Generalized Fermat's Last Theorem(3) $R^n = y_1^5 + y_2^5$

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Abstract

In this paper we prove $R^n = y_1^5 - y_2^5$ has no nonzero integer solutions for $n \geq 2$. In 1978 using this method we had proved Fermat's last theorem [1]. But on the afternoon of July 19, 1978 this proof was disproved by Chinese mathematics institute of Academia Sinica. How tragic!

We define the supercomplex number [1,2,3]

$$W = \sum_{i=1}^{5} x_1 J^{i-1} \qquad (1)$$

Where J denotes 5-th root of unity, $J^5 = 1$.

From (1) we have

$$W^n = \left(\sum_{i=1}^{5} x_i J^{i-1} \right)^n = \sum_{i=1}^{5} y_i J^{i-1} \qquad (2)$$

From (2) we have the modulus of supercomplex number

$$R^n = \begin{vmatrix} x_1 & x_5 & x_4 & x_3 & x_2 \\ x_2 & x_1 & x_5 & x_4 & x_3 \\ x_3 & x_2 & x_1 & x_5 & x_4 \\ x_4 & x_3 & x_2 & x_1 & x_5 \\ x_5 & x_4 & x_3 & x_2 & x_1 \end{vmatrix}^n = \begin{vmatrix} y_1 & y_5 & y_4 & y_3 & y_2 \\ y_2 & y_1 & y_5 & y_4 & y_3 \\ y_3 & y_2 & y_1 & y_5 & y_4 \\ y_4 & y_3 & y_2 & y_1 & y_5 \\ y_5 & y_4 & y_3 & y_2 & y_1 \end{vmatrix} \qquad (3)$$

y_i are homogeneous and irreducible polynomials.

Theorem 1. Suppose $n = 5$. From (2) we have

$$R = x_1^5 + x_2^5 + x_3^5 + x_4^5 + x_5^5 + 5(x_1 x_3^2 x_4^2 + x_2 x_4^2 x_5^2 + x_3 x_5^2 x_1^2 + x_4 x_1^2 x_2^2 + x_5 x_2^2 x_3^2 + x_1 x_2^2 x_5^2 +$$

$$+ x_2 x_3^2 x_1^2 + x_3 x_4^2 x_2^2 + x_4 x_5^2 x_3^2 + x_5 x_1^2 x_4^2) - 5(x_1 x_2 x_4^3 + x_2 x_3 x_5^3 + x_3 x_4 x_1^3 + x_4 x_5 x_2^3 +$$

$$+ x_5 x_1 x_3^3 + x_1 x_4 x_5^3 + x_2 x_5 x_1^3 + x_3 x_1 x_2^3 + x_4 x_2 x_3^3 + x_5 x_3 x_4^3) - 5 x_1 x_2 x_3 x_4 x_5$$

$$y_1 = x_1^5 + x_2^5 + x_3^5 + x_4^5 + x_5^5 + 20(x_1 x_2 x_4^3 + x_2 x_3 x_5^3 + x_3 x_4 x_1^3 + x_4 x_5 x_2^3 + x_5 x_1 x_3^3 + x_1 x_4 x_5^3 +$$

$$+ x_2 x_5 x_1^3 + x_3 x_1 x_2^3 + x_4 x_2 x_3^3 + x_5 x_3 x_4^3) + 30(x_1 x_3^2 x_4^2 + x_2 x_4^2 x_5^2 + x_3 x_5^2 x_1^2 + x_4 x_1^2 x_2^2 +$$

$$+x_5x_2^2x_3^2+x_1x_2^2x_5^2+x_2x_3^2x_1^2+x_3x_4^2x_2^2+x_4x_5^2x_3^2+x_5x_1^2x_4^2)+120x_1x_2x_3x_4x_5$$

