

## Evolutionary sequence of spacetime/intrinsic spacetime and associated sequence of geometries in a metric force field. Part IV.

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The flat two-dimensional proper intrinsic spacetime  $(\phi\rho', \phi c\phi t')$  underlying flat four-dimensional proper spacetime  $(E'^3, ct')$ , which emerged at the first stage of evolution of spacetime/intrinsic spacetime within a long-range metric force field, isolated in the first three parts of this paper, endured for no moment before transforming into a curved two-dimensional proper intrinsic spacetime with orthogonal curvilinear intrinsic dimensions on the vertical intrinsic spacetime plane, in the larger spacetime/intrinsic spacetime domain of combined positive (or our) universe and the negative universe. It therefore possesses intrinsic Lorentzian metric tensor at every point. It projects an underlying flat relativistic intrinsic spacetime  $(\phi\rho, \phi c\phi t)$  that replaces flat proper intrinsic spacetime, which is made manifest outwardly in flat four-dimensional relativistic spacetime  $(E^3, ct)$  that replaces flat proper spacetime, at the second (and final) stage of evolution of spacetime/intrinsic spacetime within a long-range metric force field. The curved 'two-dimensional' absolute intrinsic spacetime  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  with absolute intrinsic sub-Riemannian metric tensor that evolved at the first stage is brought forward to the second stage. Apart from absolute intrinsic Riemann geometry on the curved 'two-dimensional' absolute intrinsic spacetime brought forward from the first stage, intrinsic local Lorentzian geometry that involves the derivation of intrinsic local Lorentz transformation in terms of an isolated intrinsic parameter, referred to as intrinsic static speed, and validation of intrinsic local Lorentz invariance between the curved two-dimensional proper intrinsic spacetime and its projective flat two-dimensional relativistic intrinsic spacetime are established. These are then made manifest in local Lorentz transformation in terms of static speed and local Lorentz invariance between the flat proper spacetime at the first stage of evolution of spacetime/intrinsic spacetime and the flat relativistic spacetime at the second stage, within the four-world picture. The conclusion that spacetime is everywhere flat in every long-range metric force field is reached. Particularization to the gravitational field will be a straight forward process, while using the results of this paper as template.

### 1 Introduction

The first two parts of this paper [1-2] are devoted to the development of absolute intrinsic Riemann geometry on curved '2-dimensional' absolute intrinsic metric spacetime  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$ , which is underlied by its projective flat 2-dimensional proper intrinsic metric spacetime  $(\phi\rho', \phi c\phi t')$  and the outward manifestation of the latter namely, the flat four-dimensional proper metric spacetime  $(E', ct')$ . These evolve from an initial flat 'four-dimensional' absolute metric spacetime  $(\hat{E}^3, \hat{c}\hat{t})$  underlied by flat 'two-dimensional' absolute intrinsic metric spacetime  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$ , at the first stage of evolutions of spacetime and its underlying intrinsic spacetime within an absolute metric force-field and its underlying absolute intrinsic metric force-field in the positive (or our) universe.

Actually of the three co-existing metric spacetimes namely, the curved '2-dimensional' absolute intrinsic metric spacetime  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$ , its projective underlying flat 2-dimensional proper intrinsic metric spacetime  $(\phi\rho', \phi c\phi t')$  and the outward manifestation of the latter namely, the flat four-dimensional proper metric spacetime  $(E'^3, ct')$  in Fig. 4 or Fig. 11

of [2], shown to evolve from an initial flat 'four-dimensional' absolute metric spacetime  $(\hat{E}^3, \hat{c}\hat{t})$  underlied by flat 'two-dimensional' absolute intrinsic metric spacetime  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  in Fig. 6 of [2], at the first stage of evolution of spacetime/intrinsic spacetime in a long-range metric force-field, only the curved 'two-dimensional' absolute intrinsic metric spacetime  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  and its underlying projective flat two-dimensional proper intrinsic metric spacetime  $(\phi\rho', \phi c\phi t')$  are new in physics. The flat 4-dimensional proper metric spacetime  $(E'^3, ct')$  that overlies  $(\phi\rho', \phi c\phi t')$  or which is the outward (or physical) manifestation of  $(\phi\rho', \phi c\phi t')$ , as discussed in subsection 4.4 of [3], is not new in physics, being what has been known as the space of classical mechanics, assuming the absence of (relativistic) gravitational field.

The flat proper metric spacetime  $(E'^3, ct')$  containing the rest masses  $m_0$  of particles and objects in  $E'^3$  in the assumed absence of (relativistic) gravitational field, thereby making  $(E'^3, ct')$  to remain flat, has also been known in physics to support the special theory of relativity (SR) involving the motion of the rest masses  $m_0$  of material particles relative to ob-

servers. The special theory of relativity/intrinsic special theory of relativity (SR/ $\phi$ SR) have actually been developed on the flat ( $E'^3, ct'$ ) and its underlying flat ( $\phi\rho', \phi c\phi t'$ ) within the four-world picture in the context of the present theory in [3-6].

As known in physics until now, the introduction of a relativistic gravitational field into the flat four-dimensional proper metric spacetime ( $E'^3, ct'$ ), will transform it into a curved four-dimensional relativistic metric spacetime ( $E^3, ct$ ) (usually denoted by  $(x^0, x^1, x^2, x^3)$ ) in the context of the general theory of relativity (GR). It must be recalled however that although curvature of the relativistic spacetime in a metric force field is a well thought-out prescription [7, see p. 111-149], it nevertheless remains an unproven fundamental postulate of the general theory of relativity.

The special theory of relativity (SR) cannot alter the extended flat 4-dimensional proper metric spacetime ( $E'^3, ct'$ ) on which it operates in the assumed absence of gravity. The intrinsic special theory of relativity ( $\phi$ SR) can likewise not alter the extended flat two-dimensional proper intrinsic metric spacetime ( $\phi\rho', \phi c\phi t'$ ) on which it operates in the assumed absence of gravity. These, as explained under summary and conclusion in [6], is due to the fact that the spacetime/intrinsic spacetime coordinates (or spacetime/intrinsic spacetime geometry) associated with SR/ $\phi$ SR are affine spacetime/intrinsic affine spacetime coordinates (or affine spacetime/intrinsic affine spacetime geometry) with no metric quality.

It is by introducing the source of a long-range relative metric force-field (where relative metric force field shall be defined), at a point on flat three-dimensional proper metric space  $E'^3$  and consequently the source of a long-range relative intrinsic metric force-field at the same point in the straight line proper intrinsic space  $\phi\rho'$  in Fig. 4 or Fig. 11 of [2], that the extended flat proper metric spacetime ( $E', ct'$ ) and its underlying flat proper intrinsic metric spacetime ( $\phi\rho', \phi c\phi t'$ ) can be made to evolve into a different four-dimensional relativistic spacetime underlied by two-dimensional relativistic intrinsic spacetime in all neighbourhood of the source of the long-range metric force field. The geometry associated with this at the second stage of evolution of spacetime/intrinsic spacetime in a metric force-field shall be developed in the rest of this paper.

## 2 New geometrical background in the four-world picture for the theory of relativity associated with the presence of a long-range metric force field in spacetime

### 2.1 The global curved proper intrinsic metric spacetime and underlying flat relativistic intrinsic metric spacetime/flat relativistic metric spacetime in a metric force field

Let us introduce non-uniform proper intrinsic static speeds  $\phi V'_s$  along the straight line proper intrinsic metric space  $\phi\rho'$  and along the straight line proper intrinsic metric time dimen-

sion  $\phi c\phi t'$  in Fig. 11 of [2], such that  $\phi V'_s$  has its maximum magnitude at a point S in ( $\phi\rho', \phi c\phi t'$ ) and decreases continuously until it vanishes at a point O in ( $\phi\rho', \phi c\phi t'$ ) that is far removed from point S. These will be made manifest in non-uniform proper static speeds  $V'_s$  at every point in the proper Euclidean 3-space  $E'^3$  and at every point along the proper time dimension  $ct'$ , such that  $V'_s$  has its maximum magnitude at the corresponding point S in ( $E'^3, ct'$ ) and decreases in magnitude continuously until it vanishes at point O in ( $E'^3, ct'$ ) that is far removed from point S in that figure.

The foregoing are quite apart from the projective non-uniform absolute intrinsic static speeds  $\phi\hat{V}_s$  along the straight line proper intrinsic space  $\phi\rho'$  and straight line proper intrinsic metric time dimension  $\phi c\phi t'$  and the non-uniform absolute static speeds  $\hat{V}_s$  in  $E'^3$  and  $ct'$  in Fig. 11 of [2]. As discussed under the introduction above, the presence of non-uniform absolute intrinsic static speed  $\phi\hat{V}_s$  along the proper intrinsic space  $\phi\rho'$  and proper intrinsic time dimension  $\phi c\phi t'$  cannot produce curvature of  $\phi\rho'$  and  $\phi c\phi t'$  or produce any other effect on them. The presence of absolute static speeds  $\hat{V}_s$  in the proper metric space  $E'^3$  and proper metric time dimension  $ct'$  can likewise produce no effect on  $E'^3$  and  $ct'$  in Fig. 11 of [2].

Now let us recall the evolution of Fig. 11 of [2] from Fig. 6 of that paper. The introduction of non-uniform absolute intrinsic static speeds  $\phi\hat{V}_s$  along the initial straight line absolute intrinsic metric space  $\phi\hat{\rho}$  and along the initial straight line absolute intrinsic time 'dimension'  $\phi\hat{c}\hat{\phi}\hat{t}$  in Fig. 6 of [2], will cause the straight line absolute intrinsic metric space  $\phi\hat{\rho}$  to evolve into curved absolute intrinsic metric space  $\phi\hat{\rho}$ , where  $\phi\hat{\rho}$  will have maximum curvature at the point S where  $\phi\hat{V}_s$  is maximum and zero curvature at a point O that is far removed from point S, where  $\phi\hat{V}_s$  vanishes. On the other hand, the straight line absolute intrinsic time 'dimension'  $\phi\hat{c}\hat{\phi}\hat{t}$  along the vertical in Fig. 6 of [2] will remain not curved from its vertical position, thereby yielding the half-geometry of Fig. 1 of [2], which is valid with respect to 3-observers in the proper Euclidean 3-space  $E'^3$  of that figure.

The initial straight line absolute intrinsic time 'dimension'  $\phi\hat{c}\hat{\phi}\hat{t}$  in Fig. 6 of [2] remains not curved from the vertical, while the initial straight line absolute intrinsic space  $\phi\hat{\rho}$  in that figure becomes curved absolute intrinsic space  $\phi\hat{\rho}$  in Fig. 1 of [2], because the absolute time  $\hat{t}$  and the absolute intrinsic time  $\phi\hat{t}$  remain absolute (or invariant), that is, do not evolve into the proper time  $t'$  and proper intrinsic time  $\phi t'$  respectively, with respect to 3-observers in the proper Euclidean 3-space  $E'^3$  in that figure in the context of absolute physics/absolute intrinsic physics.

Since there is a perfect symmetry of state between the positive (or our) universe and the positive time-universe, the half-geometry of Fig. 2 of [2] will evolve with respect to 3-observers in the proper Euclidean 3-space  $E^{0/3}$  within the symmetry-partner region of spacetime in the positive time-universe,

simultaneously with the half-geometry of Fig. 1 of [2] in our universe. The union of Figs. 1 and 2 of [2] then gives the full geometry of Fig. 3 of [2], which is equivalent to the full geometry of Fig. 4 of Fig. 11 of [2], containing the spacetime/intrinsic spacetime dimensions of our universe solely. This and the foregoing paragraph are mere repetitions of what have been discussed in the process of development of the geometry of Fig. 4 of [2] in that paper, repeated here to serve as a reminder.

In the context of the theory of relativity/intrinsic theory of relativity associated with the presence of a long-range proper metric force-field in proper metric spacetime ( $E'^3, ct'$ ) and long-range proper intrinsic metric force-field in proper intrinsic metric spacetime ( $\phi\rho', \phi c\phi t'$ ), which establish non-uniform proper static speeds  $V'_s$  in the proper Euclidean 3-space  $E'^3$  and proper time dimension  $ct'$  and non-uniform proper intrinsic static speeds  $\phi V'_s$  in the proper intrinsic space  $\phi\rho'$  and proper intrinsic time dimension  $\phi c\phi t'$ , on the other hand, the proper time  $t'$  and the proper intrinsic time  $\phi t'$  and the three dimensions  $x'^1, x'^2, x'^3$  of the proper Euclidean 3-space  $E'^3$  and the proper intrinsic dimensions  $\phi x'$  of the proper intrinsic space  $\phi\rho'$  are all relative with respect to 3-observers in the proper Euclidean 3-space.

The implication of the foregoing paragraph is that all the four proper metric coordinates  $x'^0, x'^1, x'^2$  and  $x'^3$ ;  $x'^0 = ct'$ , of the flat proper metric spacetime ( $E'^3, ct'$ ) simultaneously transform into relativistic metric coordinates  $x^0, x^1, x^2$  and  $x^3$ ;  $x^0 = ct$ , of the relativistic metric spacetime ( $E^3, ct$ ) with respect to 3-observers in 3-space. The proper intrinsic metric coordinates  $\phi x'$  and  $\phi c\phi t'$  of the proper intrinsic metric spacetime ( $\phi\rho', \phi c\phi t'$ ) likewise transform into relativistic intrinsic metric coordinates  $\phi x$  and  $\phi c\phi t$  of the two-dimensional relativistic intrinsic spacetime ( $\phi\rho, \phi c\phi t$ ) simultaneously with respect to 3-observers in 3-space, in the context of the theory of relativity/intrinsic theory of relativity associated with the presence of a long-range proper metric force field in proper metric spacetime and long-range proper intrinsic metric force-field in proper intrinsic metric spacetime.

