A Fifth Smarandache Friendly Prime Pair

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Abstract

A Smarandache friendly prime pair (SFPP) is a pair of prime numbers (*p,q*), $p < q$, such that the product *pq* is equal to the sum of all primes from *p* to *q* inclusive. Previously four such pairs were known: (2,5), (3,13), (5,31) and (7,53). Now a fifth one is found by a brute force computer search. A heuristic approximation can be to estimate the expected number of SFPPs in a given interval. The result suggests that the probability of further pairs existing is about 0.07.

Introduction

Given any sequence $\{a_i\}$, two elements of the sequence a_n, a_m are called a Smarandache Friendly Pair with respect to the sequence if

$$a_n a_m = \sum_{i=n}^{m} a_i$$

For example, if the sequence is the natural numbers then there are an infinite number of friendly pairs [1] and all such pairs can be determined [2].

Where the sequence is the prime numbers $a_n = p_n$, the pairs are called Smarandache Friendly Prime Pairs (SFPP). Four such pairs were previously discovered by Felice Russo [3] (2,5), (3,13), (5,31) and (7,53).

Using a brute force calculation a fifth pair has been found to be (3536123, 128541727). The calculation method was similar to the one described by Russo. A pre-calculation of the partial sum of primes function was used to optimise performance of the program.

Heuristics

It is possible to estimate the number of Smarandache friendly prime pairs in a given range by using the well known asymptotic approximation for the n^{th} prime number

$$p_n \sim n \log(n)$$

We define the discrepancy function

$$\Delta_{n,m} = p_n q_m - \sum_{i=n}^{m} p_i$$

An SFPP is a solution to $\Delta_{n.m} = 0$.

The approximation gives

$$\Delta_{n,m} \sim nm\log(n)\log(m) - \frac{m^2}{2}\log(m) + \frac{n^2}{2}\log(n)$$

Assuming $\gg n$, the last term can be neglected and the solution to $\Delta_{n.m} = 0$ is given by

$$m \sim 2n\log(n)$$

At around this point $\Delta_{n,m}$ will be small. We can estimate the probability that an exact zero will be found for fixed *n* by estimating the change

$$\delta = \Delta_{n,m+1} - \Delta_{n,m} \sim n\log(n)\log(m) - m\log(m) \sim -n\,(\log(n))^2$$

Assuming a pseudo-random behaviour for the primes, this means that for large *n* the probability f_n of finding an SFPP whose first prime is p_n is given by

$$f_n \sim \frac{1}{n(\log(n))^2}$$

To estimate $e(k,l)$ the number of SFPPs with first prime between p_k and p_l for large *k* and *l* , we sum the series approximately by replacing the sum with an integral,

$$e(k,l) \sim \sum_{n=k}^{l} f_n \sim \frac{1}{\log(k)} - \frac{1}{\log(l)}$$

The number of SFPPs with first prime greater than p_n for *n* large can now be estimated as

$$e(n,\infty) \sim \frac{1}{\log(n)}$$

The convergence of the series suggests that the number of SFPPs is finite. However the convergence is slow. In the previous study[3] Felice Russo had searched up to p_n = 10000, which is *n* = 1229. Therefore the probability of finding larger primes was *a priori* 0.14.

The search has now been continued up to p_n = 29000000, *n* ~ 1700000 (note that the search is not provably exhaustive). The probability of further SMPPs is now estimated at 0.07

To get an idea of how good this estimation scheme is it is indicative to search for *near* SFPPs with $|\Delta_{n,m}| < M$ for a suitable value of *M.* This increases the expected number of finds by a factor of 2*M*. As an example, consider *M* = 200 with the first prime between 10000 and 1000000. The estimated number of finds is about 20, and the actual number found was 14. This is a reasonable confirmation given the approximations made in making the estimate.

References

[1] A. Murphy, “Smarandache friendly numbers and a few more sequences”, Smarandache Notions Journal, Vol 12 N, 1-2-3 Spring 2001

[2]Maohua Le, “The Smarandache friendly natural number pairs”, Smarandache Notions Journal, V13, Issue 1-2-3 (Spring 2002)

[3] Felice Russo, “On a problem concerning the Smarandache friendly prime pairs”, viXra:1004.0125, Smarandache Notions Journal, p56-58, 2002