The Inner Connection Between Gravity, Electromagnetism and Light

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Abstract. In this paper, we prove the existence of an inner connection between the gravitational force and the electromagnetic force using a different procedure than the standard approaches. A new quantification of the space-time expansion allows the use of a short interval as well as a large interval of time as unit, which leads to a natural extension of the laws of physics. We prove that gravity is naturally traceable to the surrounding medium as light.

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1 Introduction

1.1 The Laws of Physics

It is known that there are four apparently quite distinct ways in which matters interacted among themselves: gravitationally, electromagnetically, weakly, and strongly, where the first two interactions are long range and manifest themselves in macroscopic size and even more, meanwhile the second two interactions have very short ranges and are only important at the nuclear and subnuclear level. It is also known that the weak interaction is the one responsible for beta decay, while the strong one is responsible for the binding of protons and neutrons to form the nuclei of atoms.

Finding a way to related these four forces to each others was and remains one of the great quests of physicists. The weak force and the electromagnetic force were unified in a theory presented independently by Abdu-Salam, Weinberg and Glashow in 1967 ([7],[11],[13]). However, no success in associating the gravitational force with the others has yet been achieved despite the intense effort implemented. Most attempts at unification have been for many years within a frame associating electromagnetism with new geometrical properties of space-time ([6],[4]). In this paper we present a new approach that will focus on the existence of an inner connection between the two first interactions using a new mathematical model that incorporates a restricted notion of space expansion in the Lorentz Transformations. The incorporation of a special expansion in the Lorentz Transformations that conserves their linearity allows the study of the uniform motion of the observer relative to the source in an expanding space and rises the existence of an inner connection between the gravitational forces and electromagnetic forces.

1.2 New Tools for Investigation

It is known that our universe is in continuous evolution from the past, to the present, toward the future, and to obtain an optimal model that can describe the universe evolution from the small scale structure to the large scale structure, it is relevant to build a model in which the past and the present coexist. This coexistence will trace back the evolution of our universe from the past to the present and it will predict its future.

The age of our universe is estimated to 15 billion years, which is a huge number if we look at it as a continuous running time from one second to another. However, this huge number in seconds is just fifteen in terms of billion, fifteen quantifications in terms of large period of time as unit. Our universe is huge, and to trace back its evolution from its beginning to nowadays in an adequate model requires the consideration of large scale of space and time in the used model.

In this purpose let us consider the subdivision of 15 billion years into n time-intervals subdivision (or *steps*), such that the *step*(n) represents the present time. Suppose that during each *step* the universe is static and expanding from one *step* to another. Let us consider the quantification $\mathcal{Q}$ that approximates the universe expansion movement step by step as follow:

$\mathcal{Q}$: *if the distance between two events is equal to* L_n *at the step(n), and equal to* L_{n+1} *at the step(n+1), then* $L_{n+1} = a_{n+1} L_n$ *for all* $n \geq 0$*, where* $(a_n)_{n \geq 0}$ *is a sequence such that* $a_0 = 1$*,* $a_n > 1 \quad \forall n \geq 1$*, and* $\prod_{i=0}^{n} a_i$ *converges.*

Using the above quantification, the distance between two separated events at the present time (*step*(n)) is given by $L_n = L_0 \prod_{i=0}^{n} a_i$ which is a function of their distances from the past, and then this linear quantification allows to trace back the evolution of the distance between well separated events in time.

The square of the invariant interval between events at the *step*(n) will be given by the equation

$$ds_n^2 = C_0^2 dt_n^2 - (\prod_{i=0}^{n} a_i^2)(dx^2 + dy^2 + dz^2), \qquad \forall n \geq 0 \tag{1}$$

where C_0 is the constant velocity of light at the Big Bang. Since $\prod_{i=0}^{n} a_i$ is constant for all n, then a simple change of variables makes equation (1) equivalent to the equation

$$ds_n^2 = C_0^2 dt_n^2 - (dX_n^2 + dY_n^2 + dZ_n^2), \qquad \forall n \geq 0 \tag{2}$$

where

$$dX_n = (\prod_{i=0}^{n} a_i) dx, \quad dY_n = (\prod_{i=0}^{n} a_i) dy, \quad dZ_n = (\prod_{i=0}^{n} a_i) dz.$$

For each value of n the metric (2) remains linear and can be considered in the description of the distance between two separated events in the special relativity theory. The only difference is that the metric (1) describes the distance between two separated events *in an expanding space-time* with a special expansion that allows the study of the uniform motion of the observer relative to the source, meanwhile the metric used in special relativity allows the study the uniform motion of the observer relative to the source *in a static space-time*.

Using the line elements (1) such that for each n the equations of Newtonian mechanics hold good, a classical calculus leads straight forward to the obtention of the Lorentz transformation equations at the *step*(n) as follow:

$$T_n : \begin{cases} x' = & \dfrac{x - vt}{\sqrt{1 - (\prod_{i=0}^{n} a_i^2)\frac{v^2}{C_0^2}}} \\ y' = & y \\ z' = & z \\ t' = & \dfrac{t - (\prod_{i=0}^{n} a_i^2)\frac{vx}{C_0^2}}{\sqrt{1 - (\prod_{i=0}^{n} a_i^2)\frac{v^2}{C_0^2}}} \end{cases} \tag{3}$$

from which we derive the existence of a limiting velocity for any moving bodies in an expanding universe (with linear expansion) given by

$$v_{ln} = \frac{C_0}{\prod_{i=0}^{n} a_i} \qquad \forall n \geq 0, \tag{4}$$

where this limiting velocity corresponds to the velocity of light. The limiting velocity was equal to C_0 at the Big Bang (C_0 represents the fossil velocity of light at the $step(0)$ which is the maximum speed of signal propagation at the beginning of the universe expansion). This maximum speed of signal propagation becomes equal to C_n at the $step(n)$ (the present time), where C_n given by

$$C_n = \frac{C_0}{\prod_{i=0}^{n} a_i} \qquad \forall n \geq 0. \tag{5}$$

The new equation (5) represents the velocity of light at the $step(n)$ which corresponds to the velocity of light we measure today experimentally (in the quantification $\mathcal{Q}$ the $step(n)$ corresponds to the present), that is

$$C_n = \frac{C_0}{\prod_{i=0}^{n} a_i} = 2.99792458 \times 10^8 m/s. \tag{6}$$

The equation (6) manifests two aspects, one variable aspect together with the universe expansion, represented by the first equality, and another constant aspect given by the current experimental measurement of the velocity of light.

