

One-Way Light Speed Determination Using the Range Measurement Equation of the GPS

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Abstract. The one-way speed of light is determined using the range measurement equation of the Global Positioning System. This equation has been rigorously and extensively tested and verified in the Earth-Centred Inertial frame, a frame that moves with the Earth as it revolves around the Sun but does not share its rotation. The result is a simple demonstration of one-way light speed anisotropy depending on the direction of propagation relative to the rotating Earth that contradicts the principle of light speed constancy.

Keywords: special theory of relativity, principle of the constancy of the speed of light, GPS, range measurement equation, one-way light speed measurement.

1. Introduction

The isotropy or constancy of the speed of light in all inertial frames is a fundamental postulate of Einstein's Special Theory of Relativity and a major principle of modern physics [1-3]. While the principle is defined in inertial frames, its application in the non-inertial frame of the Earth's surface such as in the Einstein clock-synchronization procedure and the SI unit of length is accepted science since it would be virtually useless otherwise. Indeed the overwhelming majority of experiments testing the postulate including those by Brillet and Hall [4], Hils and Hall [5], Gagnon et.al. [6], Krisher et.al. [7], Riis et.al. [8], Antonini et al. [9], Hermann et al. [10] and many others [11] have been performed in the non-inertial terrestrial frame. However while light speed invariance has been experimentally established for two-way light transmission, the one-way speed of light has not been properly tested since this requires clock synchronization that it has not been possible to achieve [11]. The advent of the Global Positioning System (GPS) has changed the situation.

The GPS is a modern timing-ranging system with accurate synchronized atomic clocks that enable precise determination of position on the Earth. Wolf and Petit [12] used this system to test the isotropy of the speed of light and published a limit of $\delta c/c < 5 \times 10^{-9}$. This result however holds for the Earth-Centered Inertial frame or ECI frame in which the test was conducted and does not necessarily apply to other inertial frames moving relative to the ECI frame where the light speed may vary. Based on GPS timing, Marmet [13] observed that a light signal takes about 14 nanoseconds longer than the average time (where average time equals distance divided by c) traveling Eastward from San Francisco to New York while the signal takes about 14 nanoseconds less than the average time traveling Westward from New York to San Francisco. Kelly [14] also noted that time measurements using the GPS show that a light signal takes 207.4 nanoseconds longer to circumnavigate the Earth Eastward at the equator than the average time while a light signal takes 207.4 nanoseconds less in the Westward direction around the same path.

Both researchers concluded that these observed travel time differences in each direction arise because light travels at speed $c - v$ Eastward and $c + v$ Westward relative to the surface of the earth where v is the Earth's rotational speed at the particular latitude. They however did not publish any calculations perhaps because of the problems presented by measurement errors. Using the synchronized clocks of the GPS in a novel application, Gift [15] overcame these problems and directly confirmed light speed anisotropy $c \pm v$ in the East-West direction. In this paper we again test the principle of light speed constancy using a different feature of the GPS. Specifically, we utilise the range measurement equation of the GPS to demonstrate light speed anisotropy between two adjacent points fixed at the same latitude on the surface of the rotating Earth.

2. Light Speed Determination using the Range Measurement Equation

The range measurement equation is central to the operation of the GPS. It holds in an ECI frame which is a frame that moves with the Earth as it revolves around the Sun but does not share its rotation. It is given by [16]

$$\left| \bar{r}_r(t_r) - \bar{r}_s(t_s) \right| = c(t_r - t_s) \quad (1)$$

where t_s is the time of transmission of an electromagnetic signal from a source, t_r is the time of reception of the electromagnetic signal by a receiver, $\bar{r}_s(t_s)$ is the position of the source at the time of transmission of the signal and $\bar{r}_r(t_r)$ is the position of the receiver at the time of reception of the signal. Using elapsed time measurements determined by the GPS clocks and the light speed value c , this equation enables the accurate determination of position on the surface of the Earth. It has been extensively and rigorously tested and verified. The appearance of c in the equation is sometimes interpreted as a demonstration of the principle of light speed constancy in an inertial frame [17]. However Wang [18] used the range measurement equation operating in an ECI frame to show that the speed of light is dependent on the observer's uniform motion relative to the ECI frame. He did this by using the range measurement equation to determine elapsed time and concluded that the successful application of the range measurement equation in GPS operation is inconsistent with the principle of the constancy of the speed of light.

Following Wang's approach we use the range measurement equation to determine elapsed time for light traveling between two adjacent points fixed on the surface of the rotating Earth at the same latitude and use this time and the known distance between the two fixed points to determine the one-way speed of light. Thus, consider two adjacent GPS stations A and B at the same latitude and fixed on the surface of the Earth a distance D apart with B East of A. Since the Earth is rotating, the stations are moving eastward at speed v the Earth's surface speed at that latitude. Let D be sufficiently small such that the stations are moving uniformly in the same direction at speed v relative to the ECI frame. In such circumstances stations A and B at any instant constitute an inertial frame moving at speed v relative to the ECI frame.

