}\right) \tag{1.5}$$

Eq. (1.5) assumes use of the following inequality for a change in the HUP, [5]

$$\Delta x \geq \left[(1/\Delta p) + l_s^2 \cdot \Delta p\right] \equiv (1/\Delta p) - \alpha \cdot \Delta p \tag{1.6}$$

and that the mass of the graviton is partly due to the stretching alluded to by Fuller and Kishimoto [13], a supposition the author is investigating for a modification of a joint KK tower of gravitons, as given by Maartens **[16]** for DM. I.e., what if the following actually occurred? Assume that the stretching of early relic neutrinos that would eventually lead to the KK tower of gravitons--for when $\alpha < 0$, is [14]

$$m_n(Graviton) = \frac{n}{L} + 10^{-65} \text{ grams} \tag{1.7}$$

Also assume that to calculate the deceleration, the following modification of the HUP is used: [2] $\Delta x \geq \left[(1/\Delta p) + l_s^2 \cdot \Delta p\right] \equiv (1/\Delta p) - \alpha \cdot \Delta p$, where the LQG condition is $\alpha > 0$, and brane worlds have, instead, $\alpha < 0$. The deceleration parameter in (1.4) will have either higher-dimensional contributions, in the brane theory case, or no higher-dimensional contributions, in the LQG case. Eq. (1.10) shows typical adjustments to the Friedmann equation if brane world geometry is assumed . Eq. , Eq (1.8) and Eq. (1.9) are typical Friedman equations given for when a cosmological constant is included in the evolution of the scale factor**. [11, 17,18,]** Eq. (1.10) shows brane world adjustments to the Friedmann equation, [16] . Also Eq. (1.10) will be the starting point used for a KK tower modified version of Eq,. (1.4) [14] So

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho_{Total}}{3M_{Planck}^2} - \frac{k}{a^2} + \frac{\Lambda}{3} \tag{1.8}$$

$$\left(\frac{\ddot{a}}{a}\right) = -\frac{(\rho_{Total} + 3p_{Total})}{6M_{Planck}^2} + \frac{\Lambda}{3} \tag{1.9}$$

In the brane world theory Friedmann equations as presented as a consequence of Randall Sundrum theory,

$$\dot{a}^2 = \left[\left(\frac{\rho}{3M_4^2} + \frac{\Lambda_4}{3} + \frac{\rho^2}{36M_{Planck}^2}\right)a^2 - \kappa + \frac{C}{a^2}\right] \tag{1.10}$$

For the purpose of Randal Sundrum brane worlds, (1.10) is differentiated with respect to $d/d\tau$, and then terms from Eq. (1.9) will be used, and put into a derivable equation. which will be for a RS brane world version of $q = -\frac{\ddot{a}a}{\dot{a}^2}$. Note that Roy Maartens [**16**] states that KK modes (graviton) satisfy a 4-dimensional Klein-Gordon equation, with an effective 4-dim mass, $m_n(Graviton) = \frac{n}{L}$, with $m_0(Graviton) = 0$, and L as the stated "dimensional value" of higher dimensions. The value $m_0(Graviton) \sim 10^{-65} - 10^{-60}$ gram is almost zero. A non-zero mass for the graviton is a violation of the usual correspondence principle for spin two objects, in quantum mechanical reasoning. G Grossing **[19]** and J. Baker-Jarvis, and P. Kabos **[20]** have shown how the Schrodinger and Klein Gordon equations can be derived from classical Lagrangians, i.e., using a version of the relativistic Hamilton-Jacobi- Bohm equation, with a wave functional $\psi \sim \exp(-iS/\hbar)$, with S the action, so as to obtain working values for a tier of purported masses of a graviton from the equation , for 4-D of making the flat space approximation

$\left[g^{\alpha\beta}\partial_\alpha\partial_\beta \xrightarrow[FLAT-SPACE]{} \nabla^2 - \partial_\tau^2\right]$, and $\left[\nabla^2 - \partial_\tau^2\right]\cdot\psi_n = m_n^2(graviton)\cdot\psi_n$. This involves a small graviton mass, while assuming that a small added mass, $m_0(Graviton) \approx 10^{-65}$ grams, is a result of a semi-classical superstructure which containing the usual field theory/brane world treatment of gravitons. All this discussion will show up in **Appendix A** below to derive Eq. (1.16) For book keeping, the author uses the substitutions . $(a_0/a)^3 = (1+z)^3$, and $a \equiv a_0/(1+z)$, Then

$$\rho(z) \equiv \rho_0 \cdot (1+z)^3 - \left[\frac{m_g}{8\pi G}\right]\cdot\left(\frac{a_0^4}{14\cdot(1+z)^4} + \frac{2a_0^2}{5\cdot(1+z)^2} - \frac{1}{2}\right) \tag{1.11}$$

$$A1(z) \cong \frac{C\cdot(1+z)^3}{a_0^3}\cdot\left[1\Big/\sqrt{\Phi\left(\rho(z), a_0/(1+z), C\right)}\right] \tag{1.12}$$

$$A2(z) \cong -\left(\frac{\rho(z)}{3M_4^2} + \frac{1}{36}\cdot\frac{\rho(z)^2}{M_P^6}\right)\Big/\left[\Phi\left(\rho(z), a_0/(1+z), C\right)\right] \tag{1.13}$$

$$A3(z) \cong \frac{1}{2}\left(\cdot\left[\frac{1}{3M_4^2} + \frac{1}{18}\cdot\frac{\rho(z)\cdot}{M_P^6}\right]\Big/\left[\Phi\left(\rho(z), a_0/(1+z), C\right)\right]^{1/2}\right)\cdot$$
$$\left[3\cdot\rho_0\cdot(1+z)^3 + 4\cdot\left(\frac{a_0^4/(1+z)^4}{14} + \frac{a_0^2/(1+z)^2}{5}\right)\cdot\left(\frac{m_g}{8\pi G}\right)\right] \tag{1.14}$$

$$\Phi\left(\rho(z), a_0/(1+z)\right), C\right) = \frac{C\cdot(1+z)^4}{a_0^4} + \left(\frac{\rho(z)}{3M_4^2} + \frac{1}{36}\cdot\frac{\rho(z)^2}{M_P^6}\right) \tag{1.15}$$

So, for $4 < z \le 0$, i.e., not for the range, say $z \sim 1100$ at 380,000 years after the big bang, it would be possible to model q(z), the final version of the cosmological deceleration parameter used:

$$q(z) = A1(z) + A2(z) + A3(z) \tag{1.16}$$

Negative values for Eq. (1.16) appear probable at about $z \sim 1.5$, when (1.14) would dominate, leading to $q(z \sim 1.5)$ with a negative value. The positive value conditions rely upon contributions from the C dark radiation term in Eq. (1.15). The final result is that the deceleration parameter calculation can be done for the brane world case and KK gravitons.. Now what can one expect with LQG condition with respect to the HUP, with $\alpha > 0$? The claim is made by Beckwith that LQG, due to the starting point of the LQG Friedmann equations, similar to what Alves [5] used, duplicates deceleration results by Beckwith [13]

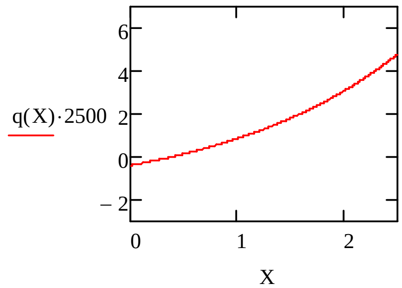

Figure 1: Motivated by a similar diagram in **A**lves [5], where X is the red shift z and q(X) is the rate of deceleration of the universe due to a small $m_{graviton} \cong 205 \times 10^{-65}$ grams.

