Prime sieve using multiplication operation table

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abstract. Ben Green and Terence Tao showed that for any positive integer *k*, there exist infinitely many arithmetic progressions of length *k* consisting only of prime numbers. [14] Four parallel proofs of Szemer´edi's theorem have been achieved; one by direct combinatorics, one by ergodic theory, one by hypergraph theory, and one by Fourier analysis and additive combinatorics. Even with so many proofs, Professor T. Tao points out that with this problem, there remains a sense that our understanding of this result is incomplete; for instance, none of the approaches were powerful enough to detect progressions in the primes, mainly due to the sparsity of the prime sequence. [22] Oliver Lonsdale Atkin introduced a prime sieve using irreducible binary quadratic forms and modular arithmetic; the algorithm enumerates representations of integers by certain binary quadratic forms. A way that uses modular arithmetic is widely known: $6n+\delta$, $12n+\delta$, $30n+\delta$, $60n+\delta$.[31] In this paper, we assert that the composite number of the $12n+1,5,7,11$ series as selected by a Modular Arithmetic and Multiplication Table are not random but consist of very structural and regular arithmetic progression groups.

1. Introduction

Look through a list of prime numbers and you'll find that it's impossible to predict when the next prime will appear. The list seems chaotic, random, and offers no clues as to how determine the next number. It is hard to guess at a formula that could generate this kind of pattern. In fact, this procession of primes resembles a random succession of numbers much more than it does a nice orderly pattern. [8] [21]

This paper starts from the question of "Can we express prime numbers and recognize them spatially such as Prime Spiral?" The prime spiral, also known as Ulam's spiral, is a plot in which the positive integers are arranged in a spiral, with primes indicated in some way along the spiral.

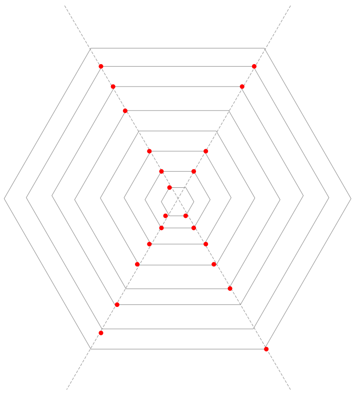


Figure – 1) Prime numbers are aligned to X-shape of 4 groups with a period of $12n$.(except 2, 3)

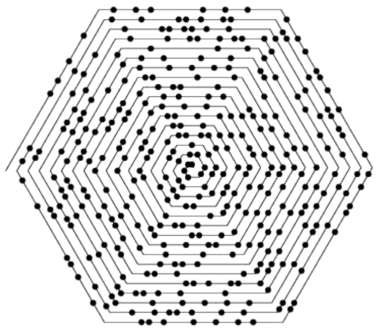


Figure – 2) A hexagonal prime spiral can also be constructed, as illustrated above (Abbott 2005, wolfram).

1.1. Primes in Modular Arithmetic

If a and d are integers, with d non-zero, then a remainder is an integer r such that a = qd + r for some integer q, and with $0 \le |r| < |d|$.[46] Putting prime numbers on the regular hexagon, every prime number except 2 and 3 is contained in the $12n+1,5,7,11$ series, is sorted into 4 kinds of remainder groups—1,5,7, and 11—and belongs to at least one of these 4 groups.

1.2 Primes in Arithmetic Progression

Dirichlet's theorem, states that for any two positive coprime integers a and d, there are infinitely many primes of the form $a+nd$, where $n \ge 0$. In other words: there are infinitely many primes which are congruent to a modulo d. The numbers of the form $a+nd$ form an arithmetic progression [37]

$$a,\ a+d,\ a+2d,\ a+3d,\ \cdots a+nd$$

and Dirichlet's theorem states that this sequence contains infinitely many prime numbers.[33] Szemerédi's theorem generalizes the statement of van der Waerden's theorem. A theorem of Szemerédi asserts that all subsets of the integers with positive upper density will contain arbitrarily long arithmetic progressions. [22] Also any given arithmetic progression of primes has a finite length. Green-Tao settled an Szemerédi' conjecture by proving the Green-Tao theorem (The primes contain arbitrarily long arithmetic progressions). It follows immediately that there are infinitely many AP-k for any k (integer $k \ge 3$, an AP-k (also called PAP-k) is k primes in arithmetic progression) [23]

2. The Specific Composite Numbers of the $12n+1,5,7,11$ series

Like prime numbers, composite numbers appear intuitively irregular and seem to be difficult to group into a pattern. But, the specific composite numbers are generated by a certain rule and that rule is that composite numbers consist of sixteen arithmetic progression groups of different lengths; each group and each arithmetic progression included in that group forms that rule.

$25 = 5\times5,\ 35 = 5\times7,\ 49 = 7\times7,\ 55 = 5\times11,\ 65 = 5\times13,\ 77 = 7\times11,$

$85 = 5\times17,\ 91 = 7\times13,\ 95 = 5\times19,\ 115 = 5\times23,\ 119 = 7\times17$

2.1 Generating the Composite Number of the $12n+1,5,7,11$ series

Let us denote the set A_n is all elements of the $12n+1,5,7,11$ series(n=0,1,2,…); the set P_n is all elements of prime numbers as comprised in A_n; the set C_n is all elements of composite numbers as comprised in A_n

1	2	3	4	5	6	7	8	9	10	11	12
13	14	15	16	17	18	19	20	21	22	23	24
[25]	26	27	28	29	30	31	32	33	34	[35]	36
37	38	39	40	41	42	43	44	45	46	47	48
[49]	50	51	52	53	54	[55]	56	57	58	59	60
61	62	63	64	[65]	66	67	68	69	70	71	72
73	74	75	76	[77]	78	79	80	81	82	83	84
[85]	86	87	88	89	90	[91]	92	93	94	[95]	96
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
$12n+1$	$12n+2$	$12n+3$	$12n+4$	$12n+5$	$12n+6$	$12n+7$	$12n+8$	$12n+9$	$12n+10$	$12n+11$	$12n+12$

Table – 1) All elements of the $12n+1,5,7,11$ series are not multiples of 2 and 3. Therefore, all prime numbers except 2 and 3 are contained in the $12n+1,5,7,11$ series. The number in the box denotes the elements of C_n.

Theorem 1.1) All prime numbers but 2 and 3 exist in forms of $12n+1,5,7,11$ with a period of $12n$.

Proof A)

(i) All natural numbers can be represented with a period of 12.

(ii) All even numbers but 2 are not prime numbers

Elements of $12n+2,4,6,8,10,12$ are all even. $(n=0,1,2....n)$
Therefore, all $12n+2,4,6,8,10,12$'s but 2 are not prime numbers.

(iii) All $12n+3$'s but 3 are not prime numbers.
$12n+3 = (3\times4)n+3$ is a multiple of 3.

(iv) $12n+9$ is not a prime number.
$12n+9 = (3\times4)n+9$ is a multiple of 3.

As in Theorem 1), every prime number but 2 and 3 is contained in the periodic A_n. So, let us denote this series as follows:

$$A_n = \begin{cases} A_1, & if\ remainder \equiv 1 \pmod{12} \\ A_5, & if\ remainder \equiv 5 \pmod{12} \\ A_7, & if\ remainder \equiv 7 \pmod{12} \\ A_{11}, & if\ remainder \equiv 11 \pmod{12} \end{cases}$$

In *Figure – 2)* *Figure – 3)*, the A_n series are multiplied infinitely and we can find 10 basics equations which falls into one of the 4 groups.

$\times$	A_1	A_5	A_7	A_{11}
A_1	$A_1 \times A_1$	$A_1 \times A_5$	$A_1 \times A_7$	$A_1 \times A_{11}$
A_5	$A_5 \times A_1$	$A_5 \times A_5$	$A_5 \times A_7$	$A_5 \times A_{11}$
A_7	$A_7 \times A_1$	$A_7 \times A_5$	$A_7 \times A_7$	$A_7 \times A_{11}$
A_{11}	$A_{11} \times A_1$	$A_{11} \times A_5$	$A_{11} \times A_7$	$A_{11} \times A_{11}$

P

Figure – 3) Multiplication Table

$\times$	$++(A_1)$	$-+(A_5)$	$--(A_7)$	$+-(A_{11})$ 12n+1,5,7,11
$++(A_1)$	$++$	$-+$	$--$	$+-$
$-+(A_5)$	$-+$	$++$	$+-$	$--$
$--(A_7)$	$--$	$+-$	$++$	$-+$
$+-(A_{11})$	$+-$	$--$	$-+$	$++$
12n+1,5,7,11				

Figure – 4) Sign Assignments in the Multiplication Table

If we assign signs, we can summarize them as follows:

Multiplication	Sign	Equation	Remainder
$A_1 \times A_1$	$++(A_1)$	$(12x+1)(12y+1)$	$1 \pmod{12}$
$A_5 \times A_5$	$++(A_1)$	$(12x+5)(12y+5)$	$1 \pmod{12}$
$A_7 \times A_7$	$++(A_1)$	$(12x+7)(12y+7)$	$1 \pmod{12}$
$A_{11} \times A_{11}$	$++(A_1)$	$(12x+11)(12y+11)$	$1 \pmod{12}$
$A_1 \times A_5$	$-+(A_5)$	$(12x+1)(12y+5)$	$5 \pmod{12}$
$A_7 \times A_{11}$	$-+(A_5)$	$(12x+7)(12y+11)$	$5 \pmod{12}$
$A_1 \times A_7$	$--(A_7)$	$(12x+1)(12y+7)$	$7 \pmod{12}$
$A_5 \times A_{11}$	$--(A_7)$	$(12x+5)(12y+11)$	$7 \pmod{12}$
$A_1 \times A_{11}$	$+-(A_{11})$	$(12x+1)(12y+11)$	$11 \pmod{12}$
$A_5 \times A_7$	$+-(A_{11})$	$(12x+5)(12y+7)$	$11 \pmod{12}$

Table – 2) The Result List of Multiplication Table

Theorem 1.2) All elements of the $A_n \times A_n$ table multiplication are contained in set A_n.

Proof B)

$$(12x+\alpha)(12y+\beta), (\alpha, \beta \in \{1,\ 5,\ 7,\ 11\}) =$$

$$144xy + 12\beta x + 12\alpha y + \alpha\beta = \begin{cases} \alpha\beta \in \{1,\ 25,\ 49,\ 121\},\ if\ remainder \equiv 1 \pmod{12} \\ \alpha\beta \in \{5,\ 77\},\ if\ remainder \equiv 5 \pmod{12} \\ \alpha\beta \in \{7,\ 55\},\ if\ remainder \equiv 7 \pmod{12} \\ \alpha\beta \in \{11,\ 35\},\ if\ remainder \equiv 11 \pmod{12} \end{cases}$$

Therefore, $\alpha\beta \in \{1,\ 5,\ 7,\ 11,\ 25,\ 35,\ 49,\ 55,\ 77,\ 121\}$

Theorem 1.3) All elements of the A_n table multiplication are contained in the set of the $A_n \times A_n$ table multiplication.

Proof C)

For any element k of the set A_n : $k \in P_n$ or $k \in C_n$,

$$\text{if } k \text{ is} \begin{cases} P_n : & P_n \times 1, \ k \in (12x+\alpha)\times 1 \text{ or } 1\times(12y+\beta) \\ C_n : & \begin{cases} P_n \times P_n, \ k \in (12x+\alpha)\times(12y+\beta) \\ P_n \times C_n, \ k \in (12x+\alpha)\times(12y+\beta) \\ C_n \times C_n, \ k \in (12x+\alpha)\times(12y+\beta) \end{cases} \end{cases}$$

Therefore, $C_n \geq 25$. Additionally, it is possible to find all values of C_n of the $12n+1,5,7,11$ series in the results of a matrix-multiplication of $A_n \times A_n$ that are greater than 5.

