# PROPOSING A NEW STRUCTURE TO THE ELECTROMAGNETIC WAVE - A NEW SOLUTION TO THE MAXWELL'S EQUATIONS IN VACUUM 

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#### Abstract

The author shows that the Maxwell's equations in vacuum have solutions which have helical structure in space while being circularly polarized at the same time. This goes against the universally accepted solutions which treat the electromagnetic wave as sinusoidal wave propagating along a linear path. He shows that the helical wave structure assumes that the fundamental state of the electromagnetic wave is the circularly polarized state and not the linearly polarized state. Since the photon is theorized to exist in a circularly polarized state, the proposed new solution is consistent with that picture. The author proposes a simple experiment using a maser to confirm the veracity of the proposed helical structure.


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## 1 Introduction

We know that the beauty of the Maxwell's equations (given below) is that while they are very simple linear equations, all aspects of electromagnetism could be explained by them.
(i) $\nabla \bullet \xi=\rho / \varepsilon_{o}$,
(ii) $\nabla \times \boldsymbol{\xi}=-\partial \mathbf{B} / \partial t$,
(iii) $\nabla \cdot \mathbf{B}=0$.
(iv) $\quad c^{2} \nabla \times \mathbf{B}=\mathbf{j} / \varepsilon_{o}+\partial \xi / \partial t$

Since we propose to study the transmission of the electromagnetic (EM) waves in vacuum, we shall take the charge density, $\rho$ and the electric current density $\mathbf{j}$ as zero which gives

$$
\begin{array}{llllll}
(i) & \nabla \bullet \xi & =0, & (i i) & \nabla \bullet \xi & =  \tag{2}\\
(i i i) \\
\nabla \bullet \mathbf{B} & =0 . & (i v) & c^{2} \nabla \times \mathbf{B} / \partial t & = & \partial \xi / \partial t
\end{array}
$$

We shall now solve these equations to see how the concept of the EM wave emerges from them. Subsequently we shall modify the approach suitably to arrive at the new solutions. We shall follow Feynman's insightful approach here in solving these equations in the conventional way [1]. We know that the first equation can be written as

$$
\begin{equation*}
\nabla \bullet \xi=\frac{\partial \xi_{x}}{\partial x}+\frac{\partial \xi_{y}}{\partial y}+\frac{\partial \xi_{z}}{\partial z}=0 \tag{3}
\end{equation*}
$$

Here we assume that there are no variations with $y$ and $z$, so that the last two terms could be taken as zero. Hence, we have

$$
\begin{equation*}
\frac{\partial \xi_{x}}{\partial x}=0 \tag{3A}
\end{equation*}
$$

This means that $\xi_{x}$ is a constant in the $x$-direction. If we study Maxwell's equation (2.iv), assuming that just as in the case of the electric field, there is no variation in $y$ and $z$ directions in the magnetic field also, it can be seen that $E_{x}$ is also a constant in time. Such a field could be conveniently taken as zero as we are interested in only dynamic fields. Therefore we may take $\mathrm{E}_{\mathrm{x}}=0$. In other words, the electric field exists only in the y and z directions which are perpendicular to the direction of propagation. As a first step, for the sake of simplicity, we may assume that the electric field has a component only in the y-direction and obtain a solution on that basis. Later we may take up the case where the electric field has a component only in the z-direction. Then, the general solution could always be expressed as the superposition of the two cases.