Figure 1 is to be compared with Figure 4, page 11, of Alves [5] The significance of figure 1 lies in that higher dimensions in themselves do not change , if a four dimensional representation of the graviton has a low value mass of $m_{graviton} \cong 205 \times 10^{-65}$ gram what is seen in four dimensions .

B . Using the LQG condition $\alpha > 0$, in Snyder geometry modified HUP

The claim is that almost all the complexity is removed with $\alpha > 0$, and what is left is a set of equations similar to the tried and true $\left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho_{Total}}{3M_{Planck}^2} - \frac{k}{a^2} + \frac{\Lambda}{3}$. To get an idea of what happens with LQG versions of the Friedman equation, one can look at Taveras' **[21]** treatment of the Friedmann equations, where he obtains, to first order, if ρ is a scalar field density,

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\kappa}{3} \cdot \rho \tag{1.17}$$

and

$$\left(\frac{\ddot{a}}{a}\right) = -\frac{2 \cdot \kappa}{3} \cdot \rho \tag{1.18}$$

The interpretation of ρ as a scalar field DENSITY, and if one does as Alves et al [5] .did, i.e, work with flat space, then with k=0, in $\left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho_{Total}}{3M_{Planck}^2} - \frac{k}{a^2} + \frac{\Lambda}{3}$, and $\rho \equiv \rho_0 \cdot \left(\frac{a_0}{a}\right)^3 - \left[\frac{m_g c^6}{8\pi G \hbar^2}\right] \cdot \left(\frac{a^4}{14} + \frac{2a^2}{5} - \frac{1}{2}\right)$

The sticking point in all of this is to interpret the role of ρ. In the LQG version by **[21],** $\left(\frac{\dot{a}}{a}\right)^2 = \frac{\kappa}{3} \cdot \rho$ may be rewritten as follows: If conjugate momentum is in many cases, "almost" or actually a constant,

$$\left(\frac{\dot{a}}{a}\right)^2 \equiv \frac{\kappa}{6} \cdot \frac{p_\phi^2}{a^6} \tag{1.19}$$

This assumes that the conjugate dimension in this case has a quantum connection specified via an effective scalar field, ϕ, obeying the relationship

$$\dot{\phi} = -\frac{\hbar}{i} \cdot \frac{\partial}{\partial \cdot p_\phi} \tag{1.20}$$

Alves research results [5] for both LQG and brane worlds means using the Friedmann equations directly as a way to computer deceleration of the universe, from the Friedmann equations, especially if the graviton has nearly zero rest mass. To do that, we delve into a LQG version of deceleration parameter behavior.

C. What 4-dimensional representations of the Graviton are in common with both models for both $\alpha < 0$ and $\alpha > 0$?

There are two hypotheses to consider here. The first is that there is an interaction between neutrinos and gravitons. Bashinsky **[22]** suggests an alleged modification of density fluctuations via neutrino-graviton interactions. A second far more radical hypothesis is that there are a few "stretched neutrinos" [4] that may span many light years, and these may affect gravitons, as suggested by Bashinsky **[22].** What is being considered is that there are graviton-neutrino interactions, as proposed by [22], and [13] asks, what are the natures of the neutrino-graviton interactions if a few of the neutrinos "stretch" by many light years? If there is a coupling between gravitons and neutrinos, as by **[18],** stretched neutrinos leading to stretched gravitons brings into question the correspondence principle, that gravitons be spin 2, with zero mass.

1. The probable effect of stretching of the neutrino on graviton wavelengths

Assume that with stretching of the neutrino, and a graviton neutrino coupling with zeroth order value of $m_0(Graviton) \approx 10^{-65}$ grams as a consequence of at least a few of the neutrino-gravitons obeying density fluctuation modified , according to **[22],** $\left[1 - 5\cdot\left(\rho_{neutrino}/\rho\right) + \vartheta\left(\left[\rho_{neutrino}/\rho\right]^2\right)\right]$. Note that according to [14] , the overall density of the evolving space-time continuum has neutrino-graviton interactions that effectively shrink the magnitude of overall space-time density. In addition, neutrino and graviton wavelengths then have the same order of magnitude as the matter wavelength values of neutrinos, with, initially

$$
\begin{aligned}
& m_{graviton}\Big|_{RELATIVISTIC} < 4.4\times 10^{-22} h^{-1} eV/c^2 \\
& \Leftrightarrow \lambda_{graviton} \equiv \frac{\hbar}{m_{graviton}\cdot c} < 2.8\times 10^{-8}\, meters
\end{aligned}
\tag{1.21}
$$

A few select gravitons, coupled to stretched neutrinos with almost infinite wavelengths, would lead to (1.21) , if the graviton wavelengths were, according to an argument ventured by Valev **[23],**

$$
\lambda_{graviton} \equiv \frac{\hbar}{m_{graviton}\cdot c} < 10^4\, meters \text{ or larger} \tag{1.22}
$$

III. THE CORRESPONDENCE PRINCIPLE AND 'T HOOFT'S SUPPOSITION OF 'DETERMINISTIC QM' AS APPLIED TO GRAVITONS

The author suggests that the stretch out of the graviton implied by (1.39) may be a sign that the correspondence principle, used by string theorists and others as a way to insist that the graviton be of zero mass, may have to be amended. After presenting why the author states that the correspondence principle needs revision, , the author will suggest a mechanism for replacement of the correspondence principle, which the author suggests is consistent with 't Hooft's **[24,25]** deterministic quantum mechanics. The final part to this paper suggests what "information" a particle like the graviton may carry. What can be stated about the "correspondence principle" and its connections to gravitons? Rothman and **Boughn [26] suggest** that it is unrealistic to envision gravitons ever being measured. However, it assumes current detector technology.. Finally, the author will look into what a graviton "construction" with a tiny mass may entail regarding instanton-anti-instantons, and its relationship to 't Hooft's **[24, 25]** deterministic quantum mechanics. To recap what they are suggesting as a way to quantify the scattering needed to observe a graviton in a detector, Rothman, and Bohn [26] suggest

$$
\tilde{n}\cdot\sigma\cdot\tilde{\lambda} \geq 1 \tag{1.23}
$$

where $\tilde{n}$ is the purported numerical density of "detector particles," σ is the detector cross-sectional area, and $\tilde{\lambda}$ is the mean "distance" a graviton would have to travel. However, **[26]** assumes the cross-sectional area for a graviton would have to be larger "than the diameter of Jupiter." Note that the variable $\tilde{n}$ is given by **[26]** to be $\tilde{n} \equiv M_{\det} / \left[m_{proton} \cdot V_{\det} \right]$. I.e., this is for a detector with gravitons interacting with some version of hydrogen, with $M_{\det}$ the "mass" of the detector, and with $V_{\det}$ the purported volume of the detector. Also, m_{proton} is the mass of protons in the detector that the gravitons may interact with. If so, the volume $V_{\det}$ being Jupiter-sized may look reasonable. However, this assume gas-based graviton detector technology, which uses collision cross sections. Electromagnetic and graviton interactions may allow for a far smaller $V_{\det}$, according to **[27],** and **[28]** , using a different numerical count procedure for gravitons in a unit of phase space. Beckwith **[27]** uses a very explicit numerical count, which can be interpreted as a phase space count from fundamental principles. Li, et al, **[28]** uses electromagnetic fields as affected by gravitons in a containment vessel, as a way to "count" incident gravitons. These two alternatives raise the question of the utility of Eq. (1.23) as the optimal way to measure gravitons in experimental devices. Eq. (1.23) implies a numerical count of gravitons detected during the lifetime of an experiment, where $L_{graviton-production}$ is the luminosity of graviton production, R as the purported distance the graviton would travel, while setting up the right-hand side with $\frac{A_{\det} \cdot \tau_B}{\in_{graviton}} \equiv$ (detector cross sectional area* time of process for the graviton source to be operating) / graviton energy . Also, the time limit is $\tau_B \le \frac{M_{graviton-generating-source}}{L}$. Here $M_{graviton-generating-source}$ is the relative mass of the graviton producing source, and L the luminosity of the source. The bounds for τ_B are effectively exceeded by kinematic considerations if the graviton production "site" is relic early universe gravitons, instead of what is cited for non-zero graviton energies, $\in_{graviton}$

$$N_{graviton=\exp-lifetime} \equiv \left[\frac{L_{graviton-production}}{4\pi R^2} \right] \cdot \left[\frac{A_{\det} \cdot \tau_B}{\in_{graviton}} \right] \tag{1.24}$$