As mentioned in section 4 of [3], affine spacetime coordinates and affine intrinsic spacetime coordinates that appear in SR/ $\phi$ SR shall have over-head tilde label as  $\tilde{x}, \tilde{y}, \tilde{z}, \tilde{ct}, \tilde{\phi x}$  and  $\tilde{\phi c\phi t}$ , while the metric spacetime coordinates and intrinsic metric spacetime coordinates that appear in the theory of relativity and theory of intrinsic relativity associated with the presence of metric force field in spacetime and intrinsic metric force field in intrinsic spacetime, shall have no over-head tilde label, appearing as  $x^0, x^1, x^2, x^3, \phi x$  and  $\phi c\phi t$ .

The implication of the penultimate paragraph is that the introduction of non-uniform proper intrinsic static speeds  $\phi V'_s$  identically along the straight line proper intrinsic metric space  $\phi\rho'$  and the straight line proper intrinsic metric time dimension  $\phi c\phi t'$  in Fig. 11 of [2], will cause both  $\phi\rho'$  and  $\phi c\phi t'$  to be identically curved simultaneously relative to the horizontal and vertical respectively, such that the curved  $\phi\rho'$  lying in

the first quadrant and curved  $\phi c\phi t'$  lying in the second quadrant in the larger spacetime domain of combined positive and negative universes, form orthogonal curvilinear intrinsic metric dimensions with respect to 3-observers in the Euclidean 3-space in our (or positive) universe. In symmetry, the proper intrinsic metric space  $-\phi\rho'^*$  and the proper intrinsic metric time dimension  $-\phi c\phi t'^*$  will be identically curved simultaneously relative to the horizontal and vertical respectively, such that the curved  $\phi\rho'^*$  lying in the third quadrant and  $-\phi c\phi t'^*$  lying in the fourth quadrant, form orthogonal curvilinear intrinsic metric dimensions with respect to 3-observers in the Euclidean 3-space in the negative universe, within a long-range relative metric force-field and its underlying long-range relative intrinsic metric force-field.

A consequence of the foregoing is that the geometry of Fig. 1 will evolve with respect to 3-observers in the relativistic Euclidean 3-spaces  $E^3$  and  $-E^{3*}$  in the positive and negative universes, as indicated, at the second stage of evolution of spacetime/intrinsic spacetime within a long-range metric force field. The non-uniform proper intrinsic static speeds  $\phi V'_s$  introduced along the straight line proper intrinsic space  $\phi\rho'$  and straight line proper intrinsic time dimension  $\phi c\phi t'$  in Fig. 11 of [2] have maximum magnitude at point  $(S, S^0)$  in  $(\phi\rho', \phi c\phi t')$  and decrease in magnitude continuously until they vanish at point O in  $(\phi\rho', \phi c\phi t')$ , which is far removed from the point  $(S, S^0)$  in that figure.

Fig. 1 has evolved from Fig. 11 of [2] upon introducing non-uniform proper intrinsic static speeds along the straight line proper intrinsic metric space  $\phi\rho'$  and straight line proper intrinsic metric time dimension  $\phi c\phi t'$  in that figure. Hence the curved 'two-dimensional' absolute intrinsic metric spacetime  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  in our universe in Fig. 11 of [2] and the corresponding curved 'two-dimensional' absolute intrinsic metric spacetime  $(-\phi\hat{\rho}^*, -\phi\hat{c}\phi\hat{t}^*)$  in the negative universe (not shown in Fig. 11 of [2]), have remained in Fig. 1.

Our main interest in this paper is in the curved proper intrinsic metric spaces  $\phi\rho'$  and  $-\phi\rho'^*$  and curved proper intrinsic metric time dimensions  $\phi c\phi t'$  and  $-\phi c\phi t'^*$ . The curved proper intrinsic metric space  $\phi\rho'$  in the first quadrant and the curved proper intrinsic metric time dimension  $\phi c\phi t'$  in the second quadrant evolve simultaneously with respect to 3-observers in the relativistic Euclidean 3-space  $E^3$  in the first quadrant (or in the positive universe) and curved proper intrinsic metric space  $-\phi\rho'^*$  in the third quadrant and the curved proper intrinsic metric time dimension  $-\phi c\phi t'^*$  in the fourth quadrant evolve simultaneously with respect to 3-observers in the relativistic Euclidean 3-space  $-E^{3*}$  in the third quadrant (or in the negative universe).

The curved proper intrinsic metric space  $\phi\rho'$  in the first quadrant projects a straight line relativistic intrinsic metric space  $\phi\rho$  along the horizontal, which is made manifest in the relativistic metric Euclidean 3-space  $E^3$ , in which 3-observers are now located, as indicated. The curved proper intrinsic metric time dimension  $\phi c\phi t'$  in the second quadrant

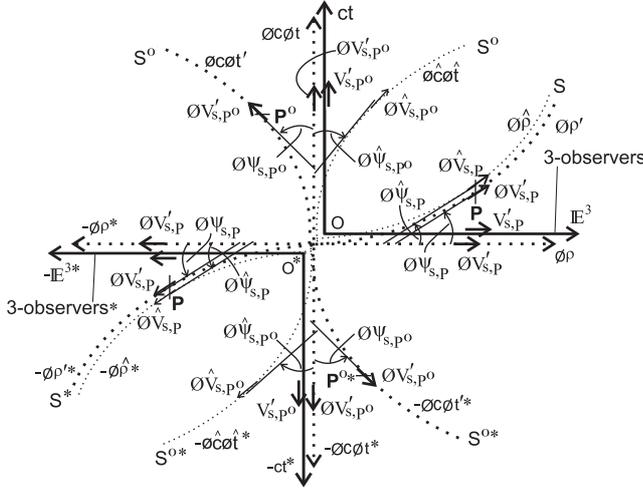


Fig. 1: The extended curved two-dimensional proper intrinsic metric spacetimes with orthogonal curvilinear intrinsic dimensions namely,  $(\phi\rho', \phi c\phi t')$  and  $(-\phi\rho'^*, -\phi c\phi t'^*)$ , with intrinsic Lorentzian metric tensor at every point, underlied by their projective flat two-dimensional relativistic intrinsic metric spacetimes  $(\phi\rho, \phi c\phi t)$  and  $(-\phi\rho^*, -\phi c\phi t^*)$  that are made manifest outwardly in flat four-dimensional relativistic metric spacetimes  $(E^3, ct)$  and  $(-E^{3*}, -ct^*)$ , with respect to 3-observers in the relativistic Euclidean 3-spaces  $E^3$  and  $-E^{3*}$  in the positive and negative universes, which evolve at the second stage of spacetimes/intrinsic spacetimes within symmetry-partner long-range metric force fields in the two universes, shown along with the curved 'two-dimensional' absolute intrinsic metric spacetimes  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  and  $(-\phi\hat{\rho}^*, -\phi\hat{c}\phi\hat{t}^*)$  with absolute intrinsic sub-Riemannian metric tensors in the two universes that are brought forward from the first stage.

likewise projects straight line relativistic intrinsic metric time dimension  $\phi c\phi t$  along the vertical, which is made manifest in the relativistic metric time dimension  $ct$ , in which 1-observers in time dimension are now located in our universe.

The curved proper intrinsic metric space  $-\phi\rho'^*$  in the third quadrant likewise projects relativistic intrinsic metric space  $-\phi\rho^*$  along the horizontal, which is made manifest in the relativistic metric Euclidean 3-space  $-E^{3*}$  in which 3-observers\* are now located in the negative universe, as indicated, and the curved proper intrinsic metric time dimension  $-\phi c\phi t'^*$  in the fourth quadrant projects relativistic intrinsic metric time dimension  $-\phi c\phi t^*$  along the vertical, which is made manifest in the relativistic metric time dimension  $-ct^*$ , in which 1-observers\* in time dimension are now located in the negative universe.

However 1-observers are not indicated to exist in the time dimensions  $ct$  and  $-ct^*$  in Fig. 1, because the geometry of Fig. 1 is valid with respect to 3-observers in the Euclidean 3-spaces  $E^3$  and  $-E^{3*}$  solely, as indicated. It is in the complementary diagram to Fig. 1, to be developed shortly, which is valid with respect to 1-observers in the time dimensions that 1-observers in  $ct$  and  $-ct^*$  will be indicated.

Thus the flat four-dimensional proper metric spacetime  $(E'^3, ct')$  underlied by flat two-dimensional proper intrinsic metric spacetime  $(\phi\rho', \phi c\phi t')$ , which evolved within a long-range metric force field at the first stage of evolution of spacetime/intrinsic spacetime in our universe in Fig. 4 or Fig. 11 of [2], evolve into flat four-dimensional relativistic metric spacetime  $(E^3, ct)$  underlied by flat two-dimensional relativistic intrinsic metric spacetime  $(\phi\rho, \phi c\phi t)$  in Fig. 1 at the second stage of evolution of spacetime/intrinsic spacetime within a long-range metric force-field. The flat 4-dimensional proper metric spacetime  $(-E'^{3*}, -ct'^*)$ , which is underlied by proper intrinsic metric spacetime  $(-\phi\rho'^*, -\phi c\phi t'^*)$ , that evolved within a long-range a metric force field at the first stage of evolution of spacetime/intrinsic spacetime in the negative universe (not shown in Fig. 4 or Fig. 11 of [2]), likewise evolve into flat four-dimensional relativistic metric spacetime  $(-E^{3*}, -ct^*)$  underlied by flat two-dimensional relativistic intrinsic metric spacetime  $(-\phi\rho^*, -\phi c\phi t^*)$  in Fig. 1, at the second stage of evolution of spacetime/intrinsic spacetime within a long-range metric force-field.

There are some other features of Fig. 1 that are important for remark. First the absolute intrinsic metric space  $\phi\hat{\rho}$  and the proper intrinsic metric space  $\phi\rho'$  are shown to be identically curved relative to the relativistic intrinsic metric space  $\phi\rho$  along the horizontal. Indeed the curved  $\phi\rho'$  should fall along the curved  $\phi\hat{\rho}$  in Fig. 1. This means that the point P along the curved  $\phi\hat{\rho}$  in Fig. 11 of [2] is the same as point P along the curved  $\phi\rho'$  in Fig. 1. Consequently the absolute intrinsic angle  $\phi\hat{\psi}_{s,P}$  of inclination of the curved  $\phi\hat{\rho}$  to the horizontal at point P along  $\phi\hat{\rho}$  in Fig. 11 of [2] and the relative intrinsic angle  $\phi\psi_{s,P}$  of inclination of the curved  $\phi\rho'$  to the horizontal at point P along the curved  $\phi\rho'$  in Fig. 1 are equal in magnitude. Consequently the absolute intrinsic static speed  $\phi\hat{V}_{s,P}$  at point P along the curved  $\phi\hat{\rho}$  in Fig. 11 of [2] and the proper intrinsic static speed  $\phi V'_{s,P}$ , (which is a relative intrinsic static speed), at point P along  $\phi\rho'$  in Fig. 1 are equal in magnitude. That is,

$$\sin |\phi\hat{\psi}_{s,P}| = \sin |\phi\psi_{s,P}| \quad (1a)$$

or

$$\left| \frac{\phi\hat{V}_{s,P}}{\phi\hat{c}} \right| = \left| \frac{\phi V'_{s,P}}{\phi c} \right| \quad (1b)$$

For relations (1a) and (1b) to hold, it must be that the source of absolute intrinsic metric force field located at the point S along the curved absolute intrinsic metric space  $\phi\hat{\rho}$  in Fig. 11 of [2], which establishes non-uniform absolute intrinsic static speeds  $\phi\hat{V}_s$  between points S and O along the curved absolute intrinsic space  $\phi\hat{\rho}$  in that figure, is 'projected' as a source of proper intrinsic metric field of identical magnitude into the corresponding point S along the projective straight line proper intrinsic space  $\phi\rho'$  in Fig. 11 of [2]. The 'projective' source of proper intrinsic metric force field

thereby establishes non-uniform proper intrinsic static speeds  $\phi V'_s$  of identical magnitudes as  $\phi \bar{V}_s$  along the straight line  $\phi \rho'$  in Fig. 11 of [2] and consequently along the curved  $\phi \rho'$  in Fig. 1.

