The Principle of Proportionality of Mass and Energy: New Version

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The essence of mass and its relationship to energy is considered. It is concluded that after radiation of energy from a system or when the system does work the mass of the system must not diminish, but increase. The opposite case is heating of bodies from external sources, when an increase in internal heat energy must be accompanied by an increase in entropy and decrease in the mass of the bodies. On the basis of strong gravitation is explained mass defect of atomic nuclei. Conclusions of general relativity and covariant theory of gravitation about mass and energy are analyzed.

Keywords: *mass; energy; principle of equivalence; tensor of entropy; mass defect; general theory of relativity; covariant theory of gravitation.*

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Knowledge of the essence of mass and energy, as well as ways of identifying them, are one of the most important problems in physics. This is due to extensive use of the law of conservation of energy and momentum in various areas and due to the possibility of calculating operating forces and flows of energy through the energy gradients in spacetime. The relationship between mass and energy should be the most simple in frame of reference in which the body rests and does not rotate, since momentum and angular momentum of the body are equal to zero and kinetic energy of linear motion of the body as a whole and rotational energy in the calculation of the mass does not participate.

Studying the relationship of mass and energy in a body at rest during the formation of the theory of relativity led to the relation:

$$\Sigma_0 = kmc^2 , \qquad (1)$$

where Σ_0 – relativistic energy at rest ,

m – body mass,

c – speed of light.

Initially it was thought that the coefficient k in (1) sufficiently close to 1, then by the efforts of O. Heaviside (1889) [1], A. Poincare (1900) [2], A. Einstein (1905) [3] and a number of other physicists it was established that $k = 1$.

From (1) does not imply that the mass and relativistic energy are synonymous, the designation of the same. On the one hand, body mass is an integral property that determines the inertia of the body upon receipt of acceleration from a force. Integrality here means that not only substance of the body make contribution to the mass but also physical fields related to this substance, as well as fields from external sources in body volume. On the other hand, the energy is associated more with the law of conservation of energy, with the ability to transfer energy from one place to another in different ways and in different forms, such as heat transfer, electromagnetic radiation, electricity, etc. The force $\boldsymbol{F}$, acting on the body, is defined as the rate of change of momentum, and mass is directly included in the momentum $\boldsymbol{P}$ as a factor. When calculating force in external field is often used the formula in which the force is the gradient of the potential energy U of the body, taken with opposite sign. This implies the following:

$$\boldsymbol{F} = \frac{d\boldsymbol{P}}{dt} = -\nabla U \,. \tag{2}$$

It is seen from (2) that although all forms of energy are contributing to the body mass (and hence to the momentum), but the force as the rate of change of momentum can only depend on gradients of certain energies. If we take the average over the entire volume, the energies without gradients and flows do not cause force and acceleration, although involved in formation of body mass. In this connection it must be assumed that the mass represents the static integral gravitational and inertial properties of the body, appearing as a result of energy fluxes that interact with the body.

As described in [3] body mass in the emission of some energy L in the form of two oppositely directed photons, should be reduced by the amount $\Delta m = L / c^2$. To verify this conclusion, we once again repeat the thought experiment with the body, which emit photons. In this case we use the formula for relativistic energy and momentum of [4]:

$$\Sigma = \frac{\Sigma_0}{\sqrt{1-V^2/c^2}} \,, \qquad \boldsymbol{P} = \frac{\Sigma_0 \boldsymbol{V}}{c^2\sqrt{1-V^2/c^2}} \,, \qquad \Sigma_0 = -E_{g0} - E_0 \,, \tag{3}$$

where Σ_0 is a positive value for the relativistic energy of a body at rest,

E_{g0} – total energy of particles of the body at the atomic level, which includes various types of energies associated with atoms and molecules near absolute zero temperature: the energy of strong interaction, which bounds substance of elementary particles and retains the nucleons in atomic nuclei; the energy of electromagnetic interaction of particles; the energy of motion of substance inside the nucleons and energy of the nucleons in nuclei and electrons in atoms; the rotational energy of atoms and molecules; the vibrational energy of atoms in molecules, etc.,

E_0 – total energy of the body at the macro level, taking into account internal thermal energy E_T in the form of kinetic energy of chaotic motion of atoms and molecules and the energy of the turbulent motion of substance flows, as well as the energies of the fundamental macroscopic fields.

By definition, the total energy E_0 can be divided into three components:

$$E_0 = E_T + U_0 + W_0 , \tag{4}$$

where U_0 and W_0 are energies of macroscopic gravitational and electromagnetic fields in substance of the body, respectively, calculated within the body and beyond. In the same energies should be included energies of fields from external sources that fall inside of the body and change the energy of the substance.

We take into account in (4) the virial theorem, according to which the modulus of potential energy of fields on average twice as much internal heat for bodies that are only under action their own gravitational and electromagnetic fields:

$$2E_T + U_0 + W_0 \approx 0 , \qquad E_0 = E_T + U_0 + W_0 \approx -E_T \approx \frac{U_0 + W_0}{2} . \tag{5}$$

For the energy E_{g0} in (5) can be written:

$$E_{g0} \approx \frac{U_{g0} + W_{g0}}{2} , \tag{6}$$

where U_{g0} is the energy of the field of strong gravitation, which is assumed at the level of elementary particles and atoms, and instead of strong interaction holds the substance of nucleons, the nucleons in atomic nuclei, as well as being one of the components that hold electrons in their orbits in atoms,

W_{g0} – electromagnetic energy in substance of elementary particles and around them in atoms.