Rothman and Boughn **[26]** give a coherent argument that for neutron stars, black holes and the like, Eq. (1.41) has an upper bound of $N_{graviton=\exp-lifetime} \approx 10^{-5}$. The author suggests that the total source luminosity L versus luminosity of graviton production process of the source $L_{graviton-production}$ may be very different from the ratio values given by Rothman, and Bohn, of $L_{graviton-production} / L = f_{graviton}$ as almost $\sim .01 - .02$. If the $f_{graviton}$ is over ten times larger, plus the life time $\tau_B \le \frac{M_{graviton-generating-source}}{L} >>$ of graviton production from black holes with a larger time due to having a value of $M_{graviton-generating-source} >> 10^{15}$ grams , with 10^{15} grams ~ mass of a black hole, then $N_{graviton=\exp-lifetime} \approx 10^{-5}$ may be way too small. Furthermore, if the stretched neutrino hypothesis, with coupling to the graviton, occurs, then, assuming that there is at a minimum $\lambda_{graviton} \equiv \frac{\hbar}{m_{graviton} \cdot c} < 10^4 \, meters$, instead of $\lambda_{graviton} \equiv \frac{\hbar}{m_{graviton} \cdot c} < 2.8 \times 10^{-8} \, meters$. Even with a non-giant planet sized detector, one would see a count of data that $N_{graviton=\exp-lifetime} >>$ $N_{Rothman-calculated-graviton-\exp er-lifetime} \approx 10^{-5}$, perhaps with $N_{graviton=\exp-lifetime}$ as nearly unity. And this is due to recalibration of the different input coefficients..The inequality given counting

$N_{graviton=\exp-lifetime} >> \quad N_{Rothman-calculated-graviton-\exp er-lifetime}$ raises a question regarding the use of standard correspondence principle to characterize gravitons, and suggests an alternative: gravitons with spin 2, but perhaps masses slightly larger than zero. Eventually, this will lead -to considering the correspondence principle, as well as "deterministic" quantum mechanics as ‘t Hooft wrote about in reference [25] as a way to consider the nature of gravitons
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A. Can the graviton have a small mass? Embedding the laws of QM regarding gravitons in a nonlinear theory.

Recently, an alternative to usual space-time gravitation theories was proposed by Shinji Mukohyama: HoYYava gravity **[29]** Robert Brandenberger **[30]** also modeled this new theory in terms of the early universe, with the claim that there was a matter bounce instead of standard inflation. This theory, ironically, depends upon a chaotic inflationary potential $V(\phi) = (1/2) \cdot m^2 \phi^2$ for its pre-bounce conditions, and uses "dark radiation" for obtaining a "bounce." **[30]** has also presented “scale-invariant, super-horizon curvature perturbations." Both **[29]** and **[30]** accept scale-free "perturbations" as long as the contraction phase does use "quantum vacuum fluctuations." So it will be necessary to wait to see if HoYYava gravity develops further, or is provided with a mechanism to transfer energy to the standard model of cosmology predictions for the radiation and matter eras. Matt Visser presented **[31]** HoYYava gravity in terms of the alleged “benefits of Lorentz symmetry breaking" with a nonphysical graviscalar spin zero "graviton" contribution [27], which, if not removed, yields mathematical artifacts with no physical content in them**.** Spin zero gravitons (gravit scalars) completely eliminate string theory as a consideration in Gravitons. Note that at that the lowest vibration of a cosmological string, a spin two particle with zero mass, can be interpreted as a graviton, Graviton modes with spin zero would sever completely the connection to quantum mechanics, and complicate making semi classical analogies to quantum mechanics Visser viewed the elimination, removal, and reduction of the gravi scalar as crucial to the proof and validity of this supposition about semi classical gravity models. Gravi scalars, if they exist would require a complete break with the correspondence principle of quantum mechanics for spin two quantum spin particles **[33]** . By way of contrast, what the author will attempt to do is to with gravitons is far more modest: referencing the construction of a graviton in terms of instanton-anti-instantons, and asking if a composition of a graviton as an "object" comprising such kink-antikinks can be tied in with ‘t Hooft's “deterministic quantum mechanics” **[24,25].** Beginning the analysis, the author will briefly review what he did with CDW in $1+\varepsilon^{+}$ dimensions, and then reference the chances for doing the same for four dimensions for gravitons, closing with a discussion of the ability of the graviton to carry information and what this says about graviton mass.

B. Brief review of S-S' in CDW, and its relevance to higher dimensional ‘objects’

The following is a presentation of Beckwith's density-wave instanton-anti-instanton construction for CDW, which has classical analogies. Beckwith's kink-anti-kink models **[32,34]** have a classical analogy with

$$\phi_{\pm}(z,\tau) = 4 \cdot \arctan\left(\exp\left\{ \pm \frac{z + \beta \cdot \tau}{\sqrt{1-\beta^2}} \right\} \right) \quad (1.25)$$

which is a solution to

$$\frac{\partial^2 \phi(z,\tau)}{\partial \tau^2} - \frac{\partial^2 \phi(z,\tau)}{\partial z^2} + \sin \phi(z,\tau) = 0 \quad (1.26)$$

A tunneling Hamiltonian version of such solutions uses a Gaussian wave functional formalism, with

$$\Psi_{i,f}\;[\phi(\mathbf{x})]\Big|_{\phi\equiv\phi_{ci,cf}} = c_{i,f}\cdot\exp\left\{-\int d\mathbf{x}\;\alpha\left[\phi_{C i,f}(\mathbf{x})-\phi_0(\mathbf{x})\right]^2\right\}, \qquad (1.27)$$

This procedure in Eq. (1.27) allowed for deriving a confirmation **[32,34]** of the fit between the false vacuum hypothesis and data obtained for current vs. applied electrical field (I-E) curves of experiments initiated in the mid-1980s by [35]. Further research by Beckwith led to the finding that the modulus of the tunneling Hamiltonian was proportional to current, with E_T a threshold pinning field **[32, 34]** . The threshold pinning field, as presented in Beckwiths PhD dissertation [32] is an absolute magnitude of applied electric field for which above a magnitude for an applied electric field there is current flow in CDW 'information' in a laboratory measurement of current versus electric field strength in $NbSe_3$--. This phenomena was presented experimentally in Miller, et al, [35] in the mid 1980s . The term pinning is an artifact of CDW terminology, and is not the important datum. I.e. pinning is an artifact of historical terminaology and is less important than the term threshold. . Below this applied electric field threshold, no CDW transport occurred. The use of instantons and judicious use of Eq (1.44) is necessary for modeling a current of CDW waves, which cannot be obtained by conventional CDW models **{32,34].** I.e. the use of instanton-anti- instanton structure is a necessary for modeling I-E behavior in quasi-one- dimensional systems. A similar situation asrises in cosmology, once evolution equations are properly defined.