$12x+1,5,7,11(except\ 1)$

×	5	7	11	13	17	19	23	25
5	25							
7	35	49						
11	55	77	121					
13	65	91	143	169				
17	85	119	187	221	289			
19	95	133	209	247	223	361		
23	115	161	253	299	391	437	529	
25	125	175	275	325	425	475	575	625

$12y+1,5,7,11(except\ 1)$

Figure – 2) Multiplication Table of $12n+1,5,7,11$ series

2.2 Prime Sieve

In the third century B.C., the scholar Eratosthenes came up with a simple algorithm for listing all the prime numbers up to a given N , referred to as the sieve of Eratosthenes. A standard improvement in the sieve of Eratosthenes is to enumerate values of *xy* not divisible by 2, 3, or 5. [30]

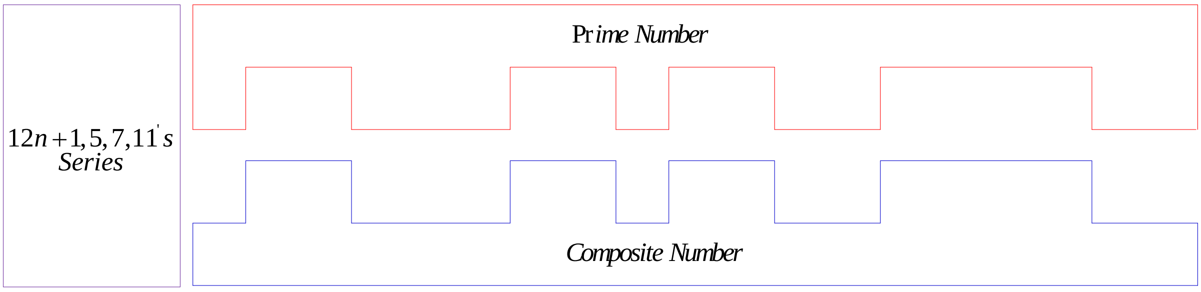


Figure 2.2.*a*) Prime numbers and composite numbers have complementary relationship regarding $12n+1,5,7,11$ series.

In the following [Attached document 1], the prime numbers under 1000 are being filtered using C_n . More specifically, you can see that arithmetic progressions, $Ga_n \sim Gp_n$, are intertwined. Of course, since the number of arithmetic progressions of each group increases as N increases, the complexity of intertwining of further arithmetic progressions increases. The composite numbers intuitively appear to be irregular and seem to be difficult to group into a pattern. However, the composite numbers are sorted into sixteen arithmetic groups $(Ga_n \sim Gp_n)$ and are generated regularly by arithmetic progressions of each group.

2.3 The Structure of a matrix-multiplication of $A_n \times A_n$

Unlike prime numbers, which are unpredictable, the composite numbers are formed by sixteen arithmetic progression groups. This means that composite numbers in principle are predictable because whole composite numbers follow this rule. However, the composite numbers are made up of sixteen arithmetic progressions and it is difficult to see the whole of the arithmetic progressions, whose number increases, without necessary computations and information media that can store the computed results. If you can find the computed results of various arithmetic progressions intuitively, you can predict the rule that governs the composite numbers. This immediately means that you will be able to find the rule that governs the prime numbers. It is not a problem of whether or not the composite numbers are predictable but a problem of human perception.

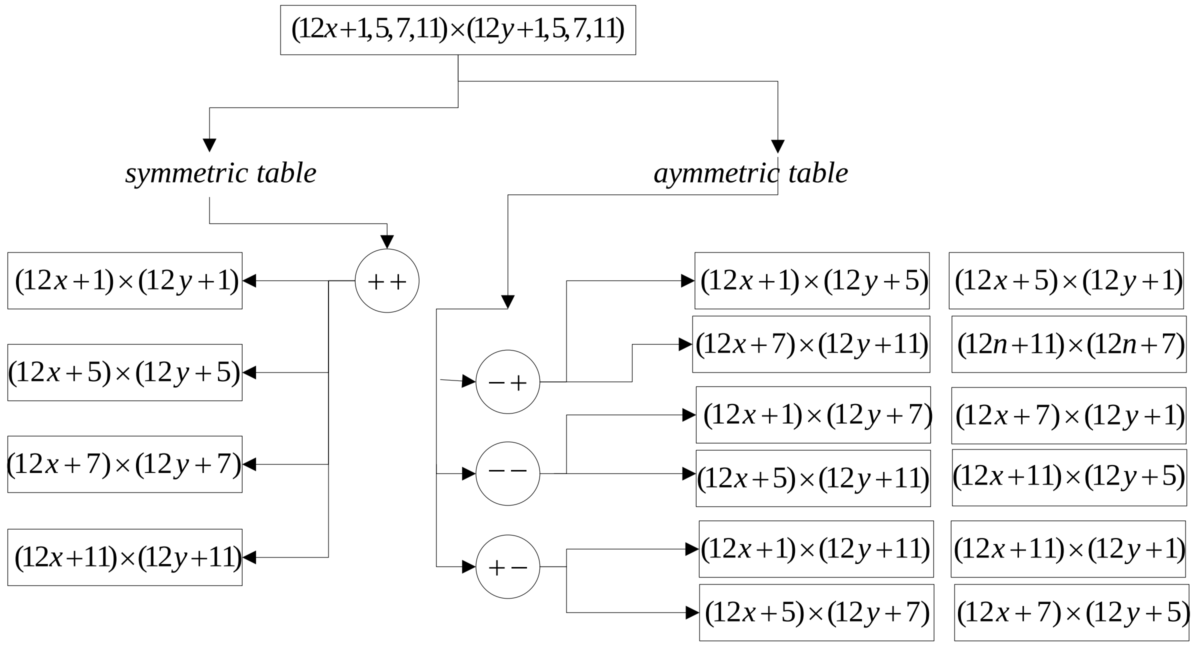


Figure – 2.3.a) Structure of Multiplication Table

⊗ G is a temporary mark used in this paper.

⊗ $a \sim p$ are marked in alphabetical order.

⊗ n is marked to clarify that composite numbers are a set of arithmetic progressions.

N	Group Name	a_1(*first term*)	d(*common difference*)	$n = 1, 2, 3..., n$
1	Ga_n	$(12x+1)(12y+1)$	$12\times(12x+1)$	$x = n, y = n$
2	Gb_n	$(12x+5)(12y+5)$	$12\times(12x+5)$	$x = n-1, y = n-1$
3	Gc_n	$(12x+7)(12y+7)$	$12\times(12x+7)$	$x = n-1, y = n-1$
4	Gd_n	$(12x+11)(12y+11)$	$12\times(12y+11)$	$x = n-1, y = n-1$
5	Ge_n	$(12x+5)(12y+1)$	$12\times(12y+5)$	$x = n-1, y = n$
6	Gf_n	$(12x+1)(12y+5)$	$12\times(12x+1)$	$x = n, y = n-1$
7	Gg_n	$(12x+11)(12y+7)$	$12\times(12x+1)$	$x = n-1, y = n-1$
8	Gh_n	$(12x+7)(12y+11)$	$12\times(12x+7)$	$x = n-1, y = n-1$
9	Gi_n	$(12x+7)(12y+1)$	$12\times(12x+7)$	$x = n-1, y = n$
10	Gj_n	$(12x+1)(12y+7)$	$12\times(12x+1)$	$x = n, y = n-1$
11	Gk_n	$(12x+5)(12y+11)$	$12\times(12x+5)$	$x = n-1, y = n-1$
12	Gl_n	$(12x+11)(12y+5)$	$12\times(12x+11)$	$x = n-1, y = n-1$
13	Gm_n	$(12x+1)(12y+11)$	$12\times(12x+1)$	$x = n, y = n-1$
14	Gn_n	$(12x+11)(12y+1)$	$12\times(12x+11)$	$x = n-1, y = n$
15	Go_n	$(12x+5)(12y+7)$	$12\times(12x+5)$	$x = n-1, y = n-1$
16	Gp_n	$(12x+7)(12y+5)$	$12\times(12x+7)$	$x = n-1, y = n-1$

Table – 3) The composite numbers of the $12n+1, 5, 7, 11$ series are sorted into a total of sixteen groups($Ga_n \sim Gp_n$).

2.4 Symmetric table and Asymmetric table

Analyzing the table of the $12n+1, 5, 7, 11$ series, by the values of horizontal axis(α) and vertical axis(β), the table divides into a symmetric table if $\alpha = \beta$, and into an asymmetric table if $\alpha \neq \beta$. Therefore, we can find the following results.

$i)$ Symmetric table, $\alpha = \beta$

$(12x+1)\times(12y+1),\ (x, y \geq 1)$
$(12x+5)\times(12y+5)$,
$(12x+7)\times(12y+7)$,
$(12x+11)\times(12y+11)$

Since $\alpha = \beta$, the results are the same, reflecting along the diagonal elements independent of the orders of α and β. So, the symmetric table has four cases.

$ii)$ Asymmetric table, $\alpha \neq \beta$

$(12x+1)\times(12y+5),\ (x \geq 1)$, $(12x+7)\times(12y+11)$
$(12x+1)\times(12y+7),\ (x \geq 1)$, $(12x+5)\times(12y+11)$
$(12x+1)\times(12y+11),\ (x \geq 1)$, $(12x+5)\times(12y+7)$

However, the commutative law does not hold if $\alpha \neq \beta$. So, depending on the orders of α and β, the results are different for the diagonal elements. An asymmetric table has twelve cases.

α	β	Sign	Equation
1	5	$-+$	$(12x+1)\times(12y+5),\ (x\geq 1)$
5	1	$-+$	$(12x+5)\times(12y+1),\ (x\geq 1)$
7	11	$-+$	$(12x+7)\times(12y+11)$
11	7	$-+$	$(12x+11)\times(12y+7)$
1	7	$--$	$(12x+1)\times(12y+7),\ (x\geq 1)$
7	1	$--$	$(12x+7)\times(12y+1),\ (x\geq 1)$
5	11	$--$	$(12x+5)\times(12y+11)$
11	5	$--$	$(12x+11)\times(12y+5)$
1	11	$+-$	$(12x+1)\times(12y+11),\ (x\geq 1)$
11	1	$+-$	$(12x+11)\times(12y+1),\ (x\geq 1)$
5	7	$+-$	$(12x+5)\times(12y+7)$
7	5	$+-$	$(12x+7)\times(12y+5)$

Table – 4) The Equation of Asymmetric Table

The composite numbers of the $12n+1,5,7,11$ series divide into a total of sixteen arithmetic progression groups in the matrix multiplication, $(12x+\alpha)\times(12y+\beta),\ (\alpha,\beta=1,5,7,11)$, depending on $\alpha=\beta$ and $\alpha\neq\beta$.

3. Composites in Arithmetic Progression

3.1 $(12x+1)(12y+1),(Ga_n)$

$\times$	13	25	37	49	61	73	85	97 $\rightarrow 12x+1$
13	169							
25	325	625						
37	481	925	1369					
49	637	1225	1813	2401				
61	793	1525	2257	2989	3721			
73	949	1825	2701	3577	4453	5329		
85	1105	2125	3145	4165	5185	6205	7225	
97	1261	2425	3589	4753	5917	7081	8245	9409
$\downarrow 12y+1$								

Figure 3.1.*a*) From the matrix multiplication, each of the diagonal elements becomes the initial terms of the arithmetic progressions.