The point  $P^0$  along the curved proper intrinsic metric time dimension  $\phi c \phi t'$  in the second quadrant is the symmetry-partner to point P along the curved proper intrinsic metric space  $\phi \rho'$  in the first quadrant in Fig. 1. Consequently the relative intrinsic angle  $\phi \psi_{s,P^0}$  of inclination of the curved  $\phi c \phi t'$  to the vertical at point  $P^0$  along the curved  $\phi c \phi t'$  and the relative intrinsic angle  $\phi \psi_p$  of inclination of the curved  $\phi \rho'$  to the horizontal at point P along the curved  $\phi \rho'$  are equal in magnitude. It then follows that the proper intrinsic static speed  $\phi V'_{s,P^0}$  and  $\phi V'_P$  are equal in magnitude. That is,

$$\sin \phi \psi_{s,P^0} = \sin \phi \psi_{s,P} \quad (2a)$$

or

$$\frac{\phi V'_{s,P^0}}{\phi c} = \frac{\phi V'_{s,P}}{\phi c} \quad (2b)$$

Finally, the proper intrinsic static speed  $\phi V'_{s,P}$  at point P along the curved proper intrinsic metric space  $\phi \rho'$  is shown to be invariantly projected as proper intrinsic static speed  $\phi V'_{s,P}$  into the relativistic intrinsic metric space  $\phi \rho$  along the horizontal and this is made manifest in proper static speed  $V'_{s,P}$  in the relativistic Euclidean 3-space  $E^3$  in Fig. 1. The proper intrinsic static speed  $\phi V'_{s,P^0}$  at point  $P^0$  along the curved proper intrinsic metric time dimension  $\phi c \phi t'$  is likewise shown to be invariantly projected as proper intrinsic static speed  $\phi V'_{s,P^0}$  into the relativistic intrinsic metric time dimension  $\phi c \phi t$ , which is made manifest in proper static speed  $V'_{s,P^0}$  in the relativistic metric time dimension  $ct$  along the vertical in Fig. 1.

On the other hand, one expects that the proper intrinsic static speed  $\phi V'_{s,P}$  along the curved proper intrinsic metric space  $\phi \rho'$  should be projected as relativistic intrinsic static speed  $\phi V_{s,P}$  (without prime label) into the relativistic intrinsic metric space  $\phi \rho$  along the horizontal and that the proper intrinsic static speed  $\phi V'_{s,P^0}$  along the curved proper intrinsic metric time dimension  $\phi c \phi t'$  should be projected as relativistic intrinsic static speed  $\phi V_{s,P^0}$  into the relativistic intrinsic metric time dimension  $\phi c \phi t$  in Fig. 1.

The fact that the proper intrinsic static speeds  $\phi V'_{s,P}$  along the curved  $\phi \rho'$  and  $\phi V'_{s,P^0}$  along the curved  $\phi c \phi t'$  are invariantly projected as proper intrinsic static speeds  $\phi V'_{s,P}$  into  $\phi \rho$  along the horizontal and  $\phi V'_{s,P^0}$  into  $\phi c \phi t$  along the vertical respectively in Fig. 1, is a graphical representation of the invariance of intrinsic static speed in the context of the intrinsic theory of relativity associated with the presence of a long-range proper intrinsic metric force field in intrinsic metric space. This invariance is stated as follows

$$\phi V_s = \phi V'_s \quad (3a)$$

Hence

$$V_s = V'_s, \quad (3b)$$

where Eqs. (3a) and (3b) have been written at an arbitrary pair of symmetry-partner points along the curved  $\phi \rho'$  and  $\phi c \phi t'$ . Proper metric force fields are relative metric force fields and proper static speeds are relative static speeds, as shall be clarified towards the end of this paper.

The invariance of proper intrinsic static speed and proper static speed, (3a) and (3b), in the context of the theory of relativity and theory of intrinsic relativity associated with the presence of a proper metric force field in proper metric spacetime and proper intrinsic metric force field in proper intrinsic metric spacetime, which involve proper static speeds and proper intrinsic static speeds respectively, established in spacetime and intrinsic spacetime by the source of long-range proper metric force field, at the second stage of evolution of spacetime/intrinsic spacetime within the metric force field shall be given formal proof elsewhere with further development. The corresponding invariance of absolute intrinsic static speed and absolute static speed expressed by Eqs. (79a) and (79b) of [2] in the context of absolute intrinsic metric theory of physics and absolute metric theory physics involving absolute intrinsic static speeds and absolute static speeds respectively, established in absolute spacetime and absolute intrinsic spacetime by the source of a long-range absolute metric force-field at the first stage of evolution of spacetime/intrinsic spacetime within the metric force field, shall likewise be given formal proofs elsewhere with further development.

The perfect symmetry of state among the four universes namely, the positive (or our) universe, the negative universe, the positive time-universe and the negative time-universe, isolated in [3-6], implies that as the geometry of Fig. 1 evolves with respect to 3-observers in the relativistic Euclidean 3-spaces  $E^3$  and  $-E^{3*}$  in our universe and the negative universe, at the second stage of evolution of spacetime/intrinsic spacetime within symmetry-partner long-range metric force fields in our universe and the negative universe, the geometry of Fig. 2 evolves simultaneously with respect to 3-observers in the relativistic Euclidean 3-spaces  $E^{03}$  and  $-E^{03*}$  in the positive time-universe and the negative time-universe, at the second stage of evolution of spacetime/intrinsic spacetime within the symmetry-partner long-range metric force fields in the positive time-universe and the negative time-universe.

Fig. 2 in the positive time-universe and the negative time-universe co-exists with Fig. 1 in the positive (or our) universe and the negative universe. It should serve as a complementary diagram to Fig. 1 towards formulating the theory of relativity associated with the presence of symmetry-partner proper (or relative) metric force fields in proper spacetimes in our universe and the negative universe. However Fig. 2 in its present form cannot serve as a complementary diagram to Fig. 1. This is so because the spacetime/intrinsic spacetime dimensions of the positive and negative time-universes in Fig. 2 are elusive

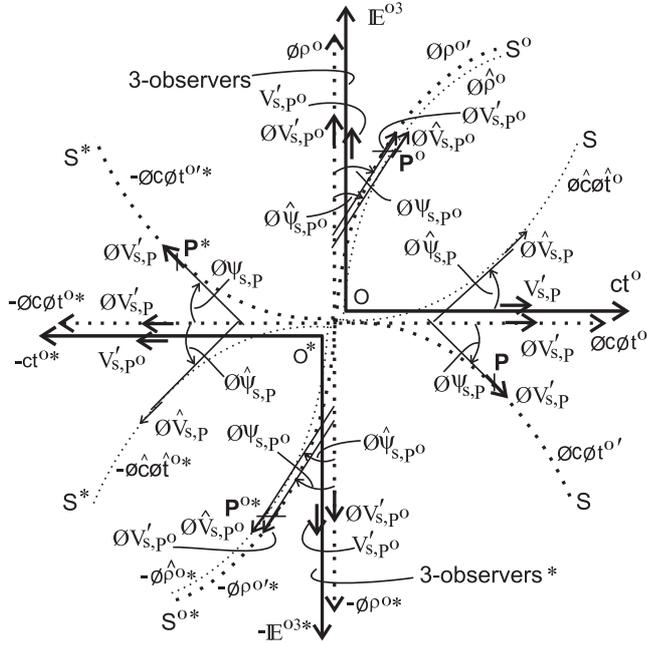


Fig. 2: The symmetrical global spacetime/intrinsic spacetime diagram in the positive time-universe and the negative time-universe, which evolve simultaneously with Fig. 1 in our universe and the negative universe at the second stage of evolution of spacetimes/intrinsic spacetimes within symmetry-partner long-range metric force fields in the positive time-universe and negative time-universe, with respect to 3-observers in the relativistic Euclidean 3-spaces in those universes.

to observers in our universe and the negative universe or cannot appear in physics in our universe and the negative universe.

In order for Fig. 2 to be able to serve as a complementary diagram to Fig. 1, the spacetime/intrinsic spacetime dimensions of the positive and negative time-universes in it must be transformed into those of our universe and the negative universe, as developed in [5-6]. This means that the following transformations of spacetime/intrinsic spacetime dimensions must be performed on Fig. 2, thereby obtaining Fig. 3.

$$\left. \begin{aligned}
 E^{03} &\rightarrow ct; -E^{03*} \rightarrow -ct^*; ct^0 \rightarrow E^3; \\
 &-ct^{0*} \rightarrow -E^{3*}; \\
 \phi\rho^0 &\rightarrow \phi c\phi t; -\phi\rho^{0*} \rightarrow -\phi c\phi t^*; \phi c\phi t^0 \rightarrow \phi\rho; \\
 &-\phi c\phi t^{0*} \rightarrow -\phi\rho^*; \\
 \phi\rho^{0'} &\rightarrow \phi c\phi t'; -\phi\rho^{0'*} \rightarrow -\phi c\phi t'^*; \phi c\phi t^{0'} \rightarrow \phi\rho'; \\
 &-\phi c\phi t^{0'*} \rightarrow -\phi\rho'^*; \\
 \phi\hat{\rho}^0 &\rightarrow \phi\hat{c}\phi\hat{t}; -\phi\hat{\rho}^{0*} \rightarrow -\phi\hat{c}\phi\hat{t}^*; \phi\hat{c}\phi\hat{t}^0 \rightarrow \phi\hat{\rho}; \\
 &-\phi\hat{c}\phi\hat{t}^{0*} \rightarrow -\phi\hat{\rho}^*
 \end{aligned} \right\} (4)$$

Fig. 3 obtained by performing the transformations of system (4) on Fig. 2, is valid with respect to 1-observers in the

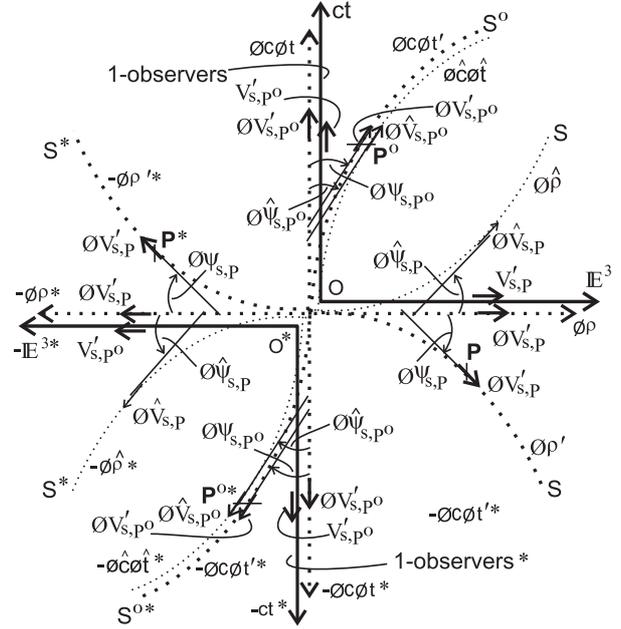


Fig. 3: The spacetime/intrinsic spacetime diagram obtained by transforming the spacetimes/intrinsic spacetimes of the positive time-universe and the negative time-universe in Fig. 2 to the spacetimes/intrinsic spacetimes of the positive and negative universes; the complementary diagram to Fig. 1, which is valid with respect to 1-observers in the relativistic metric time dimensions in our universe and the negative universe.

time dimensions  $ct$  and  $-ct^*$ , as indicated. It contains the spacetime/intrinsic spacetime dimensions of our universe and the negative universe solely. It is hence a valid complementary diagram to Fig. 1 for the purpose of formulating the theory of relativity/intrinsic theory of relativity associated with the presence of symmetry-partner proper metric force fields in proper metric spacetimes and symmetry-partner proper intrinsic metric force fields in proper intrinsic metric spacetimes in our universe and the negative universe.

The curved absolute intrinsic metric space  $\phi\hat{\rho}$  and the curved absolute intrinsic metric time 'dimension'  $\phi\hat{c}\phi\hat{t}$  in the first quadrant (or in the positive universe) in Fig. 1, are the curved absolute intrinsic time 'dimension'  $\phi\hat{c}\phi\hat{t}^0$  and the curved absolute intrinsic space  $\phi\hat{\rho}^0$  respectively of the 'two-dimensional' absolute intrinsic Riemann geometry at the first stage of evolution of spacetime/intrinsic spacetime in the positive time-universe, which become transformed into  $\phi\hat{\rho}$  and  $\phi\hat{c}\phi\hat{t}$  respectively in Fig. 3. They co-exist with the curved proper intrinsic metric time dimension  $\phi c\phi t'$  and curved proper intrinsic metric space  $\phi\rho'$  at the second stage of evolutions of spacetime and intrinsic spacetime, as shown in Fig. 3. Similarly for the curved  $-\phi\hat{c}\phi\hat{t}^*$  and  $-\phi\hat{\rho}^*$  in the third quadrant in Fig. 3.

