First Draft

Higgs-less Symmetry Breaking from Renormalization Group Theory

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Abstract

We develop here a Higgs-less model of electroweak symmetry breaking using critical behavior of infrared Yang-Mills theory. Gauge bosons and fermions acquire mass near the Wilson-Fisher point of Renormalization Group flow. The entire family structure of Standard Model is recovered using the technique of "epsilon expansion". We also find that dimensional regularization offers a straightforward solution to the cosmological constant problem. A brief discussion on how our Higgs-less model could preserve unitarity of high-energy di-boson scattering is also included.

Key words: Renormalization Group; Critical phenomena; Feigenbaum universality; Standard model

1. Introduction

The Standard Model of particle physics (SM) is a highly successful theory that has been in place for more than 35 years. It includes the $SU(3)\otimes SU(2)\otimes U(1)$ gauge model of strong and electroweak interactions along with the *Higgs mechanism* that spontaneously breaks the electroweak $SU(2)\otimes U(1)$ group down to the $U(1)$ group of electromagnetism. Despite its outstanding reliability, SM is viewed as a low-energy framework that is likely to be amended by new phenomena occurring in the Terascale sector [1, 2].

The Higgs boson is the last building block of SM that, at the time of writing, resists experimental verification. The whole theoretical and experimental consistency of SM hinges on the validity of the Higgs mechanism. The vacuum expectation value of the Higgs breaks the electroweak symmetry, giving mass to both W and Z gauge bosons and fermions. The tree level exchange of the Higgs boson contributes to the scattering amplitude of longitudinally polarized gauge bosons in a way that makes the total amplitude consistent with unitarity, provided the Higgs mass is not larger than 1 TeV [21]. The minimal Higgs sector with one Higgs doublet is automatically consistent with experimental data on flavor changing neutral currents and CP violation. Finally, radiative corrections induced by the Higgs boson affect the electroweak precision observables, notably the Peskin-Takeuchi parameters, and once again they are consistent with experiment, provided the Higgs boson mass is smaller than 145 GeV, at 2σ confidence level [21].

The discovery of the mechanism responsible for electroweak symmetry breaking (EWSB) is one of the key objectives of the Large Hadron Collider (LHC). The elementary Higgs boson picture of electroweak (EW) and flavor symmetry breaking suffers from several drawbacks. In particular [1, 2]:

- It does not provide a compelling dynamical explanation for EWSB.
- It does not account for breaking of CP symmetry.
- It appears to be highly contrived, requiring fine tuning of parameters to enormous precision.
- It has a hierarchy problem of widely different energy scales.
- It provides no insight into flavor physics.
- It is at odds with the measured value of the cosmological constant.

Similar or different drawbacks persist in supersymmetric extensions of Higgs theories (MSSM) and alternative models of EWSB such as Technicolor [3, 4]. As the Large Hadron Collider collects its first inverse femtobarns of data, with no definitive signal for the elementary Higgs boson, we inquire herein if it is possible to build a meaningful Higgs-less electroweak model that has explanatory power and falls in line with all available experimental observations.

Surprisingly, building a satisfactory alternative to the Higgs mechanism turns out to be highly non-trivial, and all proposed approaches so far face more or less severe problems. Either new contributions to electroweak precision observables are unacceptably large, or one has to accept fine-tuning at least at the one-percent level (MSSM, Little Higgs, pseudo-Goldstone Higgs), or complicated ad-hoc theoretical structures have to be added to the theory. Moreover, extensions of the SM typically face flavor and CP problems, and/or a host of model-specific problems [22].

The paper is organized in the following way: after a brief listing of main challenges raised by Yang-Mills theory in section 2, sections 3 and 4 introduce the basis of our approach along with its set of assumptions and conventions. The next two sections describe the link between Yang-Mills theory and the Landau-Ginzburg-Wilson (LGW) model of equilibrium phase transitions, as well as the relevance of its Wilson-Fisher point for the physics of EWSB. Section 6 includes a side-by-side comparison between predictions and experimental observations. Building on the same premises, section 7 develops a straightforward solution for the hierarchy problem using dimensional regularization and derives the numerical value of the cosmological constant. Section 8 contains a brief discussion on how our Higgs-less model could preserve unitarity of high-energy di-boson scattering. A summary of open questions is formulated in the last section.

