A Prediction Loophole in Bell's Theorem

John R. Dixon

Abstract: We consider the Bell's Theorem setup of Gill et. al. (2002). We present a "proof of concept" that if the source emitting the particles can predict the settings of the detectors with sufficiently large probability, then there is a scenario consistent with local realism that violates the Bell inequality for the setup.

1 Introduction

We assume the reader is familiar with the paper of Gill et. al. (2002). We argue that if the source emitting the particles can predict the detector settings with sufficiently large probability, then it can tailor its emissions so that their Bell Inequality (our inequality (2) below) is violated.

Our assumption that the source emitting the particles can predict, with some chance of being wrong, the detector settings is the only aspect of our setup which strays from their definition of "local realism". We do not seek to argue precisely *how* the source might go about the prediction. We speculate that the material particles or "quantum wavefunction" of the experimental apparatus and its surroundings might somehow effectively act analogously to a "neural network". And further, that this "neural network" has a limited ability to forecast the future. The limitation being on how many physical interactions it can project into the future before the uncertainty in the predictions gets significantly large, analogous to a weather prediction. As anecdotal support, we note that the one exceptional experiment which did not violate a Bell Inequality described in Aspect A. (2002), was also one of the longest experiments. Hence it may have exceeded the posited "neural network's" capacity to forecast the detector settings.

Our hypothesis is quite testable experimentally. We propose Bell experiments in which the detector settings are chosen by "difficult to forecast" methods of randomization. For our hypothesized "neural network", a computer generating pseudorandom settings by an algorithm starting from an arbitrary seed, or reading a list of random settings from memory appears easily predictable, since it follows from a relatively short physical process from a few inputs. Further, we find it plausible that a quantum source of randomness setting the detectors might somehow become physically "coupled" with the source emitting the particles, making it quite easy to forecast for our imagined "neural network". On the other hand, detector settings depending upon coin flips, dice rolls, and so on seem less predictable, as they exceed our own current forecast methods except under very carefully controlled circumstances. Detector settings based on real time weather variables or stock prices seem even less predictable. We speculate that experiments using such sources of randomization for the detectors may exceed the posited "neural network's" forecasting capabilities, preventing a violation of Bell's Inequality, and thus contradicting quantum mechanical laws.

2 The Proof of Concept

The emitter sends particles to the left and right with states which we shall denote by ± 1. We let E_L denote the state of the particle sent to the left, and E_R the right. For both the left and right detector, a "setting" is chosen ± 1. We let S_L and S_R denote the random setting actually chosen for the left and right detectors, respectively. The measurement results in an outcome ± 1. We will denote the outcome left by O_L and right by O_R. We assume

$$O_L = E_L \times S_L \quad \text{and} \quad O_R = E_R \times S_R. \tag{1}$$

According to Gill et. al. (2002), under the assumption of "local realism":

$$\begin{aligned} 0 &\geq P(O_L = O_R | S_L = +1, S_R = -1) \\ &- P(O_L = O_R | S_L = -1, S_R = +1) \\ &- P(O_L = O_R | S_L = +1, S_R = +1) \\ &- P(O_L = O_R | S_L = -1, S_R = -1), \end{aligned} \tag{2}$$

but for a particular quantum system $q = \sqrt{2} - 1$ in actual experiments.

Thus under local realism, for a large number of repetitions of the experiment:

$$q \approx \frac{N_{+-}^{=}}{N_{+-}} - \frac{N_{-+}^{=}}{N_{-+}} - \frac{N_{++}^{=}}{N_{++}} - \frac{N_{--}^{=}}{N_{--}}$$

where $q \leq 0$. Here $N_{+-}^{=}$ denotes the number of trials in which "$O_L = O_R$, $S_L = +1$, and $S_R = -1$", and N_{+-} denotes the number of trials in which "$S_L = +1$ and $S_R = -1$". The other terms are defined analogously.

We will denote the event that the emitter "predicts" that the settings that will be chosen are $S_L = +1$ and $S_R = -1$ by g_{+-}. Here g stands for "guess", which we use as a synonym for "predict" (to avoid possible notational confusion). Similarly we use d_{+-} to denote the event that the detector settings are chosen as $S_L = +1$ and $S_R = -1$. Finally e_{+-} denotes the event that the particles emitted left and right are in states $E_L = +1$ and $E_R = -1$. Other predictions, settings, and emissions have analogous notation.

