

Collatz problem and conjecture. A generalization of the problem

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Abstract

The aim of this article is presents an open problem of Mathematics. We will talk and present shortly Collatz problem and conjecture to make clear our motivation for new problem. Introduction to the original Collatz problem is given as in [1],[2],[3],[4]. From our point of view a very proper introduction to the main problem. A generalization is proposed as well as three questions are asked to a reader at the end of article, after definition of our problem. We thought, it is possible to develop a mathematical game based on Collatz problem. We leave this idea for future works.

Introduction

There are still numerous unsolved problems in Mathematics, Physics, Chemistry, Biology, Philosophy. Many many questions are still waiting for answers. Appropriate information for unsolved problems I can refer a reader to the web. At least I, hope that a solution of one of them can change our thinking about nature, about our life. One unsolved problem in Mathematics, which take my attention by its beauty is Collatz problem, with its conjecture. Collatz problem and Conjecture.

M. Bruschi in the introduction part of his "A generalization of the Collatz problem and conjecture"[5] article had wrote:

The Collatz problem is so well known that we refer for formulation, references and bibliography to the web [1],[2],[3]. Let us quote from ref. [1]: "The Collatz conjecture is an unsolved conjecture in mathematics. It is named after Lothar Collatz, who first proposed it in 1937. The conjecture is also known as the $3n + 1$ conjecture, as the Ulam conjecture (after Stanislaw Ulam), or as the Syracuse problem; the sequence of numbers involved is referred to as the hailstone sequence or hailstone numbers, or as wondrous numbers per Gödel, Escher, Bach [4]. We take any number n . If n is even, we halve it ($n/2$), else we do "triple plus one" and get $3n + 1$. The conjecture is that for all numbers this process converges to 1. Hence it has been called 'Half Or Triple Plus One', sometimes called HOTPO. Paul Erdős said about the Collatz conjecture: 'Mathematics is not yet ready for such problems.' He offered \$500 for its solution."

Suggested Problem

Taking as the base of our proposed problem the problem of Collatz, I want to present for Your judgement the following problem. Just for convenience and for future work let consider $(2,3,1,1,n)$ as a solution of Collatz problem. Let me explain origination of numbers inside of brackets. 2 comes from “If n is even, we halve it ($n/2$)” condition, 3 and 1 came from “else we do “triple plus one” and get $3n + 1$ ” condition. Finally, 1 is the number to which the process for all numbers converges.

Now let consider (b,k,c,d,n) numbers such that the following holds

1. If n is divisible to b, take n/b
 2. Else, take $kn+c$,
- such that this process is converges to d.

This Collatz problem’s generalization is broader one and contains the Collatz problem as partial case. Of course it is evident, that Collatz conjecture generally saying is not works. According to this situation the first question, which can arose is the following

Question 1.1

For which b and k differ from 2 and 3 accordingly, the Collatz conjecture is true?

Second question has general character

Question 1.2

Find (k,b,c,d,n) for our Collatz problem.

Remarks

In “Generalizations” section of article [5] M. Bruschi made the following relationships between k,d and b numbers

$$k = b^m + 1 \quad \text{and} \quad d = b^m - (n \bmod b^m) \quad (2)$$

and used m as a control parameter. Moreover, he did a remark

Remark of Bruschi *Clearly for $b = 2, m = 1$ one gets the original Collatz map.*

Now I would like to turn Your attention to our generalization. Our generalization does not put any extra condition and does not connect numbers between each other. From this point of view this is a new generalization according to existing old versions. Also, version of the generalization suggested in [5] can be account as a good result of imagination of author: make a relationship between 2 and 3 numbers of original Collatz problem by this way. Firstly I would to go by the same way, but then I changed my mind, I decided to have freedom and did not put any extra

conditions on (k,b,c,d,n) numbers. After all I would like to consider and ask readers to find solutions for this question with the following extra condition

Question 2.1

Find solutions for (1), such that that two numbers, not matter which two numbers of (k,b,c,d,n) are co-prime numbers

References

- [1] http://en.wikipedia.org/wiki/Collatz_conjecture#Syracuse_function
- [2] <http://mathworld.wolfram.com/CollatzProblem.html>
- [3] <http://www.cecm.sfu.ca/organics/papers/lagarias/paper/html/paper.html>
- [4] http://en.wikipedia.org/wiki/G%C3%B6del,_Escher,_Bach
- [5] M. Bruschi, A generalization of the Collatz problem and conjecture arXiv:0810.5169v1[math.NT]