An Inner Connection Between Gravity, Electromagnetism and Light

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Abstract

In this paper, we find out a clear connection between gravity and electromagnetism. We prove that the universe expansion affects gravity as well as the permittivity and the permeability of the free space. The use of a special expansion notion in the special relativity theory to study the inertial frame movement leads to the existence of an inner connection between gravity and electromagnetism. It turns out that gravity is perceptible to the surrounding medium via an invariant equation, which leads to perceive gravity as nothing else than a transverse undulation of the same medium that is the origin of electric and magnetic phenomena as well as light.

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I. INTRODUCTION

A. The Laws of Physics

The week force and the electromagnetic force were unified in a mathematically consistent fashion by Abdu-Salam, Weinberg and Glashow in 1970 ([9],[13],[15]). However, there is no convincing approach yet concerning any possible connection of all the other forces of nature (the strong force, the gravitational force and the electromagnetic force). Most attempts at unification have been for many years within a frame associating electromagnetism with new geometrical properties of space-time ([7],[4]). In this paper we present a modest approach that will focus on the existence of an inner connection between gravity and electromagnetism using a new mathematical model that incorporates a restricted notion of space expansion in the special relativity theory. The use of a special expansion notion in the special relativity theory to study the uniform motion of the observer relative to the source leads to the existence of inner connection between gravity and electromagnetism.

B. New Tools for Investigation

It is known that our universe is in continuous evolution from the past, to the present, toward the future, and to obtain a real model that can describe the universe evolution from the small scale structure to the large scale structure, it is relevant to build a model in which the past and the present coexist. This kind of coexistence will trace back the evolution of our universe from the past to the present and it will predict the future.

It is also known that the age of our universe is estimated around 15 billion years, which is a huge number if we look at it as a continuous running time from one second to another. However, this huge number in seconds is just fifteen in terms of billion, fifteen quantifications in terms of large period of time. Running from small period to large period is quite difficult in terms of analysis and study for the simple reason that almost all our tools of analysis are built for the human period of time (small and big), and nothing for the large, the very large period of time. Our universe is huge, and to trace back its evolution is not a matter of seconds, it is a matter of billion in which the small scale structure as well as the large scale structure is in permanent evolution. The study of this evolution is quite difficult if you run in the same time the small scale variable as well as the large scale variable. However, the study will be easier if one of the two variables is quantified: the time small period as unit or the time large period as unit.

Let us consider the subdivision of 15 billion years into n time-intervals subdivision (or *steps*), such that the *step*(n) represents the present time. Suppose that during each *step* the universe is static, and expanding from one *step* to another with a constant rate. Let us consider the quantification introduced in [5] that approximates the natural universe expansion movement step by step as follow:

$\mathcal{Q}$: *if the distance between two events is equal to* L_n *at the step(n), and equal to* L_{n+1} *at the step(n+1), then* $L_{n+1} = a_{n+1}L_n$ *for all* $n \geq 0$, *where* a_n *is a sequence such that* $a_0 = 1$, $a_n > 1$ $\forall n \geq 1$, *and* $\prod_{i=0}^{n} a_i$ *converges.*

Using the above quantification, the distance between two separated events at the present time (*step*(n)) is given by $L_n = L_0 \prod_{i=0}^{n} a_i$ which is a function of their distances from the past, and then this linear quantification allows to trace back the evolution of the distance between well separated events in time.

Using the above quantification, we wrote in ([5]) the Lorentz transformations and basic kinematic results of special relativity as well as the relativistic momentum and energy using the square of the invariant interval between events at the *step*(0) defined by: $ds_0^2 = C_0^2 dt_0^2 - (dx^2 + dy^2 + dz^2)$, where C_0 is the constant velocity of light at the Big Bang (which corresponds, in the quantification $\mathcal{Q}$, to the velocity of light at the *step*(0)). We rewrote the Lorentz transformations and basic kinematic results of special relativity as well as the relativistic momentum and energy using the square of the invariant interval between events at the *step*(1) defined by: $ds_1^2 = C_0^2 dt_1^2 - A_1(dx^2 + dy^2 + dz^2)$, where A_1 is a constant such that $A_1 > 1$, etc, and we repeated this procedure in [5] until we obtained the Lorentz transformations and basic kinematic results of special relativity as well as the relativistic momentum and energy using the square of the invariant interval between events at the *step*(n) defined by: $ds_n^2 = C_0^2 dt_n^2 - A_n(dx^2 + dy^2 + dz^2)$, where A_n is a constant such that $A_n > A_{n-1} > ... > A_2 > A_1 > 1$. The quantification $\mathcal{Q}$ gives that $A_n = \prod_{i=0}^{n} a_i$, where a_n is a sequence such that $a_0 = 1$, $a_n > 1$ $\forall n \geq 1$, and $\prod_{i=0}^{n} a_i$ converges. Using the line elements ds_n^2 such that for each n the law of conservation of momentum and energy remains valid, the Lorentz transformation is found, at the *step*(n) for all $n \geq 0$, to be given by:

$$T_n : \begin{cases} x' = \dfrac{x - vt}{\sqrt{1 - (\prod_{i=0}^{n} a_i^2)\frac{v^2}{C_0^2}}} \\ y' = y \\ z' = z \\ t' = \dfrac{t - (\prod_{i=0}^{n} a_i^2)\frac{vx}{C_0^2}}{\sqrt{1 - (\prod_{i=0}^{n} a_i^2)\frac{v^2}{C_0^2}}} \end{cases} \tag{1}$$

from which we derive the existence of a limiting velocity for any moving bodies in an expanding universe (with linear expansion) given by

$$v_{ln} = \frac{C_0}{\prod_{i=0}^{n} a_i} \qquad \forall n \geq 0. \tag{2}$$

This limiting velocity corresponds to the velocity of light from the *step*(0) to the *step*(n) for all $n \geq 0$ in an expanding universe with linear expansion quantified by $\mathcal{Q}$, and this velocity appears to be decreasing together with the universe expansion as:

$$c_n = \frac{C_0}{\prod_{i=0}^{n} a_i} \qquad \forall n \geq 0, \tag{3}$$

where C_0 is the constant velocity of light at the Big Bang (the fossil velocity of light [5]). The decreasing nature of the velocity of light together with the universe expansion from the Big Bang to nowadays comes from the *step* metric in which $A_n > A_{n-1} > ... > A_2 > A_1 > 1$ that approximates the *step* expansion of the universe. The *step*(n) in the used quantification represents the current universe in which the measure of the velocity of light is given experimentally by

