Big crash of basic concepts of physics of the 20th century?

Peter Sujak
Prague, Czech Republic
peter.sujak@email.cz
(July 30, 2011)

PACS
03.65.Ca. Formalism
03.65.Pm Relativistic wave equations

Abstract
This paper analyzes energy and momentum in the definition relations of relativistic energy, in the de Broglie momentum hypothesis and in Klein-Gordon, Dirac and Schrodinger equation. Results of analyzes show that Planck constant and relativistic relationships on the length contraction and increase in mass are the reflection of the same physical principle of nature, that in de Broglie momentum idea $h/\lambda = mv$ the wave of matter λ needs to be connected with real dimension l_o of a particle and that on this basis we can come to the fundamental QM equation Klein-Gordon, Dirac and Schrodinger without necessity of wave function .

1. Introduction

The relationships of quantum physics for energy $E=h\nu$ and momentum $p=h/\lambda$ of a photon and the relationship of relativistic mechanics for total energy $E_t = mc^2 = (m_o{}^2v^2c^2/(1-v^2/c^2)+m_o{}^2c^4)^{1/2}$ are the basic relationships of contemporary physics. From the quadratic form of the total energy relationship $E_t{}^2 = m^2c^4 = m_oc^4 + m^2c^2 = E_o{}^2+E_k{}^2$ we can deduce that the kinetic i.e. added energy to the rest energy $E_o = m_oc^2$ is $E_k = mvc$. We can also derive this relation from the total relativistic energy as

$$E_k = (m^2c^4-m_o{}^2c^4)^{1/2} = m_oc^2(1/(1-v^2/c^2)-1)^{1/2} = m_oc^2(c^2/(c^2-v^2)-1)^{1/2} = m_oc^2(v^2/(c^2-v^2))^{1/2} = \quad \textbf{(1)}$$
$$= m_ovc(c^2/(c^2-v^2))^{1/2} = m_ovc/(1-v^2/c^2)^{1/2} = mvc = pc \quad \text{so} \quad E_k = mvc = pc = mc^2.v/c.$$

In quantum mechanics (QM) so as in relativistic mechanics (RM) for kinetic energy we write directly $E_k=mc^2-m_oc^2=E_t-E_o$ so $E_k=mc^2-m_oc^2=mvc=pc$ in contrast to total energy in RM where we write the square root of the sum of squares $E_t = (m^2v^2c^2+m_o{}^2c^4)^{1/2} = (E_k{}^2+ E_o{}^2)^{1/2}$.
This way we get the relation for the ratio of kinetic energy to momentum as $E_k/p=mvc/mv=c$ and for the ratio of total energy to momentum as $E_t/p=mc^2/mv =c^2/v$ or $E_t/pc= mcc/mvc= c/v$. Consequently as speed of an object v approaches the speed of light c $v\rightarrow c$ its momentum multiplied by c approaches total energy $pc\rightarrow E_t$.
RM establishes the different definition of kinetic energy $E_k=mc^2-m_oc^2=mvc$ from classical mechanics (CM) $E_k = ½mv^2$. In RM the relation of classical kinetic energy is subsequently seen as an approximation of the relativistic kinetic energy relation and the classical kinetic energy relation can be found by expanding the relativistic relation into Taylor series $E_k=mc^2-m_oc^2=m_oc^2(1/(1-v^2/c^2)^{1/2} -1)= =(1+½m_ov^2 +3/8m_ov^2/ c^2..-1) \approx ½ .m_ov^2$. By this expansion we change the definition status of kinetic energy from a linear functionality in RM at the speed approaches the speed of light into a quadratic functionality in CM at a speed much slower than the light's speed in the vacuum.

2. Energy and momentum of a photon. Energy and momentum in relativistic mechanics.

The relation for relativistic kinetic energy $E_k = mc^2-m_oc^2 = mvc$ however has a general force and is active at any speed thus also at speeds much slower than the speed of light. (The first postulate of RM- all physical laws are the same form at any inertial systems of coordinates).
The relations of a photon's energy $E = h\nu$ and relativistic energy $E_t = (m^2v^2c^2 + m_o{}^2c^4)^{1/2}$ are tied together by the interaction of photons with particles such as the photoelectric effect, the scattering effect or electron –positron pair production (EPP). For EPP we write the relation between photon's

energy and kinetic energy of an electron and a positron as $h\nu - h\nu_o = ½m_e v^2 + ½m_p v^2$ and for the electron's portion we can write $½h\nu - ½h\nu_o = ½m_e v^2$. The frequency ν_o is the minimal frequency of photon's energy necessary for EPP and equals to internal energy of an electron $½h\nu_o = hc/2\lambda_o = ½m_o c^2$. This frequency ν_o corresponds to the Compton wavelength of an electron $\lambda_o = h/m_o c$ and thus we can reasonably suppose that λ_o corresponds to the maximum radius $l_o = h/m_o c^2$ of created electrons .The energy balance relation for EPP is based on the photoelectric effect explanation where the difference between incident energy of photons $h\nu$ and binding energy of electrons $h\nu_o$ equals the kinetic energy of electrons $\Delta E = h\nu - h\nu_o = ½m_o v^2$ emitted out of atoms. The classical mechanics relationship for electron's kinetic energy $E_k = ½m_o v^2$ in the photoelectric effect relation $\Delta E = h\nu - h\nu_o = ½m_o v^2$ is however approximative and according to RM we can write the relation $h\nu - h\nu_o = hc/\lambda - hc/\lambda_o = mc^2 - m_o c^2 = mvc = pc$. This relation is in accordance with the relativistic understanding of energy E=pc since by dividing $hc/\lambda - hc/\lambda_o = mvc$ by c we get the relationship for momentum at the photoelectric effect in writing as $h/\lambda - h/\lambda_o = h\nu/c - h\nu_o/c = mv = mc - m_o c = p_t - p_o = p$. This momentum relation has the same form in CM and RM and in contrast to the energy relation there is none two-faced writing or approximation of its right side hand. Following this momentum relation we can come to the idea that in the same way as we understand kinetic, total and internal energy in RM we can consider about momentum p, total momentum p_t and internal rest momentum p_o. We can imagine the internal rest momentum p_o as rotation or as the angular momentum spin installed in QM.