Let us take the Maxwell's equation (2.ii) and equate the projections along the three coordinate axes separately to obtain

$$
\begin{equation*}
(\nabla \times \xi)_{x}=\frac{\partial \xi_{z}}{\partial y}-\frac{\partial \xi_{y}}{\partial z} ; \quad(\nabla \times \xi)_{y}=\frac{\partial \xi_{x}}{\partial z}-\frac{\partial \xi_{z}}{\partial x} ; \quad(\nabla \times \xi)_{z}=\frac{\partial \xi_{y}}{\partial x}-\frac{\partial \xi_{x}}{\partial y} . \tag{4}
\end{equation*}
$$

Here $(\nabla \mathrm{x} \xi)_{\mathrm{x}}$ will be zero because the derivatives with regard to y and z are zero. Note that from (3) we have already taken $\xi_{y}$ as a constant while $\xi_{z}$ is taken as zero. $(\nabla x \xi)_{y}$ is zero because the first term which is a derivative of $\xi_{\mathrm{x}}$ which is zero while the second term is zero for reasons already stated. The only component of curl $\xi$ which is not zero is $(\nabla x \xi)_{z}$ which is equal to $\partial \xi_{y} / \partial \mathrm{x}$. Setting the three components of $(\nabla \mathbf{x} \boldsymbol{\xi})$ equal to the corresponding components of $-\partial \mathbf{B} / \partial \mathrm{t}$, we obtain

$$
\begin{equation*}
\frac{\partial B_{x}}{\partial t}=0 ; \quad \frac{\partial B_{y}}{\partial t}=0 ; \quad \frac{\partial B_{z}}{\partial t}=-\frac{\partial \xi_{y}}{\partial x} \tag{5}
\end{equation*}
$$

Since the x and y components of the magnetic field have zero time derivatives, they represent constant fields. If we study Maxwell's equation (2.ii), it can be seen that $\mathrm{B}_{\mathrm{x}}$ is also a constant In time. Such a field could be conveniently taken as zero as we are interested in only dynamic fields. Therefore, we may take $B_{x}=0$. The last equation in (5) shows that the electric field has only the $y$-component while the magnetic field has only the z-component. This means $\boldsymbol{\xi}$ and $\mathbf{B}$ are perpendicular to each other.

Let us now take the last Maxwell's equation whose components can be written as

$$
c^{2}[\nabla \times \mathbf{B}]_{x}=c^{2} \frac{\partial B_{z}}{\partial y}-c^{2} \frac{\partial B_{y}}{\partial z}=\frac{\partial \xi_{z}}{\partial t} ; c^{2}[\nabla \times \mathbf{B}]_{y}=c^{2} \frac{\partial B_{x}}{\partial z}-c^{2} \frac{\partial B_{z}}{\partial x}=\frac{\partial \xi_{y}}{\partial t}
$$

$$
\begin{equation*}
c^{2}[\nabla \times \mathbf{B}]_{z}=c^{2} \frac{\partial B_{y}}{\partial x}-c^{2} \frac{\partial B_{x}}{\partial y}=\frac{\partial \xi_{x}}{\partial t} . \tag{6}
\end{equation*}
$$

Out of these, only the term $\partial \mathrm{B}_{z} / \partial \mathrm{x}$ is not equal to zero. So these three equations gives us simply

$$
\begin{equation*}
-c^{2} \frac{\partial B_{z}}{\partial x}=\frac{\partial \xi_{y}}{\partial t} . \tag{7}
\end{equation*}
$$

Now taking partial differentiation with retard to $t$ and using the last equation in (5), we obtain the wave equations

$$
\begin{equation*}
\frac{\partial^{2} \xi_{y}}{\partial x^{2}}-\frac{1}{c^{2}} \frac{\partial^{2} \xi_{y}}{\partial t^{2}}=0 \quad ; \quad \frac{\partial^{2} B_{z}}{\partial x^{2}}-\frac{1}{c^{2}} \frac{\partial^{2} B_{z}}{\partial t^{2}}=0 . \tag{8}
\end{equation*}
$$

Similarly we can obtain the wave equation for $\xi_{z}$ and $B_{y}$ also as

$$
\begin{equation*}
\frac{\partial^{2} \xi_{z}}{\partial x^{2}}-\frac{1}{c^{2}} \frac{\partial^{2} \xi_{z}}{\partial t^{2}}=0 \quad ; \quad \frac{\partial^{2} B_{y}}{\partial x^{2}}-\frac{1}{c^{2}} \frac{\partial^{2} B_{y}}{\partial t^{2}}=0 . \tag{8A}
\end{equation*}
$$

