The fine structure constant derived from the broken symmetry of two simple algebraic identities

J. S. Markovitch

P.O. Box 752

*Batavia, IL 60510**

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Abstract

The fine structure constant is shown to arise naturally in the course of altering the symmetry of two algebraic identities. Specifically, the symmetry of the identity $M^2 = M^2$ is "broken" by making the substitution $M \to M - y$ on its left side, and the substitution $M^n \to M^n - x^p$ on its right side, where p equals the order of the identity; these substitutions convert the above identity into the equation $(M - y)^2 = M^2 - x^2$. These same substitutions are also applied to the only slightly more complicated identity $(M/N)^3 + M^2 = (M/N)^3 + M^2$ to produce this second equation $(M - y)^3/N^3 + (M - y)^2 = (M^3 - x^3)/N^3 + M^2 - x^3$. These two equations are then shown to share a mathematical property relating to dy/dx, where, on the second equation's right side this property helps define the special case $(M^3 - x^3)/N^3 + M^2 - x^3 = (10^3 - 0.1^3)/3^3 + 10^2 - 0.1^3 = 137.036$, which incorporates a value close to the experimental fine structure constant inverse.

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*Electronic address: jsmarkovitch@gmail.com

I. INTRODUCTION

The fine structure constant (FSC) is shown to arise naturally in the course of an investigation of two algebraic identities whose symmetry is altered. Specifically, the symmetry of the identities

$$M^2 = M^2$$

and

$$\left(\frac{M}{N}\right)^3 + M^2 = \left(\frac{M}{N}\right)^3 + M^2$$

will be "broken" by making the substitution

$$M \rightarrow M - y$$

on their left side, and the substitution

$$M^n \rightarrow M^n - x^p$$

on their right side, where p equals the order of each identity. The resultant equations will then be shown to share a mathematical property relating to dy/dx, where for the second equation this property gives rise to a value that is close to the experimental FSC.

II. THE SECOND-ORDER IDENTITY

Begin with the symmetric second-order identity

$$M^2 = M^2$$

and break its symmetry by making the substitution

$$M \rightarrow M - y$$

on its left side, and the substitution

$$M^n \rightarrow M^n - x^p$$

on its right side, where $p = 2$, the order of the identity. This produces

$$(M - y)^2 = M^2 - x^2 \quad , \tag{1}$$

where the constant M is assumed to be a positive integer, and x and y variables. Now for

$$x = \frac{1}{M} \tag{2a}$$

where

$$M \gg 1 \quad , \tag{2b}$$

the value for dy/dx turns out to be simply

$$\frac{dy}{dx} \approx x^p \quad , \tag{2c}$$

where $p = 2$, the order of Eq. (1) (see Appendix A for derivation).

Because Eqs. (2a)–(2c) are all that will be needed to generate the FSC in the next example, they will be termed the *FSC Conditions*.

III. THE THIRD-ORDER IDENTITY AND THE FINE STRUCTURE CONSTANT

To generate the FSC, combine the constant M^2 with the constant $(M/N)^3$ to form the expression

$$\left(\frac{M}{N}\right)^3 + M^2 \quad .$$

Set this expression equal to itself to form the new symmetric identity

$$\left(\frac{M}{N}\right)^3 + M^2 = \left(\frac{M}{N}\right)^3 + M^2$$

and apply the earlier substitutions

$$M \rightarrow M - y$$

$$M^n \rightarrow \ M^n - x^p$$

with p now equaling 3, the order of this new identity. This produces the *FSC Equation*

$$\left(\frac{M-y}{N}\right)^3 + (M-y)^2 \quad = \frac{M^3 - x^3}{N^3} + M^2 - x^3 \quad , \tag{3}$$

where the constants M and N are defined to be positive integers, and x and y are again variables. For Eq. (3) if the first two FSC Conditions—Eqs. (2a) and (2b)—are assumed, then the third FSC Condition—Eq. (2c)—is also met provided that

$$M \approx \frac{1}{3}N^3 + 1 \tag{4}$$

(see Appendix B for derivation).