$$I \propto \tilde{C}_1\cdot\left[\cosh\left[\sqrt{\frac{2\cdot E}{E_T\cdot c_V}}-\sqrt{\frac{E_T\cdot c_V}{E}}\right]\right]\cdot\exp\left(-\frac{E_T\cdot c_V}{E}\right) \qquad (1.28).$$

The phase used in Eq. (1.27) has the following graphical representation, and it is indicative of what instanton physics can be used for, i.e., this is not a substitute for a well thought out treatment of instantons, which will be connected via cited derivations which link instanton models with with appropriate metrics, as is discussed explicitly with respect to GR space time metrics in **Appendix B .** Fig. 4 in particular is a template for how the author will model a pop up effect of a S-S' pair, in a quantum mode, using S and S' pairs. Note in the case of CDW the author [32] found a current derived as being of the form $J \propto T_{if}$ I.e. a current proportional to a density wave tunneling Hamiltonian matrix element This argument actually became a modulus argument due to considering a current density proportional to $|T|$ rather than $|T|^2$ since tunneling, in this case, would involve coherent transfer of individual (first-order) bosons rather than pairs of fermions The use of $J \propto T_{if}$ instead of having current proportional to $|T|^2$ permitted the author to come up with current versus applied electric field graphs which closely matched experimental Zenier plots of CDW obtained by Miller in the mid 1980s [35]. The author rules out time independent Wheeler De Witt equations with Hamiltonians [33] for this sort of detailed analysis of gravition particle creation. In particular, there is no counter part to a tunneling Hamiltonian in cosmology which can take the place of what was done in the authors CDW studies [32], [34] However, the over lap in the case of cosmology is in the use of soliton- anti solution pairs, which allow for abrupt movement of gravitons, a topic which will be dealt with expensively later The author argues that the use of instanton- anti instanton (kink- anti kink) construction in both CDW [32,34] and its counter part in cosmology will give, as indicated in Figure 4 below vacuum nucleation pop up, in the beginning of inflation, of gravitons. As can be quoted by Mukhanov [37] for times $10^{-12} - 10^{-43}$ seconds after the big bang, and at temperatures 10 TeV to 10^{19} GeV, that topological defects may play a role in the production of new particles is possibly relevant to the authors presentation of a kink anti kink model of gravitons **.**

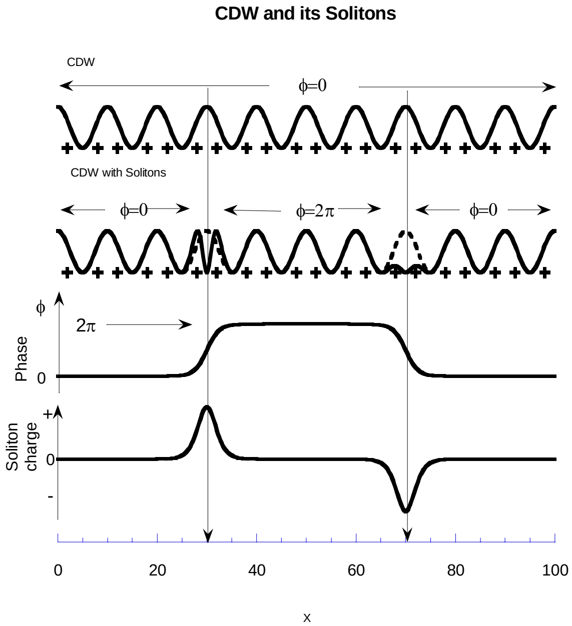

Figure 2, creation of instanton – anti instanton pairs. From Beckwith, [30]

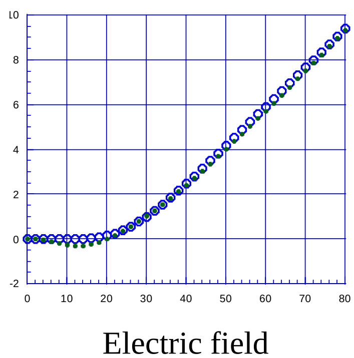

Fig 3: Results of applying Eq (1.28) as opposed to J, H. Millers Zenier plot [35]

$$I \propto G_P \cdot (E - E_T) \cdot \exp\left(-\frac{E_T}{E}\right) \quad \text{if } E > E_T \text{ ,}$$

I = 0 if $E \leq E_T$. **[35]**

The blue dots represent Eq. (1.45) whereas the black dots represent uniformly applying the non zero plot for electric fields as given by the Zener plot [31] approximation (as taken from the authors **[32], [34]**).

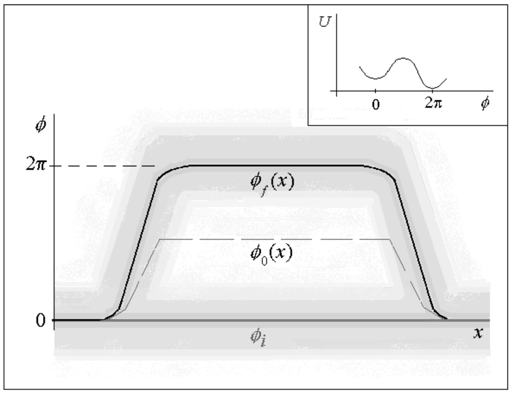

Fig. 4. The pop up effects of an intanton-anti-instanton in Euclidian space from [32,34]

In order to connect with GR, one needs to have a higher dimensional analog of $\phi_{\pm}(z,\tau) = 4 \cdot \arctan\left(\exp\left\{ \pm \frac{z+\beta\cdot\tau}{\sqrt{1-\beta^2}} \right\} \right)$ that is consistent with space-time metrics, a topic which will be presented in the next section. We need to associate Instanton structure with typical GW topologies, and the way to do for a five dimensional cosmology is indicated in Appendix A. The five-dimensional "line element" involves an instanton that is almost identical in structure to Eq. (1.42). Having mentioned this similarity in form for the instanton, it is appropriate to note that the fifth dimension of the line element may form a semi-classical conduit of information for forming a vacuum nucleation of the instanton itself.

C. Dropping in 'information' to form an instanton-anti-instanton pair, and avoiding the cosmological singularity via the 5th dimension

As suggested by Beckwith [38], there is no reliable way to reconcile the formation of an instanton-anti-instanton pair, and, for example, to avoid having an instanton disrupted by a cosmological singularity. The graphical example in [38] was suggested to consider the question: What if there were, in higher than four dimensions, a region of **space** about the four dimensional singularity at the beginning of expansion of the universe via the use of the fifth dimension in which an instanton-anti-instanton could have information transferred from a prior universe and re materialize in the onset of the big bang after the point of nucleation of a new universe, thereby avoiding the cosmic singularity ? . This question lead to , in Chongquing the author led to defining in Chouguing **[38]** the region about the singularity as a ring of space-time about the origin, but not overlapping it, with a time dimension defined [38] as

$$\Delta t \equiv 10^{\beta} \cdot t_{Planck} \tag{1.29}$$

The exact uncertainty principle in five dimensions is open to discussion, but the author in his presentation in Chongquing **[38]** suggested, as an example, a five-dimensional version of $\Delta E \Delta t \geq \hbar$. For the tiny mass specified via the $m_{graviton} \propto 10^{-65}$ grams , if one makes energy equivalent to mass, then the small mass times the speed of light, squared, in the case of instanton-anti-instanton (kink-anti-kink) would be the S-S' pair for the instanton nucleated about the cosmic singularity. The classical treatment of this problem would be to assume that the transfer of information from a prior universe to our own went through a 5th dimension with the cosmic singularity, a 4th dimensional artifact The initial 4 dimensional 'graviton' would have , $m_{graviton} \propto 10^{-65}$ grams, with, say, a top value for the graviton mass after acceleration of $m_{graviton} \propto 10^{-61}$ grams. This is abrupt acceleration, making the graviton mass at least 10^4 times heavier than initially. To understand the rationale for such a supposition, a brief review of typical field theories involving "massive" gravitons and the limit $m_{graviton} \to 0$ will be presented, with a description of why these effects may lead to semi-classical approximations.