We know the solutions for the wave equations are given by

$$
\begin{align*}
\psi_{y} & =\xi_{y} e^{-i(\omega t-k x)} ; \psi_{z}=\xi_{z} e^{-i(\omega t-k x)} .  \tag{9}\\
\psi_{y}^{\prime} & =B_{y} e^{-i(\omega t-k x)} ; \psi_{z}^{\prime}=B_{z} e^{-i(\omega t-k x)} .
\end{align*}
$$

where $\omega$ is the angular frequency and k is the wave vector. Combining both, the wave equation


The electric and the magnetic fields are perpendicular to each other and the direction of propagation. Note that the sinusoidal curve represents the field on the line of propagation.

## Figure 1

in a general direction will be given by

$$
\begin{equation*}
\psi=\xi e^{-i(\omega t-\mathbf{k} . \mathbf{r})} \tag{9A}
\end{equation*}
$$

Similarly, we may obtain the wave equation for the magnetic component also which may be written as

$$
\begin{equation*}
\psi^{\prime}=\mathbf{B} e^{-i(\omega t-\mathbf{k} . \mathbf{r})} \tag{9B}
\end{equation*}
$$

Note that the magnetic field will always be perpendicular to the electric field. Another important point to be kept in mind is that the solutions represent not a single wave, but a wave front which has its components of the electric field $\xi_{y}$ and $\xi_{y}$ (also the corresponding magnetic fields) constant along the y and z directions at any instant.

## 2 The New Solutions of Maxwell's Equation

We shall now look for the solution of the Maxwell's equations in a different way. Earlier in (3) we had taken $\partial \xi_{y} / \partial y=\partial \xi_{z} / \partial z=0$, which lead us to the result, $\partial \xi_{x} / \partial x=0$. The assumption, $\partial \xi_{y} / \partial y=\partial \xi_{z} / \partial z=0$ means that at any instant, the electric field in the transverse directions is a constant one. In fact, to satisfy the condition of zero divergence for the electric field we need not take $\xi_{y}$ and $\xi_{z}$ as constants. We may as well take

$$
\begin{equation*}
\frac{\partial \xi_{y}}{\partial y}=-\frac{\partial \xi_{z}}{\partial z} . \tag{10}
\end{equation*}
$$

Note that this relation when applied in (3) gives us the equation, $\partial \xi_{x} / \partial x=0$. Let us examine the implications of such an assumption. The relation given in (10) shows that the electric field has a gradient along the $y$ and $z$-axes. But note that when $\xi_{y}$ increases with $y, \xi_{z}$ decreases with $z$.

The simplest way to interpret (10) is to assume $\xi_{y}$ and $\xi_{z}$ to be the field at a point which undergoes simple harmonic motion (SHM) along y and $z$ directions simultaneously. We may attribute the field at the point to be directly proportional to the displacement from the x -axis. There are four ways of defining $\xi_{y}, \xi_{z}$, y and z so that (10) is satisfied. They are

$$
\begin{equation*}
\xi_{y}= \pm \alpha y \quad \text { and } \quad \xi_{z}=\mp \alpha z \tag{11}
\end{equation*}
$$

where $y=R \cos [\omega t-k x]$ and $z=\mp R \sin [\omega t-k x]$.
Of these four possibilities, we shall take up the first case given by the following set of equations for study right now.

$$
\begin{gather*}
\xi_{y}=\alpha y \quad \text { and } \quad \xi_{z}=-\alpha z  \tag{12}\\
\text { where } y=R \cos [\omega t-k x] \text { and } \quad z=-R \sin [\omega t-k x] . \tag{12A}
\end{gather*}
$$