$$c_n = \frac{C_0}{\prod_{i=0}^{n} a_i} = 2.99792458 \times 10^8 m/s. \tag{4}$$

Since our approximation deals with a finite subdivision of 15 billion years, the decreasing nature of light velocity is undetectable at the small or big period of time. It can not be detectable by the scale of seconds, or hours, or years, or maybe hundred of years, however, by billion it is quite detectable. The velocity of light is locally constant at the scale of normal time (using a short interval of time as unit) and globally variable at the scale of cosmical time (using a large interval of time as unit). This local and global behavior can be derived straight forward from the quantification $\mathcal{Q}$. Indeed, the local behavior is reached in the quantification $\mathcal{Q}$ if we use a big number of subdivisions: The bigger the used number n of steps is the shorter the time interval of steps we obtain. Thus, as n tends to infinity, we have $a_{n+1} \approx 1$ as a consequence of the convergence of the product $\prod_{i=0}^{n} a_i$, then

$$\prod_{i=0}^{n+1} a_i = (\prod_{i=0}^{n} a_i)\ a_{n+1} \approx \prod_{i=0}^{n} a_i, \qquad \forall n > A, \tag{5}$$

where A is a large positive real number. Hence the equations (3) and (5) give

$$c_n \approx c_{n+1} \qquad \forall n > A, \tag{6}$$

therefore the velocity of light is almost constant for short period of time (locally). However, the equation (6) is not valid anymore for the large period of time. Indeed, the smaller the used number n of steps is the bigger the time interval of steps we obtain, and in that case we have $a_{n+1} > 1$ for all $n < A$ and

$$\prod_{i=0}^{n+1} a_i = (\prod_{i=0}^{n} a_i)\ a_{n+1} > \prod_{i=0}^{n} a_i, \qquad \forall n < A, \tag{7}$$

hence the equations (3) and (7) lead to the following inequality:

$$c_n > c_{n+1} \qquad \forall n < A, \tag{8}$$

that is to say the velocity of light is decreasing globally together with the universe expansion from one step to another.

Discussing problems raised by varying speed of light cosmology can be found in ([1],[2],[3]) where varying the speed of light can have no effect on the foundation of relativity since one has to use the value of the velocity of light which was appropriate in each local reference frame at any given time, that is to say the velocity of light remains a locally measured invariant but its value depends on the cosmical time as it is explained in ([1]). The use of Mach's principle in the Einstein-Friedman's equation can also lead to interpret the speed of light as intimately connected to the expansion of the universe as it was pointed out in ([14]), however how it is connected to the universe expansion remains unsatisfactory due to the difficulty to use the time notion for the local period of time (normal time) as well as the large period of time (cosmical time), and making c varying with normal time as $c(t)$ is not appropriate mathematically if we don't know how it varies with the cosmical time meanwhile it remains invariant at normal time.

II. CHARACTERISTICS OF AN EXPANDING VACUUM

In general, the permeability is not the same for all medium as it can vary with the position of the medium, the frequency of the field applied, the humidity, the temperature, the composition of the medium, and other parameters, and so for the permittivity. In an expanding universe something is changing making bigger the distance between matter, affecting the temperature of the universe, its density and other parameters that will be discussed here. Using the above quantification $\mathcal{Q}$ the vacuum permittivity and vacuum permeability are related together at the *step*(n) by the equation $\mu\varepsilon c_n^2 = 1$, where c_n is the experimental measurement of the velocity of light at the present time given by (4). Since the velocity of light varies as n varies (n is a finite subdivision of 15 billion of years) and since we have

$$c_n^2 = \frac{1}{\mu\varepsilon}, \tag{9}$$

then the product of permittivity and permeability of vacuum varies together with the universe expansion. We will denote the permeability and the permittivity of vacuum at the *step*(n) by μ_n, and ε_n for all $n \geq 0$. The equation (9) must remain invariant together with the universe expansion and its invariance step by step from the Big Bang (*step*(0)) to the present time (*step*(n)) is described by the following *step* equations

$$\begin{cases} \mu_0\varepsilon_0 C_0^2 = 1, & \text{at the } \textit{step}(0) \quad \text{(the Big Bang)} \\ \vdots \quad \vdots & \quad \vdots \quad \vdots \\ \mu_n\varepsilon_n c_n^2 = 1, & \text{at the } \textit{step}(n) \quad \text{(the present time).} \end{cases} \tag{10}$$

Indeed, the Maxwell's theory is a theory of light, and this theory must remain valid from the Big Bang to nowadays together with the universe expansion, which means that the function representing the waves traveling with a velocity c_n for all $n \geq 0$ can be found through the wave equation that satisfies Maxwell's equations step by step from the Big Bang to nowadays. Therefore at the beginning of the universe expansion the velocity of the electromagnetic waves in empty space must verify: $\mu_0\varepsilon_0 C_0^2 = 1$, where C_0 is the maximum speed of signal propagation at the beginning of the universe expansion, and μ_0, ε_0 are the permeability and the permittivity of the primordial vacuum. The system (10) gives

$$\mu_n\varepsilon_n = \mu_0\varepsilon_0 \prod_{i=0}^{n} a_i^2 \qquad \forall n \geq 0 \tag{11}$$

which means that the product of permeability and permittivity of vacuum increases together with the universe expansion. The characteristics of vacuum in an expanding universe are not static and this is normal since the state of the vacuum does vary from point to point. Assuming that the permittivity and the permeability vary together with the universe expansion in the same manner, the formula (11) leads to the following equations:

$$\begin{cases} \mu_n = \mu_0 \prod_{i=0}^{n} a_i & \text{permeability of free space at the } \mathit{step}(n)\ \forall n \geq 0 \\ \varepsilon_n = \varepsilon_0 \prod_{i=0}^{n} a_i & \text{permittivity of free space at the } \mathit{step}(n)\ \forall n \geq 0. \end{cases} \tag{12}$$

That is to say that the permittivity of an expanding vacuum as well as the permeability of an expanding vacuum is increasing together with the universe expansion. As for the light velocity the permeability and the permittivity of the free space are locally constant at the scale of normal time (using a short interval of time as unit) and globally variable at the scale of cosmical time (using a large interval of time as unit). Likewise the local behavior as well as the global behavior of the permittivity and the permeability of an expanding vacuum can be justified using equations (5) and (7). This variation must be extremely small, which makes it not detectable at the normal time. The increase of permittivity of an expanding vacuum from the Big Bang to nowadays means that the extensity of the electric field in an expanding vacuum is decreasing with respect to the electric displacement. Meanwhile, the increase of the permeability of an expanding vacuum means that the extensity of the magnetic field in an expanding vacuum is decreasing with respect to the magnetic induction. The permittivity and permeability are a measure of how much the free space changes together with the universe expansion to absorb energy when subject to electric and magnetic fields, they are characteristics of an expanding free space.