2. Challenges of Yang-Mills theory

In our view, there are a couple of key roadblocks that have slowed down progress on the theoretical side of high-energy physics for the past 35 years:

- Because Yang-Mills field is self-interacting, it is inherently nonlinear and prone to undergo complex behavior [5].
- Dynamics of Yang-Mills field is strongly coupled in the infrared (IR) where perturbation theory breaks down and traditional methods of quantum field theory (QFT) fail to apply.

3. New tools: nonlinear dynamics and critical behavior

To deal with these challenges, we start from a far less explored vantage point. Specifically, we exploit the fact that both mapping theorem [6] and the LGW model of critical behavior [7, 19] are able to explain the dynamics of gauge field theory using the principles of Renormalization Group program (RG).

- **The mapping theorem**

The electroweak group $SU(2)\otimes U(1)$ is broken at a scale approximately given by $\mu_{EW} = O(G_F^{-1/2})$, in which G_F is the Fermi constant. Yang-Mills fields associated with $SU(2)$ are vectors denoted as $A_\mu^a(x)$, in which $\mu = 0,1,2,3$ is the Lorentz index and $a = 1,2,3$ is the group index. To manage the large number of equations derived from the Yang-Mills theory, it is desirable to devise a method whereby $A_\mu^a(x)$ are reduced to analog fields having less complex structure. The *mapping theorem* allows for such a reduction. The action functional of classical scalar field theory in four-dimensional space-time is defined as

$$S[\Phi] = \int d^4x [\frac{1}{2}(\partial\Phi)^2 - \frac{1}{4!} g^2 \Phi^4] \tag{1}$$

An extremum of (1) is also an extremum of the $SU(2)$ Yang-Mills action provided that:

a) g represents the coupling constant of the Yang-Mills field,

b) some components of $A_\mu^a(x)$ are chosen to vanish and others to equal each other.

In the most general case, the following approximate mapping between Yang-Mills fields and scalar $\Phi(x)$ holds [6]:

$$A_\mu^a(x) = \eta_\mu^a \Phi(x) + O(\frac{1}{\sqrt{2}g}) \tag{2}$$

where η_μ^a are properly chosen constants. Mapping becomes exact in the Lorenz gauge $\partial^\mu A_\mu^a(x) = 0$ and in the IR regime of strong coupling ($g \to \infty$).

- **<u>LGW theory near dimension four: a brief overview</u>**

Consider the Euclidean space LGW action in $D-$ dimensional space-time [7, 8, 19]

$$S[\Phi] = \int d^Dx [\frac{1}{2}(\partial\Phi)^2 + V(\Phi)] \tag{3}$$

In particular,

$$V(\Phi) = \frac{r}{2}\Phi^2 + \frac{g^2}{4!}\Phi^4 - j\Phi \tag{4}$$

in which j denotes the external current coupled to Φ and r stands for the deviation from the critical temperature ($r = T - T_c$). According to the RG program, rescaling the cutoff $\Lambda \to \Lambda' = \frac{\Lambda}{b}$, $b > 1$ and integrating out fast modes within $\Lambda' < |k| < \Lambda$, turns the original action into an *effective* action. The effective theory built with this prescription represents a lower-energy image of the original theory, namely

$$S[\Phi],\Lambda \to S_{eff}[\Phi_<],\Lambda' \tag{5}$$

Here, $\Phi_<(x)$ are the slow modes of the field ($|k| < \Lambda'$),

$$\Phi_<(x) = \int_{|k|<\Lambda'} \frac{d^D k}{(2\pi)^D} \Phi(k)\exp(ikx) \tag{6}$$

and

$$\int D_\Lambda[\Phi]\exp(-S[\Phi]) \approx \exp(-S_{eff}[\Phi_<]) \tag{7}$$

with

$$S_{eff}[\Phi_<] = \int d^D x[\frac{1}{2}(\partial\Phi_<)^2 + V_{eff}(\Phi_<)] \tag{8}$$

