Applications of Euclidian Snyder geometry to space time physics & Deceleration parameter (DM generated DE replacement?)

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## Abstract

#### A thought experiment: LQG or string theory as an initial space-time template for emergent gravity?

- Applications of deformed Euclidian space to questions about the role of string theory and/or LQG
- To what degree are the fundamental constants of nature preserved between different cosmological cycles?
- To what degree is gravity an emergent field that is partly/largely classical with extreme nonlinearity, or a QM/quantum field theory phenomenon?

#### <u>Math</u> – Physics representation of core issues of higher dimensional <u>contribution</u>

Start off with a basic statement of strength of matter
 graviton interaction, assuming KK graviton

$$\mathfrak{I} = -\frac{\kappa}{2} \cdot \sum_{n} \int d^4 x h_{uv}^n \cdot T^{uv} \sim 1/M_{PL}^2$$

The stress energy tensor comes from the standard model, and the h term is from using a KK graviton interactions model, up to the n th mode.

Has some similarity with graviton-neutrino interaction issues talked about in this PPT

#### Does the last slide hold if we make the following modification of a KK tower of gravitons ?

- Modification put in, as seen in later to mimic DE
- Suggestion to look at, here, is to consider what if

• 
$$m_n(Graviton) = \frac{n}{L} + 10^{-65}$$
 grams?

#### Issues to raise

- 1. Is there a link between a 1<sup>st</sup> inflation, ending in 10<sup>^</sup> - 35 seconds after big bang, and 2<sup>nd</sup> inflation commencing before 1 billion years ago ?
- 2. Commonality between the two?
- 3. Do gravitons, with tiny mass play a role in the 1<sup>st</sup> and 2<sup>nd</sup> inflationary episodes ?

### Look at A. Yurov; arXiv : hepth/028129 v1, 19 Aug, 2002

- Claim: Exist one emergent complex scalar field. Accounts for both 1<sup>st</sup> and 2<sup>nd</sup> inflation
- lacksquare

Potential in both cases chaotic inflation of the type

$$V = \vec{m}^2 \Phi^* \Phi$$

## Mass m is for inflaton, and $2^{nd}$ $\vec{m}$ expression has links to $5^{th}$ dim length L

$$\vec{m} \approx \sqrt{\frac{3}{8}} \cdot \left[ \sqrt{\frac{3H^2}{4\pi G}} \right|_{time \sim 10^{-35} \text{ sec}} + \sqrt{\frac{3H^2}{4\pi G}} \right|_{time \sim 10^{-44} \text{ sec}} \right]$$

$$\left|\vec{m}\right| \leq \left[\frac{l^2}{4}\right]$$

## Important simplification used

 From beginning of inflation, assume V potential energy is much smaller than H contribution

$$\frac{3H^2}{4\pi G} >> V(t)\Big|_{time \sim 10^{-44} \text{ sec}}$$

### The 5<sup>th</sup> dimensional length L is for a brane theory "arc-length"

• From Roy Maarten's Brane theory work

$$dS^{2}\Big|_{5-\dim} = \frac{l^{2}}{z^{2}} \cdot \left[\eta_{uv} dx^{\mu} dx^{\nu} + dz^{2}\right]$$

#### Find equivalence between

• 1<sup>st</sup>, 2<sup>nd</sup> equations for Friedman relations

$$H^{2} = \frac{1}{6} \cdot \left[ \dot{\phi}^{2} + \ddot{m}^{2} \phi^{2} + \frac{M^{2}}{\phi^{2}} \right] \leftrightarrow \left( \frac{\tilde{\kappa}^{2}}{3} \left[ \rho + \frac{\rho^{2}}{2\lambda} \right] \right) + \frac{m}{a^{4}}$$

$$\dot{H}^2 \cong \left[-2\frac{m}{a^4}\right] \leftrightarrow \dot{H} = V - 3H^2$$