The shape of the universe in the fractal manifold model is described by an infinite number of packed expanding universe points ([6]) (see Fig.1). The universe points are simulated to expand as balls, and the visible space of the universe is only described by the surfaces of those expanding balls where matter and geodesics are located. Any magnetic flux lines in the universe are channeled through geodesics located in the packed expanding surfaces ([6]), which makes the inner part of those universe points naturally isolated magnetically as the universe expands (natural voids). The physical interpretation of this behavior is explained by the increase of the permeability of the visible space together with the universe expansion.

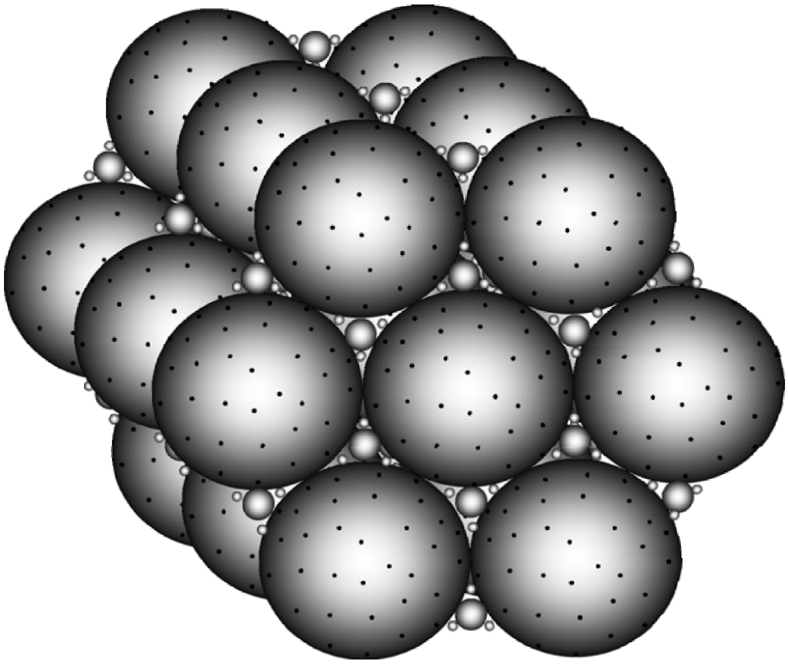

Figure 1: The space described by a fractal manifold is constituted of an infinite number of packed expanding balls (called universe points), where the visible universe is only the surface of those packed balls. Matter (simulated by dots in this illustration that might represent galaxies or any other big structure of matter) and geodesics (that describe possible matter locations) are located on the surface of those expanding spheres.

As the universe expands the magnetic flux lines are deflected from the normal propagation at the $step(0)$ to a considerable extend in a bent space with higher permeability and higher permittivity at the $step(n)$ for all $n > 0$, and the flux density sharply increases in those packed surfaces that represent the visible space of our universe. The distortion of the magnetic field following the variable geometry of the visible space is sustained by the increase of the visible space permeability to absorb all magnetic flux lines and to not allow the magnetic flux lines to penetrate the inner part of those universe points (see Fig.2). This leads to the following conclusion: the permittivity and the permeability of the inner part of those expanding universe points (voids) are less than the permittivity and permeability of their boundaries (the visible space).

In the fractal manifold model the expansion of the universe points is the cause of the recession movement of matter in our universe, and creates voids that bend the space and transform its shape. Those expanding voids simulate what we dubbed as dark energy ([6]).

Taking into account the universe expansion, the equation relating light velocity with permittivity and permeability can be formulated via equations (3) and (11) as follow

$$\varepsilon_n \mu_n c_n^2 = \varepsilon_0 \mu_0 C_0^2 = 1 \qquad \forall n \geq 0 \tag{13}$$

which means that as the universe expands the product $\varepsilon_n \mu_n c_n^2$ for all $n \geq 0$ remains invariant together with the universe expansion (for all time, global time and local time).

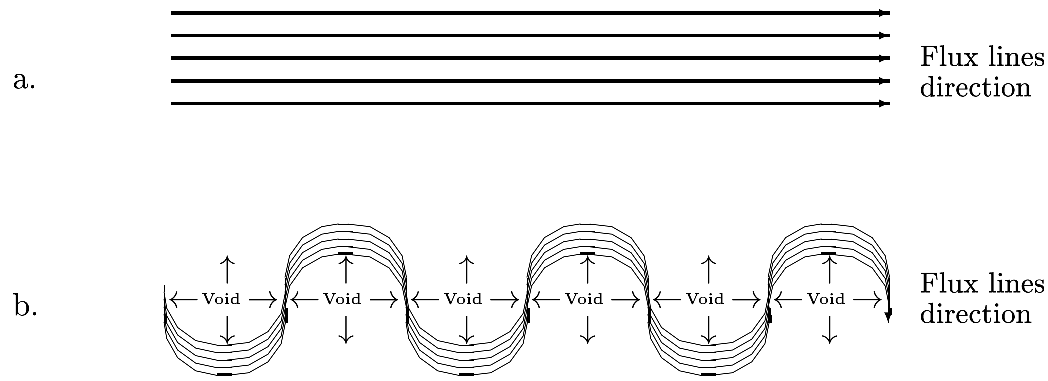

Figure 2: The figure a. represents the magnetic flux lines in a space before its expansion in a vacuum with permittivity ε_0 and permeability μ_0. Meanwhile b. represents the magnetic flux lines in an expanding free space at the $step(n)$ with permittivity ε_n and permeability μ_n. As the universe expands through the universe points expansion, the permeability of the expanding vacuum as well as the permittivity of the expanding vacuum increases together with the universe expansion and then the magnetic flux lines are deflected from the normal to the considerable extent in which the flux density sharply increases, and where the magnetic flux cannot penetrate into those expanding regions (the expanding universe points) but they travel through their boundary by fluctuating following the expanding space geodesics ([6]).

