A Variable Model of the Fine-Structure Constant

Ioannis Iraklis Haranas
Department of Physics and Astronomy
York University
314A Petrie Science Building
Toronto – Ontario
CANADA
M3J-1P3
E-mail: ioannis@yorku.ca

Abstract

Recent evidence suggests that the fine-structure constant $\alpha = \frac{e^2}{\hbar c}$, a measure of the strength of the electromagnetic interaction between photons and electrons, is slowly increasing over cosmological timescales. High-resolution measurements of quasar spectra suggest that there has been a variation $\Delta\alpha/\alpha$ =-0.72 ± 0.18×10^{-5} over the past 6-10 Gyr.To model this, we propose variability in the speed of light that produces a cosmological time variation $|\dot{\alpha}/\alpha|$ = 10^{-15} and 10^{-16} yr^{-1} at z = 3 which also agrees with the observational spectral data.

Key words: cosmology, fundamental constants, quantum-electrodynamics.

1. Introduction

The fine-structure constant α is a very important parameter in Quantum Electrodynamics (QED). Sommerfeld introduced it for the first time in 1916, so that he could describe the fine structure of the atomic levels and the corresponding resonance lines. Not very long after that it was also understood that α was equally important for the description of the structure of the atomic and molecular spectra. Today we know that any electromagnetic phenomenon may be described in terms of powers of α. In real life, α is not a true constant as quantum field theory establishes and high-energy physics confirms, simply because coupling constants depend on distance, momentum or energy. All that is because of vacuum polarization [2]. The value of the fine-structure constant α is equal to α = 1/ 137.0359895 [2], but the CODATA suggested a different value based on the 1997 adjustment of fundamental constants of physics which was equal to α = 1/ 137.03599993(52). After all we should mention that the value of α is known with a ruther high accuracy of approximately $\approx 4\times10^{-9}$. Its high accuracy might be one thing but that does not exclude the possibility that α could have been different in early cosmological times. In recent articles [4], [5], [6], [7] there is evidence to suggest that the fine-structure constant α is slowly increasing over cosmological timescales.

According to the definition the speed of light c in vacuum is given by:

$$c^2 = \frac{1}{(\varepsilon\, \mu)} \tag{1}$$

where ε and μ are the electric permitivity and magnetic permeability of the of the vacuum. We can now consider a variable speed of light c (t) given by the relation below:

$$c^2(t) = \frac{1}{[\eta(t)]} \tag{1a}$$

where $\eta(t)$ is a function of time. This way the vacuum becomes a variable medium, where the velocity of the electromagnetic waves depends on the variation and magnitude of $\eta(t)$. Quoting Moffat's work [9] we say that in particular the increase in the value of c in the early universe would be traced to a phase transition in the function $\eta(t)$, associated with a spontaneous symmetry breaking of Lorentz invariance of the vacuum. It's more fissible to assume a variable speed of light in order to explain the hyperfine constant variation since we can better resolve its implications to cosmology. It is not clear at this stage what would be the advantages of a variable e or $\hbar$.

2. Using a Variable Speed of Light Fine-Structure Constant

First let us now redifine the fine-structure constant which is known to be equal to:

$$\alpha = \frac{\mathrm{e}^2}{\hbar \mathrm{c}} \tag{1b}$$

and e is the electron charge, c the speed of light and $\hbar$ is Planck's constant. If α is meant to varry in time it could be in general a function of time $\alpha(t)$. That would also require that either all of its quantities change in time or the variation of one could also produce some kind of change. Assume now that the only one varrying in time is c and let it vary according to the relation:

$$c(t) = c_o \xi(t) \tag{2}$$

where ξ(t) is a scalar field in the preferred frame of reference [9] and c_o is the present value for the speed of light. Substituting into (1) we obtain:

$$\alpha(t) = \frac{e^2}{\hbar c_o \xi(t)} = \frac{\alpha_o}{\xi(t)}. \tag{3}$$

That makes $\dot{\alpha}/\alpha$ in general equal to:

$$\frac{\dot{\alpha}}{\alpha} = -\frac{\dot{c}}{c} \tag{3a}$$

Here we have assumed that e and $\hbar$ are not changing in time and are the known constants.