$$\begin{aligned}
y_2 &= 5(x_1x_5^4+x_2x_1^4+x_3x_2^4+x_4x_3^4+x_5x_4^4)+10(x_1^2x_3^3+x_2^2x_4^3+x_3^2x_5^3+x_4^2x_1^3+x_5^2x_2^3)+\\
&\quad +20(x_1x_3x_4^3+x_2x_4x_5^3+x_3x_5x_1^3+x_4x_1x_2^3+x_5x_2x_3^3)+\\
&\quad +30(x_1x_2^2x_3^2+x_2x_3^2x_4^2+x_3x_4^2x_5^2+x_4x_5^2x_1^2+x_5x_1^2x_2^2)\\
&\quad +60(x_1x_4x_5x_3^2+x_2x_5x_1x_4^2+x_3x_1x_2x_5^2+x_4x_2x_3x_1^2+x_5x_3x_4x_2^2)\\
y_3 &= 5(x_1x_4^4+x_2x_5^4+x_3x_1^4+x_4x_2^4+x_5x_3^4)+10(x_1^2x_5^3+x_2^2x_1^3+x_3^2x_2^3+x_4^2x_3^3+x_5^2x_4^3)+\\
&\quad +20(x_1x_2x_3^3+x_2x_3x_4^3+x_3x_4x_5^3+x_4x_5x_1^3+x_5x_1x_2^3)+\\
&\quad +30(x_1x_3^2x_5^2+x_2x_4^2x_1^2+x_3x_5^2x_2^2+x_4x_1^2x_3^2+x_5x_2^2x_4^2)+\\
&\quad +60(x_1x_2x_4x_5^2+x_2x_3x_5x_1^2+x_3x_4x_1x_2^2+x_4x_5x_2x_3^2+x_5x_1x_3x_4^2)\\
y_4 &= 5(x_1x_3^4+x_2x_4^4+x_3x_5^4+x_4x_1^4+x_5x_2^4)+10(x_1^2x_2^3+x_2^2x_3^3+x_3^2x_4^3+x_4^2x_5^3+x_5^2x_1^3)+\\
&\quad +20(x_1x_2x_5^3+x_2x_3x_1^3+x_3x_4x_2^3+x_4x_5x_3^3+x_5x_1x_4^3)+\\
&\quad +30(x_1x_2^2x_4^2+x_2x_3^2x_5^2+x_3x_4^2x_1^2+x_4x_5^2x_2^2+x_5x_1^2x_3^2)+\\
&\quad +60(x_1x_3x_4x_5^2+x_2x_4x_5x_1^2+x_3x_5x_1x_2^2+x_4x_1x_2x_3^2+x_5x_2x_3x_4^2)\\
y_5 &= 5(x_1x_2^4+x_2x_3^4+x_3x_4^4+x_4x_5^4+x_5x_1^4)+10(x_1^2x_4^3+x_2^2x_5^3+x_3^2x_1^3+x_4^2x_2^3+x_5^2x_3^3)+\\
&\quad +20(x_1x_4x_3^3+x_2x_5x_4^3+x_3x_1x_5^3+x_4x_2x_1^3+x_5x_3x_2^3)+\\
&\quad +30(x_1x_4^2x_5^2+x_2x_5^2x_1^2+x_3x_1^2x_2^2+x_4x_2^2x_3^2+x_5x_3^2x_4^2)+\\
&\quad +60(x_1x_4x_5x_2^2+x_2x_5x_1x_3^2+x_3x_1x_2x_4^2+x_4x_2x_3x_5^2+x_5x_3x_4x_1^2)
\end{aligned} \tag{4}$$