Moreover the electric and magnetic fields in an electromagnetic waves are related by an invariant value of the characteristic impedance together with the universe expansion:

$$Z_n = \mu_n c_n = \sqrt{\frac{\mu_n}{\varepsilon_n}} \qquad \forall n \geq 0, \tag{14}$$

If we denote Z_0 the characteristic impedance at the $step(0)$ (Big Bang), then the use of the equations (12), and (3) gives

$$Z_n = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \mu_0 C_0 = Z_0 = cst \qquad \forall n \geq 0. \tag{15}$$

III. GRAVITATIONAL CONSTANT IN AN EXPANDING UNIVERSE

It is known that Hydrogen is the most abundant gas in the universe accounting for 89% of all atoms. The Hydrogen atoms were formed in the first few seconds after the event that marked the beginning of the universe. An atom of Hydrogen has one proton with positive charge of mass $m_p = 1,6726 \times 10^{-27} kg$ that represents the nucleus, and one electron of mass $m_e = 9.1094 \times 10^{-31} kg$ in orbit around the nucleus at an estimated distance of $r = 0,53 \times 10^{-10} m$.

The natural attraction of the proton of the Hydrogen atom and its electron is represented by the electric force given by Coulomb law

$$\vec{F_e} = k \frac{q_e q_p}{r^2} \hat{r}, \tag{16}$$

where q_e is the electron charge and q_p is the proton charge, $k = \frac{1}{4\pi\varepsilon_n}$ is the constant of proportionality, where ε_n is the permittivity of vacuum at the *step*(n) (the present time), and $\hat{r}$ is the unit vector pointing the force direction.

Besides the gravitational attraction between the nucleus of the Hydrogen atom and its electron is given by

$$\vec{F_g} = G \frac{m_p m_e}{r^2} \hat{r} \tag{17}$$

where m_e is the electron mass, m_p is the proton mass, G is the gravitational constant of proportionality, and $\hat{r}$ is the unit vector pointing the force direction from one mass to the other. The ratio of the electrical attraction (16) to the gravitational attraction (17) is given by

$$\frac{|\vec{F_e}|}{|\vec{F_g}|} = \frac{k q_p q_e}{G m_p m_e} = \frac{q_p q_e}{4\pi\varepsilon_n G m_p m_e} = 2,3 \times 10^{39}. \tag{18}$$

Since matter is not affected by the universe expansion (matter does neither expand nor contract as the universe expands) the ratio of the electrical attraction (16) to the gravitational attraction (17) remains constant together with the universe expansion (invariant under the universe expansion). Then we have

$$\frac{|\vec{F_e}|}{|\vec{F_g}|} = \frac{q_p q_e}{4\pi\varepsilon_n G m_p m_e} = 2,3 \times 10^{39} \qquad \forall n > 0, \tag{19}$$

which gives that the product $\varepsilon_n G$ in the equation (19) must be constant as the universe expands for all n, otherwise matter will be affected (Hydrogen is not found without its electron in ordinary chemistry, and as ionized Hydrogen is highly chemically reactive). Since the permeability of vacuum varies linearly as (12) with $(\prod_{i=0}^{n} a_i)$ as coefficient of linearity, then the gravitational constant G must vary linearly with $(\frac{1}{\prod_{i=0}^{n} a_i})$ as a coefficient of linearity together with the universe expansion to maintain the constancy of the product $\varepsilon_n G$. Accordingly, we will put the gravitational constant G_n at the *step*(n) as proportional to the primordial gravitational constant G_0:

$$G_n = \frac{G_0}{\prod_{i=0}^{n} a_i}. \tag{20}$$

Therefore the gravitational value of G is affected by the universe expansion and this variation has the same interpretation as the velocity of light: it is locally constant at the scale of normal time (using a short interval of time as unit) and globally variable at the scale of cosmical time (using a large interval of time as unit). The gravitational constant is changing very slowly and this variation is undetectable over hundred of years or maybe thousand of years, meanwhile globally it is detectable over cosmical time and this variation could be detected by observations from the past.

According to the formulation of the gravitational force between matter, the gravitational constant is independent from the distance between matter, independent from the mass of matter involved. It can be measured everywhere in our universe, and it will have the same measured value everywhere at a given step of the universe expansion, however, its value is closely related to the universe expansion and its value at the *step*(n) of the universe expansion (the present time) corresponds to the current experimental value:

$$G_n = \frac{G_0}{\prod_{i=0}^{n} a_i} = 6.673 \times 10^{-11} N.m^2 kg^{-2}, \tag{21}$$

where G_n represents the measure of the gravitational force intensity at the present time (at the *step*(n) of the universe expansion) which is function of G_0 the measure of the gravitational force intensity at the primordial space (*step*(0), at the Big Bang).

Does this variation over large scale of time unit violate the law of conservation of energy in our universe? The law of conservation of energy states that *the total energy of all isolated system in our universe remains constant over time, regardless of other possible changes* *within the system.* Under the hypothesis where the universe energy is constant over time the energy of any isolated part of this universe must be constant too, if there is no exchange between the isolated part of the universe and any other part of the universe, the energy of this system remains constant as well as the energy in the remaining part of the universe. Today we know that our universe is expanding, moreover it is accelerating, and this acceleration needs an increase of the universe total energy. This acceleration is detectable over large interval of time, by observation from the past over million of light years, it is not detectable over hundred of years from the past or maybe thousand of years from the past. Likewise, the gravity variation is only detectable over large scale of time interval, hence there is no violation of the law of conservation of energy if the gravity remains constant at the normal time. However at the large scale of time interval (cosmical time) the law of conservation of energy becomes not valid since the hypothesis of the constancy of the universe energy is not valid anymore. Moreover to find an isolated system in an accelerated expanding host is not possible. Certainly we cannot detect any small difference between two experimental measures of the gravitational constant nowadays because we run into fundamental limits of experimentation, and experimental efforts need to be implemented to go beyond these limits.