Ga_1: 169(a_1) 325(a_2) 481(a_3) 637(a_4) 793(a_5) 949(a_6) 1105(a_7) 1261(a_8)
Ga_2: 625(a_1) 925(a_2) 1225(a_3) 1525(a_4) 1825(a_5) 2125(a_6) 2425(a_7)
Ga_3: 1369(a_1) 1813(a_2) 2257(a_3) 2701(a_4) 3145(a_5) 3589(a_6)
Ga_4: 2401(a_1) 2989(a_2) 3577(a_3) 4165(a_4) 4753(a_5)
Ga_5: 3721(a_1) 4453(a_2) 5185(a_3) 5917(a_4)
Ga_6: 5329(a_1) 6205(a_2) 7018(a_3)
Ga_7: 7225(a_1) 8545(a_2)
Ga_8: 9409(a_1)

Figure 3.1.*b*) We have listed the symmetric matrix results for each arithmetic progression. (a_1 =first term, d =common difference)

$$\mathrm{Ga}_1 : a_1 = (12x+1)(12y+1) = 169, \quad d = 12\times(12x+1) = 156, (x=1, y=1)$$

$$\mathrm{Ga}_2 : a_1 = (12x+1)(12y+1) = 625, \quad d = 12\times(12x+1) = 300, (x=2, y=2)$$

$$\mathrm{Ga}_3 : a_1 = (12x+1)(12y+1) = 1369, \quad d = 12\times(12x+1) = 444, (x=3, y=3)$$

$$\mathrm{Ga}_4 : a_1 = (12x+1)(12y+1) = 2401, \quad d = 12\times(12x+1) = 588, (x=4, y=4)$$

$$\mathrm{Ga}_5 : a_1 = (12x+1)(12y+1) = 3721, \quad d = 12\times(12x+1) = 732, (x=5, y=5)$$

$$\mathrm{Ga}_6 : a_1 = (12x+1)(12y+1) = 5329, \quad d = 12\times(12x+1) = 876, (x=6, y=6)$$

$$\mathrm{Ga}_7 : a_1 = (12x+1)(12y+1) = 7225, \quad d = 12\times(12x+1) = 1020, (x=7, y=7)$$

$$\mathrm{Ga}_8 : a_1 = (12x+1)(12y+1) = 9409, \quad d = 12\times(12x+1) = 1164, (x=8, y=8)$$

$$\vdots$$

$$\mathrm{Ga}_n : a_1 = (12x+1)(12y+1) = 144xy + 12x + 12y + 1, \quad d = 12\times(12x+1), (x=n, y=n)$$

Figure 3.1.c) Table multiplication results are arithmetic progressions that have initial terms and common differences and have a general formula. (a_1 =first term, d =common difference)

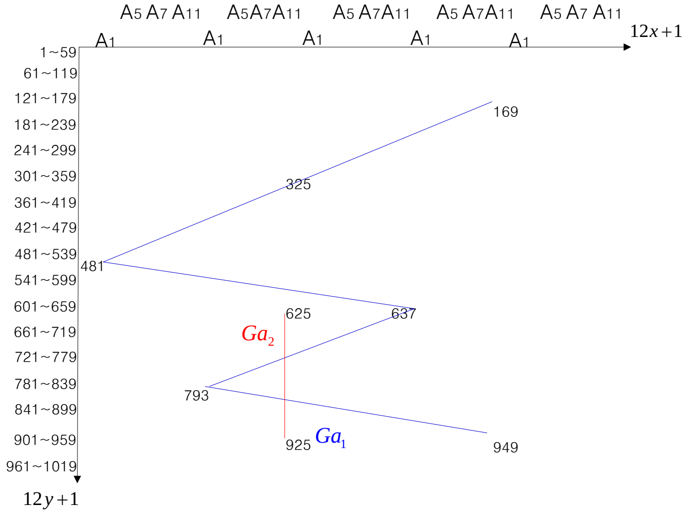


Figure 3.1.d) The contour of the arithmetic progression that belongs to Ga_n

The *Figure* 3.1.d) shows the contour of the arithmetic progression, cutting by 60 up to 1000. If N increases, more arithmetic progressions are generated. A point to note here is that they appear as different contours because they have different initial terms and common differences that belong to Ga_n .

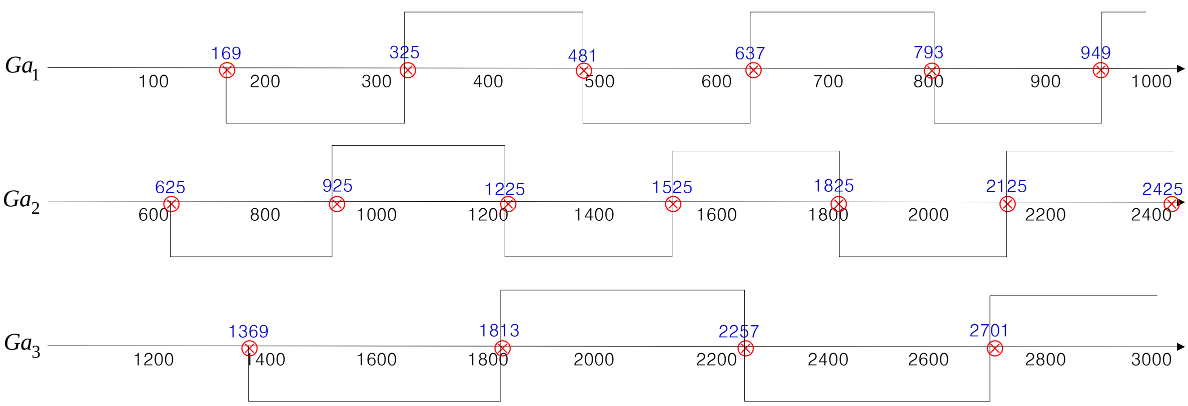


Figure 3.1.e) In Ga_n ,there exist, under rules, a number of arithmetic progressions that have different initial terms and common differences.

$Ga_1, Ga_2, Ga_3 \sim Ga_n$ make up arithmetic progression groups that have initial terms, $(12x+1)(12y+1), (x, y = n)$ and common differences, $12(12x+1), (x = n)$. Of course, as n increases, the values of the initial term and the common difference increase.

N	*current value*	*new value*
$Ga_1 : a_1(169)$	Ga_1	Ga_1
$Ga_2 : a_1(625)$	Ga_1, Ga_2	Ga_2
$Ga_3 : a_1(1369)$	Ga_1, Ga_2, Ga_3	Ga_3
$Ga_4 : a_1(2401)$	Ga_1, Ga_2, Ga_3, Ga_4	Ga_4
$Ga_5 : a_1(3721)$	$Ga_1, Ga_2, Ga_3, Ga_4, Ga_5$	Ga_5
$\vdots$	$\vdots$	$\vdots$
$Ga_n : a_1$	$Ga_1, Ga_2, Ga_3, Ga_4, Ga_5, \cdots, Ga_n$	Ga_n

Figure 3.1.f) As the natural number N, increases, new arithmetic progressions accumulate and keep increasing.

In *Figure* 3.1.f), since new arithmetic progressions accumulate as the natural number N increases, the distribution density of the composite numbers in each region becomes high. For example, the number of composite numbers of the arithmetic progression, Ga_n, is higher in the 1,000-2,000 region than in the 11,000-12,000 region. Just like a timer that rings after a certain period of time, for a certain N, the corresponding arithmetic progression operates.

Theorem 2) As N increases, the density of composite numbers becomes higher. In other words, it means that, as N increases, prime number density becomes low. (of the sparsity of the prime sequence.)

When writing the list of prime numbers, you will find that prime numbers become more and more sparse.

n	$\pi(n)$	$\pi(n)/n$
10	4	0.4
10^2	25	0.25
10^3	168	0.168
10^4	1,229	0.1229
10^5	9,592	0.09592
10^6	78,498	0.078498
10^7	664,579	0.066458
10^8	5,761,455	0.057615
10^9	50,847,534	0.050848
10^{10}	455,052,512	0.045505

Table – 5) The Sparse of Prime Numbers

The number of prime numbers between 1 and 100 is greater than that between 101 and 200. There are 4 prime numbers (40%) between 0 and 10, 25 prime numbers (25%) between 0 and 100, 168(16.8%) between 0 and 1000, 1,229 (12.3%) between 0 and 10000, 9592 (9.5%) between 0 and 100000, and 78,498 (7.8%) between 0 and 1000000. The percentage gradually decreases. [9][10]

Section	The count of prime	The count of composite	Total
$0 \sim 10^2$	25(0.25)	9(0.09)	34
$10^2 \sim 2\times10^2$	21(0.21)	13(0.13)	34
$2\times10^2 \sim 3\times10^2$	16(0.16)	17(0.17)	33
$3\times10^2 \sim 4\times10^2$	16(0.16)	17(0.17)	33
$4\times10^2 \sim 5\times10^2$	17(0.17)	17(0.17)	34
$5\times10^2 \sim 6\times10^2$	14(0.14)	19(0.19)	33
$6\times10^2 \sim 7\times10^2$	16(0.16)	17(0.17)	33
$7\times10^2 \sim 8\times10^2$	14(0.14)	20(0.20)	34
$8\times10^2 \sim 9\times10^2$	15(0.15)	18(0.18)	33
$9\times10^2 \sim 10^3$	14(0.14)	19(0.19)	33

Table – 6) Densities of Prime and Composite Numbers in Each Region

Therefore, if you know reason that the number of composite numbers increases, you will know why prime number density decreases. This is because, at least, we know the rules under which composite numbers are generated. This phenomenon is a natural result because new arithmetic progressions accumulate as a natural number, N, increases. As Ga_n, $Gb_n, Gc_n \sim Gp_n$ make up groups that all have different initial terms and common differences. Therefore, they make up sixteen groups of arithmetic progressions.

3.2 $(12x+5)(12y+5),(Gb_n)$

×	5	17	29	41	53	65	77	89
5	25							
17	85	289						
29	145	493	841					
41	205	697	1189	1681				
53	265	901	1537	2173	2809			
65	325	1105	1885	2665	3445	4225		
77	385	1309	2233	3157	4081	5005	5929	
89	445	1513	2581	3649	4717	5785	6853	7921

Gb_1: $25(a_1)$ $85(a_2)$ $145(a_3)$ $205(a_4)$ $265(a_5)$ $325(a_6)$ $385(a_7)$ $445(a_8)$

Gb_2: $289(a_1)$ $493(a_2)$ $697(a_3)$ $901(a_4)$ $1105(a_5)$ $1903(a_6)$ $1513(a_7)$

Gb_3: $841(a_1)$ $1189(a_2)$ $1537(a_3)$ $1885(a_4)$ $2233(a_5)$ $2581(a_6)$

Gb_4: $1681(a_1)$ $2173(a_2)$ $2665(a_3)$ $3157(a_4)$ $3649(a_5)$

Gb_5: $2809(a_1)$ $3445(a_2)$ $4081(a_3)$ $4717(a_4)$

Gb_6: $4225(a_1)$ $5005(a_2)$ $5785(a_3)$

Gb_7: $5929(a_1)$ $6853(a_2)$

Gb_8: $7921(a_1)$

$\text{Gb}_1 : a_1=(12x+5)(12y+5)=25,\quad d=12\times(12y+5)=60,(x=0,y=0)$

$\text{Gb}_2 : a_1=(12x+5)(12y+5)=289,\quad d=12\times(12y+5)=204,(x=1,y=1)$

$\text{Gb}_3 : a_1=(12x+5)(12y+5)=841,\quad d=12\times(12y+5)=348,(x=2,y=2)$

$\text{Gb}_4 : a_1=(12x+5)(12y+5)=1681,\quad d=12\times(12y+5)=492,(x=3,y=3)$

$\text{Gb}_5 : a_1=(12x+5)(12y+5)=2809,\quad d=12\times(12y+5)=636,(x=4,y=4)$

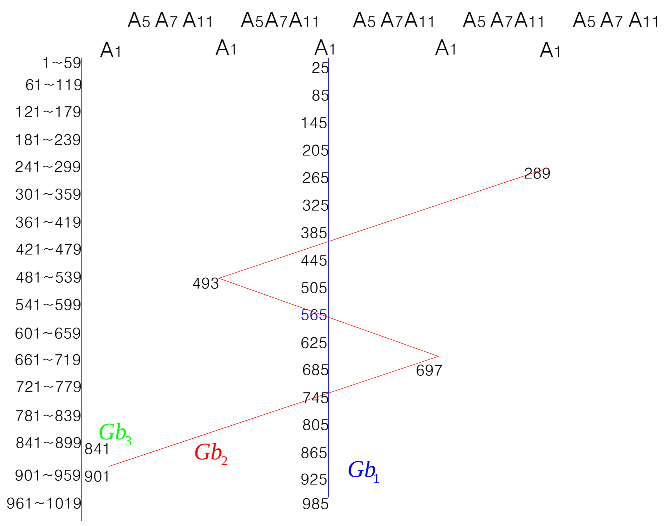
$\text{Gb}_6 : a_1=(12x+5)(12y+5)=4225,\quad d=12\times(12y+5)=780,(x=5,y=5)$

$\text{Gb}_7 : a_1=(12x+5)(12y+5)=5929,\quad d=12\times(12y+5)=924,(x=6,y=6)$

$\text{Gb}_8 : a_1=(12x+5)(12y+5)=7921,\quad d=12\times(12y+5)=1068,(x=7,y=7)$

$\vdots$

$\text{Gb}_n : a_1=(12x+5)(12y+5)=144xy+60x+60y+25,\quad d=12\times(12x+5),(x=n-1,y=n-1)$