Taking into account (5) and (6) the relativistic energy (3) of a body at rest can be written as:

$$\Sigma_0 = -E_{g0} - E_0 = -\frac{U_{g0} + W_{g0}}{2} - \frac{U_0 + W_0}{2} . \tag{7}$$

The main contribution to the energy Σ_0 brings negative energy U_{g0} of field of strong gravitation, which provides the positivity of relativistic energy and body mass, determined by the expression:

$$m = \frac{\Sigma_0}{c^2}. \tag{8}$$

We turn now to the thought experiment. Suppose there is a body at rest and two photons are emitted from the body in opposite directions, one photon with the energy E_F along axis OX, and another with the same energy against the axis OX. We assume that the photon emission occurs only due to changes in energy of macroscopic fields, then the energy balance before and after the radiation has the form:

$$\Sigma_0 = -E_{g0} - E_0 = -E_{g0} - E_1 - 2E_F = \Sigma_1 - 2E_F. \tag{9}$$

After radiation the body remains stationary, since the momentums of photons are contrary and the total momentum of the system remains zero. It is assumed in (9) that at the moment of radiation components of the energy Σ_0 are changed, and the energy of photons is taken with the minus sign, which means the body's energy loss by radiation. It follows that $E_0 - E_1 = 2E_F$, and hence $E_0 > E_1$. Both energy E_0 and E_1 are negative, so that for the modulus of energy is obtained: $|E_1| > |E_0|$. Relation (9) also shows that the relativistic energy Σ_1 is greater than the energy Σ_0. This entails an increase in body mass after the emission of photons: $m_1 = \frac{\Sigma_1}{c^2} > m = \frac{\Sigma_0}{c^2}$. Such result is consistent with the fact that if a star is radiated more energy, then more the star is compressed and heated. The total energy E_0 of such star becomes more negative and positive internal thermal energy E_T in (5) increases, which according to (7) and (8) increases the mass of the star. One of the reasons for choosing the negative sign in front of E_0 in (7) is symmetry of this expression when E_{g0} has negative sign too. In addition, the energy of gravitation in compressed star converts into internal thermal energy and radiation energy, and according to virial theorem these energies approximately equal to each other. If the energy of gravitation gives rise to the radiation energy and its mass, in the same degree the energy of gravitation can generate additional mass of the star.

The above-described justification of (9) and choosing the negative sign in front of the photon energy $2E_F$ is missing in [3]. Instead of it the mechanical model of the phenomenon is used, when the photons as some parts of the body leave this body and take away part of its mass. Accordingly, in this picture the sign of the photon energy is chosen positive, and body mass after the emission of photons should decrease. However, photons are not parts of the body because they are generated due to absolute acceleration of charges of the body without reducing the magnitude of these charges (if body mass may vary due to changes in the body's energy, the charge of the body remains till the moment when it will removed from the body or compensated by a charge of opposite sign). Therefore, loss of the relativistic energy of the body through the transfer of photon energy must be offset by an increase rather than decrease in the energy and body mass.

Let us now consider the photon emission from the body moving with velocity V along the axis OX. The photon emitted in the direction of the axis OX, will have blue shift of its frequency and increased energy, and the photon emitted in the opposite direction, will have red shift of the frequency and decreased energy. The energy of both photons according to formula for the Doppler effect will be equal to $2E'_f = \frac{2E_F}{\sqrt{1-V^2/c^2}}$, and momentum of the photons is equal to $P_F = \frac{2E_F V}{c^2\sqrt{1-V^2/c^2}}$ and is directed along the velocity of the body.

Taking into account formula (3), the balance of energy and momentum before and after the photon emission yields:

$$\Sigma = \frac{\Sigma_0}{\sqrt{1-V^2/c^2}} = \frac{\Sigma_1}{\sqrt{1-V^2/c^2}} - 2E'_f = \frac{\Sigma_1}{\sqrt{1-V^2/c^2}} - \frac{2E_F}{\sqrt{1-V^2/c^2}} ,$$

$$P = \frac{\Sigma_0 V}{c^2\sqrt{1-V^2/c^2}} = \frac{\Sigma_1 V}{c^2\sqrt{1-V^2/c^2}} - P_F = \frac{\Sigma_1 V}{c^2\sqrt{1-V^2/c^2}} - \frac{2E_F V}{c^2\sqrt{1-V^2/c^2}} . \qquad (10)$$

In (10) the energy and momentum of photons have the minus sign, since the photons carry away from the body some part of its energy and momentum. In the moment of photon emission there occur a corresponding increase in body mass, relativistic energy and momentum. After canceling identical terms (10) becomes as (9). This means that the difference between the formulas for the processes of photon emission of body at rest and the body in motion is only associated with the Lorentz transformation and determined by the factor $\sqrt{1-V^2/c^2}$.

Heating of bodies

From the above we can come to the idea that heating of the body by external sources of energy should decrease body mass. As it was found in [5] based on Lorentz-invariant thermodynamics, the amount of heat δQ, that is generated in a certain volume V_b of the body over time dt, is determined by the integral:

$$\delta Q = -dt\int \nabla \cdot (\boldsymbol{S}_{\Gamma} + \boldsymbol{S}_{P})\, dV_b = -dt\int (\boldsymbol{S}_{\Gamma} + \boldsymbol{S}_{P}) \cdot \boldsymbol{n}\, dS_b\,, \qquad (11)$$

where $\boldsymbol{S}_{\Gamma}$ – density of the flux of gravitational energy,

$\boldsymbol{S}_{P}$ – electromagnetic energy flux density (Poynting vector),

$\boldsymbol{n}$ – unit vector normal to the surface area S_b surrounding the volume V_b.