The velocity of light is locally constant (using a short interval of time as unit) and globally variable (using a large interval of time as unit). This local and global behavior can be derived straight forward from the quantification $\mathcal{Q}$. Indeed, the local behavior is reached in the quantification $\mathcal{Q}$ if we use a big number of subdivisions: the bigger the number n of steps is, the shorter the time interval of steps we obtain. Thus, for n big enough, we have $a_{n+1} \approx 1$ (consequence of the convergence of the product $\prod_{i=0}^{n} a_i$), then

$$\prod_{i=0}^{n+1} a_i = (\prod_{i=0}^{n} a_i)\ a_{n+1} \approx \prod_{i=0}^{n} a_i, \qquad \forall n > A, \tag{7}$$

where A is a large positive real number. Hence the equations (5) and (7) give

$$C_n \approx C_{n+1} \qquad \forall n > A, \tag{8}$$

therefore the velocity of light is almost constant for short period of time as unit. However, the equation (8) is not valid anymore for the large period of time as unit. Indeed, the smaller the number n of steps is, the bigger the time interval of steps we obtain, and in that case we have $a_{n+1} > 1$ for all $n < A$ and

$$\prod_{i=0}^{n+1} a_i = (\prod_{i=0}^{n} a_i)\ a_{n+1} > \prod_{i=0}^{n} a_i, \qquad \forall n < A, \tag{9}$$

hence the equations (5) and (9) lead to the following inequality:

$$C_n > C_{n+1} \qquad \forall n < A, \tag{10}$$

that is to say the velocity of light is globally decreasing together with the universe expansion from one step to another.

Discussing problems raised by varying speed of light cosmology can be found in ([1],[2],[3]) where varying the speed of light can have no effect on the foundation of relativity since one has to use the value of the velocity of light which was appropriate in each local reference frame at any given time, that is to say the velocity of light remains a locally measured invariant but its value depends on the cosmical time as it is explained in ([1]). The use of Mach's principle in the Einstein-Friedman's equation can also lead to interpret the speed of light as intimately connected to the expansion of the universe as it was pointed out in ([12]). However how it is connected to the universe expansion remains unsatisfactory due to the difficulty to use the time notion for the local period of time (short interval of time as unit) as well as the large period of time (cosmical time, or large interval of time as unit). Besides, to make c varying with time as $c(t)$ is not appropriate mathematically if we don't know how it should vary with the cosmical time, meanwhile it must remain invariant using short interval of time as unit to avoid any contradiction with the experimental measurements and deeply rooted theories.

2 Characteristics of an Expanding Vacuum

In general, the permeability is not the same for all medium as it can vary with the position of the medium, the frequency of the field applied, the humidity, the temperature, the composition of the medium, and other parameters, and so for the permittivity. In an expanding universe something is changing making bigger the distance between matter, affecting the temperature of the universe, its density and other parameters that will be discussed here. It is known that a magnetic field B moving with a speed C_n (measured at the *step*(n)) perpendicular to the field lines generates an electric field of magnitude

$$E = C_n B \tag{11}$$

in the region through which it passes. Meanwhile an electric field E moving with the speed C_n (measured at the *step*(n)), perpendicular to the field lines generates a magnetic field of magnitude

$$B = \varepsilon\mu C_n E \tag{12}$$

in the region through which it passes. If we suppose that the electric and magnetic fields of an electromagnetic wave generate each others as the wave moves at the *step*(n) with speed C_n through space, then the parameters of proportionality of equations (11) and (12) verify the following equation

$$\mu\varepsilon C_n^2 = 1. \tag{13}$$

The vacuum permittivity and vacuum permeability are related together at the *step*(n) by the equation (13), where C_n is the experimental measurement of the velocity of light at the present time given by (6). Since the velocity of light varies globally as n varies and since we have

$$C_n^2 = \frac{1}{\mu\varepsilon}, \tag{14}$$

then the product of permittivity and permeability of vacuum varies globally together with the universe expansion, meanwhile locally it remains constant. We will denote the permeability and the permittivity of vacuum at the *step*(n) by μ_n, and ε_n for all $n \geq 0$. Obviously the equation (13) remains invariant together with the universe expansion since equations (11) and (12) are valid for all $n \geq 0$. This invariance from one step to another is described by the following equations

$$\begin{cases} \mu_0\varepsilon_0 C_0^2 = 1, & \text{at the } step(0) \quad \text{(the Big Bang)} \\ \vdots \quad \vdots & \quad \vdots \quad \vdots \\ \mu_n\varepsilon_n C_n^2 = 1, & \text{at the } step(n) \quad \text{(the present time)}, \end{cases} \tag{15}$$

where at the beginning of the universe expansion the velocity of the electromagnetic waves in empty space verifies: $\mu_0\varepsilon_0 C_0^2 = 1$, with C_0 the maximum speed of signal propagation at the beginning of the universe expansion, and where μ_0, ε_0 are the permeability and the permittivity of the primordial vacuum.