Note that the helical wave defined by (12) and (12A) will be executing rotation in the clockwise direction. We may attribute to any point on the helical path a magnetic field which is perpendicular to the resultant of $\xi_{y}$ and $\xi_{z}$ at that point. This is to state that the point which is
executing the SHM has got electric and magnetic fields which are perpendicular to each other and also perpendicular to the x -axis. We may take

$$
\begin{equation*}
B_{z}=\alpha y ; \quad B_{y}=\alpha z \tag{12B}
\end{equation*}
$$

Note the difference between (12B) and (12). Here the $B_{z}$ depends on $y$, not on $z$. Likewise, $B_{y}$ depends on $z$, not $y$. Now we observe that for a given value of $x$, the locus of the point which has field components $(\xi, \mathbf{B})$ where $\xi=\left(\xi_{\mathrm{y}}+\xi_{\mathrm{z}}\right)$ and $\mathbf{B}=\left(\mathbf{B}_{\mathrm{y}}+\mathbf{B}_{\mathrm{z}}\right)$ will be a circle with radius R because we have

$$
\begin{equation*}
y^{2}+z^{2}=R^{2} \tag{12C}
\end{equation*}
$$

From (12) and (12B) one may get the impression that there is nothing to limit the magnitude of the electric and the magnetic fields. However, the fact the frequency of the SHMs in the transverse directions has to the equal to that of the wave propagation introduces a limit to the value of R to $1 / \mathrm{k}$. We shall be discussing this issue in detail in section 4 .

Let us now proceed further with this line of reasoning. Let us now take equation (4)

$$
\begin{equation*}
\frac{\partial \xi_{z}}{\partial y}-\frac{\partial \xi_{y}}{\partial z}=-\frac{\partial B_{x}}{\partial t} ; \quad \frac{\partial \xi_{x}}{\partial z}-\frac{\partial \xi_{z}}{\partial x}=-\frac{\partial B_{y}}{\partial t} ; \quad \frac{\partial \xi_{y}}{\partial x}-\frac{\partial \xi_{x}}{\partial y}=-\frac{\partial B_{z}}{\partial t} \tag{13}
\end{equation*}
$$

On the basis of (11), we observe that $\partial \xi_{z} / \partial y=\partial \xi_{y} / \partial z=0$. Further, since $\xi_{x}=0$ everywhere, its derivative will also be zero. This would leave us with

$$
\begin{equation*}
\frac{\partial B_{x}}{\partial t}=0 ; \quad \frac{\partial B_{y}}{\partial t}=\frac{\partial \xi_{z}}{\partial x} ; \quad \frac{\partial B_{z}}{\partial t}=-\frac{\partial \xi_{y}}{\partial x} \tag{13A}
\end{equation*}
$$

Let us now take the last Maxwell's equation whose components can be written as

$$
\begin{gather*}
c^{2}[\nabla \times \mathbf{B}]_{x}=c^{2} \frac{\partial B_{z}}{\partial y}-c^{2} \frac{\partial B_{y}}{\partial z}=\frac{\partial \xi_{z}}{\partial t} ; c^{2}[\nabla \times \mathbf{B}]_{y}=c^{2} \frac{\partial B_{x}}{\partial z}-c^{2} \frac{\partial B_{z}}{\partial x}=\frac{\partial \xi_{y}}{\partial t} \\
c^{2}[\nabla \times \mathbf{B}]_{z}=c^{2} \frac{\partial B_{y}}{\partial x}-c^{2} \frac{\partial B_{x}}{\partial y}=\frac{\partial \xi_{z}}{\partial t} \tag{14}
\end{gather*}
$$