2.1 Eastward Transmission

Let station A transmit a signal eastward at time t_I to station B which receives it at time t_F . On an axis fixed in the ECI frame along the line joining the two stations with the origin west of station A, let $x_A(t)$ be the position of station A at time t and $x_B(t)$ be the position of station B at time t . Then from the range measurement equation (1),

$$x_B(t_F) - x_A(t_I) = c(t_F - t_I) \quad (2)$$

where $x_B(t_F)$ is the position of station B at time t_F and $x_A(t_I)$ is the position of station A at time t_I . Since the stations are moving uniformly in the same direction at speed v relative to the ECI frame, it follows that the relation between the position $x_B(t_F)$ of station B at the time of reception of the signal and its position $x_B(t_I)$ at the time of emission of the signal is given by

$$x_B(t_F) = x_B(t_I) + v(t_F - t_I) \quad (3)$$

Substituting for $x_B(t_F)$ from (3) in (2) yields

$$x_B(t_I) - x_A(t_I) + v(t_F - t_I) = c(t_F - t_I) \quad (4)$$

This becomes

$$x_B(t_I) - x_A(t_I) = D = (c - v)(t_F - t_I) \quad (5)$$

giving the elapsed time as

$$(t_F - t_I) = \frac{D}{c - v} \quad (6)$$

Therefore the speed c_{AB} of the light traveling from station A to station B is given by separation D divided by elapsed time $(t_F - t_I)$ which using (6) is

$$c_{AB} = \frac{D}{(t_F - t_I)} = \frac{D}{D/(c - v)} = c - v \quad (7)$$

2.2 Westward Transmission

Let station B transmit a signal westward at time t_I to station A which receives it at time t_F . Then using the range measurement equation (1) and noting that $x_B(t_I) > x_A(t_F)$,

$$x_B(t_I) - x_A(t_F) = c(t_F - t_I) \quad (8)$$

where $x_B(t_I)$ is the position of station B at time t_I and $x_A(t_F)$ is the position of station A at time t_F . Since the stations are moving uniformly in the same direction at speed v relative to the ECI frame, the relation between the position $x_A(t_F)$ of station A at the time of reception of the signal and its position $x_A(t_I)$ at the time of emission of the signal is given by

$$x_A(t_F) = x_A(t_I) + v(t_F - t_I) \quad (9)$$

Substituting for $x_A(t_F)$ from (9) in (8) yields

$$x_B(t_I) - x_A(t_I) - v(t_F - t_I) = c(t_F - t_I) \quad (10)$$

This becomes

$$x_B(t_I) - x_A(t_I) = D = (c + v)(t_F - t_I) \quad (11)$$

giving the elapsed time as

$$(t_F - t_I) = \frac{D}{c + v} \quad (12)$$

Therefore the speed c_{BA} of the light traveling from station B to station A is given by separation D divided by elapsed time $(t_F - t_I)$ which using (12) is

$$c_{BA} = \frac{D}{(t_F - t_I)} = \frac{D}{D/(c + v)} = c + v \quad (13)$$

3. Discussion

The results in equations (7) and (13) indicate that light travels faster westward than eastward relative to the surface of the Earth. In particular the one-way determination of light speed using the range measurement equation of the GPS establishes in (7) that a signal sent eastward travels at speed c minus the rotational speed of the Earth v at that latitude giving $c - v$. The GPS data also shows in (13) that a signal sent westward travels at speed c plus the rotational speed of the Earth v at that latitude giving $c + v$. This is true for the short-distance inertial frame considered in this paper as well as long-distance circumnavigation of the Earth [14]. A similar light speed change $c \pm w$ was observed in the Roemer [19] and Doppler [20] experiments for the revolution of the Earth at speed w in its approximately linear motion around the Sun.

The light speeds $c \pm v$ determined using the GPS range measurement equation are different from the findings of the many light speed experiments [4-10] conducted in the terrestrial frame which all give light speed c but are exactly the results obtained using the GPS synchronized clocks [15]. These changed light speed values $c \pm v$ contradict the principle of the constancy of the speed of light since the principle requires constant light speed c for light traveling westward or eastward between the two adjacent stations. The timing differences arising from the varying light speed on the surface of the Earth are

actually programmed in the GPS computers in order to produce accurate global positioning.

Additionally, the operational accuracy of the range measurement equation in the ECI frame and the detection of light speed anisotropy $c \pm v$ in the frame of the Earth indicate that light travels at speed c in the ECI frame only and travels at different speeds in frames moving relative to the ECI frame. It follows therefore that the ECI frame is a preferred frame, a conclusion also arrived at by Wang [18]. He suggested that the ECI frame is a preferred frame only for regions close to the Earth since it is unlikely to extend to the boundaries of the solar system. In view of this and the previously reported detection of ether drift [21], further investigation is necessary in order to determine the extent of the domain of the ECI frame relative to other possible preferred frames.

4. Conclusion

In this paper, the range measurement equation of the GPS was used to determine the one-way light speed between two adjacent points at the same latitude and fixed on the surface of the rotating Earth. The result $c \pm v$ represents an unmistakable variation in the speed of light arising from the rotation of the Earth as the light travels in the east-west direction and is consistent with results previously obtained [13-15]. It is a direct repudiation of the principle of light speed constancy which is today routinely applied in the non-inertial frame of the surface of the Earth and is an integral part of orthodox science.

This confirms the light speed variation claim made previously by this author in [21] and demolishes the strident criticisms of that claim by Osborne [22], Klauber [23] and Flanagan and Terzian [24]. As Feynman said, "If it disagrees with experiment it is wrong. In that simple statement is the key to science. It does not make any difference how beautiful the guess is. It does not make any difference how smart you are, who made the guess, or what his name is-if it disagrees with experiment it is wrong. That's all there is to it." This of course is how science should be conducted but history and experience teach us otherwise.

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