The essence of the previous argument concerning the non violation of the law of conservation of energy for short period of time interval can rigorously be justified. Indeed, the use of the new Lorentz transformation T_n given by the system (1) as a substitute of the classical one (see [5] for more details) leads to the formulation of the law of the total energy of any moving body at the *step*(n) of the universe expansion within an isolated system of bodies or particles:

$$E_n(v) = m_n(v) \left(\frac{C_0}{\prod_{i=0}^{n} a_i} \right)^2 \tag{22}$$

where $m_n(v)$ is the relative mass at the *step*(n) given by the equation

$$m_n(v) = \frac{m_0}{\sqrt{1 - (\prod_{i=0}^{n} a_i^2)\frac{v^2}{C_0^2}}}. \tag{23}$$

Meanwhile the body's rest energy at the *step*(n) is given by the law

$$E_n(0) = m(0) \left(\frac{C_0}{\prod_{i=0}^{n} a_i} \right)^2, \tag{24}$$

where $m(0)$ represents the mass of the body at rest relative to the observer at the *step*(n) of the universe expansion. As it can be seen from the equations (22), (23), and (24) there is no conservation of energy from one *step* to another since the total energy, rest energy and relative mass of moving body are affected by the universe expansion. The formulation of the equations (22), (23), and (24) at the *step*(n) and at the *step*$(n-1)$ from the universe expansion reveals the equation of the evolution of the total energy from one step to another due to the universe expansion:

$$E_n(v) = E_{n-1}(v)\frac{1}{a_n^2}\frac{\sqrt{1-(\prod_{i=0}^{n-1}a_i^2)\frac{v^2}{C_0^2}}}{\sqrt{1-(\prod_{i=0}^{n}a_i^2)\frac{v^2}{C_0^2}}}. \tag{25}$$

We can take the number n of steps as big as we want to make the period of time for each step as short as we want in the considered subdivisions of the universe age, in that case we have $a_n \approx 1$ (because $\prod_{i=0}^{n} a_i$ is a convergent product), then using formula (5) there will be no significant difference between $(\prod_{i=0}^{n-1} a_i)$ and $(\prod_{i=0}^{n} a_i)$ for a large value of n, which leads in the equation (25) to the conservation of total energy from one step to another since we have

$$E_n(v) \approx E_{n-1} \qquad \forall n > A. \tag{26}$$

where A is a large positive real number. However for a large period of time there is no conservation of energy since formula (7) leads to

$$E_n(v) \neq E_{n-1} \qquad \forall n < A. \tag{27}$$

This argument confirms that the law of conservation of energy is valid only for a considered short period of time and invalid for a considered large period of time in an accelerated or decelerated universe expansion. The law of conservation of energy depends on the considered time interval in an expanding universe. A perfect isolated system in an accelerated or decelerated universe expansion is fairly accurate under the consideration of a short period of time.

IV. ELECTROMAGNETIC AND GRAVITY IN AN EXPANDING UNIVERSE

A. Gravitational Force

According to the law of universal gravitation at the *step*(n) (the present time), the force exerted by a gravitational mass m_1 on a gravitational mass m_2 separated by a distance r is given by

$$\vec{F_{12}} = -\Big(\frac{G_0}{\prod_{i=0}^{n} a_i}\Big)\frac{m_1 m_2}{r^2}\hat{r} \tag{28}$$

where $\dfrac{G_0}{\prod_{i=0}^{n} a_i}$ is the measure of the gravitational force intensity at the present time (at the *step*(n) of the universe expansion) that corresponds to the experimental measure given by (21), $\hat{r}$ is the unit vector pointing from m_1 to m_2, G_0 is the measure of the gravitational force intensity at the primordial space (*step*(0)), and a_i is the universe expanding parameter from the *step*$(i-1)$ to the *step*(i).

According to the equation (28) the gravitational force exerted by a gravitational mass m_1 on a gravitational mass m_2 is decreasing together with the universe expansion. If it is so, does this decreasing nature affect the equilibrium in the solar system for example as the universe expands? Obviously the natural equilibrium in the solar system remains valid since the rotational velocity of any moving body within the system is supposed to decrease together with the universe expansion as the limiting velocity v_{ln} (given by the equation (2)) of any moving body decreases together with the universe expansion.

B. Electric Field and Electrostatic Force

According to the value of the permittivity together with the universe expansion the electrostatic force at the *step*(n) between the electrical charges q_1 and q_2 separated by a distance r is given by

$$\vec{F_{12}} = \Big(\frac{1}{4\pi\varepsilon_n}\Big)\frac{q_1 q_2}{r^2}\hat{r} = \Big(\frac{1}{4\pi\varepsilon_0\prod_{i=0}^{n} a_i}\Big)\frac{q_1 q_2}{r^2}\hat{r} = -\vec{F_{21}} \tag{29}$$

where ε_n is the vacuum permeability at the *step*(n) given by (12), ε_0 is the vacuum permeability at the Big Bang (*step*(0)), and a_i is the universe expanding parameter from the *step*$(i-1)$ to the *step*(i).

The electric field at the *step*(n) of a point charge q_1 at any distant point M of distance r away from the charge is given by

$$\vec{E_n} = \frac{\vec{F_{12}}}{q_2} = \Big(\frac{1}{4\pi\varepsilon_0\prod_{i=0}^{n} a_i}\Big)\frac{q_1}{r^2}\hat{r} = \frac{1}{\prod_{i=0}^{n} a_i}\Big(\frac{1}{4\pi\varepsilon_0}\frac{q_1}{r^2}\hat{r}\Big) \tag{30}$$

$$\vec{E_n} = \frac{1}{\prod_{i=0}^{n} a_i}\vec{E_0} \tag{31}$$

where $\vec{E_0} = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r^2}\hat{r}$ represents the electric field of the point charge q_1 in a space with permittivity ε_0 (the primordial space) acting in the direction of the radius vector $\hat{r}$. Therefore the electric force at the *step*(n) of the universe expansion can be written as

$$\vec{F_{12}} = q_2\vec{E_n} = q_2\frac{1}{\prod_{i=0}^{n} a_i}\vec{E_0}. \tag{32}$$

According to the equation (31) the electric field of the point charge q_1 is decreasing together with the universe expansion, which leads to the decreasing nature of the electrostatic force between two distant electric charges together with the universe expansion.

C. Magnetic Induction and Magnetic Force

It is known that the magnetic induction field and magnetic force can be produced and experienced by two types of bodies: by charges in motion, or electric currents, and by magnetized bodies, such as permanent magnets. It is also known that we cannot strictly state the field produced by a charge in motion since a single charge in motion cannot produce a static magnetic induction field. Nevertheless, if the charge forms part of steady current, as if there was a natural event (supernova, or Big Bang), that creates a procession of charges following one after the other in a formation independent of time, then a simple law for the field it produces at the *step*(n) of the universe expansion can be obtained, in which the evolution of the field together with the universe expansion is traced back from the Big Bang (*step*(0)).