Thus we can talk about Einstein's definition of energy (EDE) where momentum multiplied by c equals energy E = pc. In EDE total momentum of a photon and particle $p_t = h/\lambda = h\nu/c = mc$ multiplied by c gives the total energy $E_t = p_t.c = hc/\lambda = h\nu c/c = mc.c$. Total energy of the photon begins from zero energy so zero mass then from zero frequency $\nu=0$ and from infinity photon wavelength $\lambda=\infty$.

The particle total energy begins from rest energy then rest mass $E_o = m_o c.c = p_o c = hc/\lambda_o = h\nu_o c/c$ where λ_o and ν_o are the wavelength and frequency of photon's energy needed for EPP. In EDE momentum $p = mv = mcv/c = (h/\lambda - h/\lambda_o) = h(\lambda_o - \lambda)/\lambda\lambda_o = h/\lambda .(\lambda_o - \lambda)/\lambda_o = h/\lambda .v/c$ multiplied by c gives change in energy $E_k = pc = mvc = h\nu - h\nu_o = (h/\lambda - h/\lambda_o).c = hc(\lambda_o - \lambda)/\lambda\lambda_o = mcc.v/c$.The relationship E/p=c in EDE is however valid just for the ratio of corresponding quantities that is for total quantities $E_t/p_t = h\nu/h\lambda^{-1} = \nu\lambda = mc^2/mc = c$ for the change in quantities $E_k/p = mvc/mv = (h/\lambda - h/\lambda_o).c /(h/\lambda - h/\lambda_o) = c$ and for rest quantities $E_o/p_o = h\nu_o/h\lambda_o^{-1} = c$. The ratio for non corresponding quantities is $E_t/p = mc^2/mv = h\nu/(h/\lambda - h/\lambda_o) = hc\lambda^{-1}/h(\lambda_o - \lambda)(\lambda\lambda_o)^{-1} = c\lambda_o/(\lambda_o - \lambda) = c.c/v = c^2/v$ and also $E_k / E_t = mvc/mcc = v/c$ and $p/p_t = mv/mc = v/c$. The substantial relation valid in RM as well as in QM $E_t/p = c^2/v$ resulting also from $p^2c^2 = m^2c^4 - m_o^2c^4 = m^2v^2c^2$ then pc=mvc then $pc = mc^2v/c = E_t v/c$ then $E_t/p = c^2/v$ is interpreted the way that as the speed $v \to c$ then momentum multiplied by c approaches to total energy $pc \to E_t$ .Then we can interpret the relation $E_t/pc = mcc/mvc = p_t c/pc = c/v$ so that for $v \to c$ consequently $pc \to E_t$ since kinetic energy approaches total energy $mvc \to mcc$ and since momentum approaches total momentum $mv \to mc$ so $p \to p_t$. The same way as in the relation $E_k = mc^2 - m_o c^2 = mvc$ for $v \to c$ rest energy becomes negligible and kinetic energy approach total energy $mvc \to mcc$ so in a relation $p = mc - m_o c = mv$ rest momentum becomes negligible and momentum approaches total momentum $mv \to mc$.

In RM and in QM we talk about total energy of a particle $E_t = mc^2$ and about total energy of a photon $E_t = h\nu = hc/\lambda = mc^2$. Also it is necessary to accent that in the relationship for photon's momentum $p_t = h/\lambda = mc$ we talk about photon's total momentum. Total energy and momentum of a photon as well as the mass equivalent and frequency of a photon are running from the zero values and a wavelength from an infinite value. Total relativistic energy for a particle is running from rest energy $m_o c^2 = h\nu_o = hc/\lambda_o$ by adding kinetic energy. Thus if we want to formulate the energy relation of a particle similarly to the photon's relation $E_t = h\nu = hc/\lambda = mc^2$, then kinetic energy we can write in relation $mc^2 - m_o c^2 = mvc = mc^2.v/c = h\nu - h\nu_o = hc/\lambda - hc/\lambda_o = hc\lambda^{-1}(\lambda_o - \lambda)/\lambda_o = mcc.v/c = pc$ and for momentum we can consequently get $mc - m_o c = mv = h\nu/c - h\nu_o/c = h/\lambda - h/\lambda_o = h(\lambda_o - \lambda)/\lambda\lambda_o = h\lambda^{-1}(\lambda_o - \lambda)/\lambda_o = h/\lambda.v/c = mcv/c = p$.

Thus we can believe that if we wish to transfer the photon's relation $E = h\nu$ and $p = h/\lambda$ onto an electron and if we consider $h/\lambda - h/\lambda_o = mv$ as momentum of a particle just as we take kinetic energy in relation $mc^2 - m_o c^2 = h\nu - h\nu_o = hc/\lambda - hc/\lambda_o$ and also consider as total momentum of a particle $h/\lambda = mc$ and total energy $hc/\lambda = mc^2$ then we can obtain the non-controversial ratio of energy to momentum written as $E/p = E_t/p = mc^2/mv = h\nu/(h/\lambda - h/\lambda_o) = hc\lambda^{-1}/h(\lambda_o - \lambda)(\lambda\lambda_o)\lambda^{-1} = c\lambda_o/(\lambda_o - \lambda) = c.c/v = c^2/v$ and ratio of energy to momentum $E/p = E_k/p = (mc^2 - m_o c^2)/mv = mvc/mv = (h\nu - h\nu_o)/(h/\lambda - h/\lambda_o) = c(h/\lambda - h/\lambda_o)/(h/\lambda - h/\lambda_o) = c$.