One of the advantages of variable speed light cosmology is that it can solve the problems of the horizon, flatness, as well as that of particle relics in the early universe when $\xi(t)$ has really high values [8], [10], [11], [12]. In the big bang model the horizon puzzle remains. How can be possible for regions, which were never in causal contact to have the same physical properties? This can be solved if somebody thinks that the value of $c(t)$ corresponds to light traveling faster in the early universe, also allows for the horizon to be much larger, allowing for causal contact between regions. Therefore, the flatness problem can be explained if we assume that the light speed undergoes as a sharp change in a phase transition, and decreases as the universe expands.

3. Moleling the Speed of Light

First let us assume that the scalar field function describing the light speed transition is of the form:

$$\xi(t) = \frac{1}{1 - \Psi(t)\left[\exp\left(1 - \frac{t}{T}\right)^{m/n} - 1\right]} \tag{4}$$

where Ψ(t) is some kind of slowly varying function of time *t* as *t* approaches zero, and *m, n* just positive integers. Furthermore there is a particular moment $t = t_p$ where $\Psi(t)$ can go to an increase up to 0.582 resulting in a sudden increase in c(t). This sharp increase in c(t) will now correspond to a phase transition in the function η(t) in (1a) in such a way that $\eta(t_p) \approx 0$. Next, T is the present age of the universe. We can now see that (4) will make the the speed of light equal to:

$$c(t) = \frac{c_o}{1 - \Psi(t)\left[\exp\left(1 - \frac{t}{T}\right)^{m/n} - 1\right]} \tag{5}$$

when $c\,(T) = c_0$ and if $t \to 0$ we have:

$$c(t) = \frac{c_o}{1 - \Psi(t)[\exp(1) - 1]} \approx \frac{c_o}{1 - 1.718\Psi(t)}. \tag{6}$$

From the above we then have a change in the speed of light that is equal to:

$$\frac{\Delta c}{c} = \frac{c - c_o}{c_o} = \frac{\Psi(t)\left[\exp\left(1 - \frac{t}{T}\right)^{m/n}\right]}{1 + \Psi(t)\left[\exp\left(1 - \frac{t}{T}\right)^{m/n} - 1\right]} \tag{7}$$

4 First Modelling of the Fine-Structure Constant

Using relation (5) we now write the fine-structure constant as follows:

$$\alpha(t) = \frac{\alpha_o}{1 + \Psi(t)\left[\exp\left(1 - \frac{t}{T}\right)^{m/n} - 1\right]} \tag{8}$$

where α_o is the present value of the fine structure constant or $\alpha(t_o) = \alpha_o$. Equation (8) gives the following variation in α:

$$\frac{\Delta\alpha}{\alpha} = \frac{\alpha - \alpha_o}{\alpha_o} = \Psi(t)\left[\exp\left(1 \text{-} \frac{\mathrm{t}}{\mathrm{T}}\right)^{m/n} - 1\right] \tag{9}$$

Similarly, we obtain the ratio of $\dot{\alpha}/\alpha$ to be equal to:

$$\frac{\dot{\alpha}}{\alpha}=-\frac{-\dot{\Psi}(t)+\left[\frac{\exp\left(1-\frac{t}{T}\right)^{m/n}\left[m\Psi(t)\left(1-\frac{t}{T}\right)^{m/n}+n(t-T)\dot{\Psi}(t)\right]}{n(t-T)}\right]}{1+\Psi(t)\left[\exp\left[\left(1-\frac{t}{T}\right)^{m/n}-1\right]\right]} \tag{10}$$