3.3 $(12x+7)(12y+7),(Gc_n)$

×	7	19	31	43	55	67	79	91
7	49							
19	133	361						
31	217	589	961					
43	301	817	1333	1849				
55	385	1045	1705	2365	3025			
67	469	1273	2077	2881	3685	4489		
79	553	1501	2449	3397	4345	5293	6241	
91	637	1729	2821	3913	5005	6097	7189	8281

Gc_1: $49(a_1)$ $133(a_2)$ $217(a_3)$ $301(a_4)$ $385(a_5)$ $469(a_6)$ $553(a_7)$ $637(a_8)$

Gc_2: $361(a_1)$ $589(a_2)$ $817(a_3)$ $1045(a_4)$ $1273(a_5)$ $1501(a_6)$ $1729(a_7)$

Gc_3: $961(a_1)$ $1333(a_2)$ $1705(a_3)$ $2077(a_4)$ $2449(a_5)$ $2821(a_6)$

Gc_4: $1849(a_1)$ $2365(a_2)$ $2881(a_3)$ $3397(a_4)$ $3913(a_5)$

Gc_5: $3025(a_1)$ $3685(a_2)$ $4345(a_3)$ $5005(a_4)$

Gc_6: $4489(a_1)$ $5293(a_2)$ $6097(a_3)$

Gc_7: $6241(a_1)$ $7189(a_2)$

Gc_8: $8281(a_1)$

$\mathrm{Gc}_1 : a_1=(12x+7)(12y+7)=49, \quad d=12\times(12y+7)=84,(x=0,y=0)$

$\mathrm{Gc}_2 : a_1=(12x+7)(12y+7)=361, \quad d=12\times(12y+7)=228,(x=1,y=1)$

$\mathrm{Gc}_3 : a_1=(12x+7)(12y+7)=961, \quad d=12\times(12y+7)=372,(x=2,y=2)$

$\mathrm{Gc}_4 : a_1=(12x+7)(12y+7)=1849, \quad d=12\times(12y+7)=516,(x=3,y=3)$

$\mathrm{Gc}_5 : a_1=(12x+7)(12y+7)=3025, \quad d=12\times(12y+7)=660,(x=4,y=4)$

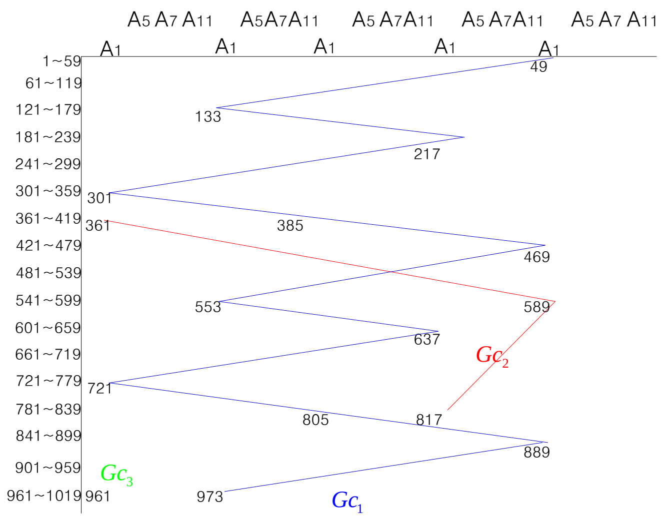
$\mathrm{Gc}_6 : a_1=(12x+7)(12y+7)=4489, \quad d=12\times(12y+7)=804,(x=5,y=5)$

$\mathrm{Gc}_7 : a_1=(12x+7)(12y+7)=6241, \quad d=12\times(12y+7)=948,(x=6,y=6)$

$\mathrm{Gc}_8 : a_1=(12x+7)(12y+7)=8281, \quad d=12\times(12y+7)=1092,(x=7,y=7)$

$\vdots$

$\mathrm{Gc}_n : a_1=(12x+7)(12y+7)=144xy+84x+84y+49, \quad d=12\times(12x+7),(x=n-1,y=n-1)$



3.4 $(12x+11)(12y+11),(Gd_n)$

×	11	23	35	47	59	71	83	95
11	121							
23	253	529						
35	385	805	1225					
47	517	1081	1645	2209				
59	649	1357	2065	2773	3481			
71	781	1633	2485	3337	4189	5041		
83	913	1909	2905	3901	4897	5893	6889	
95	1045	2185	3325	4465	5605	6745	7885	9025

Gd_1: $121(a_1)$ $253(a_2)$ $385(a_3)$ $517(a_4)$ $649(a_5)$ $781(a_6)$ $913(a_7)$ $1045(a_8)$

Gd_2: $529(a_1)$ $805(a_2)$ $1081(a_3)$ $1357(a_4)$ $1633(a_5)$ $1909(a_6)$ $2185(a_7)$

Gd_3: $1225(a_1)$ $1645(a_2)$ $2065(a_3)$ $2485(a_4)$ $2905(a_5)$ $3325(a_6)$

Gd_4: $2209(a_1)$ $2773(a_2)$ $3337(a_3)$ $3901(a_4)$ $4465(a_5)$

Gd_5: $3481(a_1)$ $4189(a_2)$ $4897(a_3)$ $5605(a_4)$

Gd_6: $5041(a_1)$ $5893(a_2)$ $6745(a_3)$

Gd_7: $6889(a_1)$ $7885(a_2)$

Gd_8: $9025(a_1)$

$\mathrm{Gd}_1: a_1=(12x+11)(12y+11)=121,\quad d=12\times(12y+11)=132,(x=0,y=0)$

$\mathrm{Gd}_2: a_1=(12x+11)(12y+11)=529,\quad d=12\times(12y+11)=276,(x=1,y=1)$

$\mathrm{Gd}_3: a_1=(12x+11)(12y+11)=1225,\quad d=12\times(12y+11)=420,(x=2,y=2)$

$\mathrm{Gd}_4: a_1=(12x+11)(12y+11)=2209,\quad d=12\times(12y+11)=564,(x=3,y=3)$

$\mathrm{Gd}_5: a_1=(12x+11)(12y+11)=3481,\quad d=12\times(12y+11)=708,(x=4,y=4)$

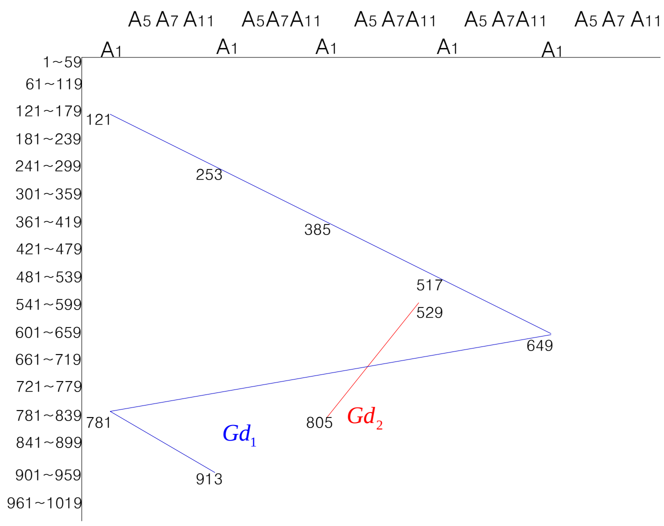
$\mathrm{Gd}_6: a_1=(12x+11)(12y+11)=5041,\quad d=12\times(12y+11)=852,(x=5,y=5)$

$\mathrm{Gd}_7: a_1=(12x+11)(12y+11)=6889,\quad d=12\times(12y+11)=996,(x=6,y=6)$

$\mathrm{Gd}_8: a_1=(12x+11)(12y+11)=9025,\quad d=12\times(12y+11)=1140,(x=7,y=7)$

$\vdots$

$\mathrm{Gd}_n: a_1=(12x+11)(12y+11)=144xy+132x+132y+121,\quad d=12\times(12y+11),(x=n-1,y=n-1)$



3.5 $(12x+1)(12y+5), (Ge_n)$

×	13	25	37	49	61	73	85	97
5	65							
17	221	425						
29	377	725	1073					
41	533	1025	1517	2009				
53	689	1325	1961	2597	3233			
65	845	1625	2405	3185	3965	4745		
77	1001	1925	2849	3773	4697	5621	6545	
89	1157	2225	3293	4361	5429	6497	7565	8633

Ge_1: $65(a_1)$ $221(a_2)$ $377(a_3)$ $533(a_4)$ $689(a_5)$ $845(a_6)$ $1001(a_7)$ $1157(a_8)$

Ge_2: $425(a_1)$ $725(a_2)$ $1025(a_3)$ $1325(a_4)$ $1625(a_5)$ $1925(a_6)$ $2225(a_7)$

Ge_3: $1073(a_1)$ $1517(a_2)$ $1961(a_3)$ $2405(a_4)$ $2849(a_5)$ $3293(a_6)$

Ge_4: $2009(a_1)$ $2597(a_2)$ $3185(a_3)$ $3773(a_4)$ $4361(a_5)$

Ge_5: $3233(a_1)$ $3965(a_2)$ $4697(a_3)$ $5429(a_4)$

Ge_6: $4745(a_1)$ $5621(a_2)$ $6497(a_3)$

Ge_7: $6545(a_1)$ $7565(a_2)$

Ge_8: $8633(a_1)$

$\mathrm{Ge}_1: a_1=(12x+1)(12y+5)=65, \quad d=12\times(12x+1)=156(x=1, y=0)$

$\mathrm{Ge}_2: a_1=(12x+1)(12y+5)=425, \quad d=12\times(12x+1)=300(x=2, y=1)$

$\mathrm{Ge}_3: a_1=(12x+1)(12y+5)=1073, \quad d=12\times(12x+1)=444(x=3, y=2)$

$\mathrm{Ge}_4: a_1=(12x+1)(12y+5)=2009, \quad d=12\times(12x+1)=588(x=4, y=3)$

$\mathrm{Ge}_5: a_1=(12x+1)(12y+5)=3233, \quad d=12\times(12x+1)=732(x=5, y=4)$

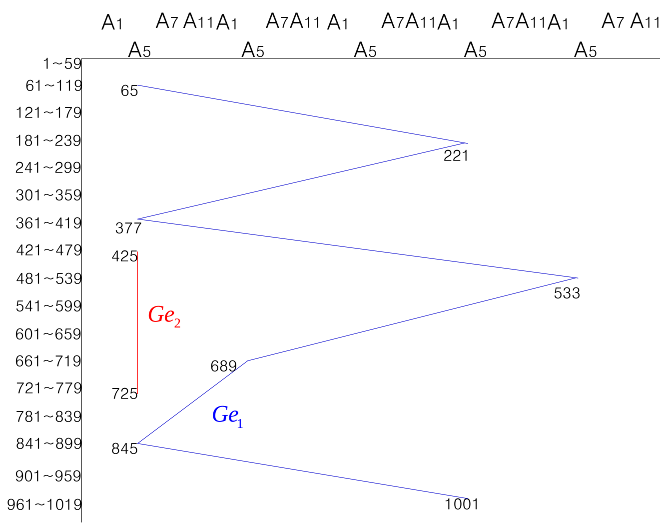
$\mathrm{Ge}_6: a_1=(12x+1)(12y+5)=4745, \quad d=12\times(12x+1)=876(x=6, y=5)$

$\mathrm{Ge}_7: a_1=(12x+1)(12y+5)=6545, \quad d=12\times(12x+1)=1020(x=7, y=6)$

$\mathrm{Ge}_8: a_1=(12x+1)(12y+5)=8633, \quad d=12\times(12x+1)=1164(x=8, y=7)$

$\vdots$

$\mathrm{Ge}_n: a_1=(12x+1)(12y+5)=144xy+60x+12y+5, \quad d=12\times(12x+1)(x=n, y=n-1)$

