A Group-Permutation Algorithm to Solve the Generalized SUDOKU

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Sudoku is a game with numbers, formed by a square with the side of 9, and on each row and column are placed the digits 1, 2, 3, 4, 5, 6, 7, 8, 9, written only one time; the square is subdivided in 9 smaller squares with the side of 3×3, which, also, must satisfy the same condition, i.e. each square to contain all digits from 1 to 9 written only once.

The Japanese company Nikoli has popularized this game in 1986 under the name of *sudoku*, meaning "single number".

Sudoku can be generalized to squares whose dimensions are $n^2 \times n^2$, where $n \geq 2$, using various symbols (numbers, letters, mathematical symbols, etc.), written just one time on each row and on each column; and the large square is divided into n^2 small squares with the side $n \times n$ and each will contain all n^2 symbols written only once.

An elementary solution of one of these generalized Sudokus, with elements (symbols) from the set

$$S = \{s_1,\ s_2, ...,\ s_n,\ s_{n+1},\ ...,\ s_{2n},\ ...,\ s_{n^\wedge 2}\}$$

(supposing that their placement represents the relation of total order on the set of elements S), is:

Row 1: all elements in ascending order

$$s_1,\ s_2, ...,\ s_n,\ s_{n+1},\ ...,\ s_{2n},\ ...,\ s_{n^\wedge 2}$$

On the next rows we will use circular permutations, considering groups of n elements from the first row as follows:

Row 2:

$$s_{n+1},\ s_{n+2}, ...,\ s_{2n};\ s_{2n+1},\ ...,\ s_{3n};\ ...,\ s_{n^\wedge 2}; s_1,\ s_2,\ ...,\ s_n$$

Row 3:

$$s_{2n+1},\ ...,\ s_{3n}; ...,\ s_{n^\wedge 2};\ s_1,\ s_2,\ ...,\ s_n;\ s_{n+1}, s_{n+2},\ ...,\ s_{2n}$$

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Row n:

$$s_{n^\wedge 2-n+1}, ...,\ s_{n^\wedge 2};\ s_1,\ ...,\ s_n,\ s_{n+1};\ s_{n+2}, ...,\ s_{2n};\ ...,\ s_{3n};\ ...,\ s_{n^\wedge 2-n}.$$

Now we start permutations of the elements of row $n+1$ considering again groups of n elements.

Row $n+1$:

$$s_2,\ ...,\ s_n, s_{n+1};\ s_{n+2},\ ...,\ s_{2n},\ s_{2n+1};\ s_{n^\wedge 2-n+2}, ...,\ s_{n^\wedge 2},\ s_1$$

Row $n+2$:

$$s_{n+2},\ ...,\ s_{2n}, s_{2n+1};\ s_{n^\wedge 2-n+2,}\ ...,\ s_{n^\wedge 2},\ s_1;\ s_2, ...,\ s_n,\ s_{n+1}$$

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Row $2n$:

s_{n^2-n+2} …, s_{n^2}, s_1; s_2, …, s_n, s_{n+1}; s_{n+2}, …, s_{2n}, s_{2n+1}

Row $2n+1$:

s_3, …, s_{n+2}; s_{n+3}, …, s_{2n+2}; s_{n^2+3}, …, s_{n^2}, s_1, s_2

and so on.

Replacing the set S by any permutation of its symbols, which we'll note by S', and applying the same procedure as above, we will obtain a new solution.

The classical Sudoku is obtained for $n = 3$.

Below is an example of this group-permutation algorithm for the classical case:

1	2	3	4	5	6	7	8	9
4	5	6	7	8	9	1	2	3
7	8	9	1	2	3	4	5	6
2	3	4	5	6	7	8	9	1
5	6	7	8	9	1	2	3	4
8	9	1	2	3	4	5	6	7
3	4	5	6	7	8	9	1	2
6	7	8	9	1	2	3	4	5
9	1	2	3	4	5	6	7	8

For a **$4^2 \times 4^2$ square** we use the following 16 symbols:

{A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P}

and use the same group-permutation algorithm to solve this Sudoku.

From one solution to the generalized Sudoku we can get more solutions by simply doing permutations of columns or/and of rows of the first solution.

A	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**	**K**	**L**	**M**	**N**	**O**	**P**
E	**F**	**G**	**H**	**I**	**J**	**K**	**L**	**M**	**N**	**O**	**P**	**A**	**B**	**C**	**D**
I	**J**	**K**	**L**	**M**	**N**	**O**	**P**	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**
M	**N**	**O**	**P**	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**	**K**	**L**
B	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**	**K**	**L**	**M**	**N**	**O**	**P**	**A**
F	**G**	**H**	**I**	**J**	**K**	**L**	**M**	**N**	**O**	**P**	**A**	**B**	**C**	**D**	**E**
J	**K**	**L**	**M**	**N**	**O**	**P**	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**
N	**O**	**P**	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**	**K**	**L**	**M**
C	**D**	**E**	**F**	**G**	**H**	**I**	**J**	**K**	**L**	**M**	**N**	**O**	**P**	**A**	**B**
G	**H**	**I**	**J**	**K**	**L**	**M**	**N**	**O**	**P**	**A**	**B**	**C**	**D**	**E**	**F**
K	**L**	**M**	**N**	**O**	**P**	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**
O	**P**	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**	**K**	**L**	**M**	**N**
D	**E**	**F**	**G**	**H**	**I**	**J**	**K**	**L**	**M**	**N**	**O**	**P**	**A**	**B**	**C**
H	**I**	**J**	**K**	**L**	**M**	**N**	**O**	**P**	**A**	**B**	**C**	**D**	**E**	**F**	**G**
L	**M**	**N**	**O**	**P**	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**	**K**
P	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**	**K**	**L**	**M**	**N**	**O**

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