The geometry of Fig. 1 and its complementary geometry of Fig. 3 in our universe and negative universe in the con-

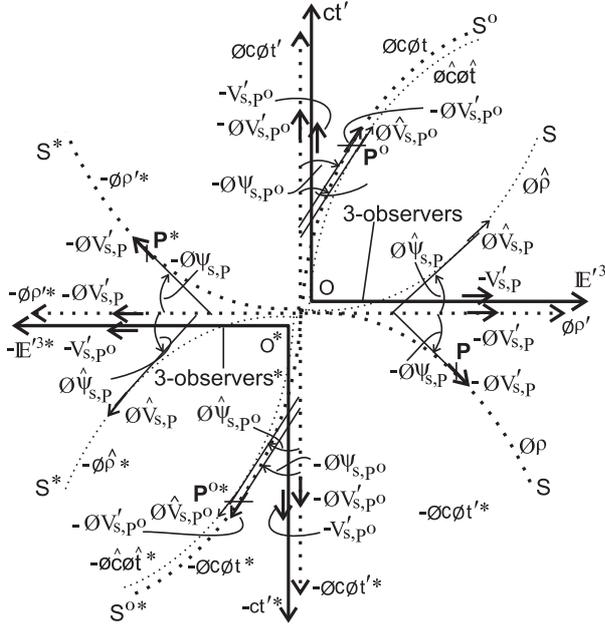


Fig. 4: The inverse to the global spacetime/intrinsic spacetime diagram of Fig. 1 that is valid with respect to 3-observers in the proper physical Euclidean 3-spaces in our universe and the negative universe.

text of the theory of relativity/intrinsic theory of relativity associated with non-uniform proper static speeds/non-uniform proper intrinsic static speeds established in spacetime/intrinsic spacetime by a proper metric force field/proper intrinsic metric force field, correspond to the geometry of Fig. 8a and its complementary geometry of Fig. 8b in [3], of the theory of relativity/intrinsic theory of relativity associated with uniform kinematical speeds/uniform intrinsic kinematical speeds namely, the special theory of relativity/intrinsic special theory of relativity (SR/ $\phi$ SR). Just as Fig. 8a of SR/ $\phi$ SR in [3] has Fig. 9a of that paper as its inverse, Fig. 4 is the inverse to Fig. 1 in the present context.

From the point of view of the theory of relativity/intrinsic theory of relativity associated with non-uniform proper static speeds/non-uniform intrinsic static speeds, the proper intrinsic metric space  $\phi\rho'$  is curved at non-uniform positive intrinsic angles  $\phi\psi_s$  relative to its projective straight line relativistic intrinsic metric space  $\phi\rho$  along the horizontal, thereby possessing maximum curvature at point S where  $\phi V'_s$  is maximum and zero curvature at point O where  $\phi V'_s$  vanishes in the first quadrant in Fig. 1. The curved  $\phi\rho'$  possesses varying positive proper intrinsic static speeds  $\phi V'_s$  and invariantly projects same into its projective relativistic intrinsic metric space  $\phi\rho$  along the horizontal in the first quadrant in that figure.

In the inverse diagram of Fig. 4, on the other hand, the relativistic intrinsic metric space  $\phi\rho$  is curved at non-uniform negative relative intrinsic angles  $-\phi\psi_s$  relative to straight line

proper intrinsic metric space  $\phi\rho'$  along the horizontal. Consequently the curved  $\phi\rho$  possesses non-uniform negative proper (or relative) intrinsic static speeds  $-\phi V'_s$  along its length and invariantly projects same into  $\phi\rho'$  along the horizontal in the first quadrant in Fig. 4.

Likewise the proper intrinsic metric time dimension  $\phi c\phi t'$  is curved at varying positive relative intrinsic angles  $\phi\psi_s$  relative to its projective straight line relativistic intrinsic metric time dimension  $\phi c\phi t$  along the vertical in the first quadrant in Fig. 1. Consequently the curved  $\phi c\phi t'$  possesses non-uniform positive proper intrinsic static speeds  $\phi V'_s$  that has maximum magnitude at point  $S^0$  and vanishes at point O along its length. It invariantly projects non-uniform positive proper intrinsic static speeds  $\phi V'_s$  into the relativistic intrinsic metric time dimension  $\phi c\phi t$  along the vertical in the first quadrant in Fig. 1.

In the inverse diagram of Fig. 4, on the other hand, the relativistic intrinsic metric time dimension  $\phi c\phi t$  is curved at non-uniform negative relative intrinsic angles  $-\phi\psi_s$  relative to straight line proper intrinsic metric time dimension  $\phi c\phi t'$  along the vertical in the first quadrant in that figure. Consequently the curved  $\phi c\phi t$  possesses non-uniform negative proper intrinsic static speeds  $-\phi V'_s$  along its length and invariantly projects same into the straight line proper intrinsic time dimension  $\phi c\phi t'$  along the vertical in the first quadrant in Fig. 4. The discussions in this and the foregoing two paragraphs obtain in the third quadrant between Fig. 1 and Fig. 4 as well.

From the point of view of the absolute intrinsic metric theory on the curved 'two-dimensional' absolute intrinsic metric spacetime  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$ , involving non-uniform absolute intrinsic static speeds  $\phi\hat{V}_s$  along the curved  $\phi\hat{\rho}$  and  $\phi\hat{c}\phi\hat{t}$ , which was developed in the preceding two parts of this paper [1-2], at the first stage of evolution of spacetime/intrinsic spacetime within a long-range metric force field, on the other hand, there is only one diagram namely, the curved absolute intrinsic spacetime  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  relative to its projective proper intrinsic spacetime  $(\phi\rho', \phi c\phi t')$ , as in Fig. 4 or Fig. 11 of [2]. Inverse diagrams and associated inverse coordinate transformations exist in relativity only. Consequently the curved  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  in the first quadrant and the corresponding curved  $(-\phi\hat{\rho}^*, -\phi\hat{c}\phi\hat{t}^*)$  in the third quadrant in Fig. 1 are retained in the inverse diagram of Fig. 4.

Now the clockwise sense of inclination of  $\phi c\phi t$  relative to  $\phi c\phi t'$  along the vertical by varying negative intrinsic angles  $-\phi\psi_s$  and the clockwise sense of inclination of  $\phi\rho$  in the fourth quadrant relative to  $\phi\rho'$  along the horizontal by varying negative intrinsic angles  $-\phi\psi_s$  in Fig. 4, is valid with respect to 3-observers in  $E'^3$  as indicated, with respect to whom anti-clockwise rotation is positive (that is, by positive intrinsic angle), hence with respect to whom clockwise rotation is negative. The inclination of  $-\phi c\phi t^*$  relative to  $-\phi c\phi t'^*$  along the vertical and inclination of  $-\phi\rho^*$  relative to  $-\phi\rho'^*$  along the horizontal is likewise valid with respect to 3-observers\*

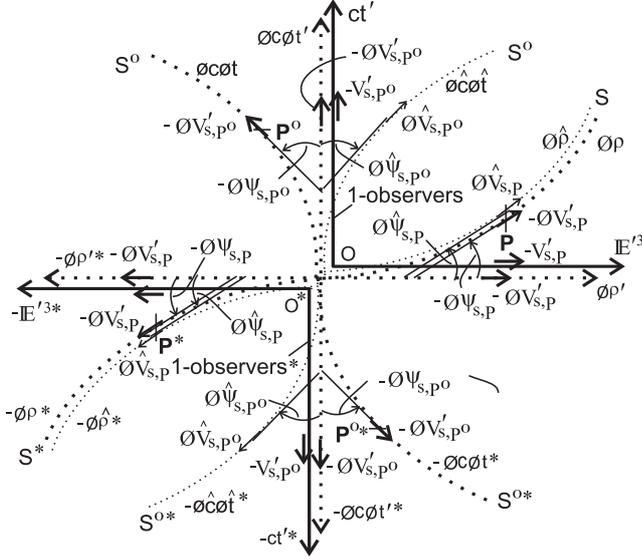


Fig. 5: The inverse to the spacetime/intrinsic spacetime diagram of Fig. 3 that is valid with respect to 1-observers in the proper physical time dimensions in our universe and the negative universe.

in  $-E'^3*$  in the third quadrant in Fig. 4, as indicated.

There is likewise an inverse to Fig. 3 shown as Fig. 5. The anti-clockwise sense of inclination of  $\phi c dt'$  relative to  $\phi c dt'$  by varying negative intrinsic angles  $-\phi\psi_s$  in Fig. 5 is valid relative to 1-observers in  $ct'$ , with respect to whom anti-clockwise rotation is negative. Likewise the anti-clockwise sense of inclination of  $\phi\rho'$  relative to  $\phi\rho'$  by varying negative intrinsic angles  $-\phi\psi_s$  is valid relative to 1-observers in  $ct'$ . The diagram of Fig. 5 is valid with respect to 1-observers in the time dimensions  $ct$  and  $-ct^*$ , as indicated.

Figs. 8a and 8b and their inverses Figs. 9a and 9b in [3] involve inclined extended straight line orthogonal primed intrinsic affine spacetime coordinates  $\phi\tilde{x}'$  and  $\phi c dt'$  relative to their projective extended orthogonal unprimed straight line intrinsic affine coordinates  $\phi\tilde{x}$  and  $\phi c dt$  (in Figs. 8a and 8b of [3]) and the inverses (in Figs. 9a and 9b of [3]). They involve constant positive intrinsic dynamical speed  $\phi v$  (in Figs. 8a and 8b) and constant negative intrinsic dynamical speed  $-\phi v$  (in Figs. 9a and 9b), in the context of the intrinsic special theory of relativity/special theory of relativity ( $\phi SR/SR$ ), as developed in [3].

On the other hand, Figs. 1 and 3 and their inverses Figs. 4 and 5 in this paper, involve extended inclined orthogonal curvilinear proper intrinsic metric spacetime dimensions  $\phi\rho'$  and  $\phi c dt'$ , which are curved relative to their projective extended straight line orthogonal relativistic intrinsic metric spacetime dimensions  $\phi\rho$  and  $\phi c dt$  (in Figs. 1 and 3) and their inverses (in Figs. 4 and 5). They involve non-uniform positive proper intrinsic static speed  $\phi V'_s$  along the curved  $\phi\rho'$  and  $\phi c dt'$  (in Figs. 1 and 3) and non-uniform negative proper intrinsic static speed  $-\phi V'_s$  along the curved  $\phi\rho$  and  $\phi c dt$  (in Figs. 4 and 5),

in the context of the intrinsic theory of relativity/theory of relativity associated with the presence of a long-range proper metric force field in spacetime and long-range proper intrinsic metric force field in intrinsic spacetime.

## 2.2 Deriving intrinsic local lorentz transformation and local lorentz transformation in terms of intrinsic static speed and static speed

Let us consider an elementary interval  $d\phi\rho'$  of the curved proper intrinsic metric space  $\phi\rho'$  about point P along the curved  $\phi\rho'$  in the first quadrant in Fig. 1. The interval  $d\phi\rho'$  possesses proper intrinsic static speed  $\phi V'_{s,P}$  and is inclined to the horizontal at intrinsic angle  $\phi\psi_{s,P}$ . It projects relativistic intrinsic metric space interval  $d\phi\rho$  along the horizontal that also possesses proper intrinsic static speed  $\phi V'_{s,P}$  with respect to 3-observers in the relativistic Euclidean 3-space  $E^3$  in Fig. 1. The corresponding elementary interval  $\phi c dt'$  of the curved proper intrinsic metric time dimension  $\phi c dt'$  about the symmetry-partner point  $P^0$  along the curved  $\phi c dt'$  in the second quadrant in Fig. 1, possesses intrinsic static speed  $\phi V'_{s,P^0}$  and is inclined at intrinsic angle  $\phi\psi_{s,P^0}$  to the vertical. It projects interval  $\phi c dt$  of relativistic intrinsic metric time dimension along the vertical that also possesses intrinsic static speed  $\phi V'_{s,P^0}$  with respect to 3-observers in  $E^3$ .

The elementary interval  $-d\phi\rho'^*$  of the curved proper intrinsic metric space  $-\phi\rho'^*$  about the symmetry-partner point  $P^*$  along the curved  $-\phi\rho'^*$  in the third quadrant in Fig. 1, possesses proper intrinsic static speed  $\phi V'_{s,P^*}$  and is inclined at intrinsic angle  $\phi\psi_{s,P^*}$  to the horizontal. It projects relativistic intrinsic space interval  $-d\phi\rho^*$  along the horizontal that also possesses proper intrinsic static speed  $\phi V'_{s,P^*}$  with respect to 3-observers\* in the relativistic Euclidean 3-space  $-E^{3*}$  in Fig. 1. The corresponding elementary interval  $-\phi c dt'^*$  of the curved proper intrinsic metric time dimension  $-\phi c dt'^*$  about the symmetry-partner point  $P^{0*}$  along  $-\phi c dt'^*$  in the fourth quadrant in Fig. 1, possesses intrinsic static speed  $\phi V'_{s,P^{0*}}$  and is inclined at intrinsic angle  $\phi\psi_{s,P^{0*}}$  to the vertical. It projects interval  $-\phi c dt$  of relativistic intrinsic metric time dimension along the vertical that also possesses intrinsic static speed  $\phi V'_{s,P^{0*}}$  with respect to 3-observers\* in  $-E^{3*}$ .

The elementary intervals of curved proper intrinsic metric spaces and curved proper intrinsic metric time dimensions  $d\phi\rho'$ ,  $-d\phi\rho'^*$ ,  $\phi c dt'$  and  $-\phi c dt'^*$  shall be considered to be indefinitely short so that they are straight short line segments within which proper intrinsic static speed has a constant value. And since the points  $P^0$  and  $P^{0*}$  along the curved  $\phi c dt'$  and curved  $-\phi c dt'^*$  and points P and  $P^*$  along the curved  $\phi\rho'$  and curved  $-\phi\rho'^*$  are symmetry-partner points, the intrinsic angle  $\phi\psi_{s,P^0}$  of inclinations of intervals  $\phi c dt'$  and  $-\phi c dt'^*$  to the vertical and the intrinsic angle  $\phi\psi_{s,P}$  of inclinations of intervals  $d\phi\rho'$  and  $-d\phi\rho'^*$  to the horizontal are equal. That is,  $\phi\psi_{s,P^0} = \phi\psi_{s,P}$ .