D. Massive Graviton field theories, and the limit $m_{graviton} \to 0$

As given by M. Maggiore [**12**], the massless equation of the graviton evolution equation takes the form

$$\partial_\mu \partial^\varpi h_{\mu\nu} = \sqrt{32\pi G} \cdot \left(T_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} T^\mu_\mu \right) \tag{1.30}$$

When $m_{graviton} \neq 0$, the above becomes

$$\left(\partial_\mu \partial^\varpi - m_{graviton}\right) \cdot h_{\mu\nu} = \left[\sqrt{32\pi G} + \delta^+\right] \cdot \left(T_{\mu\nu} - \frac{1}{3}\eta_{\mu\nu} T^\mu_\mu + \frac{\partial_\mu \partial_\nu T^\mu_\mu}{3 m_{graviton}} \right) \tag{1.31}$$

The mismatch between these two equations, when $m_{graviton} \to 0$, is largely due to $m_{graviton} h^\mu_\mu \neq 0$ as $m_{graviton} \to 0$, which in turn is due to setting $m_{graviton} \cdot h^\mu_\mu = -\left[\sqrt{32\pi G} + \delta^+\right] \cdot T^\mu_\mu$. The mismatch between these two expressions is one of several reasons for exploring what happens for semi-classical models when $m_{graviton} \neq 0, m_{graviton} \sim 10^{-65}$ grams, noting that in QM, a spin 2 $m_{graviton} \neq 0$ has five degrees of freedom, whereas the $m_{graviton} \to 0$ gram case has only two helicity states**.** Note that string theory treats gravitons as "excitations" of a closed string, as given by Keifer **[39]** , with a term added to a space-time metric, $\bar{g}_{uv}$, such that $g_{uv} \equiv \bar{g}_{uv} + \sqrt{32\pi G} f_{\mu\nu}$ with $f_{\mu\nu}$ a linkage to coherent states of gravitons. This is partly in relation to the Venziano **[40]** expression of $\Delta x \geq \frac{\hbar}{\Delta p} + \frac{l_s^2}{\hbar}\Delta p$, where $G \sim g^2 l_s^2$. Kieffer **[39]** gives a correction due to quantum gravity in page 179 of the order of $\left(\frac{m}{M_{Planck}}\right)^2$ If the mass, $m_{graviton} \sim 10^{-65}$ g**,** it will be hard to measure as an individual "particle**.**" But, if $m_{graviton} \sim 10^{-65}$ g exists, as a macro effect, it may play a role, as indicated by Fig. 1.

1. So, what about representing a graviton as a kink-anti-kink ?
How does this fit in with 't Hooft's deterministic QM?

In 2006, 't Hooft [24] used an equivalence-class argument as an embedding space for simple harmonic oscillators, as given in figure 2 in **[24].** It is also noteworthy to consider that in 2002, 't Hooft **[25]** also wrote in his introduction, that "Beneath Quantum Mechanics, there may be a deterministic theory with (local) information loss. This may lead to a sufficiently complex vacuum state." In addition, the embedding equivalence class structure may be a consequence of a family of

$$\Psi_{i,f} \left[\phi(\mathbf{x})\right]\Big|_{\phi \equiv \phi_{ci,cf}} = c_{i,f} \cdot \exp\left\{ -\int d\mathbf{x}\ \alpha \left[\phi_{Ci,f}(\mathbf{x}) - \phi_0(\mathbf{x})\right]^2 \right\},$$ solutions to a graviton state, if one is

taking the $\phi(x)$ as a kink-anti-kink combination. I.e., looking at a history plot of equivalent solutions to the graviton problem, in a 5 dimensional metric space , as outlined by Belunski and Verdaguer **[41]** and Giovannini **[42]** . This "functional" (if one assumes equivalence classes of solutions) may, with work, be part of a deterministic embedding space for the vacuum space of space-time where the graviton exists. Reformulating the above solution in terms of different values of $\phi_0(x)$ in a wave-functional representation of a graviton, is consistant with for equivalence-class structures. This would mean, for example, a considerable refinement of the metric in 5 dimensions, as stated in Appendix A, [29], $dS^2 = a(w) \cdot \left[\eta_{uv} dx^u dx^v - dw^2\right]$. The closeness of $m_{graviton} \sim 10^{-65}$ to a zero mass should not be seen as a failure of quantum physics, but a success story. However, it is suggested that establishing equivalence classes as part of a procedure to embed gravitons in space-time will require generalizing 't Hooft's equations 4.3 and 4.4 of **[24]** to the following wave functional,. Considering $c_{i,f} \cdot \exp\left\{ - \int d\mathbf{x}\ \alpha \left[\phi_{Ci,f}(\mathbf{x}) - \phi_0(\mathbf{x}) \right]^2 \right\}$, in terms of a solution using ϕ_0 as similar to the equivalence class 't Hooft [24, 25] is working with harmonic oscillators , then a connection is made to equivalence classes showing up in tHoof's 2006 manuscripts figure 2. The over lap with 't Hooft [24] comes from the fact that the functional given by $\Psi_{i,f}\ [\phi(\mathbf{x})]\big|_{\phi \equiv \phi_{ci,cf}}$ is to a good approximation using a $\phi_{Ci,f}(\mathbf{x})$ with an evolution equation similar to what happens with damped harmonic oscillators with additional energy/ information place into the damped harmonic oscillator . Notions of equivalence classes come in choices for $\phi_0(\mathbf{x})$ above. Whether the graviton can actually carry information will be considered in the next section..

IV. HOW MUCH INFORMATION NEEDS TO BE MAINTAINED TO PRESERVE THE COSMOLOGICAL CONSTANTS FROM COSMOLOGICAL CYCLE TO CYCLE?

No clear answer to this question has yet emerged. It is useful however to note that de La Peña **[43]** in 1997 proposed an order-of-magnitude estimate of information to derive a relation between Planck's constant (as a measure of the strength of the field fluctuations) and cosmological constants. If, for example, the fine structure constant has input parameter variance, as was explored by Livio et al, **[44],** with an explanation of why the fine structure constant has $\Delta\tilde{\alpha}/\tilde{\alpha} \le 10^{-5} - 10^{-6}$ when redshift values are z ~ 1.5 compared to the present eraand there is, for example, from QED, a proportional argument that $\tilde{\alpha} \equiv e^2 / \hbar \cdot c$,is

$$\tilde{\alpha} \equiv e^2 / \hbar \cdot c \equiv \frac{e^2}{d} \times \frac{\lambda}{hc} \tag{1.32}$$

However, extrapolating Eq. (1.32) after the turn on of the CMBR at z ~ 1100 380 thousand years after the big bang is inappropriate, so instead, one should look for a similar statement for what $\tilde{\alpha} \equiv e^2 / \hbar \cdot c$ would be at the onset of the big bang. The minimum length can be specified via use of the Snyder geometry inequality [6] $\Delta x \ge \left[(1/\Delta p) + l_s^2 \cdot \Delta p \right] \equiv (1/\Delta p) - \alpha \cdot \Delta p$, to how know much 'information' is stored by $\tilde{\alpha} \equiv e^2 / \hbar \cdot c$. Note that Δx is bounded by the Venziano [40] specifications of minimum length.

$$\Delta x \equiv 10^{\beta} \cdot l_P \sim \left[(1/\Delta p) + l_s^2 \cdot \Delta p \right] \equiv (1/\Delta p) - \alpha \cdot \Delta p \tag{1.33}$$

G. Veneziano [40] wrote , $10^{\beta} \cdot l_P \equiv l_{string}$, where Planck length $l_P \equiv \sqrt{\hbar G / c^3}$, and this minimum length would be a way of specifying 'transferrable information' from the past to the present universe.