#### Beckwith asserts there may be some reason to expect linkage between 1<sup>st</sup> and 2<sup>nd</sup> inflation

 Look at if the scalar field arises in 2<sup>nd</sup> inflation due to

$$dS^{2}\Big|_{5-\dim} = \frac{l^{2}}{z^{2}} \cdot \left[\eta_{uv} dx^{\mu} dx^{\nu} + dz^{2}\right] \\ \left|\vec{m}\right| \leq \left[\frac{l^{2}}{4}\right] \\ \phi_{0,-} = \sqrt{2/3} \cdot \vec{m} \cdot \left[t_{1st-EXIT} \sim 10^{-35} \text{ sec}\right] \\ \phi_{+} = \left[\phi_{0,+}^{3} - \sqrt{3/2} \cdot \frac{3M^{2}t}{\vec{m}}\right]^{1/3}$$

### Snyder formulation of HUP

1<sup>st</sup> Basic relation

 $[q, p] = i \cdot \sqrt{1 - \alpha \cdot p^{2}} \Leftrightarrow \Delta q \Delta p \ge \frac{1}{2} \cdot \left| \left\langle \sqrt{1 - \alpha \cdot p^{2}} \right\rangle \right|$ 

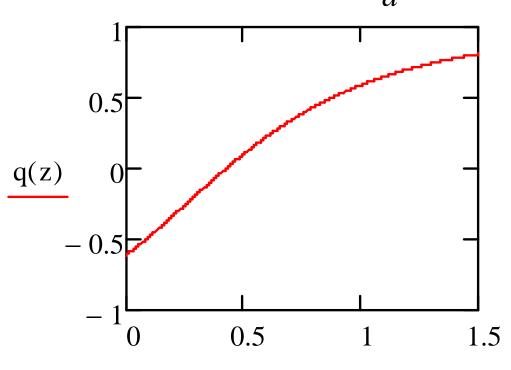
2<sup>nd</sup> Basic relation

 $\Delta q \ge \left[ \left( 1 / \Delta p \right) + l_s^2 \cdot \Delta p \right] = \left( 1 / \Delta p \right) - \alpha \cdot \Delta p$ 

 $\begin{array}{lll} \mathbf{3^{rd}\ Basic\ relation} \\ \mathsf{LQG\ has} & \alpha > 0 \\ \mathsf{Braneworld} & \alpha < 0 \end{array}$ 

#### Jerk calculation leads to

• If we define the jerk  $q = -\frac{\ddot{a}a}{\dot{a}^2}$ 



### Assuming a brane world

Z (red shift value). Change in sign for Z ~.42 is almost one billion years ago, corresponding to reacceleration of the universe, i.e

Basic results of <u>Alves</u>, et al. (2009), using their parameter values, with an additional term of "dark flow" added, corresponding to one KK additional dimensions. For brane world, the following modification of Roy Maarsten's

- KK tower assumed to have a small nonzero mass added, i.e. no zero order value for the graviton
- 4 D graviton ~  $10^{-65}$  grams

$$m_n(Graviton) = \frac{n}{L} + 10^{-65}$$

#### Issue is that a 4 dimensional Graviton with small mass violates principle of correspondence- complimentarity

# How important is such a violation ?

# For brane world, use these evolution equations

## Friedman equation, subsequently modified

$$\dot{a}^{2} = \left[ \left( \frac{\rho}{3M_{4}^{2}} + \frac{\Lambda_{4}}{3} + \frac{\rho^{2}}{36M_{Planck}^{2}} \right) a^{2} - \kappa + \frac{C}{a^{2}} \right]$$

Density equation, with nonzero graviton mass

$$\rho \equiv \rho_0 \cdot \left(\frac{a_0}{a}\right)^3 - \left[\frac{m_g c^6}{8\pi G \hbar^2}\right] \cdot \left(\frac{a^4}{14} + \frac{2a^2}{5} - \frac{1}{2}\right)$$

# For LQG, use these evolution equations

Friedman equations, assuming 'constant' momentum

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\kappa}{3} \cdot \rho \qquad \left(\frac{\dot{a}}{a}\right)^2 \equiv \frac{\kappa}{6} \cdot \frac{p_{\phi}^2}{a^6} \qquad \left(\frac{\ddot{a}}{a}\right) = -\frac{2 \cdot \kappa}{3} \cdot \rho$$