By making use of the information in the foregoing para-

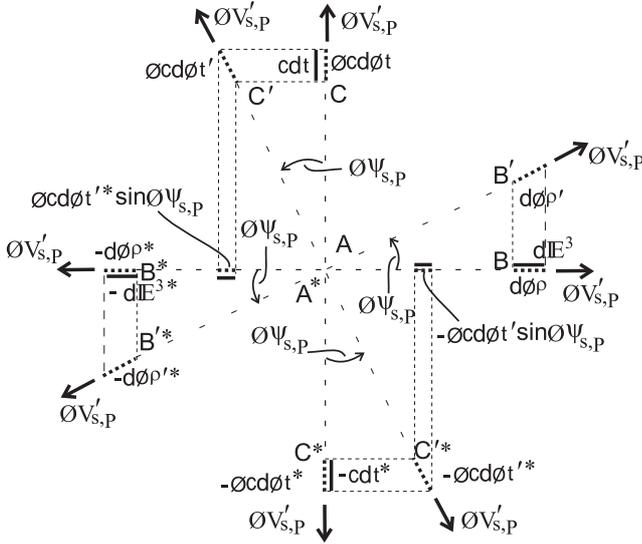


Fig. 6: The local spacetime/intrinsic spacetime diagram drawn at symmetry-partner points in spacetimes/intrinsic spacetimes in the positive (or our) universe and the negative universe, from the global diagram of Fig. 1, for deriving partial intrinsic local Lorentz transformation in terms of intrinsic static speed with respect to 3-observers in the relativistic Euclidean 3-spaces in our universe and the negative universe.

graph and drawing the inclined elementary intervals  $d\phi\rho'$ ,  $\phi cd\phi t'$ ,  $-d\phi\rho'^*$  and  $-\phi cd\phi t'^*$  relative to their projections  $d\phi\rho$ ,  $\phi cd\phi t$ ,  $-d\phi\rho^*$  and  $-\phi cd\phi t^*$  respectively from a common point we have Fig. 6.

Fig. 6 has been drawn at the symmetry-partner points P,  $P^0$ ,  $P^*$  and  $P^{0*}$  along the respective curved proper intrinsic metric dimensions  $\phi\rho'$ ,  $\phi c\phi t'$ ,  $-\phi\rho'^*$  and  $-\phi c\phi t'^*$  in Fig. 1. Hence the appearance of intrinsic angle of inclination  $\psi_{s,P}$ , (where  $\psi_{s,P} = \psi_{s,P^0}$ ), in Fig. 6. The inclined elementary intervals of proper intrinsic metric spacetime  $d\phi\rho'$ ,  $\phi cd\phi t'$ ,  $-d\phi\rho'^*$  and  $-\phi cd\phi t'^*$  possess equal proper intrinsic static speed  $\phi V'_{s,P}$  and their projective relativistic components  $d\phi\rho$ ,  $\phi cd\phi t$ ,  $-d\phi\rho^*$  and  $-\phi cd\phi t^*$  likewise possess equal proper intrinsic static speed  $\phi V_{s,P}$  as shown. It must be noted that the line segments  $AB'$ ,  $AC'$ ,  $A^*B'^*$ , and  $A^*C'^*$  are mere connecting lines and not intrinsic metric coordinates. The line segments  $AB$ ,  $AC$ ,  $A^*B^*$  and  $A^*C^*$  are likewise mere connecting lines.

The component  $d\phi\rho$  of interval of the relativistic intrinsic metric space projected along the horizontal is made manifest outwardly in an elementary volume  $dE^3$  of the relativistic Euclidean 3-space in Fig. 6. Likewise the component  $\phi cd\phi t$  of the relativistic intrinsic metric time dimension projected along the vertical is made manifest outwardly in an elementary interval  $cdt$  of relativistic metric time dimension along the vertical. In addition, the inclined negative elementary proper intrinsic metric time dimension  $-\phi cd\phi t'^*$  from the

negative universe in the fourth quadrant projects component  $-\phi cd\phi t' \sin \psi_{s,P}$  along the horizontal in the first quadrant, which is made manifest outwardly in  $-cdt' \sin \psi_{s,P}$  along the horizontal in Fig. 6. Dummy star label has been removed from the projective component  $-\phi cd\phi t'^*$  because the projective component is now an intrinsic dimension in the positive universe.

Derivation of partial intrinsic local Lorentz transformation from Fig. 6 follows the same procedure used to derive partial intrinsic Lorentz transformation from Fig. 8a of [3] in the context of intrinsic special theory of relativity ( $\phi$ SR). The procedure is applied hereunder.

Now  $d\phi\rho$  being the projective component along the horizontal of the inclined  $d\phi\rho'$ , then  $d\phi\rho = d\phi\rho' \cos \psi_{s,P}$ . Hence we can write,

$$d\phi\rho' = d\phi\rho \sec \psi_{s,P} \quad (*)$$

This is all the intrinsic metric coordinate interval transformation that should have been possible along the horizontal in the first quadrant with respect to 3-observers in the relativistic Euclidean 3-space  $E^3$  in Fig. 6, except that the inclined proper intrinsic metric time dimension interval  $-\phi cd\phi t'^*$  in the fourth quadrant also projects interval  $-\phi cd\phi t' \sin \psi_{s,P}$  along the horizontal, which must be added to the right-hand side of (\*) to have

$$d\phi\rho' = d\phi\rho \sec \psi_{s,P} - \phi cd\phi t' \sin \psi_{s,P} \quad (**)$$

But the inclined interval  $\phi cd\phi t'$  is related to its projection  $\phi cd\phi t$  along the vertical in the same Fig. 6 as  $\phi cd\phi t = \phi cd\phi t' \cos \psi_{s,P}$ , hence  $\phi cd\phi t' = \phi cd\phi t \sec \psi_{s,P}$ . By using this in (\*\*) we have

$$\left. \begin{aligned} d\phi\rho' &= d\phi\rho \sec \psi_{s,P} - \phi cd\phi t \tan \psi_{s,P}; \\ &\text{(with respect to 3-observers in } E^3) \end{aligned} \right\} (5)$$

Eq. (5) is the partial intrinsic Lorentz transformation of elementary intrinsic metric coordinate intervals that can be derived along the horizontal in the first quadrant with respect to 3-observers in  $E^3$  in Fig. 6.

The complementary diagram to Fig. 6 that can be drawn at the symmetry-partner points P,  $P^0$ ,  $P^*$  and  $P^{0*}$  along the curved proper intrinsic metric spacetimes  $\phi\rho'$ ,  $\phi c\phi t'$ ,  $-\phi\rho'^*$  and  $-\phi c\phi t'^*$  in Fig. 3; Fig. 3 being the complementary diagram to Fig. 1, is depicted in Fig. 7. The local geometry of Fig. 7 derived from the global geometry of Fig. 3 is valid with respect to 1-observers in the relativistic time dimensions  $ct$  and  $-ct^*$ , as is the case with Fig. 3.

Now  $\phi cd\phi t$  being the projective component along the vertical of the inclined  $\phi d\phi t'$  in the first quadrant in Fig. 7, then  $\phi cd\phi t = \phi cd\phi t' \cos \psi_{s,P}$ . Hence we can write,

$$\phi cd\phi t' = \phi cd\phi t \sec \psi_{s,P} \quad (***)$$



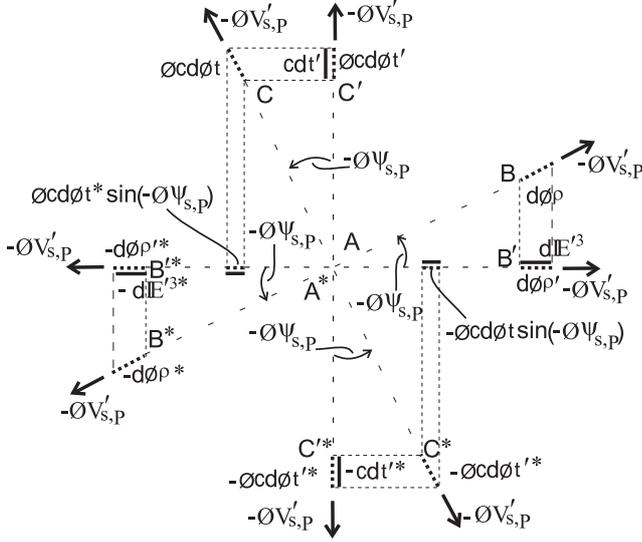


Fig. 9: The inverse to the local diagram of Fig. 7, derived from the global diagram of Fig. 5, for deriving inverse partial intrinsic local Lorentz transformation with respect to 1-observers in the proper physical time dimensions in our universe and the negative universe.

$d\phi\rho \sin \phi\psi_{s,P}$  along the vertical, which must be added to the right-hand side of (\*\*\*\*) to have

$$\phi cd\phi t = \phi cd\phi t' \sec \phi\psi_{s,P} + d\phi\rho \sin \phi\psi_{s,P} \quad (****)$$

But the inclined interval  $d\phi\rho$  in the fourth quadrant is related to its projection  $d\phi\rho'$  as  $d\phi\rho' = d\phi\rho \cos \phi\psi_{s,P}$  along the horizontal in Fig. 8. Hence  $d\phi\rho = d\phi\rho' \sec \phi\psi_{s,P}$ . By using this in (\*\*\*\*) we have

$$\left. \begin{aligned} \phi cd\phi t &= \phi cd\phi t' \sec \phi\psi_{s,P} + d\phi\rho' \tan \phi\psi_{s,P}; \\ &\text{(with respect to 3 – observers in } E'^3) \end{aligned} \right\} (8)$$

Eq. (8) is the partial inverse intrinsic Lorentz transformation of elementary intrinsic metric coordinate intervals that can be derived along the vertical in the first quadrant with respect to 3-observers in  $E'^3$  along the horizontal in Fig. 8.

Finally the inverse to Fig. 7, which can be derived from the global geometry of Fig. 5 is depicted as Fig. 9. Fig. 9 is valid with respect to 1-observers in the proper metric time dimensions  $ct'$  and  $-ct'^*$  along the vertical, as is the case with Fig. 5. The partial inverse intrinsic Lorentz transformation of elementary intrinsic metric coordinate intervals that can be derived along the horizontal in the first quadrant in Fig. 9, by following the procedure used to derive Eq. (8) from Fig. 8 is the following

$$\left. \begin{aligned} d\phi\rho &= d\phi\rho' \sec \phi\psi_{s,P} + \phi cd\phi t' \tan \phi\psi_{s,P}; \\ &\text{(with respect to 1 – observers in } ct') \end{aligned} \right\} (9)$$

By collecting Eqs. (8) and (9) we have

$$\left. \begin{aligned} \phi cd\phi t &= \phi cd\phi t' \sec \phi\psi_{s,P} + d\phi\rho' \tan \phi\psi_{s,P}; \\ &\text{(w.r.t 3 – observers in } E'^3) \\ d\phi\rho &= d\phi\rho' \sec \phi\psi_{s,P} + \phi cd\phi t' \tan \phi\psi_{s,P}; \\ &\text{(w.r.t 1 – observers in } ct') \end{aligned} \right\} (10)$$

System (10) derived from Figs. 8 and 9 is the inverse to system (7) derived from Figs. 6 and 7.

Let us consider an intrinsic event that involves interval  $\phi cd\phi t'$  of proper intrinsic metric time dimension but zero interval of proper intrinsic space ( $d\phi\rho' = 0$ ). This reduces system (10) as follows

$$\phi cd\phi t = \phi cd\phi t' \sec \phi\psi_{s,P}; \quad d\phi\rho = \phi cd\phi t' \tan \phi\psi_{s,P} \quad (11)$$

Then by dividing the second into the first equation of system (11) we have

$$\frac{d\phi\rho}{\phi cd\phi t} = \sin \phi\psi_{s,P} \quad (12)$$

But,  $d\phi\rho/d\phi t = \phi V'_{s,P}$ , is the proper intrinsic static speed of the 'primed intrinsic frame' ( $d\phi\rho', \phi cd\phi t'$ ) relative to the 'unprimed intrinsic frame' ( $d\phi\rho, \phi cd\phi t$ ). Hence

$$\sin \phi\psi_{s,P} = \phi V'_{s,P} / \phi c \equiv \beta_{s,P}(\phi V'_{s,P}) \quad (13a)$$

$$\sec \phi\psi_{s,P} = (1 - \frac{\phi V'^2_{s,P}}{\phi c^2})^{-1/2} \equiv \gamma_{s,P}(\phi V'_{s,P}) \quad (13b)$$

By using Eqs. (13a) and (13b) in systems (7) and (10) we have respectively as follows

$$\left. \begin{aligned} d\phi t' &= \gamma_{s,P}(\phi V'_{s,P})(d\phi t - \frac{\phi V'_{s,P}}{\phi c^2} d\phi\rho); \\ &\text{(w.r.t. 1 – observers in } ct) \\ d\phi\rho' &= \gamma_{s,P}(\phi V'_{s,P})(d\phi\rho - \phi V'_{s,P} d\phi t); \\ &\text{(w.r.t. 3 – observers in } E^3) \end{aligned} \right\} (14)$$

and

$$\left. \begin{aligned} d\phi t &= \gamma_{s,P}(\phi V'_{s,P})(d\phi t' + \frac{\phi V'_{s,P}}{\phi c^2} d\phi\rho'); \\ &\text{(w.r.t. 3 – observers in } E'^3) \\ d\phi\rho &= \gamma_{s,P}(\phi V'_{s,P})(d\phi\rho' + \phi V'_{s,P} d\phi t'); \\ &\text{(w.r.t. 1 – observers in } ct') \end{aligned} \right\} (15)$$

Either system (7) or its inverse (10) or the explicit form in terms of proper intrinsic static speed (14) or (15) leads to intrinsic local Lorentz invariance

$$\phi c^2 d\phi t^2 - d\phi\rho^2 = \phi c^2 d\phi t'^2 - d\phi\rho'^2 \quad (16)$$

The intrinsic local Lorentz transformation of elementary proper intrinsic metric coordinate intervals  $d\phi\rho'$  and  $\phi cd\phi t'$

into elementary relativistic intrinsic metric coordinate intervals  $d\phi\rho$  and  $\phi cd\phi t$  of system (7) or (14) and its inverse system (10) or (15), written at symmetry-partner points  $P$  and  $P^0$  along the curved proper intrinsic metric space  $\phi\rho'$  and curved proper intrinsic metric time dimension  $\phi c\phi t'$  respectively in Figs. 1 and 3 and along the curved relativistic intrinsic metric space  $\phi\rho$  and curved relativistic intrinsic metric time dimension  $\phi c\phi t$  respectively in Figs. 4 and 5, can equally be written at another symmetry-partner points  $Q$  and  $Q^0$  along those curved intrinsic metric spaces and curved intrinsic metric time dimensions, in terms of intrinsic angle  $\phi\psi_{s,Q}$  and proper intrinsic static speed  $\phi V_{s,Q}$  at the new symmetry-partner points, and this is true at every symmetry-partner points along these curved intrinsic spaces and intrinsic time dimensions.

