On the Estimated Precession of Mercury's Orbit

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Abstract.

The Sun's orbital motion around the Solar System barycentre contributes a small quadratic component to the gravitational energy of Mercury. The effect of this component has previously gone unnoticed, but it generates a significant part of the observed precession of Mercury's orbit. Consequently, the residual precession currently attributed to general relativity theory by default (43 arcsec/cy) is too large by 6.6 arcsec/cy.

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1. Introduction

For many years, the observed precession of planet Mercury's orbit has been acclaimed a confirmation of General Relativity theory because of its remarkable, close agreement. Nevertheless, in view of the enduring singularity problem and unnatural features of black-hole theory, and the unattainable unification of GR with quantum theory, it is appropriate to question the finality of this agreement. To this end, a serious omission has indeed been identified in the list of contributions which make up the total precession.

The orbit of planet Mercury has been calculated satisfactorily by several investigators; see Clemence [1], Brouwer & Clemence [2], Nobili & Will [3]. In these calculations, Newton's inverse square law has been applied to set up the differential equations of motion using the instantaneous measured distances and velocities between Mercury, the Sun and planets. Clemence [1] has listed the contributions to the motion of the perihelion of Mercury due to the individual planets, solar oblateness, general precession in longitude, and relativity.

No effect on the perihelion has ever been attributed to the Sun travelling around the Solar System barycentre, independently of Mercury. In fact, the circular motion of the Sun produces a toroidal component in the potential energy of Mercury, which generates some precession of Mercury's orbit. Such a direct physical effect on Mercury is not negated by calculating everything in heliocentric coordinates. Consequently, the residual precession attributable to general relativity theory must be less than the proclaimed 43 arcsec/cy. This effect has never been incorporated automatically, nor acknowledged in the precession calculations to date.

A thought-experiment may help to clarify this assertion as follows:

a) First, imagine Mercury alone orbiting the Sun in absolute free space, in an ideal closed elliptical orbit with no perturbations.

b) Now let the Sun be moved by some unspecified mechanism in a circle of radius equal to 1.068 solar radii. Mercury can still orbit this moving Sun, but its orbit will not be exactly elliptical as before, and its energy will be different.

c) If the period of the solar motion is very short relative to Mercury's orbital period, then Mercury will be steadily attracted towards the *centre* of this apparent 'toroidal' Sun, even though the *instantaneous* attraction is always to the Sun itself, according to Newton's inverse square law. The absolute energy of Mercury will be different from the closed ellipse energy, but more importantly, the *form* of the energy expression for the 'toroidal' Sun will cause precession of Mercury's orbit, similar to an oblate Sun.

d) If the period of solar motion around the circle is made long relative to Mercury's orbital period, then the orbit of Mercury will become more nearly elliptical as it almost accommodates the Sun's movement. The associated precession of Mercury's orbit will be correspondingly small but still remain finite because there is no reason for it to suddenly fall to zero at any particular value of Sun velocity.

e) If the period of solar motion is now set at 11.86 years, then Jupiter could be introduced to do the job of moving the Sun around the circle, with the Sun's centre of motion now at the Jupiter/Sun barycentre. Mercury will obviously continue under the influence of the moving Sun, in addition to experiencing the attraction of Jupiter in the normal way. Thus, some precession of Mercury's orbit will remain due specifically to the moving Sun, and it amounts to around 6.6 arcsec/cy. The precession due to Jupiter has always been calculated using heliocentric coordinates, but this does not automatically include the above effect of a moving Sun on Mercury's orbit.

2. Derivation of precession due to moving Sun

To quantify this stronger binding energy for Mercury, imagine that the period of Jupiter around the Sun were reduced from 11.86 years to one millisecond. An observer on some planet X, well outside the Solar System, would then see the 'Sun' blurred into a torus around the Jupiter/Sun centre of mass. The absolute potential at the observer's location would be equal to the work done in bringing planet X from infinity to that location. The *average* gravitational force, directed towards the *centre of mass*, would not be an inverse square law but would contain a small additional toroidal term, similar to the field of an oblate Sun. If planet X were allowed to move inside the orbit of Jupiter it would obviously continue to experience this toroidal field. Subsequently, if the Sun's period were increased, and planet X were to start orbiting the Sun, the toroidal field would exist at a much reduced level.