We assume the emitter targets a convergence value $q > 0$. We assume that to do this, the emitter seeks to maximize $\frac{N_{+-}^{=}}{N_{+-}}$ and minimize the three other terms. In light of (1), we assume that our emitter follows the following strategy:

$$\begin{aligned} &\text{when} \quad g_{+-} \quad \text{then} \quad e_{+-} \quad \text{or} \quad e_{-+}, \\ &\text{when} \quad g_{-+} \quad \text{then} \quad e_{--} \quad \text{or} \quad e_{++}, \\ &\text{when} \quad g_{--} \quad \text{then} \quad e_{+-} \quad \text{or} \quad e_{-+}, \quad \text{and} \\ &\text{when} \quad g_{++} \quad \text{then} \quad e_{+-} \quad \text{or} \quad e_{-+}. \end{aligned}$$

Thus for a large number of repetitions of the experiment:

$$\begin{aligned} \frac{N_{+-}^{=}}{N_{+-}} &\approx P(g_{+-}|d_{+-}) + P(g_{--}|d_{+-}) + P(g_{++}|d_{+-}) \\ \frac{N_{-+}^{=}}{N_{-+}} &\approx P(g_{+-}|d_{-+}) + P(g_{--}|d_{-+}) + P(g_{++}|d_{-+}) \end{aligned}$$

$$
\begin{aligned}
\frac{N^{=}_{++}}{N_{++}} &\approx P(g_{-+}|d_{--}) \\
\frac{N^{=}_{--}}{N_{--}} &\approx P(g_{-+}|d_{++}).
\end{aligned}
$$

We assume that after some "learning" phase, the emitter correctly predicts the detector settings with probability p. And further that given that it is wrong, the probability that its guess is any of the three incorrect settings is $\frac{1}{3}$. Thus for $(cd) = (ab)$, $P(g_{cd}|d_{ab}) = p$. And for $(cd) \neq (ab)$:

$$
\begin{aligned}
P(g_{cd}|d_{ab}) &= \frac{P(g_{cd}\&d_{ab})}{P(\text{wrong}\&d_{ab})}\frac{P(\text{wrong}\&d_{ab})}{P(d_{ab})} \\
&= P(g_{cd}|d_{ab}\&\text{wrong})P(\text{wrong}|d_{ab}) \\
&= \frac{1}{3}(1-p).
\end{aligned}
$$

Hence for a large number of repetitions of the experiment (beyond the "learning" phase, so that its contribution is negligible):

$$
\begin{aligned}
\frac{N^{=}_{+-}}{N_{+-}} &\approx p + \frac{2}{3}(1-p) \\
\frac{N^{=}_{-+}}{N_{-+}} &\approx (1-p) \\
\frac{N^{=}_{++}}{N_{++}} &\approx \frac{1}{3}(1-p) \\
\frac{N^{=}_{--}}{N_{--}} &\approx \frac{1}{3}(1-p).
\end{aligned}
$$

Thus to achieve the value q over a large number of repetitions of the experiment, the emitter must be correct in its prediction of the detector settings with probability p satisfying:

$$
q = p + \frac{2}{3}(1-p) - (1-p) - \frac{1}{3}(1-p) - \frac{1}{3}(1-p) = p - (1-p) = 2p - 1.
$$

And thus $p = \frac{q+1}{2}$. For the quantum mechanical system in which $q = \sqrt{2} - 1$, p is a valid probability: $0 < p = \frac{\sqrt{2}}{2} < 1$.

If p cannot be controlled by the emitter and is too large, it may modify its emissions with probability r according to the following scheme. The emitter generates a random variable X (a pseudorandom variable is adequate), with $P(X=1) = r$ and $P(X=0) = 1-r$, and

- when X=0 it uses the scheme as previously described:

$$
\begin{array}{llllll}
\text{when} & g_{+-} & \text{then} & e_{+-} & \text{or} & e_{-+}, \\
\text{when} & g_{-+} & \text{then} & e_{--} & \text{or} & e_{++}, \\
\text{when} & g_{--} & \text{then} & e_{+-} & \text{or} & e_{-+}, \quad \text{and} \\
\text{when} & g_{++} & \text{then} & e_{+-} & \text{or} & e_{-+}.
\end{array}
$$