Density equation

$$\rho \equiv \rho_0 \cdot \left(\frac{a_0}{a}\right)^3 - \left[\frac{m_g c^6}{8\pi G\hbar^2}\right] \cdot \left(\frac{a^4}{14} + \frac{2a^2}{5} - \frac{1}{2}\right)$$

### Can neutrinos interact with Gravitons? Part 1

Bashinsky states that the density of gravitons interacting with neutrinos causes an alteration of overall GR density via

 $\left|1-5\cdot\left(\rho_{neutrino}/\rho\right)+\mathcal{G}\left(\left[\rho_{neutrino}/\rho\right]^{2}\right)\right|$ 

# Can neutrinos interact with Gravitons? Part 2

 George Fuller and Chad Kishimoto's PRL stretched neutrino hypothesis: a neutrino could be stretched 'across the universe' leading to (if there is an interaction with gravitons):

A few select gravitons, coupled to almost infinite wavelength stretched neutrinos would lead to at least the following stretched graviton wave

$$\lambda_{graviton} \equiv \frac{\hbar}{m_{graviton} \cdot c} < 10^4 \, meters$$

# Semiclassical interpretation of giant graviton waves?

Brought up as a way to interpret the existence of a small graviton mass, which appears to violate the QM correspondence principle (shown later)

Main motivation: a field theory limit demo that shows problems with **massive** graviton field theories, and the limit

$$m_{graviton} \rightarrow 0$$

#### How to measure a graviton/ GW ?

 Look at the normalized gravitational wave density function

$$\Omega_{gw} \equiv \frac{\rho_{gw}}{\rho_c} \equiv \int_{f=0}^{f=\infty} d(\log f) \cdot \Omega_{gw}(f) \Longrightarrow h_0^2 \Omega_{gw}(f) \cong 3.6 \cdot \left[\frac{n_f}{10^{37}}\right] \cdot \left(\frac{f}{1kHz}\right)^4$$

 Note that n depends upon frequency and is stated to be part of the unit phase space Infinite Quantum statistics. From the work presented in the Paris observatory, July 2009 Start with

$$Z_N \sim \left(\frac{1}{N!}\right) \cdot \left(\frac{V}{\lambda^3}\right)^N$$
  $S \approx N \cdot \left(\log[V/N\lambda^3] + 5/2\right)$ 

 $S \approx N \cdot \left( \log \left[ V / \lambda^3 \right] + 5 / 2 \right)$   $V \approx R_H^3 \approx \lambda^3$ 

V stands for volume of nucleation regime space. "particles" nucleate from 'vacuum' in QM

For DM. V for nucleation is HUGE. Graviton space V for nucleation is tiny, well inside inflation/ Therefore, the log factor drops OUT of entropy S if V chosen properly. For small V, then

$$\Delta S \approx \Delta N_{gravitons}$$

#### Some considerations about the partition function

Glinka (2007): if we identify 
$$\Omega = \frac{1}{2|u|^2 - 1}$$

- as a partition function (with u part of a Bogoliubov transformation) due to a graviton-quintessence gas, to get information theory-based entropy  $S \equiv \ln \Omega$
- 1. Derivation by Glinka explicitly uses the Wheeler De Witt equation
- 2. 2. Is there in any sense a linkage of Wheeler De Witt equation with String theory results ?

#### PROBLEM TO CONSIDER:

Ng's result quantum counting algorithm is a **STRING theory** result.Glinka is **Wheeler De Witt equation**. **Equivalent ?** 

#### **Questions to raise**.

Can we make a linkage between Glinka's quantum gas argument, and a small space version/ application of Ng's Quantum infinite statistics ?

In addition, if the quantum graviton gas is correct, can we model emergent structure of gravity via linkage between Ng particle count, and Q.G.G argument?