It follows from the foregoing paragraph that the intrinsic local Lorentz invariance (16) obtains at every point on the global curved two-dimensional proper intrinsic metric space-time  $(\phi\rho', \phi c\phi t')$  in Figs. 1 and 3. This guarantees that the projective two-dimensional relativistic intrinsic metric space-time  $(\phi\rho, \phi c\phi t)$  is everywhere flat and consequently the outward manifestation of  $(\phi\rho, \phi c\phi t)$  namely, the global four-dimensional relativistic spacetime  $(E^3, ct)$  is everywhere flat, as illustrated in Figs. 1 and 3.

Graphically, let us consider the local geometry of Fig. 6 to be drawn at every symmetry-partner points along the global curved proper intrinsic metric space  $\phi\rho'$  and global curved proper intrinsic metric time dimension  $\phi c\phi t'$ , starting from point O in Fig. 1. Then the elementary proper intrinsic metric spacetime intervals  $d\phi\rho'$  and  $\phi cd\phi t'$  in Fig. 6 will be inclined to the horizontal and the vertical respectively at varying intrinsic angles  $\phi\psi_s$ , starting from  $\phi\psi_s = 0$  at point O in Fig. 1 and increasing continuously away from that point. Thus by stringing together the inclined  $d\phi\rho'$  in Fig. 6 at consecutive points, starting from point O in Fig. 1, one obtains the continuous curved global proper intrinsic space  $\phi\rho'$  in Fig. 1. Likewise, by stringing together the inclined  $\phi cd\phi t'$  in Fig. 6 at consecutive points, starting from point O in Fig. 1, one obtains the continuous curved global proper intrinsic metric time dimension  $\phi c\phi t'$  in that figure.

By stringing together the projective elementary relativistic intrinsic metric space interval  $d\phi\rho$  along the horizontal in Fig. 6 at consecutive points, starting from point O in Fig. 1, one obtains the continuous straight line global relativistic intrinsic metric space  $\phi\rho$  along the horizontal in Fig. 1. And by stringing together the projective elementary relativistic intrinsic metric time dimension  $\phi cd\phi t$  along the vertical in Fig. 6 at consecutive points, starting from point O in Fig. 1, one obtains the continuous straight line global relativistic intrinsic metric time dimension  $\phi c\phi t$  in Fig. 1. The straight line global relativistic intrinsic metric space  $\phi\rho$  thus obtained along the horizontal is then made manifest outwardly in the global relativistic Euclidean 3-space  $E^3$  and the straight line global relativistic intrinsic metric time dimension  $\phi c\phi t$  obtained along

the vertical is made manifest outwardly in the global relativistic metric time dimension  $ct$  with respect to 3-observers in  $E^3$  in Fig. 1.

It is important to note that in stringing together the projective intervals of relativistic intrinsic metric space  $d\phi\rho$  along the horizontal in Fig. 6 at consecutive points from point O in Fig. 1, described in the foregoing paragraph, the component  $-\phi cd\phi t' \sin \phi\psi_{s,P} = -\phi cd\phi t \tan \phi\psi_{s,P}$ , projected into the horizontal by the inclined  $-\phi c\phi t'^*$  in the fourth quadrant in Fig. 6, must be disregarded. This is so because it cannot be observed (or measured) as relativistic intrinsic space interval by 'intrinsic 1-observers' in  $\phi\rho$ .

The curved proper intrinsic metric space  $\phi\rho'$  and curved proper intrinsic metric time dimension  $\phi c\phi t'$  in Figs. 1 and 3 are relative intrinsic metric space and relative intrinsic metric time dimension. Consequently the component  $d\phi\rho = d\phi\rho' \cos \phi\psi_{s,P}$  projected along the horizontal and the component  $-d\phi\rho' \sin \phi\psi_{s,P}$  projected along the vertical by an elementary intrinsic coordinate intervals  $d\phi\rho'$  at point P along the curved  $\phi\rho'$  and  $-d\phi\rho'^*$  at point P\* along the curved  $-\phi\rho'^*$ , as illustrated in Figs. 6 and 7, being relative intrinsic coordinate intervals, are metric intrinsic coordinate intervals with respect to 3-observers in the relative 3-space  $E^3$  and 1-observers in the relative time dimension  $ct$  respectively in Figs. 1 and 3. Likewise, the projective component  $\phi cd\phi t = \phi cd\phi t' \cos \phi\psi_{s,P}$  along the vertical and the projective component  $-\phi cd\phi t' \sin \phi\psi_{s,P}$  along the horizontal of the elementary intrinsic coordinate interval  $\phi cd\phi t'$  at the symmetry-partner point P<sup>0</sup> along the curved  $\phi c\phi t'$  in Figs. 1 and 3, as illustrated in Figs. 6 and 7, being relative intrinsic coordinate intervals, are metric intrinsic coordinate intervals with respect to 3-observers in  $E^3$  and 1-observers in  $ct$  respectively.

It follows from the foregoing paragraph that there are no projective 'non-metric' intrinsic coordinate intervals to be discarded in coordinate projection (or transformation) relations derivable from Figs. 6 and 7, which should thereby yield intrinsic Riemannian metric line element and intrinsic Riemannian metric tensor on the curved proper intrinsic metric spacetime  $(\phi\rho', \phi c\phi t')$  with respect to 3-observers in  $E^3$ . Rather the geometries of Figs. 6 and 7 give rise to the intrinsic local Lorentz transformation of system (7) or (14) with respect to 3-observers in  $E^3$  and 1-observers in  $ct$ , as indicated, which implies that the curved proper intrinsic spacetime  $(\phi\rho', \phi c\phi t')$  in Figs. 1 and 3 possesses intrinsic local Lorentzian metric tensor at every point of it with respect to 3-observers in  $E^3$  and 1-observers in  $ct$  conjointly. The curved relativistic intrinsic spacetime  $(\phi\rho, \phi c\phi t)$  in Figs. 4 and 5 likewise possesses intrinsic local Lorentzian metric tensor at every point of it with respect to 3-observers in  $E'^3$  and 1-observers in  $ct'$  conjointly for the same reason.

On the other hand, the component  $\delta\phi\hat{\rho}$  projected into the proper intrinsic time dimension  $\phi c\phi t'$  along the vertical by an elementary absolute intrinsic coordinate interval  $d\phi\hat{\rho}$  along

the curved absolute intrinsic space  $\phi\hat{\rho}$  in Fig. 4 of [2], being an absolute intrinsic coordinate interval, is a ‘non-metric’ intrinsic coordinate interval with respect to 1-observers in the relative time dimension  $ct'$  and the component  $\phi\hat{c}\delta\phi\hat{t}$  projected into  $\phi\rho'$  along the horizontal by the corresponding elementary intrinsic coordinate interval  $\phi\hat{c}\delta\phi\hat{t}$  along the curved absolute intrinsic time ‘dimension’  $\phi\hat{c}\delta\phi\hat{t}$ , being an absolute intrinsic coordinate interval, is a ‘non-metric’ intrinsic coordinate interval with respect to 3-observers in the relative 3-space  $E'^3$  in that figure. The fact that the projective ‘non-metric’ intrinsic coordinate intervals must be discarded, while using the projective intrinsic metric coordinate intervals  $d\phi\rho'$  along the horizontal and  $\phi c d\phi t'$  along the vertical solely in Fig. 4 of [2] in constructing intrinsic metric line element and intrinsic metric tensor on the curved  $(\phi\hat{\rho}, \phi\hat{c}\delta\phi\hat{t})$  with respect to 3-observers in  $E'^3$  and 1-observers in  $ct'$  conjointly in that figure, leads to the absolute intrinsic sub-Riemannian metric tensor  $\phi\hat{g}_{ij}^*$  of Eq. (33) of [2] on the curved absolute intrinsic metric spacetime  $(\phi\hat{\rho}, \phi\hat{c}\delta\phi\hat{t})$  with respect to these observers, as fully derived in [2].

Now let us collect the partial intrinsic Lorentz transformations of elementary intrinsic spacetime intervals that are valid with respect to 3-observers in the Euclidean 3-spaces in systems (14) and (15) to have

$$\left. \begin{aligned} d\phi\rho' &= \gamma_{s,P}(\phi V'_{s,P})(d\phi\rho - \phi V'_{s,P}d\phi t) \\ d\phi t &= \gamma_{s,P}(\phi V'_{s,P})(d\phi t' + \frac{\phi V'_{s,P}}{\phi c^2}d\phi\rho') \end{aligned} \right\} \quad (17)$$

Now from the point of view of what can be observed and measured as intrinsic space interval with intrinsic laboratory rod and as intrinsic time interval by intrinsic laboratory clock by ‘intrinsic 1-observers’ in intrinsic space  $\phi\rho$ , the terms  $-\gamma_{s,P}(\phi V'_{s,P})\phi V'_{s,P}d\phi t$  and  $\gamma_{s,P}(\phi V'_{s,P})(\phi V'_{s,P}/\phi c^2) \times d\phi\rho'$  must be set to zero in system (17), thereby reducing system (17) as follows from the point of what can be measured with intrinsic laboratory rod and clock by ‘intrinsic 1-observers’ in intrinsic space

$$d\phi\rho = \gamma_{s,P}(\phi V'_{s,P})^{-1}d\phi\rho' = d\phi\rho'(1 - \frac{\phi V'^2_{s,P}}{\phi c^2})^{\frac{1}{2}} \quad (18)$$

and

$$d\phi t = \gamma_{s,P}(\phi V'_{s,P})d\phi t' = d\phi t'(1 - \frac{\phi V'^2_{s,P}}{\phi c^2})^{-\frac{1}{2}} \quad (19)$$

Eqs. (18) and (19) give intrinsic metric space contraction and intrinsic metric time dilation formulae explicitly in terms of intrinsic static speed. These are intrinsic length contraction and intrinsic time dilation formulae in the context of the intrinsic theory of relativity associated with the presence of a long-range intrinsic metric force field in intrinsic spacetime.