The system (15) gives

$$\mu_n\varepsilon_n = \mu_0\varepsilon_0 \prod_{i=0}^{n} a_i^2 \qquad \forall n \geq 0 \tag{16}$$

which means that the product of permeability and permittivity of vacuum increases globally together with the universe expansion. The characteristics of vacuum in an expanding universe are not static and this is normal since the state of the vacuum does vary from point to point. Assuming that the permittivity and the permeability vary together with the universe expansion in the same manner, the formula (16) leads to the following equations:

$$\begin{cases} \mu_n = \mu_0 \prod_{i=0}^{n} a_i & \text{permeability of free space at the } step(n)\ \forall n \geq 0 \\ \varepsilon_n = \varepsilon_0 \prod_{i=0}^{n} a_i & \text{permittivity of free space at the } step(n)\ \forall n \geq 0. \end{cases} \tag{17}$$

That is to say that the permittivity of an expanding vacuum as well as the permeability of an expanding vacuum is globally increasing together with the universe expansion, meanwhile they remain locally constant. Indeed, as for the light velocity, the permeability and the permittivity of the free space are locally constant using a short interval of time as unit, and globally variable using a large interval of time as unit in the quantification $\mathcal{Q}$, and similarly, the local behavior as well as the global behavior of the permittivity and the permeability of an expanding vacuum can be justified using equations (7) and (9). This variation must be extremely small, which makes it not detectable if we use a short interval of time as unit for any experimental measurement duration. The increase of permittivity of an expanding vacuum from the Big Bang to nowadays means that the extensity of the electric field in an expanding vacuum is decreasing with respect to the electric displacement. Meanwhile, the increase of the permeability of an expanding vacuum means that the extensity of the magnetic field in an expanding vacuum is decreasing with respect to the magnetic induction.

The permittivity and permeability are a measure of how much the free space changes together with the universe expansion to absorb energy when subject to electric and magnetic fields, they are characteristics of an expanding free space.

Taking into account the universe expansion, the equation relating light velocity with permittivity and permeability can be formulated via equations (5) and (16) as follow

$$\varepsilon_n \mu_n C_n^2 = \varepsilon_0 \mu_0 C_0^2 = 1 \qquad \forall n \geq 0 \tag{18}$$

which means that as the universe expands the product $\varepsilon_n \mu_n C_n^2$ for all $n \geq 0$ remains invariant together with the universe expansion (for both global and local time). Moreover, the electric and magnetic fields in an electromagnetic waves are related by an invariant value of the characteristic impedance together with the universe expansion:

$$Z_n = \mu_n C_n = \sqrt{\frac{\mu_n}{\varepsilon_n}} \qquad \forall n \geq 0, \tag{19}$$

Indeed, if we denote Z_0 the characteristic impedance at the *step*(0) (Big Bang), then the equations (5) and (17) give

$$Z_n = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \mu_0 C_0 = Z_0 = cst \qquad \forall n \geq 0. \tag{20}$$

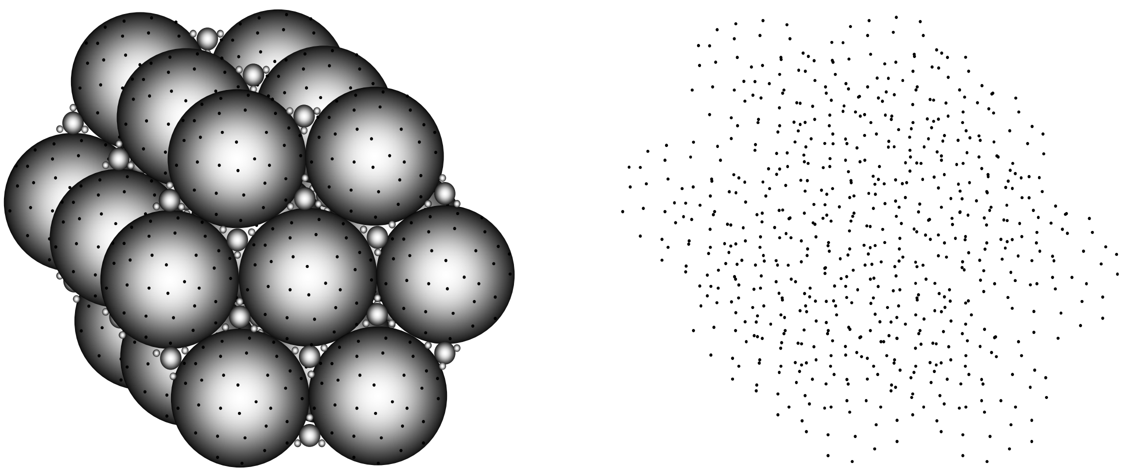

Figure 1: The space described by a fractal manifold is constituted of an infinite number of packed expanding balls (called universe points), where the visible universe is only the surface of those packed balls. Matter (simulated by dots in this illustration that might represent galaxies or any other big structure of matter) and geodesics are located on the surface of those expanding spheres. The existence of light geodesics only on the surface of those expanding points makes the space geometry invisible and matter appears to be held by invisible pillars in the sky that forbid their collapse. The left figure represents space and matter distribution at large scale with visible geometry, meanwhile the right figure represents space and matter distribution at large scale with invisible geometry.

2.1 Physical Interpretation Using Fractal Manifold Model

The metric (1) used in the special quantification $\mathcal{Q}$ is a metric found in the fractal manifold model where the shape of the universe simulated by the fractal manifold model is described by an infinite number of packed expanding universe points ([5]). The universe points are simulated to expand as balls, and the visible space of the universe is only described by the packed surfaces of those expanding balls (Fig.1). Any magnetic flux lines in the universe are channeled through geodesics located in the packed expanding surfaces ([5]), which makes the inner part of those universe points naturally isolated magnetically as the universe expands (natural voids). The physical interpretation of this behavior is explained by the increase of the permeability of the visible space together with the universe expansion. As the universe expands the magnetic flux lines are deflected from the normal propagation at the *step*(0) to a considerable extend in a bent space with higher permeability and higher permittivity at the *step*(n) for all $n \geq 0$, and the flux density sharply increases in those packed surfaces that represent the visible space of our universe. The distortion of the magnetic field following the variable geometry of the visible space is sustained by the increase of the visible space permeability to absorb all magnetic flux lines and to not allow the magnetic flux lines to penetrate the inner part of those universe points (see Fig.2). This leads to the following conclusion: the permittivity and the permeability of the inner part of those expanding universe points (voids) are less than the permittivity and permeability of their boundaries (the visible space where matter is located).

