

# The Retarded Phase Factor in Wireless Power Transmission

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## Abstract

In this paper, we present the coupling equations involving retarded phase factor in wireless power transmission. Negative resistance can be achieved in specific range of distance between two resonators. The law of energy conservation is valid when the retarded phase factor is equal to one.

**Keywords:** *wireless power transmission, retarded phase factor, negative resistance*

## Introduction

Nikola Tesla invented wireless power transmission one century ago. This technology has been awoken for modern applications [1-3]. Recently we indicated the dependence of wireless power transmission on retarded phase factor [4,5]. In this work, we solved the coupling equations of two resonators involving retarded phase factor. The characteristic of negative resistance is also revealed.

## Methods

The schematic shown in Fig.1 is very similar to the original designing of Tesla. The circuit on the left, which is called transmitter, is composed of an antenna  $C_1$ , an inductor  $L_1$ , a power source  $V$ , and a resistance  $R_1$ . The circuit on the right, which is called

receiver, is composed of an antenna  $C_2$ , an inductor  $L_2$  and a resistance  $R_2$ . The distance between the two resonators is  $D$ .

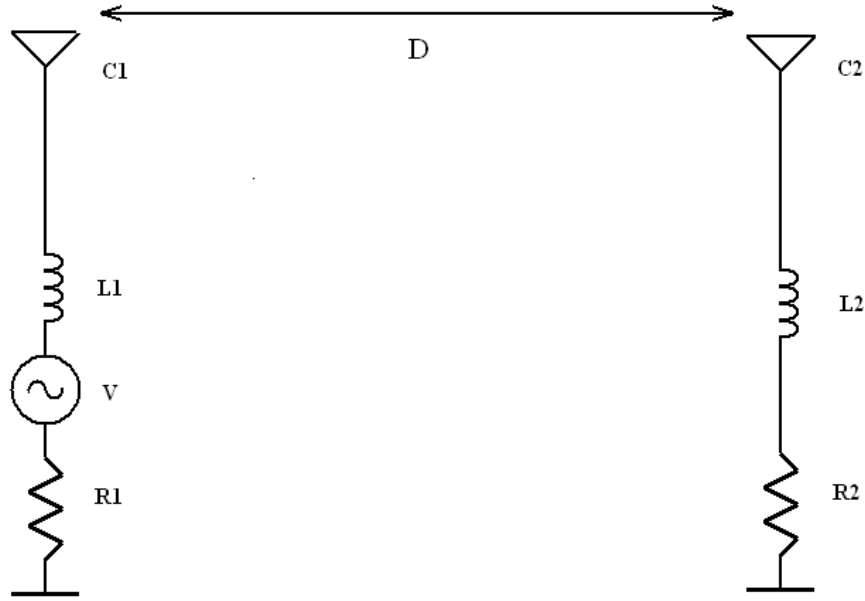


Figure 1: Schematic of two resonators separated in distance  $D$ .

The coupling equations of the system are:

$$I_1 \cdot \left( R_1 + j\omega L_1 + \frac{1}{j\omega C_1} \right) = V + j\omega M \cdot \psi \cdot I_2 \quad (1)$$

$$I_2 \cdot \left( R_2 + j\omega L_2 + \frac{1}{j\omega C_2} \right) = j\omega M \cdot \psi \cdot I_1 \quad (2)$$

where  $M$  is the mutual inductance between two resonators and  $\psi$  is the retarded phase factor:

$$\psi = e^{-i2\pi \frac{D}{\lambda}} \quad (3)$$

These equations are similar to that of Sample [3] except we include the retarded phase factor. If we assume

$$Z_1 = R_1 + j\omega L_1 + \frac{1}{j\omega C_1} \quad (4)$$

$$Z_2 = R_2 + j\omega L_2 + \frac{1}{j\omega C_2} \quad (5)$$

Then the current in the transmitter is

$$I_1 = \frac{V}{Z_1 + \frac{\psi^2 \omega^2 M^2}{Z_2}} \quad (6)$$

If the distance D is short,  $D \ll \lambda$ , then  $\psi \approx 1.0$ . We have

$$I_1 = \frac{V}{Z_1 + \frac{\omega^2 M^2}{Z_2}} \quad (7)$$

The effective impedance of the receiver coupled to the transmitter is positive so that the current  $I_1$  will be reduced after coupling of the receiver. This is what we observed in previous measurements [1-3] since their distances are much less than their wavelength. However, if the distance D is equal to one quarter of wavelength,  $D = \lambda/4$ , then  $\psi = -1.0i$ .

We have

$$I_1 = \frac{V}{Z_1 - \frac{\omega^2 M^2}{Z_2}} \quad (8)$$

The effective impedance of the receiver coupled to the transmitter is negative so that the current  $I_1$  will be increased after coupling of the receiver. This is an interesting result. The effective impedance of the transmitter is

$$Z_1' = Z_1 - \frac{\omega^2 M^2}{Z_2} \quad (9)$$

which could be negative if the value of  $Z_2$  is small enough. In that case, the oscillation amplitudes in transmitter and receiver will increase exponentially.

## Conclusion

In retarded resonance, the power transmission is determined by the retarded phase factor. The effective resistance of the receiver coupled to the transmitter could be negative. The oscillation amplitudes in transmitter and receiver would increase exponentially when the effective resistance of the transmitter is negative.

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