Now the intrinsic theory of relativity in intrinsic spacetime associated with the presence of a long-range intrinsic

metric force field in intrinsic metric spacetime, will be made manifest in the theory of relativity in metric spacetime due to the presence of a long-range metric force field in spacetime. Consequently the intrinsic local Lorentz transformation ( $\phi$ LLT) of system (7) and its inverse of system (10) in two-dimensional intrinsic metric spacetime  $(\phi\rho, \phi c\phi t)$ , will be made manifest outwardly in local Lorentz transformation (LLT) and its inverse in four-dimensional metric spacetime respectively as follows

$$\left. \begin{aligned} cdt' &= cdt \sec \psi_{s,P} - dx^1 \tan \psi_{s,P}; \\ &\text{(w.r.t. 1 – observers in } ct) \\ dx'^1 &= dx^1 \sec \psi_{s,P} - cdt \tan \psi_{s,P}; \\ &dx'^2 = dx^2; \quad dx'^3 = dx^3; \\ &\text{(w.r.t. 3 – observers in } E^3) \end{aligned} \right\} \quad (20)$$

and

$$\left. \begin{aligned} cdt &= cdt' \sec \psi_{s,P} + dx'^1 \tan \psi_{s,P}; \\ &\text{(w.r.t. 3 – observers in } E'^3) \\ dx^1 &= dx'^1 \sec \psi_{s,P} + cdt' \tan \psi_{s,P}; \\ &dx^2 = dx'^2; \quad dx^3 = dx'^3; \\ &\text{(w.r.t. 1 – observers in } ct') \end{aligned} \right\} \quad (21)$$

The explicit forms of  $\phi$ LLT (14) and its converse (15) in two-dimensional intrinsic metric spacetime are likewise made manifest in LLT and its inverse in four-dimensional spacetime respectively as follows:

$$\left. \begin{aligned} dt' &= \gamma_{s,P}(V'_{s,P})(dt - \frac{V'_{s,P}}{c^2}dx^1); \\ &\text{(w.r.t. 1 – observers in } ct) \\ dx'^1 &= \gamma_{s,P}(V'_{s,P})(dx^1 - V'_{s,P}dt); \\ &dx'^2 = dx^2; \quad dx'^3 = dx^3; \\ &\text{(w.r.t. 3 – observers in } E^3) \end{aligned} \right\} \quad (22)$$

and

$$\left. \begin{aligned} dt &= \gamma_{s,P}(V'_{s,P})(dt' + \frac{V'_{s,P}}{c^2}dx'^1); \\ &\text{(w.r.t. 3 – observers in } E'^3) \\ dx^1 &= \gamma_{s,P}(V'_{s,P})(dx'^1 + V'_{s,P}dt'); \\ &dx^2 = dx'^2; \quad dx^3 = dx'^3; \\ &\text{(w.r.t. 1 – observers in } ct') \end{aligned} \right\} \quad (23)$$

where

$$\gamma_{s,P}(V'_{s,P}) = \sec \psi_{s,P} = (1 - \frac{V'^2_{s,P}}{c^2})^{-1/2} \quad (24)$$

The dimension  $x^1$  of the relativistic Euclidean 3-space  $E^3$  is considered to be orientated along the isotropic relativistic intrinsic metric space  $\phi\rho$ , while the dimensions  $x^2$  and  $x^3$  of

$E^3$  are orientated along other directions in  $E^3$ . It follows that the dimension  $x'^1$  of the proper Euclidean 3-space  $E'^3$  was orientated along the isotropic proper intrinsic metric space  $\phi\rho'$ , while the dimensions  $x'^2$  and  $x'^3$  of  $E'^3$  were orientated along other directions in  $E'^3$ . Now the intrinsic static speed  $\phi V'_{s,P}$  lies along the isotropic intrinsic spaces  $\phi\rho'$  underlying  $E'^3$  and along  $\phi\rho$  underlying  $E^3$ . Consequently the static velocity  $\vec{V}'_{s,P}$  lies along  $x'^1$  in  $E'^3$  and along  $x^1$  in  $E^3$ . It has no component along dimension  $x'^2$  or  $x'^3$  in  $E'^3$  and no component along dimension  $x^2$  or  $x^3$  in  $E^3$ . These make systems (20) through (23) to take their forms, in which the intervals  $dx'^2$  and  $dx'^3$  transform into intervals  $dx^2$  and  $dx^3$  trivially as  $dx'^2 = dx^2$  and  $dx'^3 = dx^3$ .

Either the LLT (20) or its inverse (21) or the explicit form (22) or (23) leads to local Lorentz invariance (LLI)

$$c^2 dt^2 - (dx^1)^2 - (dx^2)^2 - (dx^3)^2 = c^2 dt'^2 - (dx'^1)^2 - (dx'^2)^2 - (dx'^3)^2 \quad (25)$$

This is the outward manifestation in the 4-dimensional spacetime of the intrinsic local Lorentz invariance ( $\phi$ LLI) (16). The local Lorentz invariance (25) is valid at every point on the four-dimensional spacetime, implying flatness everywhere in a long-range metric force field of the relativistic spacetime ( $E^3, ct$ ), as deduced from  $\phi$ LLI (16) earlier and as illustrated in Figs. 1 and 3.

Finally the intrinsic length contraction formula (18) and intrinsic time dilation (19) in intrinsic two-dimensional metric spacetime are made manifest in length contraction and time dilation formulae in four-dimensional metric spacetime as follows

$$dx^1 = \gamma_{s,P}(V'_{s,p})^{-1} dx'^1 = \left(1 - \frac{V'^2_{s,P}}{c^2}\right)^{1/2} dx'^1; \quad dx'^2 = dx^2; \quad dx'^3 = dx^3 \quad (26)$$

and

$$dt = \gamma_{s,P}(V'_{s,p}) dt' = \left(1 - \frac{V'^2_{s,P}}{c^2}\right)^{-1/2} dt' \quad (27)$$

As a summary of this section, we have derived the global curved intrinsic metric spacetime/flat spacetime geometries of Figs. 1-5 and the associated local intrinsic spacetime/spacetime geometries of Figs. 6 – 9 within the four-world picture. We have derived the intrinsic local Lorentz transformation ( $\phi$ LLT) and its inverse of systems (7) and (10) or systems (14) and (15); we have validated intrinsic local Lorentz invariance ( $\phi$ LLI) and have derived the intrinsic length contraction and intrinsic time dilation formulae (18) and (19), at a point in spacetime with the aid of Figs. 6 – 9, as must be done at every point in spacetime in a long-range metric force field, in the context of the intrinsic theory of relativity associated with the presence of an intrinsic metric force field in intrinsic spacetime.

The theory of relativity in spacetime due to the presence of a long-range metric force field in spacetime, being mere outward manifestation of the intrinsic theory of relativity in intrinsic spacetime due to the presence of intrinsic metric force field in intrinsic spacetime, the results of the theory of relativity in spacetime have been written directly from the corresponding results of intrinsic theory of relativity in intrinsic spacetime summarized above. These are the local Lorentz transformation (LLT) and its inverse of system (20) and (21) or system (22) and (23); local Lorentz invariance (25) and the length contraction and time dilation formulae (26) and (27), all of which have been written at a point in spacetime and as must be done at every point spacetime in a long-range metric force field.

The central purpose of this paper is to develop a new geometrical background for the theory of relativity associated with the presence of a long-range metric force field in spacetime within the four-world picture, in which the four-dimensional spacetime is underlied by a hidden two-dimensional intrinsic spacetime in each universe, developed in [3-6]. We deem the results derived in this section and summarized in the foregoing two paragraphs as adequate for this purpose. It must be recalled from the derivation of the concept of static intrinsic speed and static speed in part three of this paper [2], that the intrinsic static speed and static speed, which appear in the intrinsic theory of relativity and theory of relativity associated with the presence of a long-range metric force field in spacetime in this paper are pure geometrical parameters.

### 2.3 Clarifications of the concepts of relative static speed, relativity associated with static speed and relative metric force fields

It is appropriate to shine some light on the new concepts in the topic of this sub-section that are introduced in this paper. Let us start with the familiar concept (or parameter) in physics namely, the kinematical (or dynamical) velocity  $\vec{v}$  (or speed  $v$ ). It is an observable and measurable property of a particle or object in motion. The kinematical velocity is a relative parameter because its magnitude varies with the observer or frame of reference relative to which the particle is in motion. The relativity of kinematical velocity is the origin of the relativity of motion of material particles and objects described by the special theory of relativity.

On the other hand, the proper static speed  $V'_s$  is a property of space, established in space by the source of a long-range relative metric force field, irrespective of whether a particle or object is present in space or not. A particle or object of any mass located at a point P in space where the proper static speed is  $V'_{s,P}$ , will acquire  $V'_{s,P}$  but will not move relative to any observer at this speed. If it also possesses kinematical velocity  $\vec{v}$  relative to an observer while moving through point P, then it will be observed to move at the velocity  $\vec{v}$  only relative

to the observer, despite the static speed  $V'_{s,P}$  it has acquired.

The static speed established at a point in space cannot be observed or measured. It does not give rise to flow of space and consequently it does not give rise to translation in space of a material particle or object that acquired it, as said above. Further more, the static speed at a point in space is the same with respect to all observers of frames of reference. It is hence an absolute parameter from the point of view of the special theory of relativity. How then come the concepts of relative static speed and relativity associated with static speed?

In order to answer the question ending the foregoing paragraph, let us revisit the length contraction and time dilation formulae (26) and (27). Although the proper static speed  $V'_s$  at a point in space cannot be observed or measured and although its square  $V'^2_s$  cannot be observed or measured, the quantities  $(1 - V'^2_s/c^2)^{1/2} dx'^1$  and  $(1 - V'^2_s/c^2)^{-1/2} dt'$  can be observed and measured. This follows from the fact to be formally derived upon making connection to gravity elsewhere that  $V'^2_s$  is related to the classical potential  $\Phi'$  of the metric force field that establishes  $V'_s$  in space as,  $\Phi' = -\frac{1}{2} V'^2_s$  (for an attractive metric force field). The quantity  $V'^2_s$ , like the potential  $\Phi'$  at a point in space, cannot be observed or measured (as is the case with gravitational potential in particular).

As shall also be shown formally elsewhere with further development, the speed  $c$  in the factor,  $(1 - V'^2_s/c^2)^{1/2} dx'^1$  and  $(1 - V'^2_s/c^2)^{-1/2} dt'$ , is a static speed like the  $V'_s$  it divides (and not the dynamical speed of light). In other words, these factors shall appear as  $(1 - V'^2_s/c_s^2)^{1/2} dx'^1$  and  $(1 - V'^2_s/c_s^2)^{-1/2} dt'$  with further development, where  $c_s$  is the maximum over all static speeds that can be established in space or that can be acquired in space by material particles and objects, with a magnitude of  $3 \times 10^3$  m/s; (the speed of light being the maximum over all kinematical speeds of material particles and objects with equal magnitude of  $3 \times 10^3$  m/s).

Now the quantities  $(1 - V'^2_s/c_s^2)^{1/2} dx'^1 = c_s(c_s^2 - V'^2_s)^{1/2} \times dx'^1$  and  $(1 - V'^2_s/c_s^2)^{-1/2} dt' = (1/c_s)(c_s^2 - V'^2_s)^{-1/2} dt'$  can be measured, since the difference  $c_s^2 - V'^2_s$ , being equivalent to difference of potentials, can be measured. It then follows that the length contraction and time dilation formulae (26) and (27) can be observed and measured. Thus by allowing an event that involves proper time interval  $dt'$  and proper space intervals  $dx'^1$ ,  $dx'^2$  and  $dx'^3$  to occur at different positions in space within a long-range metric force field, the observed (or relativistic) time interval  $dt$  and the observed (or relativistic) dimension of 3-space  $dx^1$  of the same event will vary with position in space, while the observed dimensions  $dx^2$  and  $dx^3$  of the event will be the same at all positions within the metric force field, according to systems (26) and (27). The variation with the magnitude of the proper static speed  $V'_s$  and consequently with position in space within a long-range metric force field of the observed (or relativistic) time interval  $dt$  and the observed (or relativistic) interval of space  $dx^1$  of an

event is the concept of relativity associated with the presence of a long-range metric force field in spacetime.

In brief, the relativity associated with proper static speed in a long-range metric force field is relativity with position in space within the field (and not relativity with observer or frame of reference). Relativity of proper static speed likewise refers to variation of magnitude of proper static speed with position in space within a long-range metric force field. In other words, it refers to the fact that the proper static speeds  $V'_{s,P}$  and  $V'_{s,Q}$  at two positions P and Q of different distances  $x'^1_P$  and  $x'^1_Q$  respectively from the origin of the long-range relative metric force field have different magnitudes. It does not refer to variation of the magnitude of a static speed with observers or frames of reference. As mentioned earlier, the proper static speed at a point in space is the same with respect to all observers or frames of reference.

In the light of the foregoing, a relative (or relativistic) metric force field is the one that establishes non-zero proper static speeds in space. That is, one that establishes proper static speeds of different magnitudes (no matter how small in magnitudes in a strict sense) at different positions in the proper Euclidean 3-space  $E^3$ , which transforms invariantly as proper static speeds in the relativistic Euclidean 3-space  $E^3$  within the metric force field. The possibility of the relativity of other physical parameters, such as mass, electric and magnetic fields, energy, fluxes, temperature, entropy, potentials, etc, in the sense of the variations of their observed (or relativistic) magnitudes with proper static speed and consequently with position in space within a long-range metric force field, on the flat four-dimensional relativistic metric spacetime ( $E^3, ct$ ) (in Fig. 1) now isolated, shall be investigated upon applying the results of this paper to the gravitational field elsewhere.

Expectedly, it will be possible to derive the transformations of physical parameters and physical constants, classical and special-relativistic non-gravitational laws, as well as classical gravitational laws on flat spacetime within a long-range metric force field with the aid of the local Lorentz transformation and its inverse in terms of proper static speed of systems (22) and (23), in the context of the theory of relativity associated with the presence of a long-range metric force field in spacetime. This will be similar (or analogous) to Lorentz transformations of parameters and natural laws on flat spacetime in the context of the special theory of relativity.

### 3 Absolute intrinsic Riemann geometry on curved 'two-dimensional' absolute intrinsic spacetime at the second stage of evolution of spacetime/intrinsic spacetime in a metric force field

The '2-dimensional' absolute intrinsic metric spacetime ( $\phi\hat{p}, \phi\hat{c}\phi\hat{t}$ ) is curved relative to its projective flat proper intrinsic metric spacetime ( $\phi\rho', \phi c\phi t'$ ) in Fig. 4 or Fig. 11 of the third part of this paper [2], at the first stage of evolution of

spacetime/intrinsic spacetime within a long-range metric force field. Consequently the absolute intrinsic Riemann geometry has been formulated on the curved  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  with respect to 3-observers in the proper Euclidean 3-space  $E'^3$  that overlies the isotropic proper intrinsic space  $\phi\rho'$  in [2].