Here unlike in the case of $\xi_{y}$ and $\xi_{z}$, we have $\partial \mathrm{B}_{z} / \partial \mathrm{y}=\alpha=\partial \mathrm{B}_{\mathrm{y}} / \partial \mathrm{z}$. This will make $[\nabla \mathrm{x} \mathbf{B}]_{\mathrm{x}}=0$. Of the remaining derivatives given in (14), only $\partial \mathrm{B}_{z} / \partial \mathrm{x}$ and $\partial \mathrm{B}_{\mathrm{y}} / \partial \mathrm{x}$ will not be zero. Therefore (13) can be simplified to give

$$
\begin{equation*}
-c^{2} \frac{\partial B_{z}}{\partial x}=\frac{\partial \xi_{y}}{\partial t} ; ; \quad c^{2} \frac{\partial B_{y}}{\partial x}=\frac{\partial \xi_{z}}{\partial t} \tag{14A}
\end{equation*}
$$

Now using the above equation as well as (13A) and taking partial differentiation with retard to $t$ and we obtain the wave equations

$$
\begin{align*}
& \frac{\partial^{2} \xi_{y}}{\partial x^{2}}-\frac{1}{c^{2}} \frac{\partial^{2} \xi_{y}}{\partial t^{2}}=0 ; \frac{\partial^{2} B_{z}}{\partial x^{2}}-\frac{1}{c^{2}} \frac{\partial^{2} B_{z}}{\partial t^{2}}=0  \tag{15}\\
& \frac{\partial^{2} \xi_{z}}{\partial x^{2}}-\frac{1}{c^{2}} \frac{\partial^{2} \xi_{z}}{\partial t^{2}}=0 ; \frac{\partial^{2} B_{y}}{\partial x^{2}}-\frac{1}{c^{2}} \frac{\partial^{2} B_{y}}{\partial t^{2}}=0 \tag{15A}
\end{align*}
$$

We know that the general solution for these equations are given by (9A) itself. In other words, despite the fact that the wave represented in (15) and (15A) are helical waves existing in the physical space, we obtain the same wave function as given in (9A) as the solution.

Let us now examine the solution obtained in depth. We observe that although (15) and (15A) together represents a circularly polarized wave travelling along a helical path with its axis along the x -axis, its form is same as what we obtained for the wave been travelling along the $x$-axis. In fact, it is easy to see that the helical path will coincide with the linear path when we take the radius of the helical path to be zero. In that sense, the helical path solution is more general than the linear path solution. The fact that (15) and (15A) does not express the physical amplitude of the wave should not be taken as a serious defect in the solution. Note that the wave equation for the electric and the magnetic fields appear as separate wave equations. In a similar way, we may express the wave equations representing the spatial oscillations also by way of separate equations. For this, we substitute for $\xi_{y}$ and $\xi_{z}$ from (12) into (15) and (15A) to obtain

$$
\begin{equation*}
\frac{\partial^{2} y}{\partial x^{2}}-\frac{1}{c^{2}} \frac{\partial^{2} y}{\partial t^{2}}=0 ; \frac{\partial^{2} z}{\partial x^{2}}-\frac{1}{c^{2}} \frac{\partial^{2} z}{\partial t^{2}}=0 . \tag{15B}
\end{equation*}
$$

We should keep in mind that all these wave equations involving $\xi_{y}, \beta_{y}, \xi_{z}, B_{z}, y$ and $z$ are coupled. Since the conventional approach treats the radius of the helical wave in the real space as zero, the amplitude of the oscillations in the physical space turns out to be zero. Therefore, the wave as defined in (15B) does not make its appearance in the conventional approach.

We saw that in the case of the conventional solutions discussed in the previous section, $\xi_{y}$ and $\xi_{z}$ has no variation in the $y$ and $z$ directions. The variations in $\xi_{y}$ and $\xi_{z}$ are in the $x-$ direction and they travel as a wave front. In the case of the helical wave solution obtained now, we observe that $\xi_{y}$ and $\xi_{z}$ would vary in the $y$ and $z$ directions. In fact, they vary in proportion to the displacement from the axis. In this case, we have to examine how the wave front is formed. It is reasonable to assume that the displacement of the point along the $y$ and $z$ directions will set off disturbance in those directions. Therefore, we can imagine a wave front of helical waves moving parallel to each other just like in the conventional approach. The only difference is that here we have a front constituted by the helical waves while in the conventional approach the wave front is constituted by the plane waves.