5. Numerical Results of the First Modeling Function

Following [7] we assume that particular time t_P is long before the nucleosynthesis time ie: time $t_P << t_{NS}$ whete the redshift z was of the order of 10^9. Furthermore, we assume that $\psi(t_{BBR}) \approx \psi(t_{NS}) < 10^{-3}$ and t_{BBR} is the time of the cosmic black-body radiation. If we now assume $\psi(t) = \psi \approx$ constant and $t/T = 0.125$ corresponding to $z \approx 3$, and $T = 13.9$ Gyr, the time variation of α below gives the following numerical results when $m = 1$ and n taking all the different values in Table 1:

$$\frac{\dot{\alpha}}{\alpha}=\frac{\Psi\exp\left(1-\frac{t}{T}\right)^{m/n}\left(1-\frac{t}{T}\right)^{\frac{m}{n}-1}}{nT\left[1+\Psi\left[\exp\left(1-\frac{t}{T}\right)^{m/n}-1\right]\right]} \tag{11}$$

Table 1

m	n	Δc/c	Δα/α	$\dot{\alpha}/\alpha$ yr^{-1}
1	1	2.405×10^{-5}	8.848×10^{-6}	1.725×10^{-15}
1	2	2.551×10^{-5}	9.388×10^{-6}	9.799×10^{-16}
1	3	2.605×10^{-5}	9.583×10^{-6}	6.821×10^{-16}
1	4	2.632×10^{-5}	9.684×10^{-6}	5.229×10^{-16}
1	5	2.649×10^{-5}	9.745×10^{-6}	4.238×10^{-16}
1	6	2.660×10^{-5}	9.787×10^{-6}	3.563×10^{-16}
1	7	2.668×10^{-5}	9.817×10^{-6}	3.073×10^{-16}
1	8	2.674×10^{-5}	9.839×10^{-6}	2.702×10^{-16}

Table 2

m	n	$\Delta c/c$	$\Delta\alpha/\alpha$	$\dot{\alpha}/\alpha$ yr^{-1}
1	1	2.405×10^{-5}	8.848×10^{-6}	1.725×10^{-15}
2	1	2.160×10^{-5}	7.948×10^{-6}	2.707×10^{-15}
3	1	1.966×10^{-5}	7.233×10^{-6}	3.228×10^{-15}
4	1	1.810×10^{-5}	6.659×10^{-6}	3.464×10^{-15}
5	1	1.683×10^{-5}	6.193×10^{-6}	3.521×10^{-15}
6	1	1.579×10^{-5}	5.810×10^{-6}	3.468×10^{-15}
7	1	1.493×10^{-5}	5.494×10^{-6}	3.347×10^{-15}
8	1	1.422×10^{-5}	5.231×10^{-6}	3.186×10^{-15}

6. A Second Modeling Function

As a second trial, we can also use the following function to model the light speed transition at early times of the universe's history:

$$\xi(t) = \frac{1}{1 - \Psi(t)\cos^{-1}\left(\frac{t}{T}\right)} \tag{12}$$

where again T denotes the present age of the universe. Then again the speed of light can be written as follows:

$$c(t) = \frac{c_o}{1 - \Psi(t)\cos^{-1}\left(\frac{t}{T}\right)} \tag{13}$$

Again $c(T) = c_o$ and for $t \to 0$ we have:

$$c(t) = \frac{c_o}{1 - \frac{\pi}{2}\Psi(t)} \tag{14}$$

and, $\psi(t_P) = 2/\pi = 0.636$ results in a sudden increase in c(t). A change in the light is:

$$\frac{\Delta c}{c} = \frac{c - c_o}{c_o} = \Psi(t)\cos^{-1}\left(\frac{t}{T}\right) \tag{15}$$

7. A Second Model of the Fine Structure Constant

The fine structure constant becomes:

$$\alpha(t) = \frac{\alpha_o}{1 - \Psi(t)\cos^{-1}\left(\frac{t}{T}\right)} \tag{16}$$

Then the functional variation of the fine structure constant takes the form:

$$\frac{\Delta\alpha}{\alpha} = \frac{\alpha - \alpha_o}{\alpha_o} = -\frac{\Psi(t)}{\left[1 - \Psi(t)\cos^{-1}\left(\frac{t}{T}\right)\right]}. \tag{17}$$