Subsequently we can believe that the ratio $E/p=E_t/p=mc^2/mv\neq h\nu/h\lambda^{-1}$ is not valid and accurate are the ratios $E/p=E_t/p=mc^2/mv=h\nu/(h/\lambda-h/\lambda_o)=c^2/v$ and $E/p=E_t/p_t=mc^2/mc= h\nu/h\lambda^{-1} =c$

3. Is the de Broglie hypothesis $h/\lambda = mv$ **accurate?**

De Broglie [1] introduced the presumptions, that the photon's relationships can be transferred onto a particle as $p=h/\lambda =mv$ and $E=h\nu =mc^2$ where rest energy of a particle is associated with the frequency $h\nu_o=m_oc^2$ and particle momentum is associated with a particle's wavelength $\lambda=h/mv$ whose value is infinite $\lambda = \infty$ for zero momentum $mv = 0$. From these presumptions de Broglie comes to two different ratios of energy to momentum $E/p = h\nu/h\lambda^{-1} = hc\lambda/h\lambda = \lambda\nu =c$ and concurrently $E/p = mc^2/mv =c^2/v$. De Broglie worked out this paradox by the phase velocity $w=c^2/v=\omega/k$ and the group velocity of a particle. Consequently a phase velocity is always higher than the light's speed c and the phase velocity is infinity for a speed $v = 0$ and at a speed $v\rightarrow c$ the phase velocity approaches from infinity to c. The entire taking–over the deBroglie's formalism of the wave property of matter by QM leads to the wave function and the wave probability of particles propagation. Up to this day the meaning and role of the phase velocity and wave function is unexplained. The paradox is clearly seen if we put in $\lambda=h/mv$ into the total energy relation $E=mc^2=h\nu=hc/\lambda$ then $E=mc^2=hc/\lambda=hcmv/h=mvc$ so we get $E=mc^2=mvc$ that is valid only when $c=v$ so if total energy equals to kinetic energy and this is also valid for a free particle in QM. But from the foregoing considerations we can see that the discrepancy $c=c^2/v$ so $c=v$ results from the simultaneous apparent validity of the ratios $E/p=h\nu/h\lambda^{-1}=c$ and $E/p=mc^2/mv =c^2/v$ instead of factual validity of the ratios $E_t/p_t=mc^2/mc=h\nu/h\lambda^{-1}=hc\lambda /h\lambda=\lambda.\nu =c$ and $E_t/p=mc^2/mv=h\nu/(h/\lambda-h/\lambda_o)= =hc\lambda^{-1}/h(\lambda_o-\lambda)(\lambda\lambda_o)^{-1} = c\lambda_o/(\lambda_o-\lambda)=c.c/v =c^2/v$. The discrepancy disappears also if we put in $p_t =h/\lambda=mc$ so $\lambda=h/mc$ into $E =mc^2=h\ \nu=hc/\lambda= hcmc/h =mc^2$ and so we get $mc^2 = mc^2$. Thus we see that we can write the ratio of energy and momentum for total energy $E/p=h\nu/mv=mc^2/mv=c^2/v$ or a relation for added energy $E/p = (h\nu -h\nu_o)/mv = (mc^2- m_oc^2) /mv = mvc/mv = c$ according to taking energy from $\nu=0$ so $E= h\nu(0)= mc^2$ or from $\nu=\nu_o$ so $\Delta E = h\nu -h\nu_o= mvc$.

4. The length contraction and increase in effective mass operate inseparablely

Based upon the foregoing considerations in this paper we can come to the conviction that for transferring the photon's relation of momentum $p=h/\lambda$ and energy $E=h\nu$ onto an electron it is necessary to correctly transfer the quantities of total, added and rest energy and momentum as well as transfer the dynamics of increase in the spatial energy concentration expressed in a raising photon's frequency jointly with a shortening of the photon's dimension expressed in shortening of the photon wavelength. Thus in the same way consider about shortening in the electron dimension with increasing in electron's energy. We can regard the principle of the spatial shrinking of mass-energy with increasing mass-energy as the universal principle of the nature. So as at the photon so as in nuclear physics so as in astrophysics the greater accumulation of mass represents greater energy and leads to its less spatial localization. About this principle in fact predicate also the relativistic relationships on the length contraction and increase in effective mass with increasing energy as a result of an increasing speed. If we consider the relativistic relations of increasing in mass $m= m_o/(1-v^2/c^2)^{1/2}$ and the length contraction $l = l_o(1-v^2/c^2)^{1/2}$ then we can write $(1-v^2/c^2) = m_o^2/m^2 = l^2/l_o^2$ or rewrite it to the relationship $m_o^2c^2/m^2(c^2-v^2) = l^2c^2/l_o^2(c^2-v^2)$ or $m_o^2l_o^2c^2 = m^2l^2c^2(c^2-v^2) / (c^2-v^2)$. Thus for any difference in (c^2-v^2) the products $m_o^2l_o^2c^2=m^2c^2l^2=h^2$ remains constant so the product of mass and its spatial lay out remains constant $m_ol_o = h/c=2.21\times10^{-42}$ kg.m .For proton mass $m_o=1.673\times10^{-27}$kg this leads to Compton wavelength of proton $l_o=1.32\times10^{-15}$m. Afterwards against the calibration basis $h^2= m_o^2l_o^2c^2$ or $h^2/l_o^2=m_o^2c^2$ we can express the total value as $h^2/l^2 = m^2c^2$ and also added or change in the value as $h^2/l^2- h^2/l_o^2=m^2c^2-m_o^2c^2$.