The form of the EM wave in terms of the new solution will be as given in figure 2 . If we take any point on the helical wave with its axis along the x -axis, then the electric field will be directed perpendicular to the $x$-axis in the $y-z$ plane along the direction of the vector
representing the displacement from the x -axis while the magnetic vector at the same point will be orthogonal to both the x -axis and the electric vector. Note that in the conventional approach while the wave length and the frequency are related to each other, the magnitude of the electric vector (also the magnetic vector) is not connected to them in any manner. Going by the same reasoning, the radius of the helical wave should also be able to take on any value. Actually, this lack of clarity on the value of the electric and the magnetic vectors is quite surprising. There is no reason why only this aspect of the wave should remain undefined when

(a) shows that the field of the EM wave moves on a helical path with the electrical field always perpendicular to the direction of propagation but parallel to the radius vector while the magnetic field is orthogonal to both electric and the direction of propagation. (b) gives the transverse cross sectional view of the helical wave. Note that the electric field (magnetic field not shown) is always directed outward in line with the radius vector.

## Figure. 2

all other aspects (frequency and the wave length) are clearly defined for the wave. In section 5, we shall show that there is justification in limiting the spatial amplitude of the EM wave and attributing a definite value for the magnitude of the electric vector and consequently for the radius of the helical wave.

## 3 Solutions with the Electric Vector Oriented Differently

In the preceding discussion we have considered the case where the electric field is defined by (12) and (12A). This applies to the case where the radial vector and the electric field are in phase. In fact, the condition of the zero divergence of the electric field would be satisfied even if we had taken

$$
\begin{equation*}
\xi_{y}=-\alpha y \quad \text { and } \quad \xi_{z}=\alpha z \tag{16}
\end{equation*}
$$

where $y=R \cos [\omega t-k x]$ and $z=-R \sin [\omega t-k x]$.
In this case, the electric vector would be having a phase difference of $\pi$ with the radius vector and hence would be directed inward when the EM wave is viewed in the direction of the propagation of the EM wave (see figure 3). Note that in this case the direction of the electric
field is the reverse of the case represented by (12) and (12A). Therefore, the field created would be in the opposite direction. This phase difference does not in any way affect the


The figure gives the view of the EM wave in the direction of the progression. Here the phase of the electric field denoted by the arrow in the epicycle differs from that of the radius vector by $\pi$.

Figure. 3
behavior of the EM wave significantly. Note that when the radius of the helical wave becomes zero the, it becomes the EM wave as is understood at present.

Apart from these two, there are two other possibilities which we shall discuss now. The third possibility arises when we take

$$
\begin{equation*}
\xi_{y}=\alpha y \quad \text { and } \quad \xi_{z}=-\alpha z \tag{17}
\end{equation*}
$$

where $y=R \cos [\omega t-k x]$ and $z=R \sin [\omega t-k x]$.
This represents the case where the rotation of the electric field and the radial vector are in the

(a)

(b)
(a) shows the case where the rotation of the helical wave is in the anti-clockwise direction while the electric field rotates in the opposite direction. The field has positive component $\xi_{y}$ at $P$ on the $y$-axis and becomes $-\xi_{z}$ at $Q$, then $-\xi_{y}$ at $R$ and $\xi_{z}$ at $S$. (b) represents the case with the same direction of rotation but with the phase of of the electric vector differing from that of case (a) by $\pi$.

Figure 4
opposite directions. Here $\xi_{y}$ and $\xi_{z}$ are related to y and z just as in the first case. However, (17A) represents a wave which is rotating in the anti-clockwise direction (see figure 4.a ). Here The electric vector does not maintain its direction with regard to the radius vector.