We define the stable group [1,4]

$$G=\{g_2, g_3, g_4, g_5\} \tag{5}$$

where

$$g_2=\begin{pmatrix}1\,2\,3\,4\,5\\1\,2\,3\,4\,5\end{pmatrix},\ g_3=\begin{pmatrix}1\,2\,3\,4\,5\\1\,3\,5\,2\,4\end{pmatrix},\ g_4=\begin{pmatrix}1\,2\,3\,4\,5\\1\,4\,2\,5\,3\end{pmatrix},$$

$$g_5=\begin{pmatrix}1\,2\,3\,4\,5\\1\,5\,4\,3\,2\end{pmatrix}$$

We have

$$x_1 \to x_1, \quad x_2 \xrightarrow{g_3} x_3 \xrightarrow{g_5} x_4 \xrightarrow{g_4} x_5$$

$$y_1 \to y_1, \quad y_2 \xrightarrow{g_3} y_3 \xrightarrow{g_5} y_4 \xrightarrow{g_4} y_5$$

x_1 and y_1 are stable elements. x_i and $y_i (i = 2,3,4,5)$ are non-stable elements. $y_i (i = 2,3,4,5)$ are the same polynomials. From (3) we have a Fermat equation group

$$y_i (i = 3,4,5) = 0, \quad (6)$$

$$R^5 = y_1^5 + y_2^5 \quad (7)$$

If (6) has nozero integer solutions, then (7) has nozero integer solutions and vice versa. If (6) has no nozero integer solutions, then (7) has no nozero integer solutions, and vice versa.
We have that (7) has only trival solutions [1,5].

$$y_i (x_1, 0,0,0,0) = 0, i = 3,4,5 \quad (8)$$

We have

$$y_2 (x_1, 0,0,0,0) = 0 \quad (9)$$

Hence we prove that (7) has no nozero integer solutions.
Dirichlet and Legendre prove that (7) has no nozero integer solutions. Hence (6) has no nozero integer solutions.
From (3) there are ten Fermat's equation groups. For example

$$y_i = 0 \quad (i = 1,2,3), \quad (10)$$

$$R^5 = y_4^5 + y_5^5 \quad (11)$$

(10) and (11) have only trival solutions

$$y_i (0,0,0,0) = 0 \quad i = 1,2,3,4,5 \quad (12)$$

Theorem 2. Suppose $n \geq 2$. From (3) we have a Fermat's equation group

$$y_i (i = 3,4,5) = 0, \quad (13)$$

$$R^n = y_1^5 + y_2^5 \quad (14)$$

We have that (13) has only trival solutions

$$y_i (x_1, 0,0,0,0) = 0 \quad (15)$$

Hence (14) has no nozero integer solutions. Using our method [1-7] it is able to prove the Beal conjecture [8].

References

[1] Chen-xuan Jiang, A general proof of Fermat's last theorem, July 1978, Mimeograph papers.

[2] Chun-Xuan Jiang, Foundations of Santilli's isonumber theory with applications to new cryptograms, Fermat's theorem and Goldbach's conjecture. Inter. Acad. Press, 2002, 299-306, MR2004c:11001.(http://www.i-b-r.org/docs/jiang.pdf) (http://www.wbabin.net/math/xuan13.pdf) (http://vixra.org/pdf/1004.0027v1.pdf).

[3] Chun-Xuan Jiang, The Diophantine equations $a^2 \pm mb^2 = c^n$, $a^3 \pm mb^3 = d^2$ and $y_1^4 \pm my_2^4 = R^2$, http://www.vixra.org/pdf/1102.005lv1.pdf, http://www.wbabin.net/ntham/xuan188.pdf

[4] Chun-Xuan Jiang, The application of stable groups to biological structure, Acta Math. Sci. 5, 3(1985) 243-260.

[5] Z. I. Borevich and I. R. Shafarevich, Number theory. Academic Press, New York, 1966.

[6] Chun-Xuan Jiang, Generalized Fermat's last theorem (2) $R^n = y_1^4 - y_2^4$ http://www.vixra.org/pdf/1103.0092v1.pdf

[7] Chun-Xuan Jiang, Generalized Fermat's last theorem (1) $R^n = y_1^3 + y_2^3$. http://www.vixra.org/pdf/1103.0038v1.pdf

[8] R. Daniel Maulidin. A generalized of Fermat's last theorem: The Beal conjecture and prize problem. Notices of the AMS, 44[1997] 1436-1437.