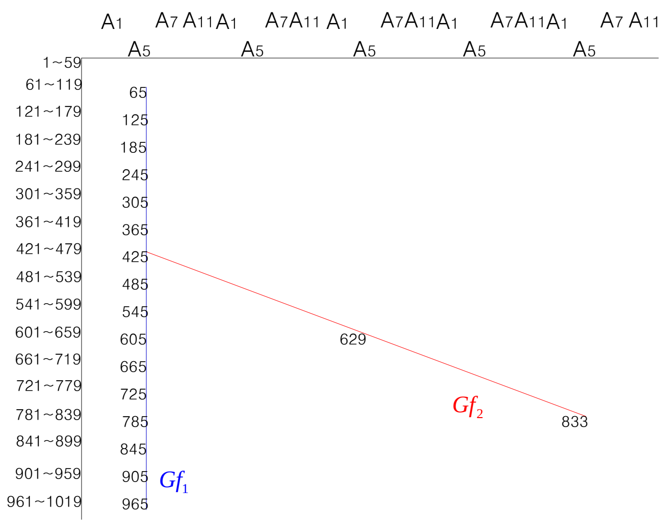


3.6 $(12x+5)(12y+1), (Gf_n)$

×	5	17	29	41	53	65	77	89
13	65							
25	125	425						
37	185	629	1073					
49	245	833	1421	2009				
61	305	1037	1769	2501	3233			
73	365	1241	2117	2993	3869	4745		
85	425	1445	2465	3485	4505	5525	6545	
97	485	1649	2813	3977	5141	6305	7469	8633

Gf_1: $65(a_1)$ $125(a_2)$ $185(a_3)$ $245(a_4)$ $305(a_5)$ $365(a_6)$ $425(a_7)$ $485(a_8)$
Gf_2: $425(a_1)$ $629(a_2)$ $833(a_3)$ $1037(a_4)$ $1241(a_5)$ $1445(a_6)$ $1649(a_7)$
Gf_3: $1073(a_1)$ $1421(a_2)$ $1769(a_3)$ $2117(a_4)$ $2465(a_5)$ $2813(a_6)$
Gf_4: $2009(a_1)$ $2501(a_2)$ $2993(a_3)$ $3485(a_4)$ $3977(a_5)$
Gf_5: $3233(a_1)$ $3869(a_2)$ $4505(a_3)$ $5141(a_4)$
Gf_6: $4745(a_1)$ $5525(a_2)$ $6305(a_3)$
Gf_7: $6545(a_1)$ $7469(a_2)$
Gf_8: $8633(a_1)$

$Gf_1 : a_1 = (12x+5)(12y+1) = 65, \quad d = 12\times(12x+5) = 60 (x=0, y=1)$
$Gf_2 : a_1 = (12x+5)(12y+1) = 425, \quad d = 12\times(12x+5) = 204 (x=1, y=2)$
$Gf_3 : a_1 = (12x+5)(12y+1) = 1073, \quad d = 12\times(12x+5) = 348 (x=2, y=3)$
$Gf_4 : a_1 = (12x+5)(12y+1) = 2009, \quad d = 12\times(12x+5) = 492 (x=3, y=4)$
$Gf_5 : a_1 = (12x+5)(12y+1) = 3233, \quad d = 12\times(12x+5) = 636 (x=4, y=5)$
$Gf_6 : a_1 = (12x+5)(12y+1) = 4745, \quad d = 12\times(12x+5) = 780 (x=5, y=6)$
$Gf_7 : a_1 = (12x+5)(12y+1) = 6545, \quad d = 12\times(12x+5) = 924 (x=6, y=7)$
$Gf_8 : a_1 = (12x+5)(12y+1) = 8633, \quad d = 12\times(12x+5) = 1068 (x=7, y=8)$
$\vdots$
$Gf_n : a_1 = (12x+5)(12y+1) = 144xy + 12x + 60y + 5, \quad d = 12\times(12y+5) (x = n-1, y = n)$



3.7 $(12x+7)(12y+11),(Gg_n)$

×	7	19	31	43	55	67	79	91
11	77							
23	161	437						
35	245	665	1085					
47	329	893	1457	2021				
59	413	1121	1829	2537	3245			
71	497	1349	2201	3053	3905	4757		
83	581	1577	2573	3569	4565	5561	6557	
95	665	1805	2945	4085	5225	6365	7505	8645

Gg_1: $77(a_1)$ $161(a_2)$ $245(a_3)$ $329(a_4)$ $413(a_5)$ $497(a_6)$ $581(a_7)$ $665(a_8)$

Gg_2: $437(a_1)$ $665(a_2)$ $893(a_3)$ $1121(a_4)$ $1349(a_5)$ $1577(a_6)$ $1805(a_7)$

Gg_3: $1085(a_1)$ $1457(a_2)$ $1829(a_3)$ $2201(a_4)$ $2573(a_5)$ $2945(a_6)$

Gg_4: $2021(a_1)$ $2537(a_2)$ $3053(a_3)$ $3569(a_4)$ $4085(a_5)$

Gg_5: $3245(a_1)$ $3905(a_2)$ $4565(a_3)$ $5225(a_4)$

Gg_6: $4757(a_1)$ $5561(a_2)$ $6365(a_3)$

Gg_7: $6557(a_1)$ $7505(a_2)$

Gg_8: $8645(a_1)$

$\mathrm{Gg}_1: a_1=(12x+7)(12y+11)=77,\quad d=12\times(12x+7)=84(x=0,y=0)$

$\mathrm{Gg}_2: a_1=(12x+7)(12y+11)=437,\quad d=12\times(12x+7)=228(x=1,y=1)$

$\mathrm{Gg}_3: a_1=(12x+7)(12y+11)=1085,\quad d=12\times(12x+7)=372(x=2,y=2)$

$\mathrm{Gg}_4: a_1=(12x+7)(12y+11)=2021,\quad d=12\times(12x+7)=516(x=3,y=3)$

$\mathrm{Gg}_5: a_1=(12x+7)(12y+11)=3245,\quad d=12\times(12x+7)=660(x=4,y=4)$

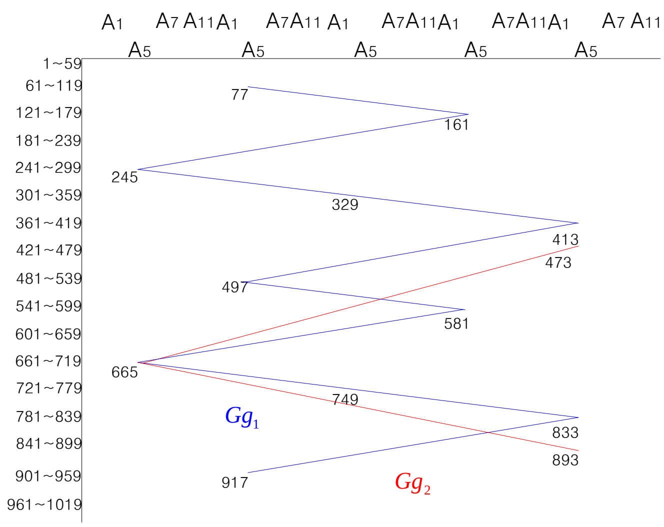
$\mathrm{Gg}_6: a_1=(12x+7)(12y+11)=4757,\quad d=12\times(12x+7)=804(x=5,y=5)$

$\mathrm{Gg}_7: a_1=(12x+7)(12y+11)=6557,\quad d=12\times(12x+7)=948(x=6,y=6)$

$\mathrm{Gg}_8: a_1=(12x+7)(12y+11)=8645,\quad d=12\times(12x+7)=1092(x=7,y=7)$

$\vdots$

$\mathrm{Gg}_n: a_1=(12x+7)(12y+11)=144xy+132x+84y+77,\quad d=12\times(12x+7)(x=n-1,y=n-1)$



3.8 $(12x+11)(12y+7),(Gh_n)$

×	11	23	35	47	59	71	83	95
7	77							
19	209	437						
31	341	713	1085					
43	473	989	1505	2021				
55	605	1265	1925	2585	3245			
67	737	1541	2345	3149	3953	4757		
79	869	1817	2765	3713	4661	5609	6557	
91	1001	2093	3185	4277	5369	6461	7553	8645

Gh_1: $77(a_1)$ $209(a_2)$ $341(a_3)$ $473(a_4)$ $605(a_5)$ $737(a_6)$ $869(a_7)$ $1001(a_8)$

Gh_2: $437(a_1)$ $713(a_2)$ $989(a_3)$ $1265(a_4)$ $1541(a_5)$ $1817(a_6)$ $2093(a_7)$

Gh_3: $1085(a_1)$ $1505(a_2)$ $1925(a_3)$ $2345(a_4)$ $2765(a_5)$ $3185(a_6)$

Gh_4: $2021(a_1)$ $2585(a_2)$ $3149(a_3)$ $3713(a_4)$ $4277(a_5)$

Gh_5: $3245(a_1)$ $3965(a_2)$ $4661(a_3)$ $5369(a_4)$

Gh_6: $4757(a_1)$ $5609(a_2)$ $6461(a_3)$

Gh_7: $6557(a_1)$ $7553(a_2)$

Gh_8: $8645(a_1)$

$\mathrm{Gh}_1 : a_1 = (12x+11)(12y+7) = 77,\quad d = 12\times(12x+11) = 132(x=0, y=0)$

$\mathrm{Gh}_2 : a_1 = (12x+11)(12y+7) = 437,\quad d = 12\times(12x+11) = 276(x=1, y=1)$

$\mathrm{Gh}_3 : a_1 = (12x+11)(12y+7) = 1085,\quad d = 12\times(12x+11) = 420(x=2, y=2)$

$\mathrm{Gh}_4 : a_1 = (12x+11)(12y+7) = 2021,\quad d = 12\times(12x+11) = 564(x=3, y=3)$

$\mathrm{Gh}_5 : a_1 = (12x+11)(12y+7) = 3245,\quad d = 12\times(12x+11) = 708(x=4, y=4)$

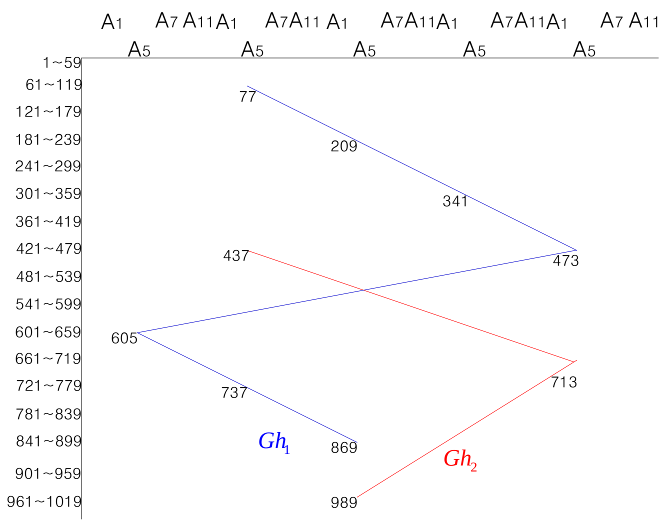
$\mathrm{Gh}_6 : a_1 = (12x+11)(12y+7) = 4757,\quad d = 12\times(12x+11) = 852(x=5, y=5)$

$\mathrm{Gh}_7 : a_1 = (12x+11)(12y+7) = 6557,\quad d = 12\times(12x+11) = 996(x=6, y=6)$

$\mathrm{Gh}_8 : a_1 = (12x+11)(12y+7) = 8645,\quad d = 12\times(12x+11) = 1140(x=7, y=7)$

$\vdots$

$\mathrm{Gh}_n : a_1 = (12x+11)(12y+7) = 144xy + 84x + 132y + 77,\quad d = 12\times(12x+1)(x=n-1, y=n-1)$



3.9 $(12x+1)(12y+7), (Gi_n)$

×	13	25	37	49	61	73	85	97
7	91							
19	247	475						
31	403	775	1147					
43	559	1075	1591	2107				
55	715	1375	2035	2695	3355			
67	871	1675	2479	3283	4087	4891		
79	1027	1975	2923	3871	4819	5767	6715	
91	1183	2275	3367	4459	5551	6643	7735	8827

Gi_1: $91(a_1)$ $247(a_2)$ $403(a_3)$ $559(a_4)$ $715(a_5)$ $871(a_6)$ $1027(a_7)$ $1183(a_8)$

Gi_2: $475(a_1)$ $775(a_2)$ $1075(a_3)$ $1375(a_4)$ $1675(a_5)$ $1975(a_6)$ $2275(a_7)$