The fourth possibility arises when we take

$$
\begin{align*}
& \xi_{y}=-\alpha y \text { and } \quad \xi_{z}=\alpha z \text {, }  \tag{18}\\
& \text { where } y=R \cos [\omega t-k x] \text { and } z=R \sin [\omega t-k x] \text {. } \tag{18A}
\end{align*}
$$

The direction of the electrical field at any point on the helical wave is given in figure 4(b). Here also the picture is similar to the third case except that there is a phase difference of $\pi$.

We can construct four more different combinations of the rotation of the helical wave and the electrical vector. Those would pertain to the states having the rotation to the helical wave and the polarization vector reverse of those which are given above. Note that in the conventional description of the EM wave, these different structures would not show up since the radius of the helical wave in the physical space is taken to be zero. This would reduce the possibilities to only two helicity states which pertains to the right handed circular polarization and the left handed circular polarization. It may be possible to set up an experiment which could establish the fact that the EM wave has a helical structure in the physical space. We shall discuss about it in section 5 .

## 4 Determining the Spatial Amplitude of the EM Wave

These new solutions of the Maxwell's equations do not alter our views regarding all the phenomena which are understood in the conventional approach. The reason why the possibility of the "new solutions" were never explored may be traced to the historical origin of the concept of the EM wave. Right from the beginning, the EM wave was thought of in terms of the linearly polarized wave and the linearly polarized state was considered as the most basic state of the wave. The circularly polarized wave was assumed to be a composite state formed by two such basic linearly polarized states with a phase difference of $\frac{1}{2} \pi$. Because of such an assumption, the solutions sought from the Maxwell's equations were for waves which are linearly polarized. And it can be easily seen from the above discussion that a linearly polarized state is not consistent with the helical structure of the wave in the physical space. It allows only such solution where the electric and the magnetic fields have no variations in the transverse directions and this is the one which is universally followed in all text books.

Let us now see if we could determine the physical amplitude of the EM wave exactly. We saw that the circularly polarized EM wave represents the locus of a point possessing electric field which undergoes two mutually orthogonal oscillations in the physical space in the transverse directions with a phase difference $\frac{1}{2} \pi$. The electric (also the magnitude of the magnetic) field at the point is directly proportional to its displacement from the x -axis, which is taken as the line of propagation of the EM wave. The path taken by such a point in the
transverse direction will be a circular one (see figure 5). The simplest case of the circular motion is obtained if we assume that it is executed at the velocity of light, c. Any other velocity would involve introduction of a new attribute to the electromagnetic wave. Since the


The figure represents transverse view of a propagating electromagnetic wave. The vertical and the horizontal lines stand for two perpendicular oscillations having phase difference of $\frac{1}{2} \pi$. Here the circular velocity is taken as $c$.

Figure. 5
wave is also progressing with the velocity c, It is obvious that by the time the EM wave progresses the distance of one wave length, it would have executed one full circle in the transverse direction. This would mean that the circumference of the circular path is also $\lambda$. Therefore, we may conclude that the radius, R of the circle would be given by

$$
\begin{equation*}
R=\lambda / 2 \pi=1 / k \tag{19}
\end{equation*}
$$

Once the value of $R$ is fixed, then we have from (12) that

$$
\begin{equation*}
\xi=\alpha R=\alpha / k=(\alpha / 2 \pi) \lambda \tag{19A}
\end{equation*}
$$

