A Different Derivation of the Calogero Conjecture

Ioannis Iraklis Haranas
Physics and Astronomy Department
York University
314 A Petrie Science Building
Toronto – Ontario
CANADA
E mail: ioannis@yorku.ca

Abstract:

In a study by F. Calogero [1] entitled " Cosmic origin of quantization" an expression was derived for the variability of *h* with time, and its consequences if any, of such an idea in cosmology were examined. In this paper we will offer a different derivation of the Calogero conjecture based on a postulate concerning a variable speed of light, [2] in conjuction with Weinberg's relationship for the mass of an elementary particle.

Introduction:

Recently, Calogero studied and reconsidered the " universal background noise which according to Nelsons's stohastic mechanics [3,4,5] is the basis for the quantum behaviour. The position that Calogero took was that there might be a physical origin for this noise. He immediately pointed his attention to the gravitational interactions with the distant masses in the universe. Therefore, he argued for the case that Planck's constant can be given by the following expression:

$$h \approx \alpha\, G^{1/2} m^{3/2} \left[R(t)\right]^{1/2} \qquad (1)$$

where α is a numerical constant = 1, G is the Newtonian gravitational constant, m is the mass of the hydrogen atom which is considered as the basic mass unit in the universe, and finally $R(t)$ represents the radius of the universe or alternatively: "part of the universe which is accessible to gravitational interactions at time t." [6]. We can obtain a rough estimate of the value of *h* if we just take into account that $R(t)$ is the radius of the universe evaluated at the present time t_0 ie. $R(t_0) \cong 10^{-28}$ cm. Taking the value of $q \approx 1$, then equation (1) gives the following value:[7]

$$h \cong 6 \times 10^{-26}\, cm^2\, g \sec^{-1} \qquad (2)$$

We should mention that in (1) the only quantity that varies with time is $R(t)$.[6]

Next, let us assume that the expanssion rate $dR(\mathrm{t})/dt$ of the radius of the universe $R(t)$, at any time t, is equal to the speed of light $c(t)$ at the same time t, and the relation can be written as follows:

$$c(t) = \dot{R}(t) = \frac{dR(t)}{dt} \qquad (3)$$

It should be noted that $R(t)$ is not a physical length, but instead, it can be thought as a universal length of space itself with a present rate of expansion given by:

$$c_0 = \frac{dR(t_0)}{dt} \qquad (4)$$

Let us now consider Weinberg's relation for the mass of an elementary particle m_π which is given by:[8]

$$m_\pi = \left(\frac{\hbar^2 H}{Gc}\right)^{1/3} \qquad (5)$$

where H is Hubble's constant, c is the speed of light, G is the gravitational constant, and $\hbar$ is Planck's constant. If we now assume that c and H are functions of time then solving for h we have:

$$h^2 = \frac{4\pi^{\,2} G m_\pi^{\,3}}{H(t)} c(t) \qquad (6)$$

Substituiting now for $c(t) = \dot{R}(t)$ and for at any time t the expression for Hubble's constant namely: $H(t) = \frac{\dot{R}(t)}{R(t)}$ and simplifying we obtain the final expression:

$$h \approx 2\pi\, \mathrm{G}^{1/2} (m_\pi)^{3/2} [R(t)]^{1/2} \qquad (7)$$

We observe that (7) is the same exprssion as the one derived by Calogero in [1] but differs by only the numerical value of the constant α which, in our case, is just different than 1 and has the value of 6.28. In addition, we can now assume that the mass of the elementary particle is the mass of the hydrogen. In [2] Geata obtains the value $h \approx 6\times10^{-}$