On the other hand, the ‘2-dimensional’ absolute intrinsic metric spacetime  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  is curved relative to the flat 2-dimensional relativistic intrinsic metric spacetime  $(\phi\rho, \phi c\phi t)$  in Fig. 1, at the second stage of evolution of spacetime/intrinsic spacetime in a long-range metric force field. It then follows that absolute intrinsic Riemann geometry must be formulated on the curved  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  with respect to 3-observers in the relativistic Euclidean 3-space  $E^3$  that overlies  $\phi\rho$  in Fig. 1 at the second stage of evolution of spacetime/intrinsic spacetime.

In order to show that absolute intrinsic Riemann geometry on curved absolute intrinsic spacetime  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  takes the same form with respect to 3-observers in the proper Euclidean 3-space  $E'^3$  solely in Fig. 4 or Fig. 11 of [2] and with respect to 3-observers in the relativistic Euclidean 3-space  $E^3$  solely in Fig. 1 of this paper, let us revisit the derivation of the absolute intrinsic metric tensor without star label from Eq. (48a-b) through Eq. (64) in [2] with the aid of Fig. 7 of that paper. Let us re-write Eq. (53) of [2] as follows

$$(d\phi s')^2 = \phi c^2 (d\phi t')^2 (\cos^2 \phi \hat{\psi}_{s,P} + \sin^2 \phi \hat{\psi}_{s,P}) - (d\phi \rho')^2 (\sec^2 \phi \hat{\psi}_{s,P} - \tan^2 \phi \hat{\psi}_{s,P}) \quad (28)$$

This is the intrinsic line element on the two-dimensional proper intrinsic spacetime  $(\phi\rho', \phi c\phi t')$  in Fig. 7 of [2], which is valid with respect to 3-observers in the proper Euclidean 3-space  $E'^3$  in that figure. Actually Eq. (28) simplifies as the intrinsic Lorentzian line element as noted in [2].

Then by applying the invariance of intrinsic line element on  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  and  $(\phi\rho', \phi c\phi t')$  in Fig. 7 of [2], expressed by Eq. (54) of that paper, which shall be reproduced here as follows,

$$\phi \hat{c}^2 (d\phi \hat{t})^2 - (d\phi \hat{\rho})^2 = \phi c^2 (d\phi t')^2 - (d\phi \rho')^2, \quad (29)$$

the  $(d\phi \rho')^2$  and  $\phi c^2 (d\phi t')^2$  in (28) were replaced by  $(d\phi \hat{\rho})^2$  and  $\phi \hat{c}^2 (d\phi \hat{t})^2$  respectively, yielding Eq. (56) of [2], which shall be reproduced here as follows

$$(d\phi \hat{s})^2 = \phi \hat{c}^2 (d\phi \hat{t})^2 (\cos^2 \phi \hat{\psi}_{s,P} + \sin^2 \phi \hat{\psi}_{s,P}) - (d\phi \hat{\rho})^2 (\sec^2 \phi \hat{\psi}_{s,P} - \tan^2 \phi \hat{\psi}_{s,P}) \quad (30)$$

The absolute intrinsic metric tensor without star label of (63) or (64) of [2] and absolute intrinsic Ricci tensor without star label of (67) or (68) of [2] were then derived with respect to 3-observers in the proper Euclidean 3-space  $E'^3$  solely in Fig. 4 or Fig. 11 of [2] from Eq. (30) above (or Eq. (56) of [2]) between Eqs. (58) and (68) of [2].

Now the intrinsic local Lorentz invariance (16) established in the context of the intrinsic theory of relativity associated with the presence of a long-range intrinsic metric force

field in intrinsic spacetime earlier in this paper, allows us to replace  $\phi c^2 (d\phi t')^2$  and  $(d\phi \rho')^2$  by  $\phi c^2 (d\phi t)^2$  and  $(d\phi \rho)^2$  respectively in Eq. (28) to have

$$(d\phi s)^2 = \phi c^2 (d\phi t)^2 (\cos^2 \phi \hat{\psi}_{s,P} + \sin^2 \phi \hat{\psi}_{s,P}) - (d\phi \rho)^2 (\sec^2 \phi \hat{\psi}_{s,P} - \tan^2 \phi \hat{\psi}_{s,P}) \quad (31)$$

While the primed intrinsic line element  $d\phi s'$  in (28) on proper intrinsic spacetime  $(\phi\rho', \phi c\phi t')$  is valid with respect to 3-observers in the proper Euclidean 3-space  $E'^3$  solely in Fig. 4 or Fig. 11 of [2], the unprimed intrinsic line element (31) on the relativistic intrinsic spacetime  $(\phi c\phi t, \phi\rho)$  is valid with respect to 3-observers in the relativistic Euclidean 3-space  $E^3$  solely in Fig. 1 of this paper.

Now by combining the intrinsic local Lorentz invariance (29) and (16) we have

$$\begin{aligned} \phi c^2 (d\phi t)^2 - (d\phi \rho)^2 &= \phi c^2 (d\phi t')^2 - (d\phi \rho')^2 \\ &= \phi \hat{c}^2 (d\phi \hat{t})^2 - (d\phi \hat{\rho})^2 \end{aligned} \quad (32)$$

Eq. (32) allows us to replace  $(d\phi \rho)^2$  and  $\phi c^2 (d\phi t)^2$  by  $(d\phi \hat{\rho})^2$  and  $\phi \hat{c}^2 (d\phi \hat{t})^2$  respectively in Eq. (31) to have Eq. (30) again. It then follows that the absolute intrinsic metric tensor of Eq. (63) or (64) and absolute intrinsic Ricci tensor of Eq. (67) or (68) of [2], derived from Eq. (30) with respect to 3-observers in the proper Euclidean 3-space  $E'^3$  solely in Fig. 4 or Fig. 11 of [2], are equally valid with respect to 3-observers in the relativistic Euclidean 3-space  $E^3$  solely in Fig. 1 of this paper.

The starred absolute intrinsic line element  $d\phi \hat{s}^*$ , starred absolute intrinsic metric tensor  $\phi \hat{g}_{ij}^*$  and starred absolute intrinsic Ricci tensor  $\phi \hat{R}_{ij}^*$  on curved ‘two-dimensional’ absolute intrinsic spacetime  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  in Fig. 4 or Fig. 11 of [2], given by Eqs. (31), (33) and (39) respectively of [2], which are valid partially with respect to 3-observers in the proper Euclidean 3-space  $E'^3$  and partially with respect to 1-observers in the proper time dimension  $ct'$  in Fig. 4 or Fig. 11 of [2], as explained in that paper, are equally valid on the curved  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  in Fig. 1 of this paper partially with respect to 3-observers in the relativistic Euclidean 3-space  $E^3$  and partially with respect to 1-observers in the relativistic time dimension  $ct$  in that figure.

Thus the formulation of absolute intrinsic Riemann geometry on curved absolute intrinsic spacetime  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  with respect to 3-observers in the relativistic Euclidean 3-space  $E^3$  (in Fig. 1 of this paper) at the second stage of evolution of spacetime/intrinsic spacetime within a long-range metric force field, follows the same procedure used to formulate absolute intrinsic Riemann geometry on the curved  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  with respect to 3-observers in the proper Euclidean 3-space  $E'^3$  in Fig. 4 or Fig. 11 of [2] at the first stage of evolution of spacetime/intrinsic spacetime within a long-range metric force field.

This means that just as done at the first stage of evolution of spacetime/intrinsic spacetime, one must write the pair of absolute intrinsic tensor equations involving starred absolute intrinsic tensors  $\phi\hat{g}_{ij}^*$  and  $\phi\hat{R}_{ij}^*$  derived on curved  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  in [1-2] and presented as Eqs. (34) and (38) of [2]. One must then solve those equations algebraically to obtain  $\phi\hat{g}_{ij}^*$  and  $\phi\hat{R}_{ij}^*$  in terms of absolute intrinsic curvature parameter  $\phi\hat{k}$  of Eqs. (64) and (68) or in terms of absolute intrinsic static speed as Eqs. (81) and (82) of [2]. The starred absolute intrinsic tensors so derived are valid partially with respect to 3-observers in the relativistic Euclidean 3-space  $E^3$  and partially with respect to 1-observers in the relativistic time dimension  $ct$  in Fig. 1 of this paper.

Then in order to obtain the absolute intrinsic metric tensor  $\phi\hat{g}_{ij}$  without star label, which is valid with respect to 3-observers in the relativistic Euclidean 3-space  $E^3$  solely, one must use the relations among the components of the starred absolute intrinsic metric tensor  $\phi\hat{g}_{ij}^*$  and the components of the absolute intrinsic metric tensor without star label  $\phi\hat{g}_{ij}$  in systems (65a) and (65b) of [2]. Once  $\phi\hat{g}_{ij}$  has been obtained, then one must apply the tensorial statement of intrinsic local Lorentz invariance (66) of [2] to derive the absolute intrinsic Ricci tensor without star label  $\phi\hat{R}_{ij}$ , which is valid with respect to 3-observers in  $E^3$  solely.

The superposition procedure developed in absolute intrinsic Riemann geometry at the first stage of evolution of spacetime/intrinsic spacetime in [2], when two or a larger number of curved absolute intrinsic metric spacetimes co-exist, is equally applicable at the second stage of evolution of spacetime/intrinsic spacetime. Clarifications of the concepts of absolute intrinsic static speed, absolute intrinsic metric tensor and absolute intrinsic metric theory of physics associated with them introduced in [2] and this section, shall be done upon making connection to gravity elsewhere.

#### 4 Summary, conclusion and direction for further investigation

The summary of the four parts of this paper essentially is that spacetime and its underlying intrinsic spacetime follow two stages of evolution in the sequence of absolute spacetime/absolute intrinsic space  $\rightarrow$  proper spacetime/proper intrinsic spacetime  $\rightarrow$  relativistic spacetime/relativistic intrinsic spacetime in every long-range metric force field, and that three theories of a metric force field are associated with each of the two stages.

The three theories of a metric force field encompassed by the geometry of Fig. 4 of [2] at the first stage of evolution of spacetime/intrinsic spacetime within the metric force field, to be developed elsewhere are: (i) an absolute intrinsic metric theory on the curved ‘two-dimensional’ absolute intrinsic metric spacetime  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  with absolute intrinsic metric tensor, (ii) a three dimensional classical metric theory in the Galileo space  $(E'^3; t')$  and (iii) an one-dimensional

classical intrinsic metric theory on the intrinsic Galileo space  $(\phi\rho'; \phi t')$ , the classical metric theory in  $(E'^3; t')$  being mere outward manifestation of the intrinsic classical metric theory in  $(\phi\rho'; \phi t')$ .

The three theories of a metric force field at the second stage of evolution of spacetime/intrinsic spacetime in a long-range metric force field encompassed by the geometries of Figs. 1 and 3 and their inverses Figs. 4 and 5 of this fourth part of this paper are: (i) the absolute intrinsic metric theory on curved ‘two-dimensional’ absolute intrinsic metric spacetime  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  with absolute intrinsic metric tensor brought forward from the first stage, (ii) a flat four-dimensional relativistic metric theory on flat 4-dimensional relativistic spacetime  $(E^3, ct)$  and (iii) a flat two-dimensional relativistic intrinsic metric theory on flat two-dimensional relativistic intrinsic metric spacetime  $(\phi\rho, \phi c\phi t)$  underlying  $(E^3, ct)$ , the metric theory on  $(E^3, ct)$  being mere outward manifestation of the intrinsic metric theory on  $(\phi\rho, \phi c\phi t)$ .

The spacetime/intrinsic spacetime geometry and the associated three theories of a metric force field that evolved at the first stage of evolution of spacetime/intrinsic spacetime in a long-range metric force field, endured for no moment before transforming into the enduring spacetime/intrinsic spacetime geometries and associated metric theory/intrinsic metric theories at the second (and final) stage. Thus the theories at the second stage, having replaced the theories at the first stage, are present in every long-range metric force field in the universe at present.

A crucial conclusion is that the four-dimensional (relativistic) spacetime and its underlying two-dimensional (relativistic) intrinsic spacetime are everywhere flat in every long-range metric force field; the only curved spacetime with Riemannian metric tensor, so to speak, being the ‘2-dimensional’ absolute intrinsic metric spacetime  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  with absolute intrinsic sub-Riemannian metric tensor, isolated in the first three parts of this paper.

For further work in the short-run, it will be necessary to particularize the spacetime/intrinsic spacetime geometries and associated metric theories/intrinsic metric theories at the two stages of evolution of spacetime/intrinsic spacetime in a long-range metric force field isolated in the four parts of this paper to the gravitational field; investigate two stages of evolution of physical parameters/intrinsic parameters that possibly accompany the two stages of evolution of spacetime/intrinsic spacetime in a gravitational field and investigate possible variations of physical parameters/intrinsic parameters with position on flat spacetime in a gravitational field, in the context of the theory of relativity/intrinsic theory of relativity associated with the presence of gravitational field in spacetime, developed for an arbitrary long-range metric force field in terms of the isolated geometrical parameters namely, static speed and intrinsic static speed in section 2 of this fourth part of this paper.

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