This shows that the magnitude of the electric vector is proportional to the wave length. Since the magnitude of the magnetic vector is equal to that of the electric vector in the case of the EM wave, we have

$$
\begin{equation*}
B=(\alpha / 2 \pi) \lambda \tag{19B}
\end{equation*}
$$

## 5 Discussion

It is quite surprising to note that that no body had noticed that the Maxwell's equations contained the helical wave solution within it. We know that photon, as it possesses unit spin, has to be treated as a circularly polarized state. In that sense, the new solutions are consistent with this treatment. But strictly speaking a photon cannot be treated as a wave as it represents the particulate aspect of light. All the same, the circular polarized sate appears to be more fundamental than the linear one. But we should keep in mind that a linearly polarized state can be obtained by the superposition of two circularly polarized states of opposite helicity. The circularly polarized state in turn could be obtained by the superposition of the
linearly polarized states which are perpendicular to each other but with a phase difference of $\frac{1}{2} \pi$. But if the new solution obtained in section 2 is correct, then any circularly polarized state, even those formed by the superposition of two linearly polarized states, could be treated as a pure state. This means that if we have a source of circularly polarized light, it could be used to verify the correctness of the proposed helical structure of the EM wave.

The problem we face when we want to measure the helical structure of the EM wave is that this property gets camouflaged by the formation of the secondary waves formed with each point on the original wave as a source. In other words, we cannot obtain a single wave for observation. If we take a source of radiation with high intensity, there the waves will interfere with each other making it lose the helical structure. The best way would be to observe the photons passing through a hollow cylinder of length, say equal to the wave length of the

> $S$ is a very weak monochromatic microwave source. $P Q$ is the cylindrical adjustable aperture while OO' represents the screen which measures the intensity of the microwave falling on it. If the helical wave hypothesis is correct, the aperture could block all waves having wave length more than $2 \pi r$.

Figure. 6
photon to be observed. The experimental set up is very simple (see figure.6). We have to have a source of EM wave preferably Maser having wavelength in the range of, say, 0.10 mtr . A metal sheet of 0.10 mtr thickness having a cylindrical aperture ( PQ ) with adjustable radius could be installed close to the Maser source. It is important to have the length of the aperture in the range of the wave length of the EM wave to be used to avoid formation of the secondary waves at Q . A screen should be place behind the metal sheet to capture the microwave energy passing through the aperture.

Let the cylindrical aperture be kept very large to begin with. The intensity of the energy received on the screen from the Maser source will be quite high. Now as we reduce the radius of the cylindrical aperture, the intensity received on the screen would keep on decreasing. If the helical wave hypothesis is correct, then when the radius of the aperture becomes less than $\lambda / 2 \pi$, there should be a sudden drop in the intensity of the energy received on the screen. In principle, there should be an absolute cut off at this aperture. But then due to interference which cannot be avoided completely, some energy may leak through the aperture. But the dip in the energy intensity received on the screen should be sharp. In the conventional approach, the aperture radius of $\lambda / 2 \pi$ does not have any significance and therefore the reduction in the intensity should be smooth.

Another foolproof method to check the veracity of the hypothesis is to use a source of the monochromatic radiation in the microwave range so weak that only one photon is emitted at an instant. This way the possibility of two waves interfering with each other and affecting its basic structure could be avoided. Since the light source is so weak that it emits only one photon in one instant, the helical structure of the wave would be maintained. Here some objection could be raised against the proposal on the ground that the moment we opt for the photon picture, we have to discard the wave picture, and treat the photon as a point particle. To avoid such controversies, we just propose that the wave train emitted by the source is a single one at a time so that there is no interference to destroy its basic structure. We shall make the radius of the cylindrical aperture adjustable. Initially, the radius of the aperture may be kept at twice the wave length of the wave as in the earlier experiment. If the basic structure of the EM radiation is a helical one in space with a radius $\lambda / 2 \pi$, then there should be a clear cut off of the radiation passing through the aperture when its radius becomes $\lambda / 2 \pi$. In terms of the conventional approach, there will not be any cut off although the intensity may fall off. So the existence of a cut off at the aperture radius of $\lambda / 2 \pi$ will be the proof for the helical structure of the EM wave.

## References:

[1] Richard Feynman, The Feynman Lectures on Physics (The Definitive Edition), Vol.2, Dorling Kindersley (India) Pvt. Ltd.,(2009). p.20.4-20.6.