Consequently for an electron with increase in its energy we have to consider about the decrease in its radius from the rest value l_o at rest energy needed for EPP so $m_oc^2= h\nu_o =hc/l_o$ so $m_oc=h/l_o$ so from Compton wavelength $l_o =h/m_oc$ in compliance with the length contraction and increase in effective mass. Then for speed approaching c the energy approaches infinity and radius approaches zero and thus the speed of the particle can not equal c since its radius would be zero. For the great concentrations of matter as so for the great concentrations of energy we nowadays accept as a natural that the dimensions approach zero e.g. for the black holes but in the same natural way we do not consider the great changes of the speed ,energies an the potentials of particles in micro-world. Up to the present-day physics has not arrived to a concrete value of an electron's dimension and in more publications (also in CODATA [2])) it states a great radius for slow electrons and a short radius for fast electrons and so we may believe in the relation between the changing of an electron's dimension with its energy. Thus we may consider that if we use rest mass in relation $h/\lambda=mc$ we get the Compton wavelength of an electron that is the rest diameter of a free electron $\lambda_o=l_o=h/m_oc=2.43\times10^{-12}$ so as for a free proton we get $\lambda_o=l_o=h/m_oc=1.32\times10^{-15}$. So if in QM rest energy of the particle is associated with the frequency $h\nu_o=m_oc^2$ then we can reasonably suppose that this rest energy is associated also with the dimension $m_oc^2=h\nu_o=hc/l_o$. This frequency $\nu_o=m_oc^2/h$ thus corresponds to the Compton wavelength of an electron $\lambda_o=h/m_oc^2$ thus to the ultimate diameter $l_o=h/m_oc^2$ of a created electron in EPP. This way the transfer of the photon's momentum relation $p=h/\lambda$ onto a particle represents a change in a particle dimension from the Compton wavelength value $h/l_o=m_oc$ following the change in momentum of a particle with increasing in its speed v as $h/l-h/l_o = h(l_o-l)/ll_o = h/l.(l_o-l)/l_o = mc.v/c = mv = p$.

According to the presented conviction of change in the electron's dimension with a change in electron's energy we can believe that the experiments demonstrating the wave property of electrons can be explained by a real change in the dimensions of electrons. In this meaning we can think about the Bragg's x-rays interference law $n\lambda= 2d\sin\theta$. So the interference of light as the interference of photons is firmly linked with the photon's wavelength λ and then with the photon's dimension. Subsequently we can believe that experiments for electrons presented to support the de Broglie wave hypothesis for example the Davisson-Germer's [3] experiment (where the relation $n\lambda= d\sin\theta$ is accounted for an explanation) can be interpreted as the real change of an electron's dimension with a change in its energy.

Thus we can reasonably suppose that in the same way as for the change in momentum of a photon the same is true for the change in electron's momentum from the rest state and we can write relationship $h/\lambda_1-h/\lambda_2=h/l-h/l_o=mv$ where l_o is the real dimension of an electron that becomes shorter as its energy increases .The classical kinetic energy of an electron consequently is expressed in in relationship $E_k=\frac{1}{2}mv^2=p^2/2m_o=h^2/2m_o\nabla^2=h^2/2m_ol^2-h^2/2m_ol_o^2= h^2/2m_o.(l_o^2-l^2)/l^2l_o^2$. Moreover if we consider the relativistic relations of the length contraction $l=l_o(1-v^2/c^2)^{1/2}$ so $(1-v^2/c^2)= l^2/l_o^2$ so $v^2/c^2=(l_o^2-l^2)/l_o^2$ and increase in mass $m= m_o/(1-v^2/c^2)^{1/2}$ we can write electron's kinetic energy as $h^2/2m_o.(l_o^2-l^2)/l^2l_o^2= =h^2/2m_ol^2.(l_o^2-l^2)/l_o^2= h^2/2m_ol^2.v^2/c^2$ and for $h= m_ol_oc$ we get

$$h^2/2m_ol^2.v^2/c^2 = m_o^2l_o^2c^2/2m_ol^2.v^2/c^2 = m_o^2c^2/2m_o.(1-v^2/c^2).v^2/c^2 = m_o^2v^2/2m_o(1-v^2/c^2) = =m^2v^2/2m_o = p^2/2m_o = \tfrac{1}{2}.mv^2. \quad \textbf{(2)}$$

If we directly put in relativistic momentum into the classical kinetic energy form $E_k=\frac{1}{2}.mv^2=p^2/2m_o = =m_o^2v^2/2m_o(1-v^2/c^2)$ then from classical kinetic energy we get classical relativistic kinetic energy. As in RM is valid $m^2v^2=m^2c^2- m_o^2c^2$ then in the same manner is valid $m^2v^2/2m_o=m^2c^2/2m_o- m_o^2c^2/2m_o$ and for total energy we can write

$$m^2v^2/2m_o + m_o^2c^2/2m_o = m_o^2v^2/2m_o(1-v^2/c^2) + m_o^2c^2/2m_o = m_o^2c^2/2m_o(v^2/(c^2-v^2)+1) = =m_o^2c^2/2m_o(1-v^2/c^2) =m^2c^2 /2m_o \quad \text{so} \quad m^2/m^2_o =1/(1-v^2/c^2) \text{ then } m/m_o =1/(1-v^2/c^2)^{1/2}. \quad (3)$$

5. Do the relations $E=h\nu$ and $E=mc^2$ express the quantity of energy?

From the foregoing considerations we can believe that the classical kinetic energy relationship $E_k= \frac{1}{2}.m_ov^2 = p^2/2m_o =m^2v^2/2m_o = m_o^2v^2/2m_o (1-v^2/c^2)= m^2c^2/2m_o-m_o^2c^2/2m_o=mc^2-m_oc^2$ has to be perceived as a limit of the relation $\mathbf{E}_k=2mE_k =m^2v^2 =p^2$ at speeds much slower than the speed of light. In RM we derive the equation of total relativistic energy from the relation $m= m_o/(1-v^2/c^2)^{1/2}$ and after bring it to the square $m^2c^2 = m^2v^2 + m_o^2c^2$ we multiply it by c^2.After applying the square root we get $E= mc^2 =(m^2v^2c^2+ m_o^2c^4)^{1/2}$.Then we can reasonably believe that the step of multiplying by c^2 is physically unfounded and is intentional in order to ensure the dimension of energy after resulting square root and just because of that we determine energy as momentum multiplied by c.