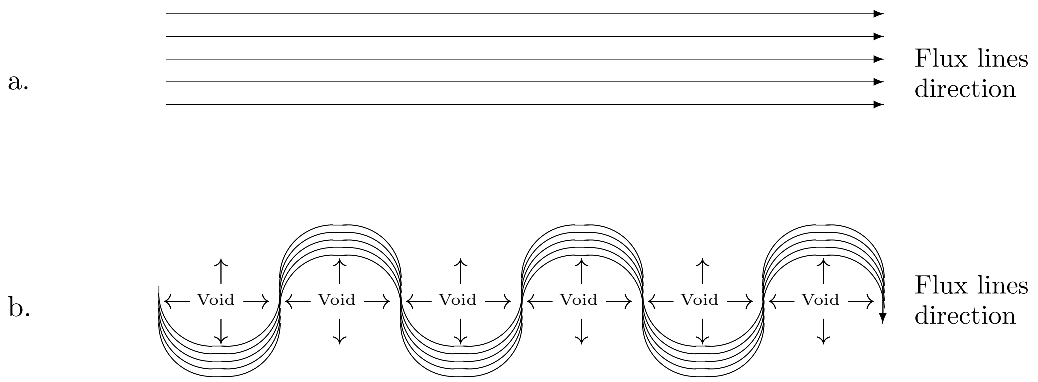

Figure 2: The figure a. represents the magnetic flux lines in a space before its expansion in a vacuum with permittivity ε_0 and permeability μ_0. Meanwhile b. represents the magnetic flux lines in an expanding free space at the *step*(n) with permittivity ε_n and permeability μ_n. As the universe expands through the universe points expansion, the permeability of the expanding vacuum as well as its permittivity increases together with the universe expansion and then the magnetic flux lines are deflected from the normal to the considerable extent in which the flux density sharply increases, and where the magnetic flux cannot penetrate into those expanding regions (the expanding universe points) but they travel through their boundary by fluctuating following the expanding space geodesics ([5]).

3 Gravitational Constant in an Expanding Universe

It is known that Hydrogen is the most abundant gas in the universe accounting for 89% of all atoms. The Hydrogen atoms were formed in the first few seconds after the event that marked the beginning of the universe. An atom of Hydrogen has one proton with positive charge of mass $m_p = 1,6726 \times 10^{-27} kg$ that represents the nucleus, and one electron of mass $m_e = 9.1094 \times 10^{-31} kg$ in orbit around the nucleus at an estimated distance of $r = 0,53 \times 10^{-10} m$. The natural attraction of the proton of the Hydrogen atom and its electron is represented by the electric force given by Coulomb law

$$\vec{F_e} = k\frac{q_e q_p}{r^2}\hat{r}, \tag{21}$$

where q_e is the electron charge and q_p is the proton charge, and k is the constant of proportionality that depends on the value of the permittivity of the space. Since the space considered here is an expanding space and since we are using the quantification $\mathcal{Q}$ that linearly quantifies the space expansion then the term of proportionality is then given by $k = \frac{1}{4\pi\varepsilon_n}$, where ε_n is the permittivity of vacuum at the $step(n)$ (the present time), and $\hat{r}$ is the unit vector pointing the force direction. The term k of proportionality is locally constant and globally variable following the short interval of time or the large interval of time we are using.

Besides the gravitational attraction between the nucleus of the Hydrogen atom and its electron is given by

$$\vec{F_g} = G\frac{m_p m_e}{r^2}\hat{r} \tag{22}$$

where m_e is the electron mass, m_p is the proton mass, G is the gravitational constant of proportionality, and $\hat{r}$ is the unit vector pointing the force direction from one mass to the other. The ratio of the electrical attraction (21) to the gravitational attraction (22) is given by

$$\frac{|\vec{F_e}|}{|\vec{F_g}|} = \frac{k q_p q_e}{G m_p m_e} = \frac{q_p q_e}{4\pi\varepsilon_n G m_p m_e}. \tag{23}$$

Since matter does neither expand nor contract as the universe expands (matter is not affected by the universe expansion) the ratio of the electrical attraction (21) to the gravitational attraction (22) remains invariant under the universe expansion, and this invariance must be true despite the use of a large interval of time as unit or a short interval of time as unit. Indeed, in order to prove the previous assertion let as assume that

$$\frac{|\vec{F_e}|}{|\vec{F_g}|} = \alpha. \tag{24}$$

and that α is variable using a large interval of time as unit (This quotient is constant using a short interval of time as unit since it is proved experimentally). Two cases are then possible with this assumption:

$$a)\begin{cases} \alpha & \text{variable} \\ |\vec{F_g}| & \text{constant} \end{cases} \qquad b)\begin{cases} \alpha & \text{variable} \\ |\vec{F_g}| & \text{variable} \end{cases}$$

a) In this case the electric force is decreasing together with the universe expansion meanwhile the gravitational force remains constant as the universe expands, and since the electric force is attractive in the Hydrogen atom then matter will expand, which is impossible.

b) If we look to this case from the *step*(n) to the *step*$(n+1)$ for example: the variation of the electric force (21) from the *step*(n) to the *step*$(n+1)$ is given by

$$\Delta F_e = |\vec{F}_{e_{n+1}}| - |\vec{F}_{e_n}|, \qquad \Longrightarrow |\vec{F}_{e_{n+1}}| = |\vec{F}_{e_n}| + \Delta F_e, \tag{25}$$

where $|\vec{F}_{e_i}|$ represents the magnitude of the electric force that represents the natural attraction of the proton of the Hydrogen atom and its electron at the *step*(i). The variation of the gravitational force (22) from the *step*(n) to the *step*$(n+1)$, if the gravitational force is supposed to be globally variable, is given by

$$\Delta F_g = |\vec{F}_{G_{n+1}}| - |\vec{F}_{G_n}|, \qquad \Longrightarrow |\vec{F}_{G_{n+1}}| = |\vec{F}_{G_n}| + \Delta F_g, \tag{26}$$

where $|\vec{F}_{G_i}|$ represents the magnitude of the gravitational force between the proton of the Hydrogen atom and its electron at the *step*(i).