The binding energy of Mercury in the field of an orbiting Sun may be calculated using Newton's law for each position of the Sun around the described orbit. First, from Figure 1, let Mercury be regarded as stationary while the Sun travels rapidly around the centre C at radius r_{SC}. Then for the Sun at distance r_1 from Mercury:

$$r_1^2 = r_{1C}^2 + r_{SC}^2 - 2r_{1C}r_{SC}\cos\theta \ . \tag{1}$$

Over a complete orbit of the Sun, $\cos\theta$ cancels out on average; so Mercury is at an average squared radius $r_a^2 = (r_{1C}^2 + r_{SC}^2)$ from this toroidal Sun. The *instantaneous* gravitational force exerted by the Sun on Mercury is given by the inverse square law,

($F_1 = - GMM_1 / r_1^2$); therefore, the force directed towards C is $F = F_1\cos\alpha$. When averaged over one orbit of the Sun, this force becomes:

$$F_{av} \approx -\left(\frac{GMM_1}{r_a^2}\right)\left[1+\left(\frac{7r_{SC}^2}{4r_a^2}\right)\right] \quad . \tag{2}$$

Thus, a small additional term distinguishes this from the inverse square law of force. The absolute potential energy of Mercury can be calculated by integrating this force from infinity:

$$PE_{av} \approx -\left(\frac{GMM_1}{r_a}\right)\left[1+\left(\frac{7r_{SC}^2}{12r_a^2}\right)\right] \quad . \tag{3}$$

This also equals the kinetic energy that Mercury would gain if it fell straight from infinity to its position r_{1C} from C. It follows that the angular momentum and kinetic energy of orbiting Mercury would be different from that around a stationary Sun. We are interested in the particular form of the force, which causes some precession of the perihelion of Mercury.

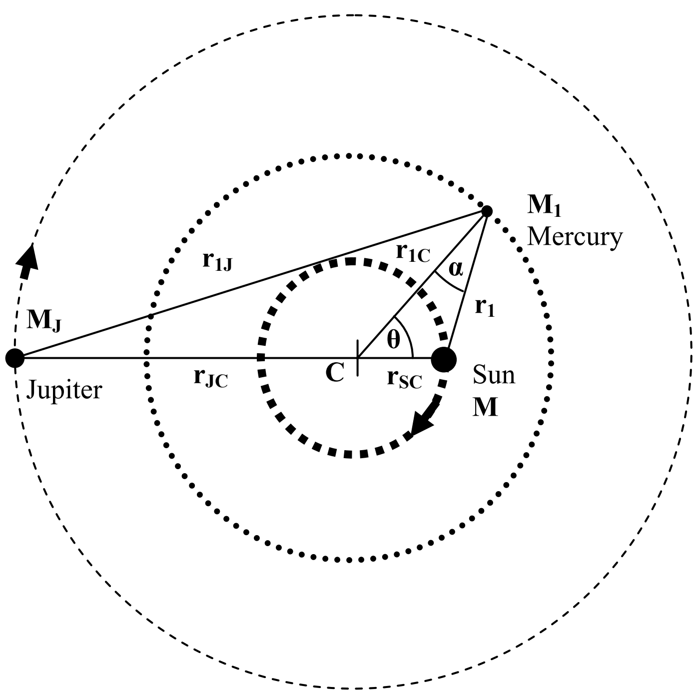

Figure 1. Schematic diagram showing Jupiter and the Sun moving rapidly around their centre of mass C. Mercury is considered to be stationary during one orbit of the Sun.