# More on the Glinka quantum gas hypothesis, part 1

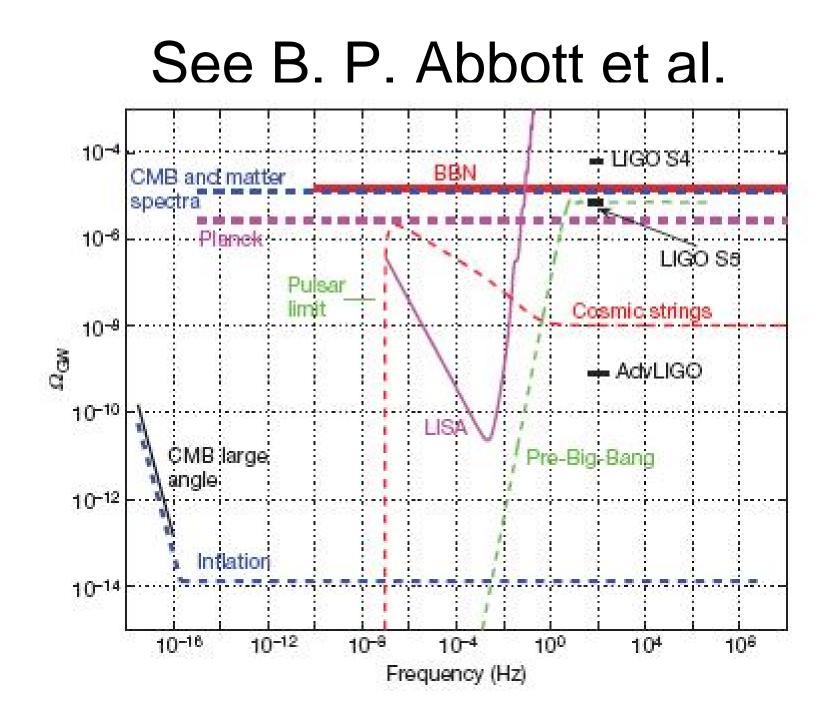
 Starting point to Glinka's Quantum gas, in terms of numerical count

$$n_{f} = \left[1/4\right] \cdot \left[\sqrt{\frac{v(a_{initial})}{v(a)}} - \sqrt{\frac{v(a)}{v(a_{final})}}\right]$$

# More on the Glinka Quantum gas hypothesis, part 2

• Relevant to understanding the role of

$$\Omega_{gw}(v) \cong \frac{3.6}{h_0^2} \cdot \left[\frac{n_f}{10^{37}}\right] \cdot \left(\frac{v}{1kHz}\right)^4$$



#### LQG, while using WdW up to a point, does not admit higher dimensions above 4 dimensions. String-Brane theory does

• Why is this relevant to a discussion of the LQG vs Brane theory discussion ?

Breakdown of field theory with respect to massive gravitons in limit  $m_{graviton} \rightarrow 0$ 

The massless equation of the graviton evolution equation takes the form

$$\partial_{\mu}\partial^{\mu}h_{\mu\nu} = \sqrt{32\pi G} \cdot \left(T_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}T_{\mu}^{\mu}\right)$$

## Consider what happens with a graviton mass

$$m_{graviton} \neq 0$$

#### From Maggiore (2008):

$$\left(\partial_{\mu}\partial^{\mu} - m_{graviton}\right) \cdot h_{\mu\nu} = \left[\sqrt{32\pi G} + \delta^{+}\right] \cdot \left(T_{\mu\nu} - \frac{1}{3}\eta_{\mu\nu}T_{\mu}^{\mu} + \frac{\partial_{\mu}\partial_{\nu}T_{\mu}^{\mu}}{3m_{graviton}}\right)$$

## The mismatch between these two equations when

 $m_{graviton} \rightarrow 0$ 

Is largely due to, even if graviton mass goes to zero

$$m_{graviton}h^{\mu}_{\mu} \neq 0$$

$$m_{graviton} \cdot h^{\mu}_{\mu} = -\left[\sqrt{32\pi G} + \delta^{+}\right] \cdot T^{\mu}_{\mu}$$

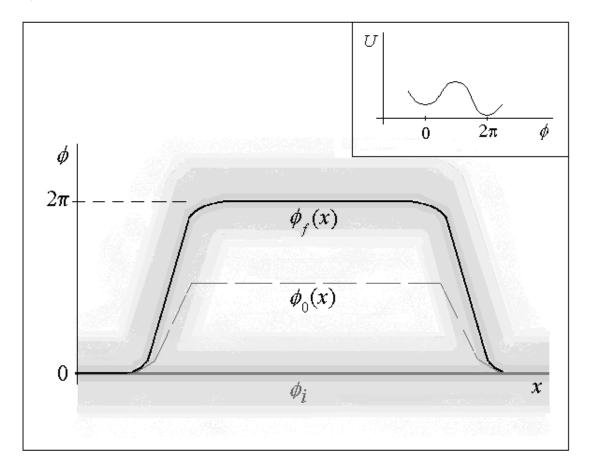
#### Try semiclassical model of graviton, as kink-anti kink pair