^{26} cm^{2}g sec^{-1}. However, even if we take the typical value of an elementary particle $m_\pi \approx 10^{-25}$ g we obtain:

$$h \approx 5.128 \times 10^{-27}\, cm^2 g \sec^{-1} \tag{8}$$

If we now use for the mass of the elementary particle the mass of the hydrogen in our equation, we get a value for h which is greater by almost a factor of ten, ie:

$$h \approx 3.500 \times 10^{-25}\, cm^2 g \sec^{-1} \tag{9}$$

Next, let us demonstrate the analytical time dependances of *h* for the different model universes. For a universe with radiation where $R(t) \propto t^{1/2}$, then a universe with matter where $R(t) \propto t^{2/3}$ and finally a De-Sitter exponentially expanding universe where $R(t) \propto e^{Ht}$ we have respectively:

$$\begin{aligned}
h &\approx 2\pi\, \mathrm{G}^{1/2} m^{3/2} \left[(2H_0)^{1/2} R_0 t^{1/2}\right]^{1/2} \\
h &\approx 2\pi\, \mathrm{G}^{1/2} m^{3/2} \left[R_o \left(\frac{3H_0}{2}\right)^{2/3} t^{2/3}\right]^{1/2} \\
h &\approx 2\pi\, \mathrm{G}^{1/2} m^{3/2} R_0^{1/2} e^{\frac{Ht}{2}}
\end{aligned} \tag{10}$$

Taking (10) in the present era $t = t_0 = 1/ H_0 = 0.98 \times 10^{10}\, h^{-1}$ y $= 3.635 \times 10^{17}$ sec, and $0.5 \leq h \leq 0.85$. [9] we obtain the following values of h rerspectively:

$$\begin{aligned}
h &\approx 6.099 \times 10^{-27}\, cm^2 g \sec^{-1} \\
h &\approx 7.007 \times 10^{-27}\, cm^2 g \sec^{-1} \\
h &\approx 8.465 \times 10^{-27}\, cm^2 g \sec^{-1}
\end{aligned} \tag{11}$$

Since we are now in the matter era, we can easily see that when taking into account the expansion of the universe we can write Calogero's result in terms of the average matter density, assuming that none of the other constants change in time as follows:

$$h \approx \frac{5.929 \times 10^{-32}}{\rho_{\mathrm{m}}^{1/6}} \tag{12}$$

Upon expansion of the universe during which the mass density drops will make the value of *h* grow. If we now put the mass of an elementary particle $m_\pi \approx 10^{-25}$ g instead of the mass of hydrogen and substituting for the average density in the universe ρ_m = 2×10^{-29} g/cm^3 we obtain the following value for h:

$$h \approx 3.599\times10^{-27}\,cm^2 g\sec^{-1} \tag{13}$$

The fact is that not only the mass of hydrogen, but also the mass of an elementary particle, gives us the value of h as derived by Calogero which could simply mean that any elementary particle might can also be considered as a basic mass unit in the universe which contributes to Calogero's origin of quantization.

Conclusions

A different derivation of Calogero's conjecture is given in this brief paper. The derivation is based on Weinberg's relation for mass of an elementary particle, and also on the notion of a variable speed of light $c(t)$. A numerical result of Planck's costant was obtained when the mass of the elementary particle was substituted for the mass of hydrogen instead. Furthermore, three different expressions were obtained in which the time dependance of h were obtained for three possible types of universes, and its value was estimated for the present era in the history of these universes. All values were within the actual value range of h that we know today. Finally an expession for h changing with matter density of the universe was obtained and showed that h changes as $\rho^{-1/6}$.

References:

[1] F., Calogero, Cosmic Origin of Quantization, Phys., Lett., A 228, 1997, 335-346.

[2] I., I., Haranas, A Simple Calculation of a Possible Variation in the Speed of Light, Journal of Theoretics, Vol # 3, No 2, 2001.

[3] E., Nelson, Derivation of the Schrodinger equation from Newtonian mechanics, Phys. Rev., 150, 1966, 1079-1085.

[4] F.Guerra, Structural Aspects of Stohastic Mechanics and Stohastic Field Theory, Phys., Rep., 77, 1981, 263-312.

[5] E., Nelson, Quantum Fluctuations,Princeton University Press, 1985.

[6] G., Gaeta, On the Cosmological Implications of the Calogero Conjecture, http:// citeseer.nj.nec.com/cachedpage/431116/2.

[7] As in Ref [6]

[8] S., Weinberg, Gravitation and Cosmology, John Wiley, New York, 1972.

[9] P., J., E., Peebles Principles of Physical Cosmology, Princeton University Press, 1993, p: 25, 102.