Afterwards according to the transfer of the relations of photon's momentum onto a particle presented in this paper the relation for CRKE $\mathbf{E}_k = m^2c^2-m_o^2c^2 = m^2v^2 = m^2c^2v^2/c^2$ harmonizes with the relation

$\mathbf{E}_k = h^2/l^2 - h^2/l_o^2 = h^2(l_o^2 - l^2)/l^2 l_o^2 = h^2/l^2.v^2/c^2$ as well as with the relation for frequency expression we get $\mathbf{E}_k = h^2\nu^2/c^2 - h^2\nu_o^2/c^2 = h^2(\nu^2-\nu_o^2)/c^2 = h^2\nu^2/c^2.v^2/c^2$ where l_o is the rest dimension and ν_o the rest spin frequency of a particle at rest mass m_o jointed in the Compton wavelength relation $h/l_o = m_oc = h\nu_o/c$. So for CRKE we can write $\mathbf{E}_k = 2mE_k = h^2/\lambda^2 - h^2\lambda_o^2 = h^2/l^2 - h^2/l_o^2 = h^2(\nu^2-\nu_o^2)/c^2 = m^2c^2 - m_o^2c^2 = m^2v^2 = p^2$ and for momentum we can write $h/\lambda - h/\lambda_o = h/l - h/l_o = h(\nu - \nu_o)/c = mc - m_oc = mv = p$. After multiplying the last relation with c $hc/\lambda - hc/\lambda_o = hc/l - hc/l_o = h\nu - h\nu_o = mc^2 - m_oc^2 = mvc = E_k$ we get the photoelectric effect explanation by Einstein.

Then for the classical relativistic kinetic energy we can write the relation

$$\mathbf{E}_k = 2mE_k = h^2/l^2 - h^2/l_o^2 = h^2\nabla_o^2 = h^2/l^2.(l_o^2-l^2)/l_o^2 = h^2/l^2.v^2/c^2 = h^2\nu^2/c^2.v^2/c^2 = m^2c^2v^2/c^2 = m^2v^2. \quad \textbf{(4)}$$

The kinetic energy i.e. the added energy written as $h^2/\nabla_o^2 = h^2/l^2.v^2/c^2 = h^2\partial\nu^2/c^2\partial t^2 = h^2\nu^2/c^2.v^2/c^2$ runs for an electron from the rest energy $m_o^2c^2$ and to this energy linked rest values $h^2/l_o^2 = h^2\nu_o^2/c^2 = m_o^2c^2$ where symbols ∇_o and $\partial_o\nu^2/\partial t^2$ means that $\nabla_o^2 = (1/l^2 - /l_o^2)$ and $\partial_o\nu^2/\partial t^2 = (\nu^2 - \nu_o^2)$ runs from l_o, ν_o. The classical kinetic energy $E_k = m^2v^2/2m_o = ½m_ov^2 = m^2c^2/2m_o - m_o^2c^2/2m_o = mc^2 - m_oc^2$ is limit of the relation $\mathbf{E}_k = 2mE_k = m^2v^2 = p^2$ and is added energy compared to the values l_o, ν_o and to rest energy $m_o^2c^2$. The difference of added energy expressed by l or ν equals zero $h^2\nabla_o^2 - h^2\partial_o\nu^2/c^2\partial t^2 = h^2/l^2.v^2/c^2 - h^2\nu^2/c^2.v^2/c^2 = 0$. If we write the relation $h^2\nabla^2 = h^2/l(\infty)^2 = h^2\partial/c^2\partial t^2 = h^2\nu(0)^2/c^2 = m(0)^2c^2$ and we mean that values of ν, m run from $\nu_o = 0$, $m_o = 0$ and value l runs from infinity $l_o = \infty$ so we talk about the total energy. The difference of total energy expressed by l or ν $h^2/\nabla^2 - h^2\partial^2/c^2\partial t^2 = h^2\nu^2/c^2 - h^2/l^2 = m^2c^2 - m^2c^2 = 0$ equals zero where all terms for energy in the last relation increase from a zero and up to the own values of an electron $m_o^2c^2 = h^2/l_o^2 = h^2\nu_o^2/c^2$ this increase means rest energy of the particle so for instance energy of the photon needed for EPP.

We can write the total classical relativistic energy (TCRE) $\mathbf{E}_t$ of a particle

$$h^2\nabla^2 = h^2\nabla(\infty)^2 = h^2/l(\infty)^2 = h^2\nabla_o^2 + h^2/l_o^2 = h^2/l^2.v^2/c^2 + m_o^2c^2 = m^2c^2v^2/c^2 + m_o^2c^2 = m^2v^2 + m_o^2c^2 = m^2c^2 \quad \textbf{(5)}$$

and for frequencies expression

$$h^2\partial^2/c^2\partial t^2 = h^2\nu(0)^2/c^2 = h^2\partial_o\nu^2/c^2\partial t^2 + h^2\nu_o^2/c^2 = h^2\nu^2/c^2.v^2/c^2 + m_o^2c^2 = m^2c^2v^2/c^2 + m_o^2c^2 = m^2c^2 \quad \textbf{(6)}$$

where l_o, ν_o, m_o are the Compton values and symbols $l(\infty)$, $\nu(0)$ mean that l runs from infinity and ν runs from zero.

