The differential coordinate transformation in the general relativity theory

Sangwha-Yi
Department of Math , Taejon University 300-716

ABSTRACT

:In the general relativity theory's the gravity field's the vacuum, save the line motive particle's coordinate systems' the differential coordinate transformation, study the star's red shift.

PACS Number:**04,04.90.+e,**04.20.-q

Key words:The general relativity theory,

The differential coordinate transformation,

The star's Red shift

e-mail address:sangwha1@**nate.com**

Tel:051-624-3953

I.Introduction

In the general relativity, Schwarzschild solution is

$$d\tau^2 = (1-\frac{2GM}{rc^2})dt^2 - \frac{1}{c^2}[\frac{dr^2}{1-\frac{2GM}{rc^2}} + r^2 d\theta^2 + r^2\sin^2\theta d\phi^2] \quad (1)$$

If $d\theta = d\phi = 0$ is

$$d\tau^2 = (1-\frac{2GM}{rc^2})dt^2 - \frac{1}{c^2}\frac{dr^2}{1-\frac{2GM}{rc^2}}, d\theta = d\phi = 0 \quad (2),$$

$$d\tau^2 = (1-\frac{2GM}{rc^2})dt^2 - \frac{1}{c^2}\frac{dr^2}{1-\frac{2GM}{rc^2}} = (1-\frac{2GM}{rc^2})dt^2[1-(\frac{dr}{dt})^2/c^2(1-\frac{2GM}{rc^2})^2]$$

$$= (1-\frac{2GM}{rc^2})dt^2[1-\overline{V}^2/c^2], \quad \overline{V} = \frac{dr}{dt}\frac{1}{(1-\frac{2GM}{rc^2})} \quad (3)$$

II.Additional chapter

In this case, think that a particle move line motion in the $d\theta = d\phi = 0$,in the r -axis.And think the coordinate systems by the particle's point and velocity.In this time, think the coordinate systems' transformation likely the special relativity theory.In the formula (2), in the r -axis, on the coordinate system $S(t,r)$, move the other coordinate system $S'(t',r')$ by the velocity u ,exist a differential coordinate transformation of the coordinate system $S(t,r)$ and the other coordinate system $S'(t',r')$

$$\sqrt{1-\frac{2GM}{rc^2}}dt = \gamma(\sqrt{1-\frac{2GM}{r'c^2}}dt' + \frac{\overline{u}}{c^2}\frac{dr'}{\sqrt{1-\frac{2GM}{r'c^2}}})$$

$$\frac{dr}{\sqrt{1-\frac{2GM}{rc^2}}} = \gamma(\frac{dr'}{\sqrt{1-\frac{2GM}{r'c^2}}} + \overline{u}dt'\sqrt{1-\frac{2GM}{r'c^2}}) \quad ,$$

$$\overline{u} = u/(1-\frac{2GM}{r'c^2}) \quad , \quad \gamma = \frac{1}{\sqrt{1-\frac{\overline{u}^2}{c^2}}} \quad (4)$$

The formula (4) insert the formula (2),

$$d\tau^2 = (1-\frac{2GM}{rc^2})dt^2 - \frac{1}{c^2}\frac{dr^2}{1-\frac{2GM}{rc^2}}$$

$$= \gamma^2 (\sqrt{1-\frac{2GM}{r'c^2}}dt' + \frac{\bar{u}}{c^2}\frac{dr'}{\sqrt{1-\frac{2GM}{r'c^2}}})^2 - \frac{1}{c^2}\gamma^2(\frac{dr'}{\sqrt{1-\frac{2GM}{r'c^2}}} + \bar{u}dt'\sqrt{1-\frac{2GM}{r'c^2}})^2$$

$$= \gamma^2(1-\frac{\bar{u}^2}{c^2})[(1-\frac{2GM}{r'c^2})dt'^2 - \frac{1}{c^2}\frac{dr'^2}{1-\frac{2GM}{r'c^2}}]$$

$$= (1-\frac{2GM}{r'c^2})dt'^2 - \frac{1}{c^2}\frac{dr'^2}{1-\frac{2GM}{r'c^2}} \quad , \ \bar{u} = u/(1-\frac{2GM}{r'c^2}), \gamma = \frac{1}{\sqrt{1-\frac{\bar{u}^2}{c^2}}}, \ d\theta = d\phi = 0 \quad (5)$$