Three possible subcases exist within this case:

Subcase 1: If $\Delta F_e > \Delta F_g$ then matter will expand. Indeed, since the electric force decreases together with the universe expansion due to equation (17), then equation (25) gives $\Delta F_e < 0$ which means that $\Delta F_g < 0$ too. The gravitational force will then decrease together with the universe expansion via equation (26). Since the gravitational force as well as the electric force is an attractive force in the Hydrogen atom and they are decreasing together with the universe expansion, then the Hydrogen atoms will expand together with the universe expansion.

Subcase 2: If $\Delta F_e < \Delta F_g$ then matter will contract if $|\vec{F}_g|$ increases and it will expand if $|\vec{F}_g|$ decreases. Indeed, if the gravitational force will increase in the Hydrogen atom as the universe expands then matter will contract since the two forces are attractive in the Hydrogen atom, and the increase of the gravitational force is more important than the decrease of the electric force. However, if the gravitational force will decrease as the universe expands then matter will expand since the two attractive forces in the Hydrogen atom decrease together with the universe expansion.

Subcase 3: If $\Delta F_e = \Delta F_g$ then matter will expand. Indeed, since the electric force decreases together with the universe expansion due to equation (17), then $\Delta F_e < 0$ which means that $\Delta F_g < 0$ too, and then the gravitational force will decrease together with the universe expansion via equation (26). The two attractive forces are decreasing together with the universe expansion in the Hydrogen atom, then matter will expand.

The only case where matter is not affected by the universe expansion is the case where α is constant using a large interval of time as unit as well as a short interval of time as unit (that is locally and globally constant), which means that the ratio of the electrical attraction to the gravitational attraction (24) remains constant together with the universe expansion, and then we have

$$\frac{|\vec{F}_e|}{|\vec{F}_g|} = \frac{q_p q_e}{4\pi\varepsilon_n G m_p m_e} = 2,3 \times 10^{39} \qquad \forall n > 0, \tag{27}$$

which gives that the product $\varepsilon_n G$ in the equation (27) must be constant as the universe expands for all n (constant globally). Since the permeability of vacuum is globally variable, and this variation is linear with $(\prod_{i=0}^{n} a_i)$ as coefficient of linearity as (17), then the gravitational constant G must vary linearly with $(\frac{1}{\prod_{i=0}^{n} a_i})$ as coefficient of linearity together with the universe expansion. Accordingly, we will put the gravitational constant G_n at the *step*(n) as proportional to the primordial gravitational constant G_0:

$$G_n = \frac{G_0}{\prod_{i=0}^{n} a_i}. \tag{28}$$

Therefore the gravitational term of proportionality in the gravitational force (22) is then given by (28) and this term is affected by the universe expansion. This variation has the same interpretation as the velocity of light: it is locally constant using a short interval of time as unit and globally variable using a large interval of time as unit. The gravitational term of proportionality is decreasing very slowly together with the universe expansion and this decrease is undetectable using a short interval of time as unit, meanwhile it should be globally detectable using an adequate large interval of time as unit. This variation could be detected via observations from the past.

According to (28) the gravitational term of proportionality of the gravitational force (22) is independent from the distance between matter, independent from the mass of matter involved, locally constant, and globally dependent on the universe expanding parameter. It can be measured everywhere in our universe, and it will have the same measured value everywhere at a given step of the universe expansion. However, its value is closely related to the universe expansion and presents two different aspects: G_n is locally constant using a short interval of time as unit meanwhile it is globally variable using a large interval of time as unit. Locally the measure of the gravitational force intensity at present time (at the *step*(n)) is given by:

$$G_n = \frac{G_0}{\prod_{i=0}^{n} a_i} = 6.673 \times 10^{-11} N.m^2 kg^{-2}, \tag{29}$$

which is a function of G_0 the measure of the gravitational force intensity at the primordial space (*step*(0), or Big Bang) and of the universe expanding parameter $(\prod_{i=0}^{n} a_i)$.

4 Electromagnetic and Gravitational Forces

4.1 Gravitational Force

Using the equation (29) at the *step*(n) and according to the law of universal gravitation, the force exerted by a gravitational mass m_1 on a gravitational mass m_2 separated by a distance r is given in a quantified expanding universe by

$$\vec{F_{12}} = -\Big(\frac{G_0}{\prod_{i=0}^{n} a_i}\Big)\frac{m_1 m_2}{r^2}\hat{r} \tag{30}$$

where $\frac{G_0}{\prod_{i=0}^{n} a_i}$ is the measure of the gravitational force intensity at the present time (*step*(n)), $\hat{r}$ is the unit vector pointing from m_1 to m_2, G_0 is the measure of the gravitational force intensity at the primordial space (*step*(0)), and a_i is the universe step-expanding parameter from the *step*$(i-1)$ to the *step*(i).

According to the equation (30) the gravitational force exerted by a gravitational mass m_1 on a gravitational mass m_2 is decreasing together with the universe expansion. This decrease is locally not detectable using a short interval of time as unit, however the gravitational force (30) is globally variable using a large interval of time as unit.

4.2 Electric Field and Electrostatic Force

According to the value of the permittivity together with the universe expansion (17) the electrostatic force at the *step*(n) between two electrical charges q_1 and q_2 separated by a distance r is given by:

$$\vec{F_{12}} = \Big(\frac{1}{4\pi\varepsilon_n}\Big)\frac{q_1 q_2}{r^2}\hat{r} = \Big(\frac{1}{4\pi\varepsilon_0\prod_{i=0}^{n} a_i}\Big)\frac{q_1 q_2}{r^2}\hat{r} = -\vec{F_{21}} \tag{31}$$

where ε_n is the vacuum permeability at the *step*(n), ε_0 is the vacuum permeability at the Big Bang (*step*(0)), and a_i is the universe step-expanding parameter from the *step*$(i-1)$ to the *step*(i). The electric field at the *step*(n) of a point charge q_1 at any distant point M of distance r away from the charge is given by

$$\vec{E_n} = \frac{\vec{F_{12}}}{q_2} = \Big(\frac{1}{4\pi\varepsilon_0\prod_{i=0}^{n} a_i}\Big)\frac{q_1}{r^2}\hat{r} = \frac{1}{\prod_{i=0}^{n} a_i}\Big(\frac{1}{4\pi\varepsilon_0}\frac{q_1}{r^2}\hat{r}\Big) \tag{32}$$

which gives

$$\vec{E_n} = \frac{1}{\prod_{i=0}^{n} a_i}\vec{E_0} \tag{33}$$

where $\vec{E_0} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1}{r^2}\hat{r}$ represents the electric field of the point charge q_1 in a space with permittivity ε_0 (the primordial space) acting in the direction of the radius vector $\hat{r}$. Therefore the electric force at the *step*(n) of the universe expansion can be written as

$$\vec{F_{12}} = q_2\vec{E_n} = q_2\frac{1}{\prod_{i=0}^{n} a_i}\vec{E_0}. \tag{34}$$

According to the equation (33) the electric field of the point charge q_1 is decreasing together with the universe expansion (globally, using a large interval of time as unit), which leads to the decreasing nature of the electrostatic force between two distant electric charges together with the universe expansion. However, this decrease is locally not detectable using a short interval of time as unit.