Invoking the limit of infinitesimal scaling $b = 1 + dt$, $dt << 1$ along with the local potential approximation leads to [7, 8, 19],

$$S_{eff}[\Phi_<] = S[\Phi_<] + \frac{Q}{2}\int d^D x \log[\Lambda^2 + \frac{\partial^2 V[\Phi_<]}{\partial\Phi_<^2}] \tag{9}$$

where

$$Q = \frac{\Lambda^D dt}{(2\pi)^D} \frac{2\pi^{D/2}}{\Gamma(D/2)} \tag{10}$$

When applied to (4), the logarithmic correction on the right hand side of (9) may be expanded as

$$\log[1 + \frac{\partial^2 V}{\partial\Phi_<^2}] = \log[1+r] + \frac{g^2}{2(1+r)}\Phi_<^2 - \frac{g^4}{8(1+r)}\Phi_<^4 + ... \tag{11}$$

in which Λ has been normalized to unity ($\Lambda = 1$). On account of (11), sufficiently small deviations from criticality ($r << 1$) produce the following approximations

$$S_{eff}[\Phi_<] \sim S[\Phi], \qquad V_{eff}[\Phi_<] \sim V[\Phi] \tag{12}$$

4. Assumptions and conventions

4.1) As previously stated, the mapping theorem applies when comparing Yang-Mills fields with classical scalar fields. We extend this ansatz and assume that the theorem holds sufficiently well for *quantum* scalar field theory. This assumption may be motivated by considering the close analogy between quantum field theory (QFT) and statistical systems *near criticality* [9]. On this basis, we assume that the Yang-Mills model is reasonably well approximated by the LGW theory of equilibrium critical behavior.

4.2) From (4.1) it follows that the dimensional parameter of LGW theory and dimensional regulator of Yang-Mills theory $\varepsilon = 4 - D$ are identical entities. This identity is made explicit in the first row of Tab. 1 below.

4.3) We analyze on the IR regime of Yang-Mills theory in which μ_{EW} stands for the EW scale, μ for the running scale and the ultraviolet (UV) scale $\Lambda = \Lambda_{UV} > \mu > \mu_{EW}$ for the cutoff. The dimensional parameter is then given by [10, 13, 24],

$$\varepsilon \approx \frac{1}{\log(\frac{\Lambda_{UV}^2}{\mu^2})} > 0 \tag{13}$$

Moreover, to streamline the derivation and make it more transparent, it is convenient to take advantage of the large numerical disparity between the two scales entering the logarithm and substitute (13) with

$$\varepsilon \sim \frac{\overline{\mu}^2}{\Lambda_{UV}^2} \tag{14a}$$

Here, the reduced UV scale ($\overline{\Lambda_{UV}}$) and the reduced running scale ($\overline{\mu}$) are defined through

$$\log(\frac{\Lambda_{UV}^2}{\mu^2}) = (\frac{\overline{\Lambda_{UV}}}{\overline{\mu}})^2 \tag{14b}$$

It is seen from (14a) that,

- maximal deviation from $D = 4$ occurs near the limit $\overline{\mu} \to \overline{\Lambda_{UV}}$. This finding is consistent with quantum gravity theories asserting that space-time turns 2+1 dimensional at very large energy scales [11, 25].
- minimal deviation from $D = 4$ ($\varepsilon \to 0$) occurs as $\overline{\mu}$ approaches the reduced EW scale, that is, when $\overline{\mu} \to \overline{\mu_{EW}}$.

4.4) The reduced UV cutoff is not uniquely determined but smeared out by high-energy noise [12]. It spans a range of values

$$\overline{\Lambda_{UV}} \in \delta\overline{\Lambda_{UV}} \tag{15}$$

(15) implies that, at any given $\overline{\mu}$ and $\overline{\Lambda_{UV}}$, dimensional parameter ε falls in the range

$$|\delta\varepsilon| = 2\overline{\mu}\frac{\delta\overline{\Lambda_{UV}}}{\overline{\Lambda_{UV}}} \tag{16}$$

5. Dynamics of RG flow equations

Elaborating from these premises leads to the following side-by-side comparison between parameters of LGW and the reduced parameters of Yang-Mills theory:

Landau –Ginzburg -Wilson theory	**Yang-Mills theory**
Dimensional parameter ($\varepsilon = 4 - D$)	Dimensional regulator ($\varepsilon = 4 - D$)
Momentum cutoff (Λ)	Ultraviolet cutoff ($\overline{\Lambda_{UV}}$)