Finally the ratio of $\dot{\alpha}/\alpha$ becomes:

$$\frac{\dot{\alpha}}{\alpha} = \frac{\dfrac{\Psi(t)}{\sqrt{1-\left(\frac{t}{T}\right)^2}} - T\dot{\Psi}(t)\cos^{-1}\left(\frac{t}{T}\right)}{T\left[1 + \Psi(t)\cos^{-1}\left(\frac{t}{T}\right)\right]} \tag{18}$$

8. Numerical Results of the Second Modeling Function

Using the same parameters and assumptions as before we obtain:

$$\frac{\Delta c}{c} = 1.445 \times 10^{-5} \tag{19}$$

$$\frac{\Delta\alpha}{\alpha} = -1.000 \times 10^{-5} \tag{20}$$

$$\frac{\dot{\alpha}}{\alpha} = 7.251 \times 10^{-16} \text{ yr}^{-1} \tag{21}$$

From (19) we can see that there is an increase of 1 part in 10^5 in light's present speed c_o = 2.99792458×10^8 m/sec [3]. In analysis presented elsewhere [13], one of the most reliable estimate of the possible deviation of the fine-structure constant at z = 2-4 from its present z = 0 is:

$$\frac{\Delta\alpha}{\alpha} = \left(-4.6 \pm 4.3[stat] \pm 1.4[syst]\right) \times 10^{-5} \tag{22}$$

Thus, an upper bound can be derived at present for a long-term variability of α:

$$\left|\frac{\dot{\alpha}}{\alpha}\right| < 1.4\times10^{-14}\, yr^{-1} \tag{23}$$

6. Conclusions

We have used a varying speed of light model to explain a recently reported variation in the fine structure constant. Two functions were used and the parameters were calculated first for the different values of the indices m and n of the first function and the results were tabulated. Second, another trigonometric function was used and results were calculated. To agree with the Cosmic Microwave Background of the standard model the assumption of $t_P << t_{NS}$ was made at a redshift $z = 10^9$. Thus, our simple model of a varying fine structure constant demonstrated that the use of a variable speed of light can be in agreement with modern cosmological data.

References

[1] Davies P., C., W., Davis Tamara, M., Lineweaver, Charles, H., *Nature* vol. 418, 8 August 2002, p: 602-603.

[2] Okun L., B., Fundamental constants of Nature. In Proc. 15th Int. Conf. On At. Physics: Zeeman–effect centenary. Van der Waals-Zeeman Lab., University of Amsterdam (in Press) hep-ph/9612249.

[3] Herbert L., Anderson, The Second Edition of Physics Vade Mecum, A Physicist's Desk Reference, American Institute of Physics, 1989, p; 4.

[4] Webb, J., K et al., *Phys. Rev. Lett*., 82, 884-887, 1999.

[5] Sognalia, A., and Cowle, L., L., *Nature,* 398, 667-888, 1999.

[6] Webb, J., K., et al., *Phys., Rv., Lett*., 87, 091301, 2001.

[7] Murphy M., T., et al Mon. Not. R. Astron., Soc., 327, 1208-1222, 2001.

[8] Moffat J., W., *International Journal of Modern Physics*, D2, 351, 1993

[9] Moffat, J., W., astro-ph/ 0109350, v2, 11 Oct 2001.

[10] Albrecht, A., Magueijo, J., *Phys. Rev.,* D 59, 043516, 1999, astro-ph/9811018.

[11] Barrow, J., D., *Phys. Rev.* D 59, 043515, 1999.

[12] Clayton, M., A., Moffat, J., W., *Phys. Letters* B460, 263.

[13] Varsalovich D., A., Poteklin A., Y., Ivanchik, A., V., in Proc. 18th Int., Conf., on X-Ray and Inner-Shell Processes X-99, AIP Press, Chicago 2000, in press.