4.3 Magnetic Induction and Magnetic Force

It is known that the magnetic induction field and magnetic force can be produced and experienced by two types of bodies: by charges in motion, or electric currents, and by magnetized bodies, such as permanent magnets. It is also known that we cannot strictly state the field produced by a charge in motion since a single charge in motion cannot produce a static magnetic induction field. Nevertheless, if the charge forms part of steady current, as if there was a natural event (supernova, or Big Bang) that creates a procession of charges following one after the other in a formation independent of time, then a simple law for the field it produces at the *step*(n) of the universe expansion can be obtained, in which the evolution of the field together with the universe expansion is traced back from the Big Bang (*step*(0)).

Indeed, using equation (17) the magnetic induction resulting from a charge q_1, moving at the *step*(n) of an expanding space with a velocity v, at the point M of a distance r away from the charge is given by:

$$\vec{B_n} = \frac{\mu_n}{4\pi} q_1 \frac{v \times r}{|r|^3} = \prod_{i=0}^{n} a_i \Big(\frac{\mu_0}{4\pi} q_1 \frac{v \times r}{|r|^3} \Big) \tag{35}$$

which gives

$$\vec{B_n} = (\prod_{i=0}^{n} a_i) \vec{B_0} \tag{36}$$

where $\vec{B_0} = \frac{\mu_0}{4\pi} q_1 \frac{v \times r}{|r|^3}$ represents the magnetic induction resulting from the same charge q_1, moving in a space with permeability μ_0 (the primordial space) with the velocity v at the same distance $|r|$ away from the charge provided that the moving charge forms part of a current distribution independent of time, where r is a vector pointing from the charge to the point in space where the field is being found.

The magnetic force, at the *step*(n) of the universe expansion, on another charge Q moving at the point M with a velocity u is then given by

$$F = Qu \wedge \vec{B_n} = Qu \wedge (\prod_{i=0}^{n} a_i) \vec{B_0} \tag{37}$$

where B_n is given by formula (36).

4.4 The Electromagnetic Force in an Expanding Universe

The total electromagnetic force, at the *step*(n) of the universe expansion, on the charge Q moving at the point M with velocity u from the point charge q_1 can then be written as

$$F_n = Q(E_n + u \wedge \vec{B_n}) \tag{38}$$

and by using formula (33) and (36)

$$F_n = Q\Big(\frac{1}{\prod_{i=0}^{n} a_i} \vec{E_0} + u \wedge (\prod_{i=0}^{n} a_i) \vec{B_0} \Big). \tag{39}$$

Every point M in an expanding space is then characterized by two vector quantities which determine the force on any charge Q:

i) There is the electric force which gives a force component independent of the motion described by the electric field $\vec{E_n}$. This force is decreasing together with the universe expansion since the electric field $\vec{E_n}$ of a point charge q_1 at any point M of a distance r away from the charge is decreasing together with the universe expansion. The decrease of the electric field $\vec{E_n}$ together with the universe expansion is due to the increase of the permittivity of the free space, and this variation is locally not detectable using a short interval of time as unit and globally detectable using a large interval of time as unit.

ii) There is the magnetic force, which depends on the velocity of the charge Q. This force is increasing together with the universe expansion since the magnetic induction $\vec{B_n}$

resulting from a charge q_1, moving in an expanding space with a velocity v, at any point M of a distance r away from the charge is increasing together with the universe expansion. The increase of the magnetic induction $\vec{B_n}$ together with the universe expansion is due to the increase of the permeability of the free space, and this variation is locally not detectable using a short interval of time as unit and globally detectable using a large interval of time as unit.

4.5 The Inner Connection

The incorporation of a special expansion in the special relativity theory rises the fact the limiting velocity (4) of any moving body presents double aspects: one local aspect if we use a short interval of time as unit in the universe quantification $\mathcal{Q}$, and one global aspect if we use a large interval of time as unit in the universe quantification $\mathcal{Q}$. The local aspect is constant meanwhile the global one is decreasing together with the universe expansion.

The changes brought in the formulation of the electromagnetic force as well as in the formulation of the gravitational force are manifested by the appearance of the universe expanding parameter

$$(\prod_{i=0}^{n} a_i), \tag{40}$$

that characterizes and approximates the universe expansion from the *step*(0) (Big Bang) to the *step*(n) (the present time). The use of the expanding parameter (40) in the formulation of the electromagnetic and gravitational forces allows to trace back their nature in an expanding universe. The inner connection between electromagnetic and gravitational forces appears to be the universe expanding parameter itself that represents the dynamic of the universe. This seems to be coherent since the dynamic of the host of matter might affect the matter dynamics.

J. C. Maxwell had made one of the great unification of physics. From static measurement (by measuring the force between two units of charges and between two units of currents) he found that the velocity of propagation of electromagnetic influences is equal to the velocity of light ([9]). In front of this mysterious coincidence Maxwell said: *we can scarcely avoid the inference that light consists in the transverse undulations of the same medium which is the cause of electric and magnetic phenomena* ([10]).

The equation that makes light traceable to the surrounding medium is given by (13). The invariance of this equation together with the universe expansion is no longer a coincidence, since the validity of this equation step by step from the Big Bang to nowadays given by (18) makes it invariable together with the universe expansion and reinforces the prediction of Maxwell. The equation (18) is a fundamental equation that relates the velocity of light to the surrounding medium of an expanding universe.