Thus we can see that for added values in the relation $h^2\partial_o\nu^2/c^2\partial t^2 - h^2\nabla_o^2 = h^2\nu^2/c^2v^2/c^2 - h^2/l^2.v^2/c^2 = m^2c^2v^2/c^2 - m^2c^2v^2/c^2 \neq m_o^2c^2$ as well as for total value $h^2\nabla^2 - h^2\partial^2/c^2\partial t^2 = h^2\nu^2/c^2 - h^2/l^2 \neq m_o^2c^2$. Then in the classical relativistic energy definition for the particle total energy we can write

$$h^2\nabla^2 - h^2\partial^2/c^2\partial t^2 = (h^2\partial_o\nu^2/c^2\partial t^2 + m_o^2c^2) - (h^2\nabla_o^2 + m_o^2c^2) = (h^2\nu^2/c^2.v^2/c^2 + m_o^2c^2) - (h^2/l^2.v^2/c^2 + m_o^2c^2) =$$
$$=(m^2c^2v^2/c^2 + m_o^2c^2) - (m^2c^2v^2/c^2 + m_o^2c^2) = (m^2v^2 + m^2_oc^2) - (m^2v^2 + m_o^2c^2) = m^2c^2 - m^2c^2 = 0\ . \quad \textbf{(7)}$$

Consequently for the difference of total and added energy $h^2\partial^2/c^2\partial t^2 - h^2\nabla_o^2$ or $h^2\partial_o\nu^2/c^2\partial t^2 - h^2\nabla^2$ we can write

$$h^2\partial^2/c^2\partial t^2 - h^2\nabla_o^2 = (h^2\partial_o\nu^2/c^2\partial t^2 + m_o^2c^2) - h^2\nabla_o^2 = h^2\nu(0)^2/c^2 - h^2/l^2.v^2/c^2 = m^2c^2 - m^2c^2v^2/c^2 = \quad \textbf{(8)}$$
$$= m^2c^2 - m^2v^2 = m_o^2c^2$$

or

$$h^2\partial_o\nu^2/c^2\partial t^2 - h^2\nabla^2 = h^2\partial_o\nu^2/c^2\partial t^2 - (h^2\nabla_o^2 + m_o^2c^2) = h^2\nu^2/c^2.v^2/c^2 - (h^2/l^2.v^2/c^2 + m_o^2c^2) = \quad \textbf{(9)}$$
$$= h^2\nu^2/c^2.v^2/c^2 - h^2/l^2 = m^2c^2v^2/c^2 - m^2c^2 = m^2v^2 - m^2c^2 = -m_o^2c^2.$$

From the last relation $h^2\nu^2/c^2.v^2/c^2 - h^2/l^2 = m^2c^2v^2/c^2 - m^2c^2 = m^2v^2 - m^2c^2 = -m_o^2c^2$ we see that if we substitute ν by $\omega^2 = \nu^2/c^2.v^2/c^2$ so $\omega = \nu.v/c^2$ thus if we substitute connection $\nu l = c$ by connection $\omega l = v/c^2$ we can write $h^2\omega^2/c^2 - h^2/l^2 = -m_o^2c^2$.

6. Where is energy and momentum in the Klein-Gordon and Dirac equation?

So for $\nu.l= \omega/k=c$ we can write the relation $h^2\nu^2/c2.v^2/c^2 - h^2/l^2= m^2c^2v^2/c^2 - m^2c^2 = - m_o^2c^2$ or we can write the relationship $h^2\nu^2/c^2 - h^2/l^2.v^2/c^2 = m^2c^2 - m^2c^2v^2/c^2 = m_o^2c^2$ but we can not write relationship as $\hbar^2\omega^2/c^2 - \hbar^2k^2 = h^2\nu^2/c^2 - h^2/l^2 = h^2\partial^2/c^2\partial t^2 - h^2\nabla^2 \neq m_o^2c^2$. But if we take $\omega/k=c^2/v$ we can write the relation $\hbar^2\omega^2/c^2 - \hbar^2k^2 = h^2\nu^2/c^2.v^2/c^2 - h^2/l^2 = -m_o^2c^2$ or we can write $\hbar^2\omega^2/c^2 - \hbar^2k^2 = h^2\nu^2/c^2 - h^2/l^2.v^2/c^2 = m_o^2c^2$ But as in QM the wave function Ψ provides ratio $c^2/v = \nu\lambda$ so $\nu^2v^2/c^4 = 1/\lambda^2$ then using the wave function we can write $\hbar\partial^2/c^2\partial t^2\Psi - \hbar^2\nabla^2\Psi = - m_o^2c^2\Psi$ what is writing of the Klein-Gordon (K-G) equation. So we can believe that we can write then the Klein-Gordon equation without the wave function Ψ in the form $\pm h^2\nu^2/c^2.(v^2/c^2 \text{ or } 1) \pm h^2/l^2.(1 \text{ or } v^2/c^2) = \pm o\ m_o^2c^2$ according to our request start from $\nu = \nu_o$ and $l_o = \infty$ (de Broglie anticipation) then $m^2_oc^2 = -m_o^2c^2$ or $\nu=0$ and $l=l_o$ then $m_o^2c^2=m_o^2c^2$ and if we start from $\nu=0$ and $l=\infty$ or $\nu = \nu_o$ and $l=l_o$ then $m^2_oc^2 = 0$.

Thus we can believe that if ν and l or alternatively t and x run from mutually corresponding values than the d'Alembertian is always zero $\Box = 0$. The K-G equation for a free particle with the wave function $\Box\Psi = -(m_o^2c^2/\hbar^2)\Psi$ can be written if ω and k or alternatively t and x do not run from the mutually corresponding value and than the value of energy expressed by ω and k or alternatively t and x are mutually shifted with the constant $m_o^2c^2$. Using the wave function Ψ we perform correction v^2/c^2 whereby we subtract or the correction c^2/v^2 whereby we add value $m_o^2c^2$ to values expressed by ω and k or alternatively t and x .