V. Conclusions

A. A reliable algorithm is needed in order to store information in a graviton

A way to obtain traces of information exchange , from prior to present universe cycles is finding a linkage between information and entropy. If such a parameterization can be found and analyzed, then Seth Lloyd's **[45]** shorthand for entropy,

$$I = S_{total} / k_B \ln 2 = [\#operations]^{3/4} = \left[\rho \cdot c^5 \cdot t^4 / \hbar \right]^{3/4} \tag{1.34}$$

could be utilized as a way to represent information which can be transferred from a prior to the present universe.. Eq. (1.3) is basic, but that there could be a variation of different intial values of information content placed into, $\hbar$ based upon arguments at and after Eq. **(1.32).**. If, for example, one views gravitons according to the idea refined by Beckwith **[27] (**from Y.J. Ng **[46])** -- that a counting algorithm for entropy is required -- then if, say, the total number of gravitons in inflation is of the order of $n \sim 10^{20}$ gravitons

$\approx 10^{20}$ entropy counts, Eq. (1.34) implies up to $\approx 10^{27}$ operations. If so, there is at least a one-to-one relationship between an operation and a bit of information, so a graviton has at least one bit of information.

B. Sensitivity limits for graviton detectors need to be improved

Note that the initial question posed in the beginning of the paper was: does the graviton have a mass? The stretch-out of a graviton wave, perhaps greater than the size of the solar system, gives, according to [4], an upper limit of a graviton mass of $\lambda_{graviton} > 300 \cdot h_0 kpc \Leftrightarrow m_{graviton} < 2 \times 10^{-29} h_0^{-1} eV$. I .e., a massively stretched graviton wave, at ultra-low frequency, may lead to a low mass limit. However, more careful and actually artificially narrow limits due to experimental searches, as presented by A. Buonanno [47] have narrowed the upper limit to about $10^{-20} h_0^{-1} eV$. Giovanni's [42] Classical and quantum gravity letter is important due to its embedding of an instanton structure in a 5 dimensional GR line element. The author chose a 5th dimensional conduit for information in a more classical model as a way to get about the singularity existing in four dimensions. One which Dr. Li and the author discussed at length in Chongquing that no instanton or instanton- anti instanton structure would survive a cosmic singularity.[48] However, it is possible gravitons have higher frequencies **[27],** which would lead to smaller regions of space needed for high frequency gravitons nucleating in the beginning of the big bang. [27]. A counting algorithm, discussed in Appendix A, may be a way for information to be transferred from a prior universe to our present universe . Note that in $\tilde{\alpha} \equiv e^2 / \hbar \cdot c \equiv \frac{e^2}{d} \times \frac{\lambda}{hc}$, most of the information probably will be packed in the wavelength given as λ which is part of how , $\tilde{\alpha}$, is defined and that the amount of information packed into this wavelength λ is packed into gravitons, as given in **Appendix B**.. However, in order to obtain information put in information/ data for $\tilde{\alpha} \equiv \frac{e^2}{d} \times \frac{\lambda}{hc}$, experiments may require better sensitivity limits than what was assumed with advanced LIGO. Advanced LIGO chooses [48], [49] $h \sim 10^{-23}$ as the maximum sensitivities for higher frequency GW. Eq. (1.35) is a formula for strain values for 1000 Hz GW.

$$h \sim \frac{\lambda}{Lb\sqrt{N\tau}} \sim 10^{-23} \tag{1.35}$$

where L is interferometer length, N is the number of quanta per second at a beam splitter, and τ is the integration time. For LIGO systems the strain value given by Eq (1.54) appears to be $h \sim 10^{-23}$. If one wishes to measure $\tilde{\alpha} \equiv e^2 / \hbar \cdot c \equiv \frac{e^2}{d} \times \frac{\lambda}{hc}$, one may find that $h \sim 10^{-23}$ is inadequate **[28, 48]**

C. Further research questions for investigative inquiry and how to link our inquiry to the overall geometry of the universe

The problem of reconciling the existence of a graviton mass with quantum mechanics , in spin two particles usually having zero mass appears to be resolvable, and may imply linkage between DE and DM in ways richer than the Chapygin gas models [11] By a problem to be solved, the author means the correspondence principle of quantum mechanics ; i.e. ,that spin 2 particles have no mass.. Furthermore, the radius of the universe problem, given by Roos [11], will yield applications of the Friedmann equations used, once there are experimental criteria for determining the Hubble Parameter , and $\Omega \equiv \rho(t) / \rho_{critical}$.

$$r_U \equiv \frac{1}{H \cdot \sqrt{|\Omega - 1|}} \tag{1.36}$$

Combining experimental confirmation of Eq. (1.36) with observations and use of different choices for $H = \frac{\dot{a}}{a}$ and $\Omega \equiv \rho(t) / \rho_{critical}$ will be tied in, with analysis of the diagram of Figure 5 below

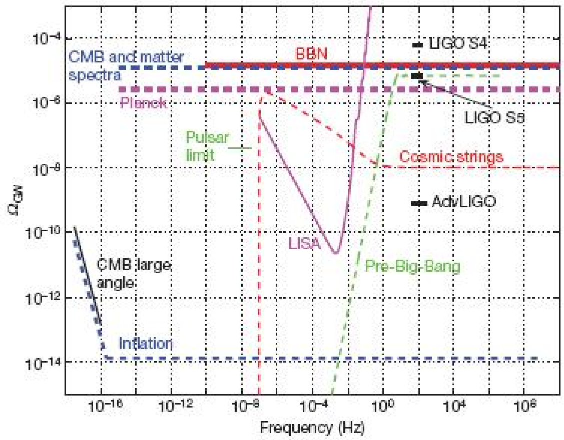

Figure 5. This figure from.B. P. Abbott et al. **[50]** (2009) shows the relation between Ω_g and frequency. The relation between Ω_g and the spectrum $h(v_g, \tau)$ is written by Grishchuk, **[51]** , as

$$\Omega_g \approx \frac{\pi^2}{3}\left(\frac{v}{v_H}\right)^2 h^2(v,\tau), \tag{1.37}$$

The author looks to an interplay between Eq. (1.36) and Eq. (1.37) as a way to resolve questions as to if the universe is open or closed and shed some light as to the existence of relic GW. Answering questions about the inter play between Eq. (1.36) and Eq. (1.37) will use **Appendix C's** view of entropy to modify Eqn (1.37) inputs . Also, try to make refinements to chaplygin gas **[52]** joint DM-DE models

Appendix A : . **An analysis of how graviton mass, assuming branes, can influence expansion of the universe**