- How does this fit in with t'Hooft's deterministic QM?
- From a 1+ dimensional kink-antikink

$$\Psi_{i,f} \left[ \phi(\mathbf{x}) \right]_{\phi = \phi_{ci,cf}} = c_{i,f} \cdot \exp\left\{ -\int d\mathbf{x} \ \alpha \left[ \phi_{Ci,f}(\mathbf{x}) - \phi_0(\mathbf{x}) \right]^2 \right\},\$$

# From density wave physics, 1+ dimensions

<u>Kink-antikinks lead to a vacuum wave function. The LHS</u> is a kink; the RHS is an antikink.



#### The wave functional should have t'Hooft equivalence class structure added, in 4 to 5 dimensions

- T'Hooft used in 2006 an equivalence class argument as an embedding space for simple harmonic oscillators, as given in his Figure 2, on page 8 of his 2006 article.
- "Beneath Quantum Mechanics, there may be a deterministic theory with (local) information loss. This may lead to a sufficiently complex vacuum state." t'Hooft
- The author submits, that a kink-anti kink formulation of the graviton, when sufficiently refined, may indeed create such a vacuum state, as a generalization of Fig 2.

# One to four-five dimensions in instanton, anti-instaton construction

For one dimension, the semiclassical treatment has (CDW) a kink given by

Beckwith(2001) as  

$$\phi_{+}(z,\tau) = 4 \cdot \arctan\left\{\exp\left\{\frac{z+\beta\cdot\tau}{\sqrt{1-\beta^2}}\right\}\right\}$$

$$\frac{\partial^2 \phi(z,\tau)}{\partial \tau^2} - \frac{\partial^2 \phi(z,\tau)}{\partial z^2} + \sin \phi(z,\tau) = 0$$

## In five dimensions, M. Giovannini (2006) has constructed

For a five dimensional line element,

$$dS^{2} = a(w) \cdot \left[ \eta_{uv} dx^{u} dx^{v} - dw^{2} \right]$$
$$\phi = \tilde{v} + \arctan((bw)^{v})$$

### Supposition to get about the singularity in 4 dimensions, in early universe models

- Dropping in of 'information' to form an instantonanti-instanton pair, and avoiding the cosmological singularity via the 5th dimension?
- This lead to the author presenting in Chongqing, 11/15/2009 the region about the GR singularity is definable via a ring of space-time about the origin, but not overlapping it, with a time dimension defined

$$\Delta t \equiv 10^{\beta} \cdot t_{Planck}$$

#### The small mass of the graviton would be for energy in $\Delta E \Delta t \ge \hbar$

 Having said this, the author is fully aware of the String theory HUP variant

$$\Delta x \ge \frac{\hbar}{\Delta p} + \frac{l_s^2}{\hbar} \Delta p$$

• The idea would be to possibly obtain a way to look at counting for GW detectors

$$h_0^2 \Omega_{gw}(f) \cong \frac{3.6}{2} \cdot \left[ \frac{n_f [graviton] + n_f [neutrino]]}{10^{37}} \right] \cdot \left( \frac{\langle f \rangle}{1 k H z} \right)^4$$

#### The following is claimed:

If n (graviton) is obtained, then higher dimensional geometry may be relevant to transmitting information via gravitons from prior to present universes

- How much information can be carried by an individual graviton?
- Assume  $\Delta S \approx \Delta N_{gravitons}$
- Use Seth Lloyd's

$$I = S_{total} / k_B \ln 2 = \left[ \# operations \right]^{3/4} = \left[ \rho \cdot c^5 \cdot t^4 / \hbar \right]^{3/4}$$

# 10<sup>20</sup> relic gravitons yields almost 10<sup>27</sup> operations!

This value implies that per graviton, as nucleated at least 4 dimensions, there is at least **one unit** of information associated with the graviton (assuming there is at least **some relationship** between an operation and information)

 $\Delta S \approx \Delta N_{gravitons} \approx 10^{20} \Leftrightarrow 10^{20}$  or higher amounts of prior universe information transmitted to our cosmos?