Similarly we can believe that for the momentum of a particle we can write $mc-mv=m_oc$ then $p_t-p = p_o$ so $h\partial/c\partial t - h\nabla_o = m_oc$ or $h\partial/c\partial t - h\nabla = \pm o\ m_oc$ where x and t runs from diverse values and with the wave function we can write $h\partial/c\partial t\,\Psi \pm h\nabla\Psi \pm o\ m_oc\Psi = 0$ what represents the Dirac equation [4]. In the matrix form of the Dirac equation $\hbar\partial/c\partial t\,\Psi + \boldsymbol{\alpha}\hbar\nabla\Psi + \beta imc\Psi = 0$ the wave function provides shift over m_oc one or both of the terms $\hbar\nabla$, $\hbar\partial/\partial t$ to $\hbar\nabla_o$, $\hbar\partial\nu_o/c\partial t$ or reverse and matrix $\boldsymbol{\alpha}$ offers a relevant algebraic sign + or - and matrix β offers a relevant algebraic sign for m_oc to – or + or offers $m_oc = 0$.

7. Do kinetic energy of electrons in the Photoelectric effect equal $E_k = mvc$ or $E_k = ½mv^2$?

From the foregoing reasoning in this paper we can come to the belief that the momentum of a photon represents the relation $p=h/\lambda=h\nu/c=mc$ where λ runs from infinity and ν,m run from zero. The momentum of a particle is $p=h/l-h/l_o=h/l.v/c=h(\nu-\nu_o)/c= h\nu/c.v/c=mcv/c=mv$ where l, ν, m run from $h/\lambda_o=h\nu_o/c=m_oc$. The total energy (TCRE) of a photon represents the relation $\mathbf{E}_t = h^2/\lambda^2=h^2\nu^2/c^2=m^2c^2$ where λ runs from infinity and ν,m run from zero and for the total energy of a particle we can write $\mathbf{E}_t = h^2/l^2.v^2/c^2+h^2/l_o^2 = h^2\nu^2/c^2.v^2/c^2+h^2\nu_o^2/c^2 = m^2c^2v^2/c^2+m_o^2c^2 = m^2v^2+ m_o^2c^2$ where l, ν ,m run from $h/\lambda_o=h\nu_o/c=m_oc$.

As the relation $\mathbf{E}_t=2mE_t=h^2\nu^2/c^2$ represents the photon's energy so for the photoelectric effect we have to write the relation $h^2\nu^2/c^2-h^2\nu_o^2/c^2=m^2c^2-m_o^2c^2=m^2v^2$ which for momentum equals the relation written as $h\nu/c-h\nu_o/c = mc-m_oc=mv$. The last relation multiplied by c and (in EDE E=pc) then this relation represents Einstein's writing for energy balance at the photoelectric effect $h\nu - h\nu_o = mc^2-m_oc^2 = mvc$. We can receive this last writing for energy at the photoelectric effect also by multiplying the Dirac equation $h\partial/c\partial t - h\nabla_o - m_oc=0$ (the equation for momentum $mc-mv= m_oc$) with c what then results in $h\partial/\partial t - hc\nabla_o - m_oc.c=h\nu(0)-(hc/l-hc/l_o)-m_oc^2= mc^2- mvc - m_oc^2= 0$.

Millikan [5] who for many years disagreed with Einstein's understanding of the photoelectric effect found in his experiments the proportional increase in kinetic energy of electrons released from a metal surface with the linear frequency increase of photons striking on that surface. Thus we believe that Millikan's experiments should be interpreted that way that with linear increase in the frequency of photons their energy increases quadratically and this energy equals to the quadratic increase in kinetic energy of electrons

$$\mathbf{E}_k = 2mE_k = h^2\nabla_o^2 = h^2/l^2-h^2/l_o^2 = h^2/l^2v^2/c^2 = h^2\nu^2/c^2-h\nu_o^2/c^2 = h^2\nu^2/c^2v^2/c^2 = \quad \textbf{(10)}$$
$$= m^2c^2-m_o^2c^2 = m^2c^2v^2/c^2 = m^2v^2 = p^2.$$

Classical kinetic energy $E_k = m^2v^2/2m_o = ½m_ov^2 = m^2c^2/2m_o - m_o^2c^2/2m_o = mc^2 - m_oc^2$ is limit of the relation $\mathbf{E}_k = 2mE_k = m^2v^2 = p^2$ at speeds much slower than the speed of light. The same way as in Millikan's experiments we observe a linear increase of photon's frequency while the energy of the photon

increases quadratically we in classical physics also observe a linear increase of the speed v of an electron while its energy increases quadratically as $½m_ov^2$.