Indeed, the magnetic induction resulting from a charge q_1, moving at the *step*(n) of an expanding space with a velocity v, at the point M of a distance r away from the charge is

$$\vec{B_n} = \frac{\mu_n}{4\pi}q_1\frac{v \times r}{|r|^3} = \prod_{i=0}^{n} a_i\left(\frac{\mu_0}{4\pi}q_1\frac{v \times r}{|r|^3}\right) \tag{33}$$

$$\vec{B_n} = (\prod_{i=0}^{n} a_i)\vec{B_0} \tag{34}$$

where $\vec{B_0} = \frac{\mu_0}{4\pi}q_1\frac{v \times r}{|r|^3}$ represents the magnetic induction resulting from the same charge q_1, moving in a space with permeability μ_0 (the primordial space) with the velocity v at the same distance $|r|$ away from the charge provided that the moving charge forms part of a current distribution independent of time, where r is a vector pointing from the charge to the point in space where the field is being found.

The magnetic force, at the *step*(n) of the universe expansion, on another charge Q moving at the point M with a velocity u is then given by

$$F = Qu \wedge \vec{B_n} = Qu \wedge (\prod_{i=0}^{n} a_i)\vec{B_0} \tag{35}$$

where B_n is given by formula (34).

D. The Electromagnetic Force in an Expanding Universe

The total electromagnetic force, at the *step*(n) of the universe expansion, on the charge Q moving at the point M with velocity u from the point charge q_1 can then be written as

$$F_n = Q(E_n + u \wedge \vec{B_n}) \tag{36}$$

and by using formula (31) and (34)

$$F_n = Q\Big(\frac{1}{\prod_{i=0}^{n} a_i}\vec{E_0} + u \wedge (\prod_{i=0}^{n} a_i)\vec{B_0}\Big). \tag{37}$$

Every point M in an expanding space is then characterized by two vector quantities which determine the force on any charge Q:

i) There is the electric force which gives a force component independent of the motion described by the electric field $\vec{E_n}$. This force is decreasing together with the universe expansion since the electric field $\vec{E_n}$ of a point charge q_1 at any point M of a distance r away from the charge is decreasing together with the universe expansion. The decrease of the electric field $\vec{E_n}$ together with the universe expansion is due to the increase of the permittivity of the free space.

ii) There is the magnetic force, which depends on the velocity of the charge Q. This force is increasing together with the universe expansion since the magnetic induction $\vec{B_n}$ resulting from a charge q_1, moving in an expanding space with a velocity v, at any point M of a distance r away from the charge is increasing together with the universe expansion.

The increase of the magnetic induction $\vec{B_n}$ together with the universe expansion is due to the increase of the permeability of the free space.

What is the physics brought in an expanding universe by an increasing magnetic field as (34) and decreasing electric field as (31)? The increase of the magnetic field together with the universe expansion leads to the increase of the magnetic force on any charged particle with momentum P at the $step(n)$ for example. Since the magnetic force is purely a deflexion force, then in an expanding universe with variable curvature an increasing magnetic field is needed to deflect the motion direction of any charged particle locally and globally (as producing special particle trajectory in a high energy particle accelerator). This trajectory is channeled on the new geometries following the variable curvature of the expanding space. Indeed an electric particle moving perpendicular to the magnetic field in the space will describe a circle with radius inversely proportional to the magnetic induction. However if the particle inters the field with an initial velocity $\vec{v_0}$ at an incline to the direction of the field, it will be deflected with a frequency given by

$$\omega_n = \frac{qB_n}{mc_n}, \quad \forall n \geq 0, \tag{38}$$

and by using the increase nature of the magnetic induction as (34) in an expanding universe measured at the $step(n)$ and formula (4), it turns out that the frequency ω_n is increasing together with the universe expansion as

$$\omega_n = \frac{qB_0}{mC_0} \prod_{i=0}^{n} a_i^2, \quad \forall n \geq 0. \tag{39}$$

Assuming we resolve the velocity vector $\vec{v_0}$ into $v_{||}$ a parallel component to the magnetic field and $v_{\perp}$ a perpendicular to the magnetic field, the particle will move following the magnetic field direction in a spiral with radius

$$R_n = \frac{mv_{\perp}c_n}{qB_n} \tag{40}$$

where using (34) and (4) gives a decreasing radius together with the universe expansion as

$$R_n = \frac{mv_{\perp}C_0}{qB_0} \frac{1}{\prod_{i=0}^{n} a_i^2}, \quad \forall n \geq 0. \tag{41}$$

The variation of the local deflection together with the universe expansion is due to the increasing Lorentz deflecting force together with the universe expansion as

$$f_n = \frac{1}{c_n} qv_{\perp}B_n = \frac{1}{C_0} qv_{\perp}B_0 \prod_{i=0}^{n} a_i^2 \tag{42}$$

that insures the circular movement of the particle, meanwhile the global deflection is described by the angle between the initial movement direction and the magnetic field direction in the expanding space.

From another side the decreasing nature of the electric field in an expanding universe as (31) might serve to slow down the speed of any charged particle with momentum P together with the universe expansion inversely proportional to expanding parameter $\prod_{i=0}^{n} a_i$ (due to the decreasing nature of the limiting velocity v_{ln} of any moving body (2) in the expanding universe). Meanwhile the decrease of the velocity of the particle together with the universe expansion under the effect of the decreasing nature of the electric field in the space will induce the increase of the radius (41) of the helix together with the universe expansion proportional to the universe expanding parameter $\prod_{i=0}^{n} a_i$.

Under the action of both a decreasing electric field and an increasing magnetic field the particle with momentum P in an expanding universe will move with an increasing frequency (39) in a spiral with a decreasing radius R_n inversely proportional to the universe expanding parameter $\prod_{i=0}^{n} a_i$ following the magnetic field direction.

E. The Inner Connection

The incorporation of a special expansion in the special relativity theory rises the fact that the velocity of light is decreasing in an expanding universe as a function of cosmical time. The changes brought in the formulation of the electromagnetic force as well as in the formulation of the gravitational force are manifested by the appearance of the term

$$(\prod_{i=0}^{n} a_i), \tag{43}$$

which represents the expanding parameter of the universe that characterizes and approximates the universe expansion from the $step(0)$ (Big Bang) to the $step(n)$ (the present time). The use of the expanding parameter (43) in the formulation of the electromagnetic and gravitational forces allows to trace back their nature in an expanding universe with variable curvature and shape.