In practice, the Sun's period around C due to Jupiter is much longer than Mercury's period, so the toroidal component of potential energy must be decreased. Given that Jupiter is causing this effect, through r_{SC} , the attenuation coefficient will be set in terms of the influence of Jupiter upon the ideal Sun-Mercury system; namely, set at the ratio of gravitational work done on Mercury by Jupiter relative to work done by the Sun. Then from Eq.(2), the elemental increase in Mercury's potential energy at r_a , attributable to the Sun's actual motion around C, would be:

$$dE_1 \approx -\left[\left(\frac{GMM_1}{r_a^{\ 2}}\right)\left(\frac{7r_{SC}^{\ 2}}{4r_a^{\ 2}}\right)\right]\left[\frac{GM_J M_1/r_{1J}}{GMM_1/r_a}\right]dr_a \tag{4}$$

$$\approx -\left(\frac{GMM_1}{r_a^{\ 3}}\right)\left(\frac{7r_{SC}^{\ 3}}{4r_{JC}^{\ 2}}\right)dr_a \quad , \tag{5}$$

where $(M_J/M) = (r_{SC}/r_{JC})$ and $r_{1J} \approx r_{JC}$ on average. This expression may be integrated to determine the corresponding work done in bringing Mercury into the Solar System from infinity. After adding the orthodox potentials we get the total *averaged* coordinate gravitational potential at Mercury,

$$P_1 \approx -\left(\frac{GM}{r_a}\right)-\left(\frac{GM}{r_a^{\ 2}}\right)\left(\frac{7r_{SC}^{\ 3}}{8r_{JC}^{\ 2}}\right)-\left(\frac{GM_J}{r_{1J}}\right) \quad . \tag{6}$$

Although the above choice of attenuation coefficient looks arbitrary, it is reasonable because if we keep r_{SC} fixed but let $M_J \to M$, and $r_{1J} \to r_a$ in Eq.(4), then the attenuation coefficient is equal to unity, and M would become 2M. Choice of this coefficient is limited by using only the available potential energy factors.

The additional term in Eq.(6) is very dependent upon the mass of Jupiter, and contributes to the precession of Mercury's perihelion as follows. Kurth [4] has shown how an arbitrary variation in gravitational field of the Sun, away from Newton's law, such as:

$$F = -\left(\frac{GM}{r^2}\right)\left(1+\frac{k}{r}\right) \quad , \tag{7}$$

would rival general relativity theory by producing 43 arcsec/cy precession in Mercury's elliptical orbit, when $k \approx 6.26(GM/c^2)$. Integration of this function yields a gravitational potential of similar form to Eq.(6), (without the Jupiter term):

$$P = -\left(\frac{GM}{r}\right) - \left(\frac{GM}{r^2}\right)\left(\frac{k}{2}\right) \quad . \tag{8}$$

Consequently, by substituting our constants in place of k, Eq.(6) should lead to around 5.8 arcsec/cy precession for Mercury's elliptical orbit.

If Saturn is substituted for Jupiter in Eq.(4), the appropriate changes in r_{SC} and r_{JC} would lead to a precession in Mercury's orbit of 0.28 arcsec/cy. Similarly, if the elemental potential energies for the planets can be summed as scalar quantities, then the total precession for Mercury increases to 6.3 arcsec/cy. Inclusion of an eccentricity factor $(1\text{-}e^2)$ would make this become 6.6 arcsec/cy, and only 36.4 arcsec/cy would remain to be attributed to GR theory.

Precessions attributable to general relativity effects in the orbits of Venus, Earth and Icarus, (Shapiro et al [5], Lieske & Null [6], Sitarski [7]) are likewise decreased.

3. Conclusion

Motion of the Sun, around the Solar System barycentre, adds a small quadratic term to the gravitational binding energy of Mercury. This term has been overlooked previously, but it is responsible for 6.6 arcsec/cy precession in the orbit of Mercury. The residual part of the total measured precession is therefore this much less than the acclaimed 43arcsec/cy. Fortunately, Einstein's general relativity theory is capable of describing the world without singularities, and with a real prospect of unification [8]. This is very welcome news for physicists because singularities have invariably been due to a breakdown of theory.

Acknowledgements

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