Based on (1.10), with inputs from Friedmann equations, $\hbar = c = 1$ so then [14]

$$q = A1 + A2 + A3 + A4 \tag{A.1}$$

where

$$A1 = \frac{C}{a^3}\cdot\left[1\Big/\sqrt{\frac{C}{a^4} - \frac{\kappa}{a^2} + \left(\frac{\rho}{3M_4^2} + \frac{\Lambda_4}{3} + \frac{1}{36}\cdot\frac{\rho^2}{M_P^6}\right)}\right] \tag{A.2}$$

$$A2 = -\left(\frac{\rho}{3M_4^2} + \frac{\Lambda_4}{3} + \frac{1}{36}\cdot\frac{\rho^2}{M_P^6}\right)\Big/\left[\frac{C}{a^4} - \frac{\kappa}{a^2} + \left(\frac{\rho}{3M_4^2} + \frac{\Lambda_4}{3} + \frac{1}{36}\cdot\frac{\rho^2}{M_P^6}\right)\right] \tag{A.3}$$

$$A3 = -\frac{1}{2}\cdot\left[\frac{(d\rho/d\tau)}{3M_4^2} + \frac{(d\Lambda_4/d\tau)}{3} + \frac{1}{18}\cdot\frac{\rho\cdot(d\rho/d\tau)}{M_P^2}\right]\Big/\left[\frac{C}{a^4} - \frac{\kappa}{a^2} + \left(\frac{\rho}{3M_4^2} + \frac{\Lambda_4}{3} + \frac{1}{36}\cdot\frac{\rho^2}{M_P^6}\right)\right]^{3/2} \tag{A.4}$$

$$A4 = \frac{\kappa}{a^3}\cdot\left[\frac{(da/d\tau)}{3}\right]\Big/\left[\frac{C}{a^4} - \frac{\kappa}{a^2} + \left(\frac{\rho}{3M_4^2} + \frac{\Lambda_4}{3} + \frac{1}{36}\cdot\frac{\rho^2}{M_P^6}\right)\right]^{3/2} \tag{A.5}$$

Furthermore, if we are using density according to whether or not 4 dimensional graviton mass is used, then

$$\rho \equiv \rho_0\cdot\left(\frac{a_0}{a}\right)^3 - \left[\frac{m_g c^6}{8\pi G\hbar^2}\right]\cdot\left(\frac{a^4}{14} + \frac{2a^2}{5} - \frac{1}{2}\right) \tag{A.6}$$

So, then one can look at $d\rho/d\tau$, obtaining

$$d\rho/d\tau = -\left(\frac{\dot{a}}{a}\right)\cdot\left[3\cdot\rho_0\cdot\left(\frac{a_9}{a}\right)^3 + 4\cdot\left(\frac{a^4}{14}+\frac{a^2}{5}\right)\cdot\left(\frac{m_g c^6}{8\pi G\hbar^2}\right)\right] \quad \text{(A.7)}$$

Here, use $\left(\frac{\dot{a}}{a}\right) = \sqrt{\frac{C}{a^4}-\frac{\kappa}{a^2}+\left(\frac{\rho}{3M_4^2}+\frac{\Lambda_4}{3}+\frac{1}{36}\cdot\frac{\rho^2}{M_P^6}\right)}$ and assume (A.6) covers ρ, then

b $d\rho/d\tau = -\sqrt{\frac{C}{a^4}-\frac{\kappa}{a^2}+\left(\frac{\rho}{3M_4^2}+\frac{\Lambda_4}{3}+\frac{1}{36}\cdot\frac{\rho^2}{M_P^6}\right)}\cdot\left[3\cdot\rho_0\cdot\left(\frac{a_9}{a}\right)^3 + 4\cdot\left(\frac{a^4}{14}+\frac{a^2}{5}\right)\cdot\left(\frac{m_g}{8\pi G}\right)\right]$ (A.8)

Now, if, to first order, $d\Lambda_4/d\tau \sim 0$ and, also, we neglect Λ_4 as of being not a major contributor

$$d\rho/d\tau \cong -\sqrt{\frac{C}{a^4}-\frac{\kappa}{a^2}+\left(\frac{\rho}{3M_4^2}+\frac{1}{36}\cdot\frac{\rho^2}{M_P^6}\right)}\cdot\left[3\cdot\rho_9\cdot\left(\frac{a_0}{a}\right)^3 + 4\cdot\left(\frac{a^4}{14}+\frac{a^2}{5}\right)\cdot\left(\frac{m_g}{8\pi G}\right)\right] \quad \text{(A.9)}$$

$$A3 \cong \frac{1}{2}\left(\cdot\left[\frac{1}{3M_4^2}+\frac{1}{18}\cdot\frac{\rho\cdot}{M_P^6}\right]\Big/\left[\frac{C}{a^4}-\frac{\kappa}{a^2}+\left(\frac{\rho}{3M_4^2}+\frac{1}{36}\cdot\frac{\rho^2}{M_P^6}\right)\right]^{1/2}\right)\cdot \quad \text{(A.10)}$$
$$\left[3\cdot\rho_0\cdot\left(\frac{a_0}{a}\right)^3 + 4\cdot\left(\frac{a^4}{14}+\frac{a^2}{5}\right)\cdot\left(\frac{m_g}{8\pi G}\right)\right]\cdot$$

Also, then, set the curvature equal to zero, i.e. $\kappa = 0$. So then $A4 = 0$, and

$$A3 \cong \frac{1}{2}\left(\cdot\left[\frac{1}{3M_4^2}+\frac{1}{18}\cdot\frac{\rho\cdot}{M_P^6}\right]\Big/\left[\frac{C}{a^4}+\left(\frac{\rho}{3M_4^2}+\frac{1}{36}\cdot\frac{\rho^2}{M_P^6}\right)\right]^{1/2}\right)\cdot \quad \text{(A.11)}$$
$$\left[3\cdot\rho_0\cdot\left(\frac{a_0}{a}\right)^3 + 4\cdot\left(\frac{a^4}{14}+\frac{a^2}{5}\right)\cdot\left(\frac{m_g}{8\pi G}\right)\right]\cdot$$

Then

$$A2 \cong -\left(\frac{\rho}{3M_4^2}+\frac{1}{36}\cdot\frac{\rho^2}{M_P^6}\right)\Big/\left[\frac{C}{a^4}+\left(\frac{\rho}{3M_4^2}+\frac{1}{36}\cdot\frac{\rho^2}{M_P^6}\right)\right] \quad \text{(A.12)}$$

$$A1 \cong \frac{C}{a^3}\cdot\left[1\Big/\sqrt{\frac{C}{a^4}+\left(\frac{\rho}{3M_4^2}+\frac{1}{36}\cdot\frac{\rho^2}{M_P^6}\right)}\right] \quad \text{(A.13)}$$

Pick, here, $\rho \equiv \rho_0\cdot\left(\frac{a_0}{a}\right)^3 - \left[\frac{m_g}{8\pi G}\right]\cdot\left(\frac{a^4}{14}+\frac{2a^2}{5}-\frac{1}{2}\right)$, after $\hbar = c = 1$, and also set

$$\Phi(\rho,a,C) = \frac{C}{a^4}+\left(\frac{\rho}{3M_4^2}+\frac{1}{36}\cdot\frac{\rho^2}{M_P^6}\right) \quad \text{(1.14)}$$

For the sake of continuity , and book keeping, the author uses the substitutions . $(a_0/a)^3 = (1+z)^3$, and $a \equiv a_0/(1+z)$, Then

$$\rho(z) \equiv \rho_0\cdot(1+z)^3 - \left[\frac{m_g}{8\pi G}\right]\cdot\left(\frac{a_0^4}{14\cdot(1+z)^4}+\frac{2a_0^2}{5\cdot(1+z)^2}-\frac{1}{2}\right) \quad \text{(A.15)}$$

$$A1(z) \cong \frac{C\cdot(1+z)^3}{a_0^3}\cdot\left[1\Big/\sqrt{\Phi\left(\rho(z),a_0/(1+z),C\right)}\right] \quad \text{(A.16)}$$

$$A2(z) \cong -\left(\frac{\rho(z)}{3M_4^2} + \frac{1}{36}\cdot\frac{\rho(z)^2}{M_P^6}\right) \Big/ \left[\Phi\left(\rho(z), a_0/(1+z), C\right)\right] \qquad \text{(A.17)}$$

$$A3(z) \cong \frac{1}{2}\left(\cdot\left[\frac{1}{3M_4^2} + \frac{1}{18}\cdot\frac{\rho(z)\cdot}{M_P^6}\right] \Big/ \left[\Phi\left(\rho(z), a_0/(1+z), C\right)\right]^{1/2}\right)\cdot$$
$$\left[3\cdot\rho_0\cdot(1+z)^3 + 4\cdot\left(\frac{a_0^4/(1+z)^4}{14} + \frac{a_0^2/(1+z)^2}{5}\right)\cdot\left(\frac{m_g}{8\pi G}\right)\right] \qquad \text{(A.18)}$$

$$\Phi\left(\rho(z), a_0/(1+z)\right), C\right) = \frac{C\cdot(1+z)^4}{a_0^4} + \left(\frac{\rho(z)}{3M_4^2} + \frac{1}{36}\cdot\frac{\rho(z)^2}{M_P^6}\right) \qquad \text{(A.19)}$$