According to (11), increase the heat can be described by the incoming flow of energy of the fundamental fields – either through the integral of the divergence of energy fluxes by volume or by using the Gauss theorem for the integral of the energy flows through the area. Equation (11) is easier to understand if one considers the following formula:

$$\nabla \cdot \boldsymbol{S}_{\Gamma} = -\frac{\partial U^{00}}{\partial t} - \boldsymbol{J} \cdot \boldsymbol{G}\,, \qquad \nabla \cdot \boldsymbol{S}_{P} = -\frac{\partial W^{00}}{\partial t} - \boldsymbol{j} \cdot \boldsymbol{E}\,, \qquad (12)$$

where U^{00} and W^{00} are the energy density of gravitational and electromagnetic fields in the form of timelike components of stress–energy tensors,

$\boldsymbol{J}$ and $\boldsymbol{j}$ – the densities of mass and electric current, respectively,

$\boldsymbol{G}$ and $\boldsymbol{E}$ – strengths of gravitational and electromagnetic fields (gravitational acceleration and electric intensity).

If we substitute (12) in (11), we see that the heat in volume of the body increases when the energy of field is increasing, as well as when work $\boldsymbol{J} \cdot \boldsymbol{G} + \boldsymbol{j} \cdot \boldsymbol{E}$ is done in the unit volume per unit time. The differential of entropy is given by:

$$dS = \frac{\delta Q}{T}\,, \qquad (13)$$

where T is the absolute Kelvin temperature.

According to (13), by heating the body from external sources, the entropy of the body increases. If the energy is radiated from the body in the process of gravitational contraction and heating of the substance, the total energy of the body is reduced by δQ and the increment of entropy dS is negative. This is due to the fact that although substance under compression and reducing of its volume is heated and entropy of the substance increases, but negative entropy of the gravitational field of the body is changed even more, so that the total entropy of substance and field is negative. For the entropy of spherical body, we derived the formula [5]:

$$S = -\int \frac{\boldsymbol{r}\cdot\nabla(U^{00} + W^{00} + L - P_0)}{T} dV_s , \tag{14}$$

where the radius-vector $\boldsymbol{r}$ measured from the center of the body,

P_0 – pressure in the comoving reference frame,

$L = \int \frac{P_0}{\rho_0} d\rho_0$ is a function of compression, calibrated so that the energy density of substance at rest was equal to the value $\rho_0 c^2$,

ρ_0 – density of substance at rest.

In (14) the integration is over the entire volume V_s of space, both inside and outside the body. The main contribution to negative entropy of the body make gradient of the gravitational field energy density U^{00} and gradient of pressure P_0. Evaluation of entropy per particle of ideal gas in gravitationally bound ball at a constant temperature of the volume, gives a value $\approx -7,2k$, where k is the Boltzmann constant.

If the energy radiates from the body the entropy of the body becomes more negative, the entropy of the outgoing radiation is positive, resulting in the total entropy of the body and the radiation is zero. This conclusion follows from the virial theorem and from (13), in which δQ means both the heat content of the body as a result of its gravitational compression, and the energy carried away by the outgoing radiation. Zero entropy was at the beginning of the formation of the body too when the substance at infinity was at rest and in the dispersed state.

In [6], we derived Lorentz-invariant expression of the first law of thermodynamics, have found tensor function of the chemical potential, tensor function of the work-energy of system, as well as tensor function of heat δQ^{ik} :

$$\delta Q^{ik} = V_e d(U^{ik} + W^{ik}) + \frac{V_e}{c^2} d(P_0 u^i u^k) - V_e \eta^{ik} d\left(\frac{P_0}{1 - V^2/c^2}\right), \tag{15}$$

where V_e is the invariant volume of a small element of substance or a small volume of space occupied by the field in the absence of substance,

U^{ik} and W^{ik} – stress–energy tensors of gravitational and electromagnetic fields,

u^i – 4-velocity of substance,

η^{ik} – metric tensor of Minkowski space-time.

From (15) follows that at constant volume V_e of element of substance increment of heat comes from the increments of the density energy-momentum of fields and changes of internal pressure P_0, which depends on the 4-velosity for an outside observer. All terms are listed in (15), may directly increase the kinetic temperature of the element of substance and therefore are a part of δQ^{ik}. To obtain the amount of heat of the body as a set of elements of the substance δQ^{ik} should be summed over all volume elements. Entropy increment tensor is defined as in (13):

$$dS^{ik} = \frac{\delta Q^{ik}}{T}.$$

Symmetric tensor of entropy is the integral over the volume:

$$S^{ik} = \int \frac{\dfrac{\eta^{ik}\rho_0 c^2}{\sqrt{1-V^2/c^2}} - \rho_0 u^i u^k \sqrt{1-V^2/c^2} - \eta^{ik}\mathbf{r}\cdot\left[\nabla(L-P_0)+\rho_0\mathbf{G}+\rho_{0q}\boldsymbol{E}\right]}{T} dV_s \,, \qquad (16)$$

where ρ_{0q} is the charge density.

For an element of substance of gravitationally bound body after a number of simplifications, the formula for the timelike component of the entropy is:

$$S^{00} = -\frac{NR\Delta\rho_0}{\rho_0},$$

where $\Delta\rho_0 \ge 0$ is the change in the density of substance on the length of the element of substance, R – gas constant.

We can show that not only S^{00}, but other components of tensor S^{ik} are negative. As it follows from (16), the entropy of element of substance is proportional to the ratio modulus of ordered energy in this element and the energy of random thermal motion of particles of substance, taken with the minus sign. Under the ordered energy means the energy of directional motion of the element of substance, energy of pressure compression and potential energy of the element of substance in gravitational and electromagnetic fields. Entropy is a function of system state, because if the system state is given by a number of physical quantities, then in each such state is carried out, after some relaxation time, usually only one definite correlation between the ordered and disordered system energies that is independent on the way of transition in this state. This relation is fixed by the concept of entropy.

In theory of infinite hierarchical nesting of matter is supposed that the source of order and orderly energy of bodies are flows of gravitons, whose properties are similar to those of photons and neutrinos, as well as high-energy charged particles. These field quanta and particles, appearing in lower levels of matter, due to their relatively high energy compared with their mass have the highest orderliness in our world and carry it into space.