Gi_3: $1147(a_1)$ $1591(a_2)$ $2035(a_3)$ $2479(a_4)$ $2923(a_5)$ $3367(a_6)$

Gi_4: $2107(a_1)$ $2695(a_2)$ $3283(a_3)$ $3871(a_4)$ $4459(a_5)$

Gi_5: $3355(a_1)$ $4087(a_2)$ $4819(a_3)$ $5551(a_4)$

Gi_6: $4891(a_1)$ $5767(a_2)$ $6643(a_3)$

Gi_7: $6715(a_1)$ $7735(a_2)$

Gi_8: $8827(a_1)$

$Gi_1 : a_1 = (12x+1)(12y+7) = 91, \quad d = 12\times(12x+1) = 156 (x=1, y=0)$

$Gi_2 : a_1 = (12x+1)(12y+7) = 475, \quad d = 12\times(12x+1) = 300 (x=2, y=1)$

$Gi_3 : a_1 = (12x+1)(12y+7) = 1147, \quad d = 12\times(12x+1) = 444 (x=3, y=2)$

$Gi_4 : a_1 = (12x+1)(12y+7) = 2107, \quad d = 12\times(12x+1) = 588 (x=4, y=3)$

$Gi_5 : a_1 = (12x+1)(12y+7) = 3355, \quad d = 12\times(12x+1) = 732 (x=5, y=4)$

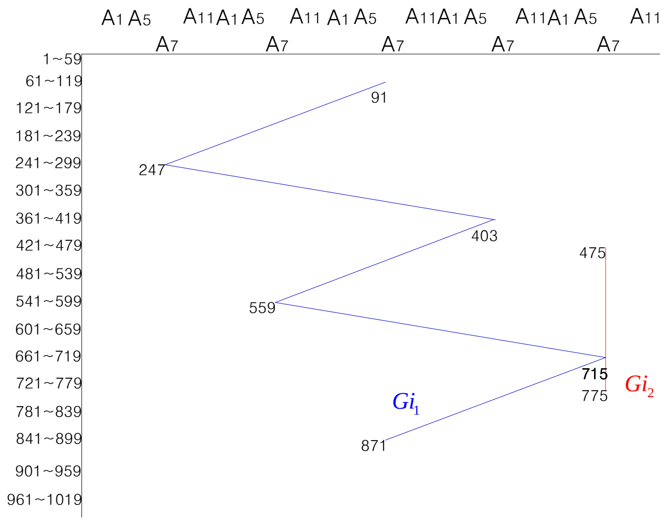
$Gi_6 : a_1 = (12x+1)(12y+7) = 4891, \quad d = 12\times(12x+1) = 876 (x=6, y=5)$

$Gi_7 : a_1 = (12x+1)(12y+7) = 6715, \quad d = 12\times(12x+1) = 1020 (x=7, y=6)$

$Gi_8 : a_1 = (12x+1)(12y+7) = 8827, \quad d = 12\times(12x+1) = 1164 (x=8, y=7)$

$\vdots$

$Gi_n : a_1 = (12x+1)(12y+7) = 144xy + 84x + 12y + 7, \quad d = 12\times(12x+1) (x=n, y=n-1)$



3.10 $(12x+7)(12y+1),(Gj_n)$

×	7	19	31	43	55	67	79	91
13	91							
25	175	475						
37	259	703	1147					
49	343	931	1519	2107				
61	427	1159	1891	2623	3355			
73	511	1387	2263	3139	4015	4891		
85	595	1615	2635	3655	4675	5695	6715	
97	679	1843	3007	4171	5335	6499	7663	8827

Gj_1: $91(a_1)$ $175(a_2)$ $259(a_3)$ $343(a_4)$ $427(a_5)$ $511(a_6)$ $595(a_7)$ $679(a_8)$

Gj_2: $475(a_1)$ $703(a_2)$ $931(a_3)$ $1159(a_4)$ $1387(a_5)$ $1615(a_6)$ $1843(a_7)$

Gj_3: $1147(a_1)$ $1519(a_2)$ $1891(a_3)$ $2263(a_4)$ $2635(a_5)$ $3007(a_6)$

Gj_4: $2107(a_1)$ $2623(a_2)$ $3139(a_3)$ $3655(a_4)$ $4171(a_5)$

Gj_5: $3355(a_1)$ $4015(a_2)$ $4675(a_3)$ $5335(a_4)$

Gj_6: $4891(a_1)$ $5695(a_2)$ $6499(a_3)$

Gj_7: $6715(a_1)$ $7663(a_2)$

Gj_8: $8827(a_1)$

$Gj_1: a_1=(12x+7)(12y+1)=91, \quad d=12\times(12x+7)=84(x=0,y=1)$

$Gj_2: a_1=(12x+7)(12y+1)=475, \quad d=12\times(12x+7)=228(x=1,y=2)$

$Gj_3: a_1=(12x+7)(12y+1)=1147, \quad d=12\times(12x+7)=372(x=2,y=3)$

$Gj_4: a_1=(12x+7)(12y+1)=2107, \quad d=12\times(12x+7)=516(x=3,y=4)$

$Gj_5: a_1=(12x+7)(12y+1)=3355, \quad d=12\times(12x+7)=660(x=4,y=5)$

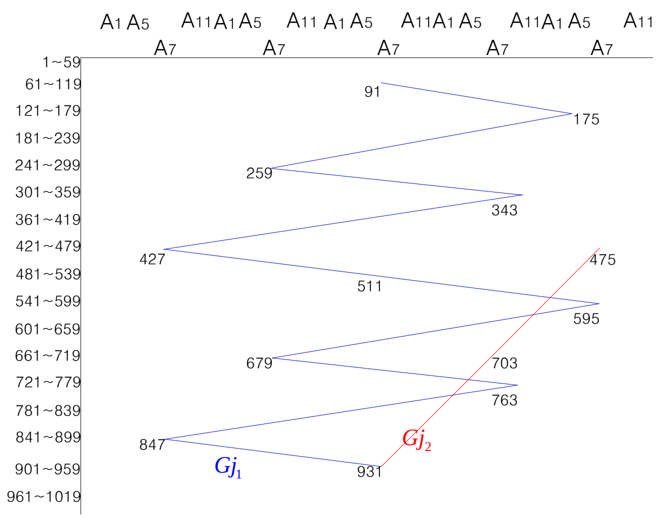
$Gj_6: a_1=(12x+7)(12y+1)=4891, \quad d=12\times(12x+7)=804(x=5,y=6)$

$Gj_7: a_1=(12x+7)(12y+1)=6715, \quad d=12\times(12x+7)=948(x=6,y=7)$

$Gj_8: a_1=(12x+7)(12y+1)=8827, \quad d=12\times(12x+7)=1092(x=7,y=8)$

$\vdots$

$Gj_n: a_1=(12x+7)(12y+1)=144xy+12x+84y+7, \quad d=12\times(12x+7)(x=n-1,y=n)$



3.11 $(12x+5)(12y+11),(Gk_n)$

×	5	17	29	41	53	65	77	89
11	55							
23	115	391						
35	175	595	1015					
47	235	799	1363	1927				
59	295	1003	1711	2419	3127			
71	355	1207	2059	2911	3763	4615		
83	415	1411	2407	3403	4399	5395	6391	
95	475	1615	2755	3895	5035	6175	7315	8455

Gk_1: $55(a_1)$ $115(a_2)$ $175(a_3)$ $235(a_4)$ $295(a_5)$ $355(a_6)$ $415(a_7)$ $475(a_8)$

Gk_2: $391(a_1)$ $595(a_2)$ $799(a_3)$ $1003(a_4)$ $1207(a_5)$ $1411(a_6)$ $1615(a_7)$

Gk_3: $1015(a_1)$ $1363(a_2)$ $1711(a_3)$ $2059(a_4)$ $2407(a_5)$ $2755(a_6)$

Gk_4: $1927(a_1)$ $2419(a_2)$ $2911(a_3)$ $3403(a_4)$ $3895(a_5)$

Gk_5: $3127(a_1)$ $3763(a_2)$ $4399(a_3)$ $5035(a_4)$

Gk_6: $4615(a_1)$ $5395(a_2)$ $6175(a_3)$

Gk_7: $6391(a_1)$ $7315(a_2)$

Gk_8: $8455(a_1)$

$\mathrm{Gk}_1 : a_1 = (12x+5)(12y+11) = 55, \quad d = 12\times(12x+5) = 60(x=0, y=0)$

$\mathrm{Gk}_2 : a_1 = (12x+5)(12y+11) = 391, \quad d = 12\times(12x+5) = 204(x=1, y=1)$

$\mathrm{Gk}_3 : a_1 = (12x+5)(12y+11) = 1015, \quad d = 12\times(12x+5) = 348(x=2, y=2)$

$\mathrm{Gk}_4 : a_1 = (12x+5)(12y+11) = 1927, \quad d = 12\times(12x+5) = 492(x=3, y=3)$

$\mathrm{Gk}_5 : a_1 = (12x+5)(12y+11) = 3127, \quad d = 12\times(12x+5) = 636(x=4, y=4)$

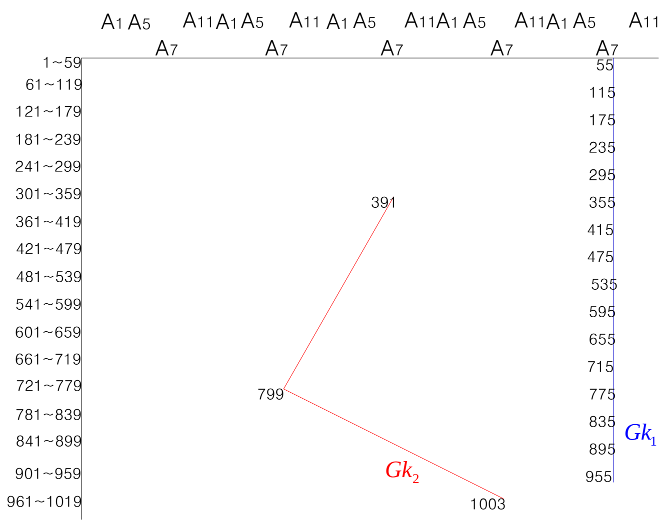
$\mathrm{Gk}_6 : a_1 = (12x+5)(12y+11) = 4615, \quad d = 12\times(12x+5) = 780(x=5, y=5)$

$\mathrm{Gk}_7 : a_1 = (12x+5)(12y+11) = 6391, \quad d = 12\times(12x+5) = 924(x=6, y=6)$

$\mathrm{Gk}_8 : a_1 = (12x+5)(12y+11) = 8455, \quad d = 12\times(12x+5) = 1068(x=7, y=7)$

$\vdots$

$\mathrm{Gk}_n : a_1 = (12x+5)(12y+11) = 144xy+132x+60y+55, \quad d = 12\times(12x+5)(x=n-1, y=n-1)$



3.12 $(12x+11)(12y+5),(Gl_n)$

×	11	23	35	47	59	71	83	95
5	55							
17	187	391						
29	319	667	1015					
41	451	943	1435	1927				
53	583	1219	1855	2491	3127			
65	715	1495	2275	3055	3835	4615		
77	847	1771	2695	3619	4543	5467	6391	
89	979	2047	3115	4183	5251	6319	7387	8455

Gl_1: 55(a_1)   187(a_2)   319(a_3)   451(a_4)   583(a_5)   715(a_6)   847(a_7)   979(a_8)

Gl_2: 391(a_1)   667(a_2)   943(a_3)   1219(a_4)   1495(a_5)   1771(a_6)   2047(a_7)

Gl_3:1015(a_1)   1435(a_2)   1855(a_3)   2275(a_4)   2695(a_5)   3115(a_6)

Gl_4:1927(a_1)   2491(a_2)   3055(a_3)   3619(a_4)   4183(a_5)

Gl_5:3127(a_1)   3835(a_2)   4543(a_3)   5251(a_4)

Gl_6:4615(a_1)   5467(a_2)   6319(a_3)

Gl_7:6391(a_1)   7387(a_2)

Gl_8:8455(a_1)

$Gl_1 : a_1 = (12x+11)(12y+5) = 55, \quad d = 12\times(12x+11) = 132 (x=0, y=0)$

$Gl_2 : a_1 = (12x+11)(12y+5) = 391, \quad d = 12\times(12x+11) = 276 (x=1, y=1)$