So, for $4 < z \leq 0$, i.e., not for the range, say $z \sim 1100$ at 380,000 years after the big bang, q(z) is then

$$q(z) = A1(z) + A2(z) + A3(z) \qquad \text{(A.20)}$$

APPENDIX B: INSTANTONS IN GENERAL RELATIVITY WITH REGARDS TO SPACE-TIME METRICS

The best, physically consistent models of GR-admissible solitons appears to be given by Belunski, and Verdaguer **[**41], in work that ties in the instanton formulation for gravitation with specific metrics in space-time physics. In addition, Givannini [42] gives a kink-anti-kink construction, , Ibanez and Verdaguer **[53]** suggest Instantons only reach speeds up to nearly light speed in nearly infinite distance travel. This instanton- anti instanton nearly fits with QM, and correspondence mass zero values for the graviton in 't Hooft's Deterministic QM The author suggests that a kink-anti-kink construction for the graviton. will be useful to avoid trouble with instantons traveling initially extremely slowly as noted by Ibanez and Verdager [53] . Having a kink-anti kink construction for the graviton would be a way to avoid having low velocities for relic gravitons (if just modeled by kinks- instantons) at their moment of creation. Updating Fig. 4, **[41]** gives an example of how to generalize an instanton from the metric g, with $g \equiv diag\{t\cdot\exp(\phi), t\cdot\exp(-\phi)\}$ when put into the Einstein equations leads to

$$\phi \equiv d\cdot\ln t + \sum_{k=1}^{s} h_k \ln(\mu_k/t) \qquad \text{(B.1)}$$

The 2nd part of this equation roughly corresponds to $\phi_+(z,\tau) = 4\cdot\arctan\left(\exp\left\{\frac{z+\beta\cdot\tau}{\sqrt{1-\beta^2}}\right\}\right)$. Further work by Belunski, and Verdaguer [**41**] yields instanton-anti-instanton solutions that are elaborations of Eq (B1)above. The tie in with Belunski and Verdauer [41] will be in the similarities between Beckwith [30] use of $\arctan\left(\exp\left\{\frac{z+\beta\cdot\tau}{\sqrt{1-\beta^2}}\right\}\right)$ and Belunski and Verdauer [41] using the formula $\arctan\left((bw)^\nu\right)$, as part of their solution containing instantons, as explained in Eq (B.2). The justification [41] for instantons in cosmology is that a kink (anti kink) can be embedded in metrics which are used in General Relativity. As to the warning given by J. Ibanez, and E. Verdaguer **[53]** that instantons travel at speeds very much smaller at the speed of light, in cosmology and reach peak velocities only much later on, at 'infinite; distance from a source. It is important now to reference a kink (instanton) solution presented by Giovannini, **[42]** 2006 , namely from a least action version of the Einstein – Hilbert action for 'quadratic' theories of gravity involving Euler- Gauss-Bonnet. Then Giovannini's [42] equation 6 corresponds to

$$\phi = \tilde{\nu} + \arctan\left((bw)^\nu\right) \qquad \text{(B.2)}$$

Givannini **[42]** also has a representation of Eq (1.47) as a kink, and makes references to an anti-kink solution, in Fig. 1 of his article in **[42].** Furthermore the similarity between Eq. (B.2) and

$\phi_+(z,\tau) = 4 \cdot \arctan\left(\exp\left\{\frac{z+\beta\cdot\tau}{\sqrt{1-\beta^2}}\right\}\right)$ in [30] is obvious. If $\arctan((bw)^v)$ overlaps in behavior with $\sum_{k=1}^{s} h_k \ln(\mu_k/t)$ the problem is amendable to analysis. If a graviton is a kink-anti-kink combination, arising from a 5 dimensional line element**, [37]**

$$dS^2 = a(w)\cdot\left[\eta_{uv}dx^u dx^v - dw^2\right] \tag{B.3}$$

then how the graviton may be nucleated in this space is important.All this is presented to give equivalence between the degree for correspondence between QM, if assumes instanton- anti instanton pairs have thin walls, ie. Nearly box like shapes, as opposed to slopes, as seen in Figure 2 and Figure 4 would indicate a very close fit with QM, and support a non linear theory involving instantons as being similar to results obtained with QM. All this is a better model than elaborate brane models of the graviton i.e. , J. Leach, and W.Lesame.**[54]** Final elaboration of this model is similar to what was done by Ruutu, et al,.**[55]**

APPENDIX C : ENTROPY GENERATION VIA NG'S INFINITE QUANTUM STATISTICS

The author brings up entropy development due to the convergence of the instanton structure brought up in Appendix B, which is also in common with string theory. Furthermore, information counting ties in with information packing as brought up in the use of small graviton creation volume, V, for relic gravitons of a high frequency (short wave length) right after the big bang would be consistent Graviton volume V for nucleation is tiny, well inside inflation. So the log factor drops out of entropy S if V is chosen properly for both Eq. (C.1) and Eq. (C.2). Ng's [46] result begins with a modification of the entropy/partition function Ng used in an approximation of temperature and its variation with respect to a spatial parameter, starting with a given early temperature $T \approx R_H^{-1}$ (R_H can be thought of as a representation of the region of space of the particles in question). Furthermore, assume that the volume of space to be analyzed is of the form $V \approx R_H^3$ and look at a preliminary numerical factor we shall call $N \sim (R_H/l_P)^2$, where the denominator is Planck's length (on the order of 10^{-35} centimeters). We also specify a "wavelength" $\lambda \approx T^{-1}$. So the value of $\lambda \approx T^{-1}$ and of R_H are the same order of magnitude. Note Jack Ng [46] changed conventional statistics: he outlined how to get $S \approx N$, or $S \approx < n >$ (where <n> is graviton density). Begin with

$$Z_N \sim \left(\frac{1}{N!}\right)\cdot\left(\frac{V}{\lambda^3}\right)^N \tag{C.1}$$

This, according to Ng, leads to entropy of the limiting value of, if $S = (\log[Z_N])$

$$S \approx N \cdot \left(\log\left[V/N\lambda^3\right] + 5/2\right) \xrightarrow[Ng-\inf inite-Quantum-Statistics]{} N \cdot \left(\log\left[V/\lambda^3\right] + 5/2\right) \approx N \tag{C.2}$$

But $V \approx R_H^3 \approx \lambda^3$, so unless N in Eq. (C.2) above is about 1, S (entropy) would be < 0, which is a contradiction. Now **Eq. (C.2)** is where [46] introduces removing the N! term in Eq. (C.1) above, removing the expression of N inside the Log expression in Eq. (C.2). Since string theory as presented by Ng [46] is based upon involves instanton like structures, i.e. branes; the quantum infinite statistics, which Beckwith used [27] for gravitons is a limiting case of Appendix I . I.e. the two structures compliment each other.

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