The stream of ordering is received by a gravitational system with a flow of gravitons, generates negentropy in the system, as the outgoing from the system flow of gravitons has a lower temperature at nearly the same energy as the energy of the incoming flow of gravitons. This negentropy will reduce the entropy of the system to negative values. In addition, the outgoing radiation from the system, typically electromagnetic, has its own entropy, so that approximately one half of the negentropy of the flow of gravitons is spent on the system entropy loss due to outgoing radiation.

In accordance with the above-mentioned and [4], we believe that the observed warming of an object due to gravitational contraction leads to an increase in mass of the object. This process is accompanied by the emission of photons from the object with energy equal to the relativistic energy of the object, excluding the rest energy, and equal to the modulus of total energy (binding energy). At the same time the total energy and entropy of the object have negative sign. In reverse process external radiation heats the object and increases total energy and entropy, and hence reduce the relativistic energy and associated with it mass of the object.

Nuclear energy

In modern physics is supposed that for determination the relativistic energy of the body is necessary to sum rest energy of its constituent particles and the total energy of the body, taking into account the mechanical energy of particles and energy fields. For the fundamental forces the total energy is usually negative, so that the relativistic energy and body mass obtained are less than the amount of energy and mass of all particles of the body, separated from each other. In the theory of infinite hierarchical nesting of matter, there are infinitely many levels of the matter which have objects of respective masses. If at some basic level of matter to take a lot of objects and start putting them into more massive objects, due to a negative total energy the relative mass of objects will be less and less at each subsequent level of matter, in relation to the total mass of the primary objects.

According to our assumptions, the total energy in the gravitational field is included in relativistic energy with the negative sign, which leads not to a decrease but to an increase in the relative mass of objects with increasing mass of these objects. If we consider the question from a philosophical point of view, the conclusions about the likely decrease or increase the relative mass of objects as we move to higher levels of matter seem to be equally valid. Apparently, the choice can be made by comparison with experimental data.

Most clearly the relationship between mass and energy is evident in the case of fusion of light nuclei and the decay of massive nuclei, where small differences in the masses of initial and final reaction products are accompanied by the release of large amounts of energy. In Table 1, according to [7], [8], the masses of some nuclei are in comparison with the sum of mass of individual protons and neutrons, of which one could compose these nuclei.

Table 1

Nucleus	Number of neutrons, N_n	Number of protons, N_p	$N_n M_n$, mass of neutrons, 10^{-27} kg	$N_p M_p$, mass of protons, 10^{-27} kg	M_N, mass of nucleus, 10^{-27} kg	$N_n M_n +$ $+N_p M_p -$ $-M_N$, 10^{-27} kg
${}^{2}_{1}H$	1	1	1.674 927 351	1.672 621 777	3.343 583 48	0.003 965 65
${}^{62}_{28}Ni$	34	28	56.947 529 93	46.833 409 75	102.808 9	0.972 04
${}^{238}_{92}U$	146	92	244.539 393	153.881 203	395.208 8	3.211 8

According to Table 1, the mass of any nucleus is less than the total mass of nucleons, of which may be formed the nucleus. Mass defect, shown in the last column of Table 1 is that the decrease in mass of the nucleus can reach almost 1%. In the standard model is supposed that by combining of nucleons their total mass is reduced by the negative total energy of the nucleus. If, however, proceed from our assumptions, as in (3) for the relativistic energy and mass of the nucleus at rest should be written:

$$\Sigma_N = -E_{gn} - E_{gp} + E_N , \qquad M_N = \frac{\Sigma_N}{c^2} = -\frac{E_{gn} + E_{gp} - E_N}{c^2} = N_n M_n + N_p M_p + \frac{E_N}{c^2} , \qquad (17)$$

where E_{gn} – total energy of the free neutrons necessary for the formation of the nucleus,

E_{gp} – total energy of the free protons that make up the nucleus,

E_N – total energy of the nucleus.

Note that in (17) we put the plus sign to the total energy E_N, in contrast to the minus sign, standing in front of the total energy E_0 in (3). This is due to the fact that the gravitational contraction energy of the gravitational field converts in radiation in the environment, and in the heating of substance, thus creating a lot of radiation and additional mass, as it is evident from (3). But situation with formation of an atomic nucleus from nucleons is different. For the emergence of the nucleus it is necessary to heat up nucleons from an external source to a temperature sufficient to initiate fusion of the nuclei, or to do some work on the nucleons. If a system radiate photons during the gravitational collapse, then in contrast to it for nuclear fusion is necessary in some way to introduce in the system some extra energy. This is reminiscent of the effect of thermal heating in the previous section and in our opinion leads to a decrease in mass of the system.

From a formal point of view, the relation (3) describes the process of creating a mass of photons in the ambient space of the system and creation an additional mass of the system in the form $\Delta m = -\frac{E_0}{c^2}$. To describe the formation of the nucleus and changes in its mass can be assumed that the interaction between nucleons leads to a negative mass of photons (photons are not born, but rather absorbed by the system; or some work is done on the system) and to a certain total energy, taken with the minus sign. Substitution in (3) instead of E_0 the total energy E_N, but taken with the minus sign, gives the change in mass $\Delta m = \frac{E_N}{c^2}$ and the plus sign in front of E_N in (17). Since the total energy E_N by itself is negative, then in (17) mass of the nucleus M_N is less than the total mass of protons and neutrons that make up the nucleus.