The product of the formulas (17) and (28) leads to a similar invariant relation together with the universe expansion: the characteristics of an expanding free space (permeability and permittivity) are related together with the gravitational constant in an invariant equation together with the universe expansion given by:

$$\varepsilon_n \mu_n G_n^2 = \varepsilon_0 \mu_0 G_0^2 \qquad \forall n \geq 0. \tag{41}$$

The product in (41) is an universal constant since it remains invariant together with the universe expansion from the Big Bang (*step*(0)) to the present time (*step*(n)) and we have

$$\varepsilon_n\mu_n G_n^2 = \varepsilon_{n-1}\mu_{n-1}G_{n-1}^2 = \ldots\ldots = \varepsilon_0\mu_0 G_0^2 = K_1 \qquad \forall n \geq 0. \tag{42}$$

This constant is given by $K_1 = 4.9542622588953008921669371753729 \times 10^{-38} \simeq 5 \times 10^{-38}$, it corresponds to what we measure today experimentally. This constant is extremely small and remains invariant together with the universe expansion since ε_0, μ_0, and G_0 are fundamental constants, characteristics of the primordial universe (the Big Bang).

The justification of the invariance of the equation (42) together with the universe expansion refutes any coincidence on it and requires the adoption of the same interpretation as for equation (13) that relates light to the local medium; gravity is no longer something else, it consists in the transverse undulations of the same medium which is the cause of electric, magnetic phenomena and light. Gravity is just another form of electricity and magnetism.

Light and gravity share the same medium and they have to be well described in an invariant equation under the universe expansion. Indeed, the multiplication of the formulas (5), (17), and (28) gives us the following invariant equation together with the universe expansion from the Big Bang (*step*(0)) to the present time (*step*(n))

$$\mu_n\varepsilon_n c_n G_n = \mu_0\varepsilon_0 C_0 G_0 \qquad \forall n \geq 0 \tag{43}$$

which means that the product $\mu_n\varepsilon_n c_n G_n$ is an universal constant invariant under the universe expansion since ε_0, μ_0, G_0 are fundamental constants, characteristics of the primordial universe (the Big Bang), and C_0 is the maximum speed of signal propagation at the beginning of the universe expansion. Their primordial product can be evaluated by what we measure today

$$\mu_n\varepsilon_n c_n G_n = \mu_{n-1}\varepsilon_{n-1}c_{n-1}G_{n-1} = \ldots = \mu_0\varepsilon_0 C_0 G_0 = \sqrt{K_1} \quad \forall n \geq 0 \tag{44}$$

where K_1 is given by the equation (42). The use of the two equations (18) and (43) gives a clear relation between velocity of light and gravity in an expanding universe described by the following invariant equation under the universe expansion

$$G_n = c_n\sqrt{K_1} \qquad \forall n \geq 0 \tag{45}$$

which comforts how their mutual nature are intimately related and that gravity is no longer something else: the gravitational interaction and the electromagnetic interaction are two manifestations of the same phenomena that resides in a transverse undulation of the surrounding medium. Each manifestation is significant according to the bodies heaviness, the bodies charges, and the distance between them.

Taking into account the variation of the electric field (33) as well as the magnetic induction (36) together with the universe expansion, it is not difficult to derive the invariance of their cross product under the universe expansion and we have

$$\vec{E_n} \wedge \vec{B_n} = \vec{E_0} \wedge \vec{B_0} \qquad \forall n \geq 0 \tag{46}$$

which means that the cross product of $\vec{E_n}$ and $\vec{B_n}$ is invariant under the universe expansion. However, the flux of energy (the amount of energy crossing unit area perpendicular the

Poynting's vector, per unit time) is not invariant under the universe expansion. Indeed, if we denote the flux of energy at the *step*(n) of the universe expansion by $\vec{S_n}$, we have

$$\vec{S_n} = \frac{1}{\mu_n}\vec{E_n} \wedge \vec{B_n} \tag{47}$$

and if we denote the flux of energy at the primordial universe (at the *step*(0)) by $\vec{S_0}$ such that

$$\vec{S_0} = \frac{1}{\mu_0}\vec{E_0} \wedge \vec{B_0} \tag{48}$$

as the universe expands we have (46), and then

$$\vec{S_n} = \frac{1}{\mu_n}\vec{E_n} \wedge \vec{B_n} = \frac{1}{\mu_n}\vec{E_0} \wedge \vec{B_0} \tag{49}$$

which gives if we take into account formula (48) and (17)

$$\vec{S_n} = \frac{1}{\prod_{i=0}^{n} a_i}\vec{S_0}. \tag{50}$$

This means that the amount of energy crossing unit area perpendicular the Poynting's vector per unit time is decreasing together with the universe expansion (the energy transportation by an electromagnetic wave in an expanding space is decreasing together with the universe expansion).

4.6 Conclusion

The quantification introduced in this paper uses a subdivision of the time interval of our universe from the big bang to present time into steps, where the duration of one step depends on the number of subdivisions. The bigger the number of subdivisions (or steps) is, the shorter the steps time interval we have. Local information is obtained using a big number of subdivisions, meanwhile global information is obtained using a small number of subdivisions. The new formalism obtained rises two different aspects, one local aspect where the limiting velocity of any moving body (including light) remains constant, and one global aspect where the limiting velocity of any moving body, as well as the permittivity and permeability of empty space, and gravity are affected by the universe expansion. The new extension of the Lorentz transformation equations allows to trace back the effect of the universe dynamic on fundamental constants and laws, and rises locally no contradiction with our rooted understanding of laws of physics. Meanwhile globally an inner connection appears in the formulation of laws of physics that makes gravity traceable to the surrounding medium as well as electric and magnetic phenomena. The global aspect imposes itself without being in contradiction with experience, since all our experimental measurements of fundamental constants are local in space and time with respect to the universe size and age. This new approach provides an interesting global picture in which the laws of physics have an inner connection that leads straight forward to their unification, meanwhile the local picture is in perfect concordance with experimental measurements. The new extend provides us with new tools of analysis for the universe global aspect that divulges the limit of our local information in space and time. All our physical experiences are local in space and time and this is a limpid reality, with special exception for some observations from the past that reveal some violation of our rooted local understanding (the faraway galaxies appear receding from us faster than the speed of light, since their receding velocities are proportional to their distance from us).