8. Where is energy and momentum in the Schrodinger equation?

Atomic physics on the bases of an observation of quadratic changes at hydrogen atomic line emission spectra formulated in the Rydberg formula $1/λ=RH(1/n_1^2-1/n_2^2)$ came to the solution that the differences $hc/λ=Ry(1/n_1^2-1/n_2^2)$ represent the transition between different energy levels of an atom and that for energy levels compared to the maximum energy level the relation $En= Eh/n^2=Ry/n^2 =hc/n^2λ_H = hν_h/n^2$ is valide ([6] N.Bohr). This quadratic changes in energy levels of atoms was explained in QM (neglecting changes in energy of a proton, which is 1836 times greater in mass than an electron) by quadratic changes in electron's energy of atoms. This explanation was formulated in [7] the stationary Schrodinger equation (SchrE) $-h^2\nabla^2Ψ=2m(E-V)Ψ$ where the classical kinetic energy relation $½mv^2=p^2/2m=h^2/2m_oλ^2$ and de Broglie hypothesis $p=h/λ=mv$ are used. Obviously the same conditions are valid also for absorption spectra so that a quadratic change in the wavelength $1/λ^2$ or frequency $ν^2/c^2$ of incident photons give rise to the quadratic change in electron's energy of the atom $h^2/2m_oλ^2=p^2/2m_o= ½.mv^2$. But we can see absorption spectra like a first stage of the photoelectric effect. So that quadratic changes in energy of an incident photon equal to the quadratic changes in electron's energy before its emission out of an atom. Also for these reasons we can consider as unreasonable to change the relation of classic kinetic energy in the photoelectric effect from the equation $hν- hν_o =½m_ov^2$ into $hν- hν_o = mc^2-m_oc^2=mvc$ with the view of conservation a linearity in the equation. Also because of non-linearity of the time dependent SchrE $iħ\partial φ/\partial t = -ħ^2\nabla^2φ/2m_o$ so non-linearity of the relation $h\partial_oν/ \partial t=h^2\nabla_o^2/2m$ that can be written as $hν-hν_o=mc^2-m_oc^2=mvc=p^2/2m_o= ½mv^2$ we declare SchrE as non-relativistic. Consequently we perform reformation in right hand side of the SchrE into the linearized term $ħ\partial/c\partial t\ Ψ= ħ\nabla Ψ$ (change of momentum $hν/c=h/λ$!) what results in the Dirac equation. On the contrary we can believe that $hν - hν_o$ or $ħ\partial φ/\partial t$ does not represent a change in energy but the change in momentum multiplied by c (in EDE E=pc). Thus we can reasonable believe that a change in energy at the photoelectric effect just as a change in energy at SchrE represents the relation $h^2ν^2/c^2 - h^2ν_o^2/c^2 = h^2\partial^2/c^2\partial t^2 = h^2\nabla_o^2 = m^2c^2- m_o^2c^2 = m^2v^2$ and this relation is seen in classical physics so at speeds much slower than the speed of light c as the classical limit of kinetic energy $h^2ν^2/2m_oc^2-h^2ν_o^2/2m_oc^2=h^2_o\partial^2/2m_oc^2\partial t^2=m^2c^2/2m_o-m_o^2c^2/2m_o=h^2\nabla_o^2/2m_o=m^2v^2/2m_o=p^2/2m_o=½mv^2$.
Consequently we believe that if we want to talk about energy than we have to change the left hand side of the photoelectric effect equation $hν- hν_o = ½mv^2$ just in the same manner as the left hand side of the ShrE $iħ\partial φ/\partial t = -h^2/2m_o\nabla^2$ from $h\partial_oν/\partial t = hν- hν_o$ representing energy in EDE where E=pc into $h^2\partial^2/2m_oc^2\partial t^2 = h^2ν^2/2m_oc^2- h^2ν_o^2/2m_oc^2$ or $2m(E_1-E_0) = h^2ν^2/c^2 - h^2ν_o^2/c^2$ respectively. But after that we are talking about the K-G equation for energy $h^2\partial^2/c^2\partial t^2 -h^2\nabla_o^2 = m_o^2c^2$ then about the RM equation $m^2c^2 - m^2v^2 = m_o^2c^2$ or about the classical kinetic energy relation $m^2c^2/2m_o-m_o^2c^2/2m_o = h^2\nabla_o^2/2m_o = =m^2v^2/2m_o=p^2/2m_o=½mv^2$ as limit of the relation $\mathbf{E}_k=2mE_k =m^2v^2 =p^2$ at speeds much slower than the speed of light.. If we want to persist in the energy definition by EDE (E=pc) then we have to change right side hand of the photoelectric effect equation and also SchrE $½mv^2=h^2/2m_o\nabla^2$ into $h\partial/c\partial t=mv=h\nabla$ and consequently we get the Dirac equation $ħ\partial/c\partial t\ Ψ+\boldsymbol{α}ħ\nabla Ψ+βimcΨ =0$ so $h\partial/c\partial t -h\nabla= ±o\ m_oc$ for momentums so $h\partial ν(0)/c\partial t -h\nabla_o=m_oc$ so $mc-mv=m_oc$. Last relation after multiplying by c represents the equation for energy in the system of EDE so where $mc^2-mvc=m_oc^2$ or $mc^2 = mvc + m_oc^2$.

9. Conclusion

From the foregoing reasoning in this paper we can believe that

- Planck constant and relativistic relationships on the length contraction firmly and increase in effective mass is the reflection of the same physical principle of nature expressed in relation $h/l = mc$ so in relation $h =mlc = m_ol_oc(1-v^2/c^2)^{1/2}/(1-v^2/c^2)^{1/2} = m_ol_oc$
- the wave of matter λ introduced in the de Broglie hypothesis $h/λ = mv$ needs to be connected with real dimension of particle $λ = l_o$ in rest state $h/λ_o = m_oc = hν_o/c$

-on this basis if we carefully consider relation among total, added, rest energies and momentums we can derive the fundamental equation of QM that is the Klein-Gordon, Dirac and Schrodinger equation without necessity of wave function

-the Klein –Gordon equation represents the equation for energy of particles also as of photons
-the Dirac equation represents the equation for momentum of particles also as of photons
-classical kinetic energy $E_k = m^2v^2/2m_o = ½m_ov^2 = m^2c^2/2m_o - m_o^2c^2/2m_o = mc^2 - m_oc^2$ is limit of the relation $\mathbf{E}_k = 2mE_k = m^2v^2 = p^2$ at speeds much slower than the speed of light.
-energies in RM as mc^2, mvc, m_oc^2 and energy of a photon hν do not represents quantity of energy but quantity of momentum multiplied by c so mc.c ,mv.c , m_oc.c, hν/c.c and dimension of such quantities equals in dimension with the quantity of energy.

References

[1]L.V. de Broglie, On the Theory of Quanta, (1925)
[2] CODATA Recommended Values
[3]C.Davisson, L.H.Germer, Diffraction of electrons by a crystal of nickel, (1927)
[4]P.A.M.Dirac, The quantum theory of the electron, (1928)
[5]R.A.Millikan, A direct photoelectric determination of Plansks "h", (1916)
[6]N.Bohr, On the Constitution of Atoms and Molecules, (1913)
[7]E.Schrodinger, An undulatory theory of the mechanics of atoms and molecules, (1926)
[8]A.S. Davidov, Quantum Mechanics, (1973)
[9] The Feynman Lectures on Physics, Vol 1.2.3., (1964)