How the nucleons in atomic nuclei are held? In [6] we gave some simple models of nuclei and described nuclear forces, by which the nucleons in a nucleus can be in equilibrium. Similarly, in order to describe the stability of some of the hadrons, we in [10] have developed their models based on the interaction of nucleons and light mesons. The solidity of the nuclei is due to the large forces acting between nucleons. If we assume that between nucleons in a nucleus acts strong gravitation and force of attraction, then there must be also powerful forces of repulsion. These forces arise from the torsion fields of rapidly rotating nucleons. Typically, the force of the torsion field is weaker than the force of gravitational attraction of masses. Similarly, magnetic forces are generally weaker than electrical forces, since in the formula for the magnetic force is velocity of light squared, which reduces the value of the force. As the magnetic forces, the forces of the torsion field considerably grow at velocity close to the speed of light, and begin to level off in value to the electric and gravitational forces, respectively. Thus, in order that spins of nucleons in a nucleus can effectively repel each other, is necessary a very fast rotation of the nucleons, which generates the field of torsion.

As an illustration, we present here a formula for the total energy of deuterium, the simplest nucleus, consisting of a neutron and a proton, according to [6]:

$$E_D = U_g + 2(U - U_p) + \eta U_o + E_r, \qquad (18)$$

where $U_g = -\frac{0.26 \Gamma M_n M_p}{R}$ – gravitational energy of the interaction of neutrons and protons (coefficient 0.26 reflects a decrease in the interaction force due to the high density and is calculated in the upgraded model of gravitation Fatio-Lesage [5], [10] as a consequence of the exponential decay of flow gravitons in substance, at low density of substance this coefficient tends to 1, and the formula for U_g takes Newtonian form),

$\Gamma = \frac{e^2}{4\pi\varepsilon_0 M_p M_e} = 1.514 \cdot 10^{29}$ m^3·kg^{-1}·s^{-2} is strong gravitational constant according to [5],

e – elementary electric charge,

ε_0 – vacuum permittivity,

M_e – electron mass,

R – distance between the centers of neutron and proton,

$2(U - U_p) = -\frac{83\,\Gamma(L^2 - L_p^2)}{126\, c_g^2 R_p^3}$ – change in the energy of torsion field of strong gravitation two nucleons, which occurs due to the increase of the spin (angular momentum) of each of the nucleons from the value L_p to L,

R_p – proton radius, approximately equal to the radius of neutron,

c_g – propagation speed of gravity which is close to the speed of light,

$\eta U_o = \frac{\eta \Gamma L^2}{c_g^2 R^3}$ – energy of interaction between the spins of two nucleons in their torsion fields,

$\eta = 2.8$ – coefficient, which reflects an increase in the spin of the nucleons as compared with the value for the angular momentum of the ball in classical physics, and arises as a consequence of relativistic calculation of rotation, increasing the mass and momentum,

$E_r = \frac{L^2 - L_p^2}{I}$ – increasing the rotational energy of nucleons in their binding in the nucleus,,

I – moment of inertia of a nucleon.

Our assumption that rotation of nucleons in their fusion to the nucleus should be increased, follows from the fact that only in this case, the repulsive force of the spins will be sufficient to counteract the attraction of the nucleons under the influence of strong gravitation. Spin orientation in a nucleus of deuterium is the same for both nucleons so that this produces a repulsive force, and during the convergence of nucleons there is an increase of rotation of the nucleons with increasing angular momentum due to the effect of gravitational induction. As a result the nucleons can spin rapidly and reach the maximum possible angular momentum.

For the deuteron total energy is $E_D = -2.224$ MeV, respectively, the binding energy as modulus of the total energy is 2.224 MeV. For more massive nuclei with an increased number of protons formula for the total energy instead of (18) can be written as:

$$E_N = U_g + A(U - U_p) + \eta U_o + E_r + W \ , \tag{19}$$

where A specifies the number of nucleons in a nucleus,

gravitational energy U_g, energy of interaction between the spins ηU_o and change the rotational energy E_r are calculated for all the nucleons,

$W = \frac{k z^2 e^2}{4\pi \varepsilon_0 R_N}$ – electrical energy of proton in nucleus for the case of uniform distribution by volume of the nucleus, when $k \approx 0.6$,

R_N – average radius of the nucleus,

z – charge number of nucleus or the number of protons.

In the literature, as a rule is considered specific binding energy, or modulus of the total energy per nucleon, i.e., the quantity $\frac{|E_N|}{A}$, and its dependence on A. For light nuclei the main contribution to (19) brings the energy of strong gravitation U_g. Assuming that radius of nucleus is approximated by the usual formula $R_N = R_0 A^{1/3}$, where $R_0 = (1.2 - 1.5) \cdot 10^{-15}$ m, and mass of the nucleus $M_N \approx A M_p$, we as in [6] can write a proportional relation:

$$\frac{|E_N|}{A} \sim \frac{|U_g|}{A} = \frac{k \Gamma M_N^2}{A R_N} \sim A^{2/3} .$$

This dependence describes well the growth of specific binding energy of nuclei up to $A \approx 20$. Then there is a saturation of energy of strong gravitational energy, the energy of nuclei do not varies as the square of nuclear mass, but much weaker. As was shown in [10], cross section of interaction of gravitons with nucleons is such that enough to put in the way of the flow of gravitons three nucleons in order to significantly reduce the flow (approximately 2.718 times, this number is the base of natural logarithms). When the number of nucleons is more 17–23 then addition of new nucleons gives less and less increases of gravitational energy per nucleon.

At the same time adding protons in a nucleus with increasing mass and charge of the nucleus leads to a marked increase in positive electric energy that is beginning to compensate the change of negative gravitational energy. As a result, at $A = 62$ for ${}^{62}_{28}\mathrm{Ni}$ there is the maximum of the dependence $\frac{|E_N|}{A}$ on A, and then the specific binding energy begins to decrease with the increase A. Thus, the formulas for strong gravitation and for the electromagnetic force and energy can describe the equilibrium of nucleons in nucleus, and also explain the dependence of specific binding energy on the mass number. The decrease in the mass of atomic nuclei, compared with a sum of the individual masses of the constituent nucleons, is a consequence of opposite flows of energy necessary for the emergence of the binding energy, compared with the case of the usual gravitational contraction of matter.