The inner connection between electromagnetic and gravitational forces appears to be the universe expanding parameter itself that represents the dynamic of the universe. This seems to be coherent since the dynamic of the host of matter might affect the matter dynamic.

At the beginning of the universe expansion (at the *step*(0) where the expanding parameter was $a_0 = 1$) the inner connection was equal to one at that time and had no effect on the electromagnetic force as well as on the gravitational force. However it became greater than one starting from the *step*(1), which affected the formulation of the physical laws. This leads to derive the following conclusion that all forces of nature were independent at the Big Bang and became dependent after the Big Bang. This picture is the inverse of the current thinking in theoretical physics (there is a believe that in the very early universe when the temperature was very high, the gravitational, weak, electromagnetic, and strong forces were unified into a single force, and only when the temperature dropped did these forces separate from each other). The major difficulty to find inner connection between forces of nature was sustained by the difficulty to incorporate special expansion that might simulate the universe expansion in the special relativity theory.

Maxwell had made one of the great unification of physics, light was no longer "something else" but was only electricity and magnetism in the following new form: little pieces of electric and magnetic fields which propagate through space on their own ([8]). Just by experiments with charges and currents scientists found the constant k^2 that appears in the equation of electrostatic and magnetostatics, which turns out to be the square of the velocity of propagation of electromagnetic influences. From static measurement (by measuring the force between two units of charges and between two units of currents) they found that the velocity of propagation of electromagnetic influences is equal to the velocity of light ([11]). In front of this mysterious coincidence Maxwell said: *we can scarcely avoid the inference that light consists in the transverse undulations of the same medium which is the cause of electric and magnetic phenomena* ([12]). The formula (9) is no longer a coincidence since one can justify how it remains invariant under the universe expansion (13), and this invariability together with the universe expansion reinforce the prediction of Maxwell and gives the equation (13) the status to be a fundamental equation of an expanding universe that makes light traceable to the expanding medium.

The product of the formulas (12) and (20) leads to the following invariant relation together with the universe expansion: the characteristics of an expanding free space (permeability

μ_n and permittivity ε_n) are related together with the gravitational constant in an invariant equation under the universe expansion given by:

$$\varepsilon_n \mu_n G_n^2 = \varepsilon_0 \mu_0 G_0^2 \qquad \forall n \geq 0. \tag{44}$$

The product in (44) is an universal constant since it remains invariant together with the universe expansion from the Big Bang (*step*(0)) to the present time (*step*(n)) and we have

$$\varepsilon_n \mu_n G_n^2 = \varepsilon_{n-1} \mu_{n-1} G_{n-1}^2 = \ldots\ldots = \varepsilon_0 \mu_0 G_0^2 = K_1 \qquad \forall n \geq 0 \tag{45}$$

where $K_1 = 4.954262258895300892166937175372 9 \times 10^{-38} \simeq 5 \times 10^{-38}$ is an extremely small constant invariant under the universe expansion since ε_0, μ_0, and G_0 are constant from the Beginning of the universe expansion.

What kind of comment and interpretation one can derive from the equation (45)? Maxwell would say: we can scarcely avoid the inference that gravity consists in the transverse undulations of the same medium which is the cause of electric, magnetic phenomena and light, and Feynman would say that gravity is no longer "something else" but it is only electricity and magnetism in the following new form: little pieces of electric and magnetic fields which propagate through space on their own. I can not add more than these comments on the conclusion derived from the invariance of the equation (45) under the universe expansion. However, light and gravity share the same medium and they have to be well described in an invariant equation under the universe expansion. Indeed, the multiplication of the formulas (3), (12), and (20) gives us the following invariant equations together with the universe expansion from the Big Bang (*step*(0)) to the present time (*step*(n))

$$\mu_n \varepsilon_n c_n G_n = \mu_0 \varepsilon_0 C_0 G_0 \qquad \forall n \geq 0 \tag{46}$$

which means that the product $\mu_n \varepsilon_n c_n G_n$ is an universal constant invariant under the universe expansion since ε_0, μ_0, G_0, and C_0 are constant from the Beginning of the universe expansion and their primordial product can be evaluated by what we measure today

$$\mu_n \varepsilon_n c_n G_n = \mu_{n-1} \varepsilon_{n-1} c_{n-1} G_{n-1} = \ldots = \mu_0 \varepsilon_0 C_0 G_0 = \sqrt{K_1} \quad \forall n \geq 0 \tag{47}$$

where K_1 is given by the equation (45). The use of the two equations (13) and (46) gives a clear relation between velocity of light and gravity in an expanding universe described by the following invariant equation under the universe expansion:

$$G_n = c_n\sqrt{K_1} \qquad \forall n \geq 0 \tag{48}$$

which comforts how their mutual nature are intimately related and that gravity is no longer something else: the gravitational interaction and the electromagnetic interaction are two manifestations of the same phenomena that resides in a transverse undulation of the surrounding medium, each manifestation is significant according to the bodies heaviness, the bodies charges, and the distance between them.

1. *Reformulation of Maxwell's Equation in an Expanding Universe*

Maxwell's equations ([10]) furnished a theory of light, and light exists in an expanding universe from the Big Bang to nowadays. The validity of Maxwell's theory in an expanding universe leads to make it valid step by step in the process of the approximation of the universe expansion. The Maxwell's equations can be written as invariant equations under the universe expansion as follow taking into account the approximation process of the universe expansion:

$$\begin{aligned}
&\nabla\vec{E}_n = \frac{\rho_t}{\varepsilon_n} && \forall n \geq 0\\
&\nabla\vec{B}_n = 0 && \forall n \geq 0\\
&\nabla \wedge \vec{E}_n = -\frac{\partial\vec{B}_n}{\partial t} && \forall n \geq 0\\
&\nabla \wedge \vec{B}_n = \mu_n J_t + \mu_n\varepsilon_n\frac{\partial\vec{E}_n}{\partial t} && \forall n \geq 0,
\end{aligned} \tag{49}$$

where the electromagnetic force in an expanding universe is given by

$$F_n = Q\Big(\vec{E}_n + u \wedge \vec{B}_n\Big) \qquad \forall n \geq 0, \tag{50}$$

in which n represents the steps number of the expansion approximation. In an expanding region without electric charges, Maxwell's equations become:

$$\begin{aligned}
&\nabla\vec{E}_n = 0 && \forall n \geq 0\\
&\nabla\vec{B}_n = 0 && \forall n \geq 0\\
&\nabla \wedge \vec{E}_n = -\frac{\partial\vec{B}_n}{\partial t} && \forall n \geq 0\\
&\nabla \wedge \vec{B}_n = \mu_n\varepsilon_n\frac{\partial\vec{E}_n}{\partial t} && \forall n \geq 0.
\end{aligned} \tag{51}$$