- When X=1, it "flips" the scheme:

$$\begin{array}{llllll} \text{when} & g_{+-} & \text{then} & e_{--} & \text{or} & e_{++}, \\ \text{when} & g_{-+} & \text{then} & e_{+-} & \text{or} & e_{-+}, \\ \text{when} & g_{--} & \text{then} & e_{--} & \text{or} & e_{++}, \quad \text{and} \\ \text{when} & g_{++} & \text{then} & e_{--} & \text{or} & e_{++}. \end{array}$$

Thus the emitter targets

$$q = (1-r)c_1 + rc_2$$

where c_1 is the convergence value of

$$\frac{N^{=}_{+-}}{N_{+-}} - \frac{N^{=}_{-+}}{N_{-+}} - \frac{N^{=}_{++}}{N_{++}} - \frac{N^{=}_{--}}{N_{--}} \tag{3}$$

given $X = 0$, and c_2 is its convergence value given $X = 1$.

We showed above that $c_1 = 2p - 1$. Given $X = 1$:

$$\begin{aligned} \frac{N^{=}_{+-}}{N_{+-}} &\approx P(g_{-+}|d_{+-}) = \frac{1}{3}(1-p) \\ \frac{N^{=}_{-+}}{N_{-+}} &\approx P(g_{-+}|d_{-+}) = p \\ \frac{N^{=}_{++}}{N_{++}} &\approx 1 - P(g_{-+}|d_{++}) = 1 - \frac{1}{3}(1-p) \\ \frac{N^{=}_{--}}{N_{--}} &\approx 1 - P(g_{-+}|d_{--}) = 1 - \frac{1}{3}(1-p). \end{aligned}$$

And so:

$$c_2 = \frac{1}{3}(1-p) - p - 2 + \frac{2}{3}(1-p) = -1 - 2p$$

Thus the emitter targets

$$q = (1-r)(2p-1) + r(-2p-1) = 2p - 1 - 2pr + r - 2pr - r = 2p - 1 - 4pr.$$

Let us examine this equation when $q = \sqrt{2} - 1$, which is the special case considered previously. It is easy to see that if p is exactly the value required, that is if $p = \frac{\sqrt{2}}{2}$, then $r = 0$ as we would expect: no modification is necessary. And if $p = 1$, which is as far as it can get too large for the target, then we have that r satisfies $\sqrt{2} - 1 = 1 - 4r$. And so $r = \frac{\sqrt{2}-2}{-4}$, which is a valid probability $0 < \frac{\sqrt{2}-2}{-4} < 1$. Finally, note that the solution for r as a function of p:

$$f(p) = \frac{q - 2p + 1}{-4p}$$

has derivative with respect to p:

$$f'(p) = \frac{8p + 4(q - 2p + 1)}{16p^2} = \frac{4q + 4}{16p^2} > 0,$$

and so it increases from 0 to $\frac{\sqrt{2}-2}{-4}$ as p increases from $\frac{\sqrt{2}}{2}$ to 1. Hence any prediction probability which is too large can be adjusted to the desired value q using this scheme.

3 Conclusion

We have shown that a Bell experiment described by Gill et. al. (2002) may violate its corresponding Bell Inequality under local realism plus the assumption that the detector settings are predictable with a sufficiently large probability. The probability of correct prediction does not have to be 1 to achieve results like those implied by quantum mechanics. We speculate that the experimental apparatus and its surroundings may have predictive ability by acting analogously to a neural network. And further, we propose that experiments be conducted which use randomization mechanisms in setting the detectors which seem difficult to forecast for the imagined "neural network". We end by noting that quantum mechanical phenomena observed in experiments that some consider most palatably interpreted as due to the future influencing the present, may find a more palatable interpretation as due to the present (imperfectly) forecasting the future.

REFERENCES

Gill R.D., Weihs G., Zeilinger A., and Zukowski M. (2002) *Comment on Exclusion of time in the theorem of Bell by K. Hess and W. Philipp.* arXiv:quant-ph/0204169v1.

Aspect A., *Bell's theorem : the naive view of an experimentalist.* in "Quantum [Un]speakables From Bell to Quantum information", edited by R. A. Bertlmann and A. Zeilinger, Springer (2002).