General theory of relativity

The axiomatics of general theory of relativity (GTR) is associated with recognition of gravitational field as some form of metric field, and with geometric difference between a curved Riemannian space-time and the flat Minkowski space-time. Currently, GTR is the most famous and developed theory of gravitation. As the basis of the theory can be considered Hilbert-Einstein equations for the metric:

$$R_{\mu\nu}-\frac{R}{2}g_{\mu\nu}+\Lambda g_{\mu\nu}=\frac{8\pi\gamma}{c^4}\left(\phi_{\mu\nu}+W_{\mu\nu}\right), \qquad (20)$$

where $R_{\mu\nu}$ − Ricci tensor,

R − scalar curvature,

$g_{\mu\nu}$ − metric tensor,

Λ − cosmological constant,

γ − gravitational constant,

c − speed of light,

$\phi_{\mu\nu}$ − stress–energy tensor of substance,

$W_{\mu\nu}$ − stress–energy tensor of electromagnetic field and other non-gravitational fields.

If we ignore the cosmological constant, consider the metric around a spherical, uncharged, non-rotating mass with the density of its substance ρ_0 and the tensor $\phi_{\mu\nu}=\rho_0 u_\mu u_\nu$, where u_μ is 4-velocity, then in spherical 4-coordinates $x^0=ct$, $x^1=r$, $x^2=\theta$, $x^3=\varphi$, the metric tensor as the solution of equation (20) has the following components:

$$g_{\mu\nu}=\begin{vmatrix} 1-\frac{2\gamma M}{rc^2} & 0 & 0 & 0 \\ 0 & -\frac{1}{1-\frac{2\gamma M}{rc^2}} & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2\sin^2\theta \end{vmatrix}. \qquad (21)$$

This is well-known Schwarzschild solution for the metric around a massive point body of mass M, which depends only on the angle θ and the distance r between the attracting center and observation point.

The equation of motion of GTR for the test body around an attractive mass M is as follows:

$$\frac{d}{ds}\left(\frac{dx^{\nu}}{ds}\right)+\Gamma^{\nu}_{\mu\rho}\frac{dx^{\mu}}{ds}\frac{dx^{\rho}}{ds}=0, \tag{22}$$

where $ds = c\,d\tau$ is invariant interval,

$d\tau$ − differential of proper time of the test body,

dx^{ν} − 4-vector of coordinate differential,

$\Gamma^{\nu}_{\mu\rho}=\frac{1}{2}g^{\nu\sigma}(\partial_{\mu}g_{\sigma\rho}+\partial_{\rho}g_{\sigma\mu}-\partial_{\sigma}g_{\mu\rho})$ − Christoffel symbol, which is expressed through the metric tensor and its derivatives on the coordinates.

If to use the metric tensor (21) to solve (22) for the time component of dx^{ν}, when $\nu = 0$, we get the following:

$$\frac{d}{d\tau}\left(c\,g_{00}\frac{dt}{d\tau}\right)=0. \tag{23}$$

Let multiply (23) on the value mc ,where m is mass of test body, and look at the situation at infinity. Here g_{00} because of the large value of r tends to 1, and the differential of proper time has the same form as in special theory of relativity: $d\tau = dt\sqrt{1-V_{\infty}^2/c^2}$, where V_{∞} denotes the velocity of the test body at infinity. Then (23) becomes an equality for infinity:

$$\frac{1}{\sqrt{1-V_{\infty}^2/c^2}}\frac{d}{dt}\left(\frac{mc^2}{\sqrt{1-V_{\infty}^2/c^2}}\right)=0.$$

In parentheses of the equality is relativistic energy of a body with mass m, which is moving at infinity with speed V_{∞}. Consequently, (23) can be treated as the law of conservation of energy of a test body in a gravitational field (in free fall, the energy of gravitational field converted into kinetic energy, and the sum of negative energy of field and positive kinetic energy is zero). After multiplying (23) on the value mc and integrating we obtain the relativistic energy:

$$\Sigma_m = mc^2 g_{00} \frac{dt}{d\tau} = \frac{mc^2}{\sqrt{1-V_\infty^2/c^2}} = const. \qquad (24)$$

According to (24), a fall of a test body in an attractive center and changing the radial distance r changes $g_{00} = 1 - \frac{2\gamma M}{rc^2}$, as well as the differential of proper time $d\tau$ with respect to the differential of the coordinate time dt, but the relativistic energy of the test body remains unchanged.

Assume now that the particles of the substance of the test body at infinity were once scattered in such a way that their speed V_∞ was near zero, and then the particles will approach a massive body and collide with each other. If in the collision the particles lose part of their total angular momentum, and convert part of their energy into thermal energy of the collision E_f, which is radiated from the system, then a steady rotation of substance around the center of attraction is possible. Condition for this is the fulfillment of the virial theorem, for which modulus of total energy of the system must be equal to energy emitted from systems: $|E_m| = E_f$. As a result the relativistic energy of a test body falling from infinity with zero initial velocity in a gravitational field, reduced by the amount E_f:

$$\Sigma = \Sigma_m - E_f = \Sigma_m - |E_m| = mc^2 + E_m. \qquad (25)$$

Thus, in general theory of relativity substance of the mass m, that rotates in a steady state around the center of attraction, must reduce their energy due to the contribution of negative total energy E_m. The same conclusion will be, if the attractive center is due to the collapse of a massive cloud of substance, which reduces over time its angular momentum through electromagnetic radiation. Equation (25) in its meaning does not coincide with (3), in which the total energy E_0 is not added but subtracted from the rest energy.