The elimination of one field or the other leads to the wave equations:

$$\Delta\vec{E_n} - \mu_n\varepsilon_n\frac{\partial\vec{E_n}}{\partial t} = 0 \qquad \forall n \geq 0 \tag{52}$$

$$\Delta\vec{B_n} - \mu_n\varepsilon_n\frac{\partial\vec{B_n}}{\partial t} = 0 \qquad \forall n \geq 0, \tag{53}$$

where the fields $\vec{E_n}$ and $\vec{B_n}$ for all $n \geq 0$ are traveling with velocities that verify an invariant equation under the universe expansion given by:

$$c_n = \frac{1}{\sqrt{\mu_n\varepsilon_n}} = \frac{\sqrt{K_1}}{G_n\mu_n\varepsilon_n} \qquad \forall n \geq 0, \tag{54}$$

and where the velocity of the electromagnetic wave is measured today (at the *step*(n)) to be equal to

$$c_n = \frac{1}{\sqrt{\mu_n\varepsilon_n}} = \frac{\sqrt{K_1}}{G_n\mu_n\varepsilon_n} = 2.99792458 \times 10^8 m/s \quad . \tag{55}$$

Taking into account the variation of the electric field (31) as well as the magnetic induction (34) together with the universe expansion, it is not difficult to derive the invariance of their cross product under the universe expansion and we have

$$\vec{E_n} \wedge \vec{B_n} = \vec{E_0} \wedge \vec{B_0} \qquad \forall n \geq 0 \tag{56}$$

which means that the cross product of $\vec{E_n}$ and $\vec{B_n}$ is invariant under the universe expansion. However, the flux of energy (the amount of energy crossing unit area perpendicular the poynting's vector, per unit time) is not invariant under the universe expansion. Indeed, if we denote the flux of energy at the *step*(n) of the universe expansion by $\vec{S_n}$, we have

$$\vec{S_n} = \frac{1}{\mu_n}\vec{E_n} \wedge \vec{B_n} \tag{57}$$

and if we denote the flux of energy at the primordial universe (at the *step*(0)) by $\vec{S_0}$ such that

$$\vec{S_0} = \frac{1}{\mu_0}\vec{E_0} \wedge \vec{B_0} \tag{58}$$

as the universe expands we have (56) and then

$$\vec{S_n} = \frac{1}{\mu_n}\vec{E_n} \wedge \vec{B_n} = \frac{1}{\mu_n}\vec{E_0} \wedge \vec{B_0} \tag{59}$$

which gives if we take into account formula (58) and (12)

$$\vec{S_n} = \frac{1}{\prod_{i=0}^{n} a_i} S_0. \tag{60}$$

This means that the amount of energy crossing unit area perpendicular the Poynting's vector per unit time is decreasing together with the universe expansion (the energy transportation by an electromagnetic wave in an expanding space is decreasing together with the universe expansion).

To inquire whether the attraction of gravitation is not also traceable, as the light, to the action of the surrounding medium was investigated forth by Professor James Clerk Maxwell (1864). At that time the assumption that gravitation arises from the action of the surrounding medium in the way pointed out by Maxwell leads to the conclusion that every part of the medium possesses, when undisturbed, an enormous intrinsic energy, and that the presence of dense bodies influences the medium so as to diminish this energy wherever there is a resultant attraction. Maxwell declared: *As I am unable to understand in what way a medium can posses such properties I cannot go any further in this direction in searching for the cause of gravitation* ([11]). Nowadays, we know that our universe is expanding regardless of matter density and gravitational attraction. In 1996 something unexpected happened, observations of very distant supernovas required a shocking change in picture. The universe's expansion is not slowing down, it is accelerating. Something, not like matter and not like ordinary energy, is pushing the galaxies apart. This has been dubbed dark energy and estimated to represents 73% of the universe.

Using the assumption that the velocity of light is decreasing as function of cosmic time together with the universe expansion is insufficient to derive whether or not gravity is traceable to the action of the surrounding medium. However knowing how the light decreases with the cosmic time together with the universe expansion leads directly to the required unification.

To the question whether the attraction of gravitation is traceable to the action of the surrounding medium my answer is yes indeed, and if it is so then there is a lot of work to be developed experimentally as well as theoretically in order to obtain an optimal picture of the phenomena in the light of the new information.

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[1] H. Arzelies, *Relativistic kinematics,* Pergamon, New York, (1966).

[2] P.P. Avelino, and C.J. Martins, Phy. Letter, B 459, 468, (1999).

[3] A. Albrecht, and J. Magueijo, Physical Review D 59, 043516, (1999).

[4] L. Bass, and E. Schrodinger, Proc. Roy. Soc. A 232, (1955).

[5] F. Ben Adda, *Mass and total energy of moving bodies in a quantified expansion*, arXiv:0808.3525v1.

[6] F. Ben Adda, *New understanding of the dark energy in the light of new space-time*, Accepted for publication AIP proceedings, (2010).

[7] A. Einstein, *Geometry and experience sidelights on relativity*, Denver (1922).

[8] R.P. Feynman, R.B. Leighton, M. Sands, *The Feynman Lectures on Physics*, Volume **II**, Addison-Wesley, (1977).

[9] L. Glashow, Rev. Mod. Phys., 52, p539, (1980).

[10] J.C. Maxwell, *A treatise on electricity and magnetism*, Dover (1954).

[11] J.C. Maxwell, *A dynamical theory of electromagnetic field*, Phylosophical Transaction of the Royal Society of London, **155**, p 459-512, (1864).

[12] J.C. Maxwell, *On physical lines of force*, Phylosophical Magazine and Journal of Sciences, Fourth series, (1861).

[13] A. Salam, Rev. Mod. Phys., **52**, p525, (1980).

[14] J.M. Vigoureux, P. Vigoureux, B. Vigoureux, *The Einstein constant c in light of Mach's principle, cosmological application*, Poundation of Physics Letters, Vol **16**, N 2, (2003).

[15] S. Weinberg, Rev. Mod. Phys., **52**, p515, (1980).