Therefore, the formula (2) treat the light,

$$d\tau^2 = (1-\frac{2GM}{rc^2})dt^2 - \frac{1}{c^2}\frac{dr^2}{1-\frac{2GM}{rc^2}} = 0, d\theta = d\phi = 0$$

$$cdt = \frac{dr}{(1-\frac{2GM}{rc^2})}, \ ct = r + \frac{2GM}{c^2}\ln|r - \frac{2GM}{c^2}| \quad (6)$$

In the r-axis, in the motive coordinate system $S(t,r)$ and $S'(t',r')$, treat the light's formula (6),

$$cdt = \frac{dr}{(1-\frac{2GM}{rc^2})}, \ cdt' = \frac{dr'}{(1-\frac{2GM}{r'c^2})}$$

$$cdt\sqrt{1-\frac{2GM}{rc^2}} = \frac{dr}{\sqrt{1-\frac{2GM}{rc^2}}}, cdt'\sqrt{1-\frac{2GM}{r'c^2}} = \frac{dr'}{\sqrt{1-\frac{2GM}{r'c^2}}}$$

$$c\sqrt{1-\frac{2GM}{rc^2}}dt = \gamma(\sqrt{1-\frac{2GM}{r'c^2}}cdt' + \frac{\bar{u}}{c}\frac{dr'}{\sqrt{1-\frac{2GM}{r'c^2}}})$$

$$= \gamma(\sqrt{1-\frac{2GM}{r'c^2}}cdt' + \frac{\bar{u}}{c}cdt'\sqrt{1-\frac{2GM}{r'c^2}})$$

$$= \gamma cdt'\sqrt{1-\frac{2GM}{r'c^2}}(1+\frac{\bar{u}}{c})$$

$$= cdt'\sqrt{1-\frac{2GM}{r'c^2}}\frac{\sqrt{1+\bar{u}/c}}{\sqrt{1-\bar{u}/c}}, \ \bar{u} = u/(1-\frac{2GM}{r'c^2}), \gamma = \frac{1}{\sqrt{1-\frac{\bar{u}^2}{c^2}}} \quad (7)$$

III.Conclusion

Therefore in the r-axis, in the motive coordinate system $S(t,r)$'s and $S'(t',r')$'s light's frequency

υ, υ' is

$$dt = \frac{1}{\upsilon} \ , \ dt' = \frac{1}{\upsilon'} \quad (8)$$

The formula (8) insert the formula (7),Hence light's Red shift is

$$\upsilon' = \upsilon \frac{\sqrt{1 - \frac{2GM}{r'c^2}}}{\sqrt{1 - \frac{2GM}{rc^2}}} \frac{\sqrt{1 + \bar{u}/c}}{\sqrt{1 - \bar{u}/c}} = \upsilon \frac{\sqrt{1 - \frac{2GM}{r'c^2}}}{\sqrt{1 - \frac{2GM}{rc^2}}} \frac{\sqrt{1 + u/c(1 - \frac{2GM}{r'c^2})}}{\sqrt{1 - u/c(1 - \frac{2GM}{r'c^2})}}$$

$$\bar{u} = u/(1 - \frac{2GM}{r'c^2}) \quad (8)$$

Reference

[1]S.Weinberg,Gravitation and Cosmology(John wiley & Sons,Inc,1972)

[2]C.H.Lee,The genaral relativity theory(Minumsa pub. Co.Inc.,Daewoo Academic book Publishing Natural science 38,Korea,Seoul,1986),Chapter 7

[3]J.brault,"The Gravitational Red shift in the Solar Spectrum",doctoral dissertation,Princeton University(1962)

[4]C.Misner, K,Thorne and J. Wheeler, Gravitation(W.H.Freedman & Co.,1973)

[5]S.Hawking and G. Ellis,The Large Scale Structure of Space-Time(Cam-bridge University Press,1973)

[6]R.Adler,M.Bazin and M.Schiffer,Introduction to General Relativity(McGraw-Hill,Inc.,1965)