Covariant theory of gravitation

In contrast to the general theory of relativity, covariant theory of gravitation (CTG) is based on axioms of Lorentz-invariant theory of gravitation [6], [11], and is a covariant generalization to curved Riemannian space-time. Gravitation in CTG relies not fictitious geometric, but actual physical force, and can be justified using Fatio-Le Sage's theory of gravitation. In CTG substance through a 4-vector density of momentum J^{μ} generates a gravitational field with a 4-potential D^{μ}, satisfying the wave equation in Riemannian space-time:

$$\Box^2 D^{\mu} = \frac{\partial^2 D^{\mu}}{c_g^2 \partial t^2} - \nabla^2 D^{\mu} + R^{\mu}{}_{\nu} D^{\nu} = -\frac{4\pi\gamma J^{\mu}}{c_g^2}, \qquad (26)$$

where c_g – propagation speed of gravity which is close to the speed of light,

$\Box^2$ is 4-d'Alembert operator,

$R^{\mu}{}_{\nu}$ is the Ricci tensor with mixed indices,

γ – gravitational constant.

4-vector density of momentum J^{μ} is determined by the product of the density of substance ρ_0, found in the frame of reference of the substance element, at the 4-velocity: $J^{\mu} = \rho_0 u^{\mu}$. If we use the approximation of weak fields and small velocities, when the CTG transforms in Lorentz-invariant theory of gravitation, the 4-velocity is:

$$u^{\mu} = \frac{dx^{\mu}}{d\tau} \approx \left(\frac{c_g}{\sqrt{1-V^2/c_g^2}}, \frac{\mathbf{V}}{\sqrt{1-V^2/c_g^2}} \right). \qquad (27)$$

The same expression of the 4-velocity (27) with condition $c_g = c$ is adopted in general relativity. In Riemannian space can be introduced [6] operator of differentiation on proper time τ :

$$\frac{D}{D\tau} = u^{\nu} \nabla_{\nu}, \qquad (28)$$

where the symbol D denotes the total differential in curved space-time,

∇_{ν} is the covariant derivative.

When the operation of covariant antisymmetric tensor product of gradient operator on the covariant 4-vector potential D_{μ} is used the gravitational field strength tensor has the form:

$$\Phi_{\mu\nu} = \nabla_{\mu} D_{\nu} - \nabla_{\nu} D_{\mu} = \partial_{\mu} D_{\nu} - \partial_{\nu} D_{\mu} ,$$

In view of $\Phi_{\mu\nu}$ the relationship between substance and field (26) is as follows:

$$\frac{4\pi\gamma J^{\mu}}{c_g^2}=\nabla_{\nu}\Phi^{\mu\nu}. \quad (29)$$

Covariant 4-vector of potential is defined as:

$$D_{\mu}=\left(\frac{\psi}{c_g},-\boldsymbol{D}\right),$$

where ψ – scalar potential,

$\boldsymbol{D}$ – vector potential.

Intrinsic properties of gravitational field, independent of material sources, are given by the relation:

$$\nabla_{\rho}\Phi_{\mu\nu}+\nabla_{\mu}\Phi_{\nu\rho}+\nabla_{\nu}\Phi_{\rho\mu}=\partial_{\rho}\Phi_{\mu\nu}+\partial_{\mu}\Phi_{\nu\rho}+\partial_{\nu}\Phi_{\rho\mu}=0. \quad (30)$$

Relations (29) and (30) have the form in which equations of gravitational field of CTG are covariant in any frame of reference.

The field in turn affects the substance, creating a gravitational force. 4-vector density of the gravitational force is given by:

$$\underset{g}{f}^{\mu}=\Phi^{\mu}{}_{\nu}J^{\nu}=-\nabla_{\nu}U^{\mu\nu}, \quad (31)$$

where $U^{\mu\nu}=\frac{c_g^2}{4\pi\gamma}\left(-g^{\mu\rho}\Phi_{\rho\sigma}\Phi^{\sigma\nu}+\frac{1}{4}g^{\mu\nu}\Phi^{\sigma\rho}\Phi_{\rho\sigma}\right)$ is stress–energy tensor constructed with the help of tensor of gravitational field $\Phi_{\mu\nu}$ and relations (29) – (31). The presence of the tensor $U^{\mu\nu}$ distinguishes CTG compared with general relativity, in which an exact expression for the stress–energy tensor of gravitational field is absent.

The general definition of force in KTG is found through (28):

$$f^{\mu}=\frac{DJ^{\mu}}{D\tau}=u^{\nu}\nabla_{\nu}J^{\mu}=u^{\nu}(\partial_{\nu}J^{\mu}+\Gamma^{\mu}_{\nu\rho}J^{\rho})=\frac{dJ^{\mu}}{d\tau}+\Gamma^{\mu}_{\nu\rho}u^{\nu}J^{\rho}. \quad (32)$$

Electromagnetic force is given by:

$$f_e^{\mu} = F^{\mu}_{\ \nu} j^{\nu} = -\nabla_{\nu} W^{\mu\nu} ,$$

where $F^{\mu}_{\ \nu}$ – electromagnetic tensor, $j^{\nu} = \rho_{0q} u^{\nu}$ – 4-current, ρ_{0q} – charge density in the reference frame where the charge is in rest, $W^{\mu\nu}$ – electromagnetic stress–energy tensor.

If there are only two fundamental fields, gravitational and electromagnetic, which creating forces, the equation of motion of element of substance takes the form:

$$\frac{dJ^{\mu}}{d\tau} + \Gamma^{\mu}_{\nu\rho} u^{\nu} J^{\rho} = -\nabla_{\nu} U^{\mu\nu} - \nabla_{\nu} W^{\mu\nu} . \qquad (33)$$

As it was shown in [11], the equation of motion in general relativity is derived from (33) as a special case.