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5 Appendix

What is the physics brought in an expanding universe by an increasing magnetic field as (36) and decreasing electric field as (33)? The increase of the magnetic field together with the universe expansion leads to the increase of the magnetic force on any charged particle with momentum P at the $step(n)$ for example. Since the magnetic force is purely a deflexion force, then in an expanding universe with variable curvature an increasing magnetic field is needed to deflect the motion direction of any charged particle locally and globally (as producing special particle trajectory in a high energy particle accelerator). This trajectory is channeled on the new geometries (Fig. 1) following the variable curvature of the expanding space. Indeed an electric particle moving perpendicular to the magnetic field in the space will describe a circle with radius inversely proportional to the magnetic induction. However if the particle inters the field with an initial velocity $\vec{v_0}$ at an incline to the direction of the field, it will be deflected with a frequency given by

$$\omega_n = \frac{qB_n}{mC_n}, \quad \forall n \geq 0, \tag{51}$$

and by using the increase nature of the magnetic induction as (36) in an expanding universe measured at the $step(n)$ and formula (6), it turns out that the frequency ω_n is increasing together with the universe expansion as

$$\omega_n = \frac{qB_0}{mC_0} \prod_{i=0}^{n} a_i^2, \quad \forall n \geq 0. \tag{52}$$

Assuming we resolve the velocity vector $\vec{v_0}$ into $v_{\parallel}$ a parallel component to the magnetic field and $v_{\perp}$ a perpendicular to the magnetic field, the particle will move following the magnetic field direction in a spiral with radius

$$R_n = \frac{mv_{\perp}c_n}{qB_n} \tag{53}$$

where using (36) and (6) gives a decreasing radius together with the universe expansion as

$$R_n = \frac{mv_{\perp}C_0}{qB_0} \frac{1}{\prod_{i=0}^{n} a_i^2}, \quad \forall n \geq 0. \tag{54}$$

The variation of the local deflection together with the universe expansion is due to the increasing Lorentz deflecting force together with the universe expansion as

$$f_n = \frac{1}{C_n} qv_{\perp}B_n = \frac{1}{C_0} qv_{\perp}B_0 \prod_{i=0}^{n} a_i^2 \tag{55}$$

that insures the circular movement of the particle, meanwhile the global deflection is described by the angle between the initial movement direction and the magnetic field direction in the expanding space.

However, the decreasing nature of the electric field in an expanding universe as (33) serve to slow down the speed of any charged particle with momentum P together with the universe expansion inversely proportional to expanding parameter $\prod_{i=0}^{n} a_i$.

Under the action of both a decreasing electric field and an increasing magnetic field the particle with momentum P in an expanding universe will move with an increasing frequency (52) in a spiral with a decreasing radius R_n inversely proportional to the universe expanding parameter $\prod_{i=0}^{n} a_i^2$ following the magnetic field direction, it will be deflected into a conical helix path in the magnetic field direction.

5.0.1 Reformulation of Maxwell's Equation in an Expanding Universe

Maxwell's equations ([8]) furnished a theory of light, and light exists in an expanding universe from the Big Bang to nowadays. The validity of Maxwell's theory in an expanding universe leads to make it valid step by step in the process of the approximation of the universe expansion. The Maxwell's equations can be written as invariant equations under the universe expansion as follow taking into account the approximation process of the universe expansion:

$$\begin{aligned}
&\nabla \vec{E_n} = \frac{\rho_t}{\varepsilon_n} && \forall n \geq 0 \\
&\nabla \vec{B_n} = 0 && \forall n \geq 0 \\
&\nabla \wedge \vec{E_n} = -\frac{\partial \vec{B_n}}{\partial t} && \forall n \geq 0 \\
&\nabla \wedge \vec{B_n} = \mu_n J_t + \mu_n \varepsilon_n \frac{\partial \vec{E_n}}{\partial t} && \forall n \geq 0,
\end{aligned} \tag{56}$$

where the electromagnetic force in an expanding universe is given by

$$F_n = Q\Big(\vec{E_n} + u \wedge \vec{B_n}\Big) \qquad \forall n \geq 0, \tag{57}$$

in which n represents the steps number of the expansion approximation. In an expanding region without electric charges, Maxwell's equations become:

$$\nabla \vec{E_n} = 0, \qquad \nabla \vec{B_n} = 0, \qquad \nabla \wedge \vec{E_n} = -\frac{\partial \vec{B_n}}{\partial t}, \qquad \nabla \wedge \vec{B_n} = \mu_n \varepsilon_n \frac{\partial \vec{E_n}}{\partial t} \qquad \forall n \geq 0. \tag{58}$$

The elimination of one field or the other leads to the wave equations:

$$\Delta \vec{E_n} - \mu_n \varepsilon_n \frac{\partial \vec{E_n}}{\partial t} = 0, \qquad \Delta \vec{B_n} - \mu_n \varepsilon_n \frac{\partial \vec{B_n}}{\partial t} = 0, \qquad \forall n \geq 0 \tag{59}$$

where the fields $\vec{E_n}$ and $\vec{B_n}$ for all $n \geq 0$ are traveling with velocities that verify an invariant equation under the universe expansion given by:

$$C_n = \frac{1}{\sqrt{\mu_n \varepsilon_n}} = \frac{\sqrt{K_1}}{G_n \mu_n \varepsilon_n} \qquad \forall n \geq 0, \tag{60}$$

and where the velocity of the electromagnetic wave is measured today (at the $step(n)$) to be equal to

$$C_n = \frac{1}{\sqrt{\mu_n \varepsilon_n}} = \frac{\sqrt{K_1}}{G_n \mu_n \varepsilon_n} = 2.99792458 \times 10^8 m/s \quad . \tag{61}$$