$Gl_3 : a_1 = (12x+11)(12y+5) = 1015, \quad d = 12\times(12x+11) = 420 (x=2, y=2)$

$Gl_4 : a_1 = (12x+11)(12y+5) = 1927, \quad d = 12\times(12x+11) = 564 (x=3, y=3)$

$Gl_5 : a_1 = (12x+11)(12y+5) = 3127, \quad d = 12\times(12x+11) = 708 (x=4, y=4)$

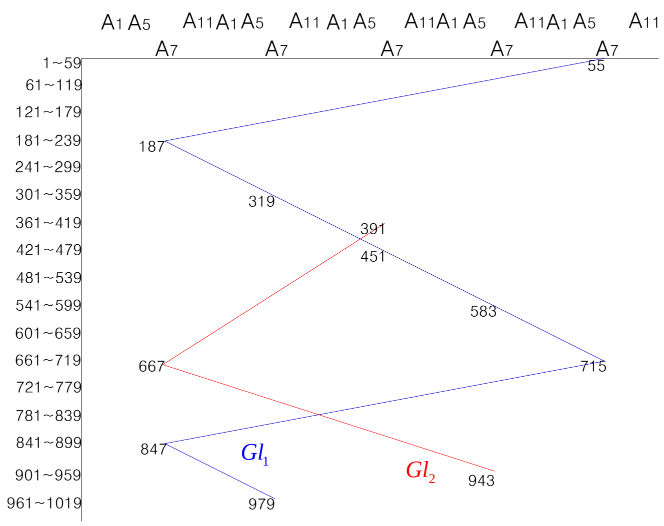
$Gl_6 : a_1 = (12x+11)(12y+5) = 4615, \quad d = 12\times(12x+11) = 852 (x=5, y=5)$

$Gl_7 : a_1 = (12x+11)(12y+5) = 6391, \quad d = 12\times(12x+11) = 996 (x=6, y=6)$

$Gl_8 : a_1 = (12x+11)(12y+5) = 8455, \quad d = 12\times(12x+11) = 1140 (x=7, y=7)$

⋮

$Gl_n : a_1 = (12x+11)(12y+5) = 144xy + 60x + 132y + 55, \quad d = 12\times(12x+11) (x = n-1, y = n-1)$



3.13 $(12x+1)(12y+11),(Gm_n)$

×	13	25	37	49	61	73	85	97
11	143							
23	299	575						
35	455	875	1295					
47	611	1175	1739	2303				
59	767	1475	2183	2891	3599			
71	923	1775	2627	3479	4331	5183		
83	1079	2075	3071	4067	5063	6059	7055	
95	1235	2375	3515	4655	5795	6935	8075	9215

Gm_1: $143(a_1)$ $299(a_2)$ $455(a_3)$ $611(a_4)$ $767(a_5)$ $923(a_6)$ $1079(a_7)$ $1235(a_8)$

Gm_2: $575(a_1)$ $875(a_2)$ $1175(a_3)$ $1475(a_4)$ $1775(a_5)$ $2075(a_6)$ $2375(a_7)$

Gm_3: $1295(a_1)$ $1739(a_2)$ $2183(a_3)$ $2627(a_4)$ $3071(a_5)$ $3515(a_6)$

Gm_4: $2303(a_1)$ $2891(a_2)$ $3479(a_3)$ $4067(a_4)$ $4655(a_5)$

Gm_5: $3599(a_1)$ $4331(a_2)$ $5063(a_3)$ $5795(a_4)$

Gm_6: $5183(a_1)$ $6059(a_2)$ $6935(a_3)$

Gm_7: $7055(a_1)$ $8075(a_2)$

Gm_8: $9215(a_1)$

$Gm_1: a_1=(12x+1)(12y+11)=143,\quad d=12\times(12x+1)=156(x=1,y=0)$

$Gm_2: a_1=(12x+1)(12y+11)=575,\quad d=12\times(12x+1)=300(x=2,y=1)$

$Gm_3: a_1=(12x+1)(12y+11)=1295,\quad d=12\times(12x+1)=444(x=3,y=2)$

$Gm_4: a_1=(12x+1)(12y+11)=2303,\quad d=12\times(12x+1)=588(x=4,y=3)$

$Gm_5: a_1=(12x+1)(12y+11)=3599,\quad d=12\times(12x+1)=732(x=5,y=4)$

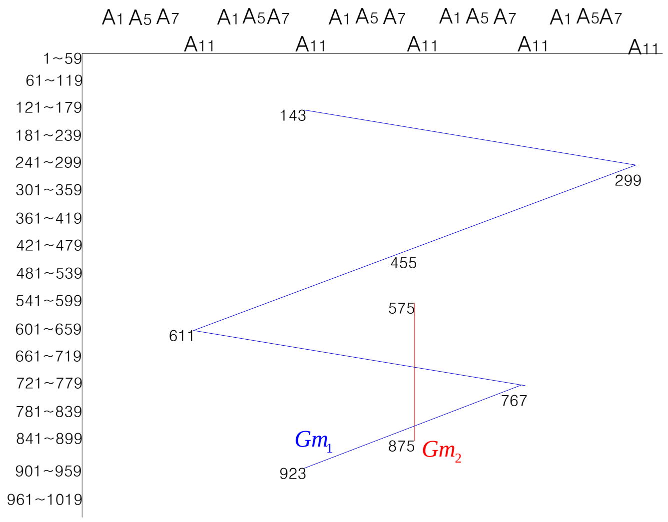
$Gm_6: a_1=(12x+1)(12y+11)=5183,\quad d=12\times(12x+1)=876(x=6,y=5)$

$Gm_7: a_1=(12x+1)(12y+11)=7055,\quad d=12\times(12x+1)=1020(x=7,y=6)$

$Gm_8: a_1=(12x+1)(12y+11)=9215,\quad d=12\times(12x+1)=1164(x=8,y=7)$

$\vdots$

$Gm_n: a_1=(12x+1)(12y+11)=144xy+132x+12y+11,\quad d=12\times(12x+1)(x=n,y=n-1)$

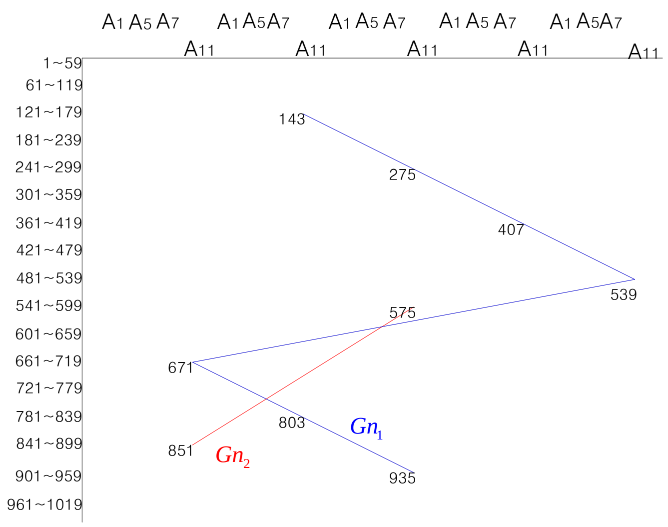


3.14 $(12x+11)(12y+1), (Gn_n)$

×	11	23	35	47	59	71	83	95
13	143							
25	275	575						
37	407	851	1295					
49	539	1127	1715	2303				
61	671	1403	2135	2867	3599			
73	803	1679	2555	3431	4307	5183		
85	935	1955	2975	3995	5015	6035	7055	
97	1067	2231	3395	4559	5723	6887	8051	9215

Gn_1: 143(a_1) 275(a_2) 407(a_3) 539(a_4) 671(a_5) 803(a_6) 935(a_7) 1067(a_8)
Gn_2: 575(a_1) 851(a_2) 1127(a_3) 1403(a_4) 1679(a_5) 1955(a_6) 2231(a_7)
Gn_3: 1295(a_1) 1715(a_2) 2135(a_3) 2555(a_4) 2975(a_5) 3395(a_6)
Gn_4: 2303(a_1) 2867(a_2) 3431(a_3) 3995(a_4) 4559(a_5)
Gn_5: 3599(a_1) 4307(a_2) 5015(a_3) 5723(a_4)
Gn_6: 5183(a_1) 6035(a_2) 6887(a_3)
Gn_7: 7055(a_1) 8051(a_2)
Gn_8: 9215(a_1)

$Gn_1: a_1 = (12x+11)(12y+1) = 143, \quad d = 12\times(12x+11) = 132 (x=0, y=1)$
$Gn_2: a_1 = (12x+11)(12y+1) = 575, \quad d = 12\times(12x+11) = 276 (x=1, y=2)$
$Gn_3: a_1 = (12x+11)(12y+1) = 1295, \quad d = 12\times(12x+11) = 420 (x=2, y=3)$
$Gn_4: a_1 = (12x+11)(12y+1) = 2303, \quad d = 12\times(12x+11) = 564 (x=3, y=4)$
$Gn_5: a_1 = (12x+11)(12y+1) = 3599, \quad d = 12\times(12x+11) = 708 (x=4, y=5)$
$Gn_6: a_1 = (12x+11)(12y+1) = 5183, \quad d = 12\times(12x+11) = 852 (x=5, y=6)$
$Gn_7: a_1 = (12x+11)(12y+1) = 7055, \quad d = 12\times(12x+11) = 996 (x=6, y=7)$
$Gn_8: a_1 = (12x+11)(12y+1) = 9215, \quad d = 12\times(12x+11) = 1140 (x=7, y=8)$
⋮
$Gn_n: a_1 = (12x+11)(12y+1) = 144xy + 12x + 132y + 11, \quad d = 12\times(12x+11) (x=n-1, y=n)$



3.15 $(12x+5)(12y+7),(Go_n)$

×	5	17	29	41	53	65	77	89
7	35							
19	95	323						
31	155	527	899					
43	215	731	1247	1763				
55	275	935	1595	2255	2915			
67	335	1139	1943	2747	3551	4355		
79	395	1343	2291	3239	4187	5135	6083	
91	455	1547	2639	3731	4823	5915	7007	8099

Go_1: $35(a_1)$ $95(a_2)$ $155(a_3)$ $215(a_4)$ $275(a_5)$ $335(a_6)$ $395(a_7)$ $455(a_8)$

Go_2: $323(a_1)$ $527(a_2)$ $731(a_3)$ $935(a_4)$ $1139(a_5)$ $1343(a_6)$ $1547(a_7)$

Go_3: $899(a_1)$ $1247(a_2)$ $1595(a_3)$ $1943(a_4)$ $2291(a_5)$ $2639(a_6)$

Go_4: $1763(a_1)$ $2255(a_2)$ $2747(a_3)$ $3239(a_4)$ $3731(a_5)$

Go_5: $2915(a_1)$ $3551(a_2)$ $4187(a_3)$ $4823(a_4)$

Go_6: $4355(a_1)$ $5135(a_2)$ $5915(a_3)$

Go_7: $6083(a_1)$ $7007(a_2)$

Go_8: $8099(a_1)$

$Go_1 : a_1 = (12x+5)(12y+7) = 35, \quad d = 12\times(12x+5) = 60(x=0, y=0)$

$Go_2 : a_1 = (12x+5)(12y+7) = 323, \quad d = 12\times(12x+5) = 204(x=1, y=1)$

$Go_3 : a_1 = (12x+5)(12y+7) = 899, \quad d = 12\times(12x+5) = 348(x=2, y=2)$

$Go_4 : a_1 = (12x+5)(12y+7) = 1763, \quad d = 12\times(12x+5) = 492(x=3, y=3)$

$Go_5 : a_1 = (12x+5)(12y+7) = 2915, \quad d = 12\times(12x+5) = 636(x=4, y=4)$

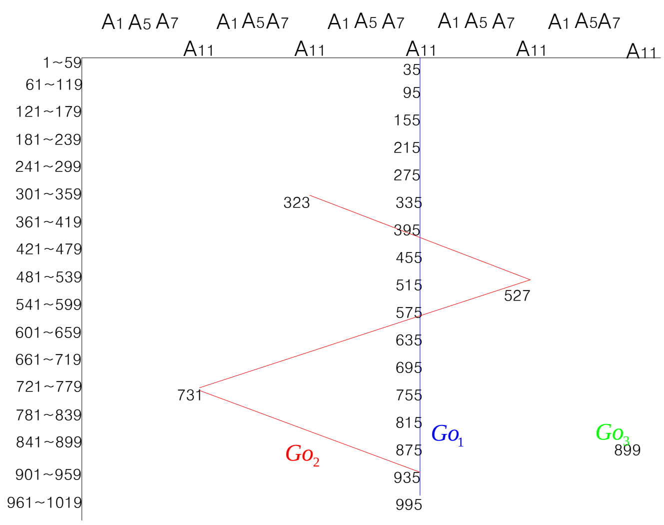
$Go_6 : a_1 = (12x+5)(12y+7) = 4355, \quad d = 12\times(12x+5) = 780(x=5, y=5)$