To determine the space-time metric are used Hilbert-Einstein equations:

$$R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi\gamma\beta}{c_g^4} \left(\phi_{\mu\nu} + U_{\mu\nu} + W_{\mu\nu} \right). \qquad (34)$$

In contrast to (20), in CTG gravitational field, along with the electromagnetic field is involved in definition the metric, so the right-hand side of (34) contains the stress–energy tensor $U_{\mu\nu}$ of gravitational field. The stress–energy tensor of substance $\phi_{\mu\nu}$ in CTG is constructed so that the covariant derivative of this tensor, taken with contravariant indices, gave the force density (32) $f^{\mu} = \nabla_{\nu} \phi^{\mu\nu}$. If we take the covariant derivative of (34), the left-hand side vanishes because of the properties of the metric tensor. This again implies equality for the density of forces (33):

$$f^{\mu} = \nabla_{\nu} \phi^{\mu\nu} = -\nabla_{\nu} U^{\mu\nu} - \nabla_{\nu} W^{\mu\nu} = \frac{dJ^{\mu}}{d\tau} + \Gamma^{\mu}_{\nu\rho} u^{\nu} J^{\rho} .$$

The solution of the equation for the metric (34) with condition $\Lambda = 0$ around an uncharged ball at rest gives components of metric tensor in spherical 4-coordinates $x^0 = ct, \quad x^1 = r, \quad x^2 = \theta, \quad x^3 = \varphi$, [6] :

$$g_{\mu\nu} = \begin{vmatrix} 1+\frac{\gamma M\alpha}{rc^2}-\frac{\beta\gamma^2 M^2}{r^2c^4} & 0 & 0 & 0 \\ 0 & -\frac{1}{1+\frac{\gamma M\alpha}{rc^2}-\frac{\beta\gamma^2 M^2}{r^2c^4}} & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2\sin^2\theta \end{vmatrix}. \qquad (35)$$

The coefficients α and β in (35) from equations (34) are not defined and should be specified for each particular system of bodies.

Using the metric tensor (35), we can find a solution to the equations of motion (33) for the time component of J^{μ}, when $\mu = 0$. In the case of a weak field and at constant density ρ_0 of element of substance is obtained:

$$\frac{d}{d\tau}\left(g_{00}\frac{dt}{d\tau}\right) = -\frac{\gamma M}{c^2 r^2}\frac{dr}{d\tau}, \qquad \text{или} \qquad g_{00}\frac{dt}{d\tau} = \frac{\gamma M}{c^2 r} + C_1. \qquad (36)$$

At infinity g_{00} tends to 1, $d\tau = dt\sqrt{1-V_\infty^2/c^2}$, where V_∞ denotes the velocity of the test body at infinity, and $C_1 = \frac{1}{\sqrt{1-V_\infty^2/c^2}}$. Suppose $V_\infty = 0$, then, after multiplication by mc^2 (36) can be written as follows:

$$\Sigma_m = mc^2 g_{00}\frac{dt}{d\tau} = mc^2 + \frac{\gamma M m}{r} = mc^2 - U. \qquad (37)$$

According to (37) substance, which had at infinity relativistic energy mc^2, during a fall in gravitational field increases its energy by an amount equal to the modulus of the potential energy field $U = -\frac{\gamma M m}{r}$. Although there is difference in CTG of expressions g_{00} and $d\tau$ from the corresponding expressions in general relativity, in (37) holds an approximate equality between the modulus of change of potential energy of gravitational field and the change in kinetic energy of motion of substance.

If for the system is true virial theorem, for which is necessary to decrease the angular momentum of the falling substance, to radiate energy E_f from system and to increase the kinetic energy of the substance on the value $E_k = E_f$, then relativistic energy is equal to:

$$\Sigma = \Sigma_m - E_f = mc^2 - U - E_k = mc^2 - E_m , \tag{38}$$

where $E_m = U + E_k$ is the total energy of mass m in gravitational field.

If the gravitation created by a stationary system with a mass m, the energy E_m in (38) will characterize the change in relativistic energy of the system that has occurred through the action of gravitational field, the interaction of particles of substance and radiation from the system. Relation (38) has the same form as (3), where in front of total energy E_0 is the negative sign. The difference between the results of CTG and general relativity is due to differences in the equations of motion (33) and (22).

Conclusion

Having examined some cases of mass-energy relation, we made the assumption that if the system loses energy in the form of radiation or to do work on the surrounding bodies, the total energy of the particles of the system must be subtracted from the rest energy of the particles making up the system. For fundamental forces the total energy is negative, which leads to an increase in mass of the particles system compared with the sum of the masses of the particles separately. In particular, the mass of a star in accordance with the covariant theory of gravitation can be larger than the total mass of fragments of stellar matter. In another case, when for formation of system is necessary to add energy to it or to do work, the total energy of particles in the system should be added to the rest energy of the particles making up the system. In some cases this leads to a reduction of the relativistic energy and mass of the system (an example is the formation of nuclei of the nucleons).

Our assumptions are essentially the opposite of the standard view, for which a suitable form of the total energy is always just added to the rest energy of the particles that make up the system. As for stars in the general theory of relativity and as for atomic nuclei, this leads to a decrease in their mass compared to the rest mass of the particles making up these objects, and heating of the body increases its mass. Apparently, in such situation is required additional confirmation, if there is in fact increase, or decrease the inert and gravitational mass of massive complex objects as compared with the sum of masses of its parts.

In this connection, should consider the following. If we calculate the share of the gravitational binding energy in relation to the rest energy of substance for a typical neutron star, this share could reach 6%. The same amount is expected for increase (or decrease) the gravitational mass of the star, and hence the force acting on a test body around the star. On the other hand, the force acting on the body, according to the Fatio-Le Sage's theory of gravitation, depends also on the density of the body. For two bodies of low density with sufficient accuracy the law of Newton's gravitational force is satisfied, but when the density of the interacting bodies reaches the density of neutron stars, the force decreases in magnitude and equal to 26% of the Newtonian force. As can be seen, the effect of changing the gravitational mass may depend not only on the total energy of bodies, but also on other parameters, which may complicate experimental verification of the theory.

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