As it turns out, the smallest positive integers fulfilling these conditions are

$$M = 10$$

and

$$N = 3 \quad .$$

Substituting these integers into the right side of the FSC Equation gives

$$\frac{10^3 - 10^{-3}}{3^3} + 10^2 - 10^{-3} = 137.036 \quad . \tag{5a}$$

The above values, in turn, determine that the left side of the FSC Equation equals

$$\left(\frac{10}{3} - \frac{1}{3 \times 29\,999.932166\,\ldots}\right)^3 + \left(10 - \frac{1}{29\,999.932166\,\ldots}\right)^2 = 137.036 \quad . \tag{5b}$$

The value 137.036 differs from the 2006 CODATA value of 137.035 999 679 by about 2.3 parts per billion [1]. In this way the FSC arises naturally from the analysis of the broken symmetry of two simple algebraic identities.

REFERENCES

[1] P. J. Mohr, B. N. Taylor, and D. B. Newell (2007), "The 2006 CODATA Recommended Values of the Fundamental Physical Constants" (Web Version 5.0). This database was developed by J. Baker, M. Douma, and S. Kotochigova. Available: http://physics.nist.gov/constants [2007, April19]. National Institute of Standards and Technology, Gaithersburg, MD 20899.

APPENDIX A

Proof that given Eq. (1), if Eqs. (2a) and (2b) are true, so is Eq. (2c).

Equation (1)

$$(M - y)^2 = M^2 - x^2$$

simplifies to

$$2My - y^2 = x^2 \quad ,$$

so that

$$(2M - 2y)dy = 2xdx$$

$$\frac{dy}{dx} = \frac{x}{M - y} \quad .$$

Equation (2a) provides that $x = 1/M$, so that

$$\frac{dy}{dx} = \frac{x^2}{1 - xy} \quad .$$

Given Eqs. (2a) and (2b), the cross-terms on the left side of Eq. (1) guarantee that $y < x^2$, while Eqs. (2a) and (2b) also determine that $\ll 1$; accordingly,

$$\frac{dy}{dx} \approx x^2 \quad .$$

In this way Eq. (2c) is recovered for $p = 2$.

APPENDIX B

Proof that if Eq. (3)'s values for M and N are consistent with Eq. (4), then if Eqs. (2a) and (2b) are true, so is Eq. (2c).

If its higher-order terms are ignored then Eq. (3), the FSC Equation

$$\left(\frac{M-y}{N}\right)^3 + (M-y)^2 = \frac{M^3 - x^3}{N^3} + M^2 - x^3$$

can be simplified to

$$\frac{3M^2 y}{N^3} + 2My \approx \frac{x^3}{N^3} + x^3$$

$$3M^2 y + 2N^3 My \approx x^3 + \ N^3 x^3$$

$$(3M + 2N^3)My \approx \ (N^3 + 1)x^3$$

$$y \approx \frac{N^3 + 1}{3M + 2N^3} \times \frac{x^3}{M}$$

$$y \approx \frac{N^3 + 1}{M + \frac{2}{3}N^3} \times \frac{x^3}{3M} \quad ,$$

so that

$$dy \approx \frac{N^3 + 1}{M + \frac{2}{3}N^3} \times \frac{x^2}{M} dx$$

$$\frac{dy}{dx} \approx \frac{N^3+1}{M+\frac{2}{3}N^3} \times \frac{x^2}{M} \quad . \tag{B1}$$

Although Eq. (B1) is approximate, the terms it ignores are small for small y.

In order to assure that Eq. (2c)

$$x^p \approx \frac{dy}{dx} \tag{B2}$$

holds for Eq. (3), combine Eqs. (B1) and Eq. (B2), where $p = 3$, the order of Eq. (3), to get

$$x^p \approx \frac{dy}{dx} \approx \frac{N^3+1}{M+\frac{2}{3}N^3} \times \frac{x^2}{M}$$

$$x^3 \approx \frac{N^3+1}{M+\frac{2}{3}N^3} \times \frac{x^2}{M}$$

$$x \approx \frac{N^3+1}{M+\frac{2}{3}N^3} \times \frac{1}{M} \quad .$$

Equation (2a) provides that $x = 1/M$, so that

$$\frac{1}{M} \approx \frac{N^3+1}{M+\frac{2}{3}N^3} \times \frac{1}{M} \quad ,$$

gives

$$1 \approx \frac{N^3 + 1}{M + \frac{2}{3}N^3} \quad ,$$

or

$$M + \frac{2}{3}N^3 \approx N^3 + 1 \quad ,$$

which recovers Eq. (4)

$$M \approx \frac{1}{3}N^3 + 1 \quad .$$

Hence, Eq. (4) does constrain Eq. (3)'s values for M and N so that, if Eqs. (2a) and (2b), the first two FSC Conditions, are true, then so is Eq. (2c), the third FSC Condition.