$Go_7 : a_1 = (12x+5)(12y+7) = 6083, \quad d = 12\times(12x+5) = 924(x=6, y=6)$

$Go_8 : a_1 = (12x+5)(12y+7) = 8099, \quad d = 12\times(12x+5) = 1068(x=7, y=7)$

$\vdots$

$Go_n : a_1 = (12x+5)(12y+7) = 144xy+84x+60y+35, \quad d = 12\times(12x+5)(x=n-1, y=n-1)$



3.16 $(12x+7)(12y+5),(Gp_n)$

×	7	19	31	43	55	67	79	91
5	35							
17	119	323						
29	203	551	899					
41	287	779	1271	1763				
53	371	1007	1643	2279	2915			
65	455	1235	2015	2795	3575	4355		
77	539	1463	2387	3311	4235	5159	6083	
89	623	1691	2759	3827	4895	5963	7031	8099

Gp_1: $35(a_1)$ $119(a_2)$ $203(a_3)$ $287(a_4)$ $371(a_5)$ $455(a_6)$ $539(a_7)$ $623(a_8)$

Gp_2: $323(a_1)$ $551(a_2)$ $779(a_3)$ $1007(a_4)$ $1235(a_5)$ $1463(a_6)$ $1691(a_7)$

Gp_3: $899(a_1)$ $1271(a_2)$ $1643(a_3)$ $2015(a_4)$ $2387(a_5)$ $2759(a_6)$

Gp_4:$1763(a_1)$ $2279(a_2)$ $2795(a_3)$ $3311(a_4)$ $3827(a_5)$

Gp_5:$2915(a_1)$ $3575(a_2)$ $4235(a_3)$ $4895(a_4)$

Gp_6:$4355(a_1)$ $5159(a_2)$ $5963(a_3)$

Gp_7:$6083(a_1)$ $7031(a_2)$

Gp_8:$8099(a_1)$

$\mathrm{Gp}_1 : a_1 = (12x+7)(12y+5) = 35, \quad d = 12\times(12x+7) = 84(x=0, y=0)$

$\mathrm{Gp}_2 : a_1 = (12x+7)(12y+5) = 323, \quad d = 12\times(12x+7) = 228(x=1, y=1)$

$\mathrm{Gp}_3 : a_1 = (12x+7)(12y+5) = 899, \quad d = 12\times(12x+7) = 372(x=2, y=2)$

$\mathrm{Gp}_4 : a_1 = (12x+7)(12y+5) = 1763, \quad d = 12\times(12x+7) = 516(x=3, y=3)$

$\mathrm{Gp}_5 : a_1 = (12x+7)(12y+5) = 2915, \quad d = 12\times(12x+7) = 660(x=4, y=4)$

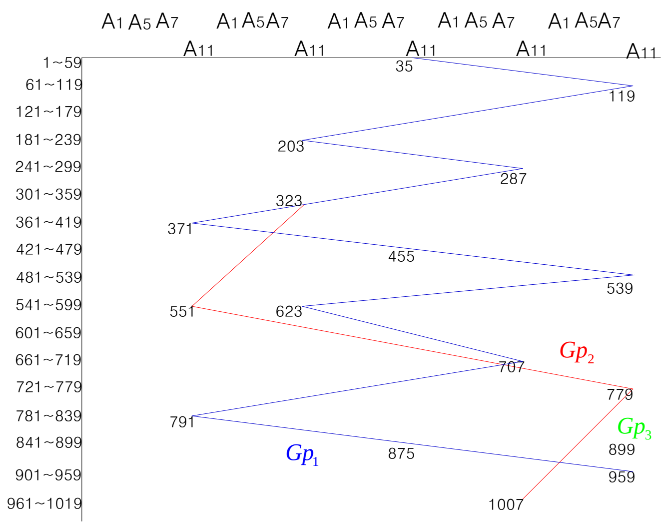
$\mathrm{Gp}_6 : a_1 = (12x+7)(12y+5) = 4355, \quad d = 12\times(12x+7) = 804(x=5, y=5)$

$\mathrm{Gp}_7 : a_1 = (12x+7)(12y+5) = 6083, \quad d = 12\times(12x+7) = 948(x=6, y=6)$

$\mathrm{Gp}_8 : a_1 = (12x+7)(12y+5) = 8099, \quad d = 12\times(12x+7) = 1092(x=7, y=7)$

$\vdots$

$\mathrm{Gp}_n : a_1 = (12x+7)(12y+5) = 144xy+60x+84y+35, d = 12\times(12x+7)(x=n-1, y=n-1)$



4. Conclusion

This paper asserts that the composite numbers of the $12n+1,5,7,11$ series make up sixteen arithmetic progression groups. We were able to indirectly deduce the uncertainty of prime numbers through the composite numbers. It is true that, unlike the prime numbers, the composite numbers are governed by a rule that is structural and regular.]

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<attached document 1>

5	7	11	13	17	19	23	25	29	31	35	37	41	43	47	49	53	55	59
							25(Gb_1)			35(Go_1,Gp_1)					49(Gc_1)		55(Gk_1,Gl_1)	
5	7	11	13	17	19	23		29	31		37	41	43	47		53		59

61	65	67	71	73	77	79	83	85	89	91	95	97	101	103	107	109	113	115	119
	65(Ge_1,Gf_1)				77(Gg_1,Gh_1)			85(Gb_1)		91(Gi_1,Gj_1)	95(Go_1)							115(Gk_1)	119(Gp_1)
61		67	71	73		79	83		89			97	101	103	107	109	113		

121	125	127	131	133	137	139	143	145	149	151	155	157	161	163	167	169	173	175	179
121(Gd_1)	125(Gf_1)			133(Gc_1)			143(Gm_1,Gn_1)	145(Gb_1)			155(Go_1)		161(Gg_1)			169(Ga_1)		175(Gj_1,Gk_1)	
		127	131		137	139			149	151		157		163	167		173		179

181	185	187	191	193	197	199	203	205	209	211	215	217	221	223	227	229	233	235	239
	185(Gf_1)	187(Gl_1)					203(Gp_1)	205(Gb_1)	209(Gh_1)		215(Go_1)	217(Gc_1)	221(Ge_1)					235(Gk_1)	
181			191	193	197	199				211				223	227	229	233		239

241	245	247	251	253	257	259	263	265	269	271	275	277	281	283	287	289	293	295	299
	245(Gf_1,Gg_1)	247(Gi_1)		253(Gd_1)		259(Gj_1)		265(Gb_1)			275(Gn_1,Go_1)				287(Gp_1)	289(Gb_2)		295(Gk_1)	299(Gm_1)
241			251		257		263		269	271		277	281	283			293		

301	305	307	311	313	317	319	323	325	329	331	335	337	341	343	347	349	353	355	359
301(Gc_1)	305(Gf_1)					319(Gl_1)	323(Go_2,Gp_2)	325(Ga_1,Gb_1)	329(Gg_1)		335(Go_1)		341(Gh_1)	343(Gj_1)				355(Gk_1)	
		307	311	313	317					331		337			347	349	353		359

361	365	367	371	373	377	379	383	385	389	391	395	397	401	403	407	409	413	415	419
361(Gc_2)	365(Gf_1)		371(Gp_1)		377(Ge_1)			385(Gb_1,Gc_1,Gd_1)		391(Gk_2,Gl_2)	395(Go_1)			403(Gi_1)	407(Gn_1)		413(Gg_1)	415(Gk_1)	
		367		373		379	383		389			397	401			409			419

421 425 427 431 433 437 439 443 445 449 451 455 457 461 463 467 469 473 475 479

425($Gf_1Ge_2Gf_2$) 427(Gj_1) 437(Gg_2)(Gh_2) 445(Gb_1) 451(Gl_1) 455($Gm_1Go_1Gp_1$) 469(Gc_1) 473(Gh_1) 475($Gk_1Gi_2Gj_2$)

421 431 433 439 443 449 457 461 463 467 479

481 485 487 491 493 497 499 503 505 509 511 515 517 521 523 527 529 533 535 539

481(Ga_1) 485(Gf_1) 493(Gb_2) 497(Gg_1) 505(Gb_1) 511(Gj_1) 515(Go_1) 517(Gd_1) 527(Go_2) 529(Gd_2) 533(Ge_1) 535(Gk_1) 539(Gn_1Gp_1)

487 491 499 503 509 521 523

541 545 547 551 553 557 559 563 565 569 571 575 577 581 583 587 589 593 595 599

545(Gf_1) 551(Gp_2) 553(Gc_1) 559(Gi_1) 565(Gb_1) 575($Gm_2Gn_2Go_1$) 581(Gg_1) 583(Gl_1) 589(Gc_2) 595($Gj_1Gk_1Gk_2$)

541 547 557 563 569 571 577 587 593 599

601 605 607 611 613 617 619 623 625 629 631 635 637 641 643 647 649 653 655 659

605(Gf_1Gh_1) 611(Gm_1) 623(Gp_1) 625(Ga_2Gb_1) 629(Gf_2) 635(Go_1) 637(Ga_1Gc_1) 649(Gd_1) 655(Gk_1)

601 607 613 617 619 631 641 643 647 653 659

661 665 667 671 673 677 679 683 685 689 691 695 697 701 703 707 709 713 715 719

665($Gf_1Gg_1Gg_2$) 667(Gl_2) 671(Gn_1) 679(Gj_1) 685(Gb_1) 689(Ge_1) 695(Go_1) 697(Gb_2) 703(Gj_2) 707(Gp_1) 713(Gh_2) 715($Gi_1Gk_1Gl_1$)

661 673 677 683 691 701 709 719

721 725 727 731 733 737 739 743 745 749 751 755 757 761 763 767 769 773 775 779

721(Gc_1) 725(Gf_1Ge_2) 731(Go_2) 737(Gh_1) 745(Gb_1) 749(Gg_1) 755(Go_1) 763(Gj_1) 767(Gm_1) 775(Gi_2Gk_1) 779(Gp_2)

727 733 739 743 751 757 761 769 773

781 785 787 791 793 797 799 803 805 809 811 815 817 821 823 827 829 833 835 839

781(Gd_1) 785(Gf_1) 791(Gp_1) 793(Ga_1) 799(Gk_2) 803(Gn_1) 805($Gb_1Gc_1Gd_2$) 815(Go_1) 817(Gc_2) 833(Gg_1Gf_2) 835(Gk_1)

787 797 809 811 821 823 827 829 839

841 845 847 851 853 857 859 863 865 869 871 875 877 881 883 887 889 893 895 899

841Gb_3 845(Ge_1Gf_1) 847(Gj_1Gl_1) 851(Gn_2) 865(Gb_1) 869(Gh_1) 871(Gi_1) 875($Go_1Gp_1Gm_2$) 889(Gc_1) 893(Gg_2) 895(Gk_1) 899(Go_3Gp_3)

853 857 859 863 877 881 883 887

901 905 907 911 913 917 919 923 925 929 931 935 937 941 943 947 949 953 955 959

901(Gb_2) 905(Gf_1) 913(Gd_1) 917(Gg_1) 923(Gm_1) 925(Gb_1Ga_2) 931(Gj_1Gj_2) 935($Gn_1Go_1Go_2$) 943(Gl_2) 949(Ga_1) 955(Gk_1) 959(Gp_1)

907 911 919 929 937 941 947 953

961 965 967 971 973 977 979 983 985 989 991 995 997

961Gc_3 965(Gf_1) 973(Gc_1) 979(Gl_1) 985(Gb_1) 989(Gh_2) 995(Go_1)

967 971 977 983 991 997

Figure) If you knew the composite numbers less than 1000, you can filter out the prime numbers.