#### Cosmological parameters and information from prior to present cosmos ?

• The fine structure constant would probably be a place to start, in terms of information

$$\widetilde{\alpha} \equiv e^2 / \hbar \cdot c \equiv \frac{e^2}{d} \times \frac{\lambda}{hc}$$

What the author thinks, is that <u>higher dimensional models</u> of gravity need to be developed, investigated, which may allow for such a counting algorithm.

Resolutions of questions about cosmological constants ?

- <u>1st Conclusion</u>, one needs a reliable information packing algorithm! I.e. for a wave length , as input into the fine structure constant, we need spatial / information limits defined for geometry
   ΔS ≈ ΔN<sub>gravitons</sub> ≈ 10<sup>20</sup> is only a beginning
  - <u>2<sup>nd</sup> Conclusion, assumed GW</u> detector sensitivity limits need a comprehensive look over, re do

#### Important consideration for review - Is there a linkage between neutrinos and gravitons? What about gravitons and E & M (photons)?

- From a 3 page article submitted to the 12<sup>th</sup> Marcel Grossman conference proceedings
- STRETCHED NEUTRINOS, AND THE SUPPOSED LINK TO Gravity / Gravitational waves data SETS
- ANDREW WALCOTT BECKWITH
- The issue of whether or not a correlation exists between neutrino physics and gravitational wave data sets/gravitons is raised anew. Particular emphasis is placed on analysis of the Fuller and Kishimoto scenario, suggesting that the wave function of a relic neutrino may span up to billions of light years across galaxies because of its low energy and particles traveling at different speeds.

#### • QUESTION ASKED :

 If there is an initial close relationship between gravitational waves /gravitons and relic neutrinos in early-universe nucleation, is there a corresponding "stretch-out" of gravitons? If so, what would this imply for improved graviton/gravity wave detectors?

## IF so, then what can we say about the following energy density ?

• We first start off with

$$\Omega_{gw} \equiv \frac{\rho_{gw}}{\rho_c} \equiv \int_{f=0}^{f=\infty} d(\log f) \cdot \Omega_{gw}(f)$$
$$\Rightarrow h_0^2 \Omega_{gw}(f) \cong 3.6 \cdot \left[\frac{n_f}{10^{37}}\right] \cdot \left(\frac{f}{1kHz}\right)^4$$

If neutrino-graviton coupling is possible, what also about photons coupled with gravitons? etc ?

• How reasonable then are the following ?

$$n_{f} \propto n_{f} [graviton] + n_{f} [neutrinos]$$
$$n_{f} \propto n_{f} [graviton] + n_{f} [neutrinos] + n_{f} [photons - E \& M]$$

Final inquiry, making sense of the supposed <u>"radius of the Universe" calculation</u>

 Matt Roos, has put in a foundational way of testing, via experiment, how to calculate a supposed 'radius of the universe'

$$r_U \equiv \frac{1}{H \cdot \sqrt{|\Omega - 1|}}$$

#### **Tweaking parameters of H, and** $\Omega \equiv \rho(t)/\rho_{critical}$ **from our inquiry**

- The choice of *H*, and of density , as in the
- equation below will allow the dynamics of
- how the universe expands mesh with a fuller
- understanding of structure formation.

$$\Omega \equiv \rho(t) / \rho_{critical}$$

#### Acknowledgements

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- **Dr. Christian Corda** is thanked for encouraging the author to explore semiclassical issues in GW and in GR, which lead to a meeting on foundational issues of gravitation in a symposium in Crete, September 2009; and forwarded the Alves et al. document to the author to expand the author's scientific horizons.

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- <u>(Please look at most recent version ! –</u>
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### Finally a good book summary with up to date summaries

- From "Series in High Energy physics, Cosmology and Gravitation" - Taylor and Francis (publishers)
- - Particle and Astroparticle Physics (2008)
- By Utpal Sarkar