Temperature (T)	Energy scale ($\overline{\mu_{EW}} < \overline{\mu} < \overline{\Lambda_{UV}}$)
Critical temperature (T_c)	EW scale ($\overline{\mu_{EW}}$)
Temperature parameter (r)	Deviation from the EW scale ($\delta\overline{\mu} = \overline{\mu} - \overline{\mu_{EW}}$)
Coupling parameter (u)	Coupling constant (g^2)
External field (h)	Fermion current (j)

Tab. 1: Comparison between LGW and Yang-Mills theories

Under these circumstances, RG flow equations for $r = \delta\overline{\mu}$, $u = g^2$ and fermion current $j = j_f$ read, respectively [13]

$$\frac{\partial(\delta\overline{\mu})}{\partial t} = (\delta\overline{\mu})(2 + bg^2) + ag^2$$

$$\frac{\partial g^2}{\partial t} = \varepsilon g^2 - 3b(g^2)^2 \tag{17}$$

$$\frac{\partial j_f}{\partial t} = (3 - \frac{\varepsilon}{2}) j_f$$

Here,

$$a = 3K_4 \overline{\Lambda_{UV}}^2, \qquad b = 3K_4, \qquad K_4 = (8\pi^2)^{-1} \tag{18}$$

On account of (12), the Wilson-Fisher (WF) fixed point of (17) is defined by the pair

$$(\delta\overline{\mu})^* = -\frac{a}{6b}\varepsilon, \qquad (g^2)^* = \frac{\varepsilon}{3b} \tag{19}$$

(19) acts as a non-trivial attractor of the RG flow. Since it resides on the critical line $\overline{\mu} = \overline{\mu_{EW}}$, it describes by definition a *massless* field theory ($r = \delta\overline{\mu} = 0$) [19]. The non-vanishing vacuum of Φ at the WF point results from minimization of (4), that is,

$$v^* = \pm\sqrt{\frac{6(-\delta\overline{\mu})^*}{(g^2)^*}} = \pm 3(K_4)^{1/2}\overline{\Lambda_{UV}} \tag{20}$$

(19) and (20) show how massive gauge bosons develop at the WF point from critical behavior near $D = 4$. Let **Error! Objects cannot be created from editing field codes.** denote the mass acquired by the gauge boson. Combining (14), (18), (19) and (20) yields

$$\begin{gathered}\boxed{(g^*)^2 M^2 \sim \overline{\mu_{EW}}^2 = const} \\ \boxed{(g^*)^2 \sim m_f^* \sim \varepsilon}\end{gathered} \tag{21}$$

in which $m_f^* = O(j_f)$ stands for the normalized fermion mass as defined in [13]. On account of assumptions 4.3), 4.4) and (21), the WF attractor (20) changes from a *single isolated* point to a *distribution* of points. Our next step is to explore the link between the structure of the WF attractor and the parameters of SM.

6. Wilson-Fisher point as source of particle masses and gauge charges

We are now ready to analyze the dynamics of (17) using the standard methods employed in the study of nonlinear systems [14]. To this end, we first note that the last equation in (17) is uncoupled to the first two. This enables us to reduce (17) to a planar system of differential equations. We next cast (17) in the form of a two-dimensional map, namely

$$(g^2)_{n+1} = (1+\varepsilon\Delta t)(g^2)_n - 3b\Delta t(g^2)_n \tag{22a}$$

$$(\delta\overline{\mu})_{n+1} = (\delta\overline{\mu})_n[1 + 2\Delta t + b\Delta t(g^2)_n] + a\Delta t(g^2)_n \tag{22b}$$

where Δt represents the increment of the sliding scale. Linearizing (22) and computing its Jacobian J gives

$$J = 1 + (2+\varepsilon)\Delta t > 1 \tag{23}$$

Thus map (23) is dissipative for $\varepsilon \neq 0$ and asymptotically conservative in the limit $\varepsilon = \Delta t = 0$. Invoking universality arguments [14, 18] we conclude that, near criticality, (22) shares the same universality class with the quadratic map. Furthermore, in the neighborhood of Feigenbaum's attractor, ε approaches $\varepsilon_\infty = 0$ according to:

$$\varepsilon_n - \varepsilon_\infty \approx a_n \cdot \overline{\delta}^{-n} \tag{24}$$

Here, $n >> 1$ is the index counting the number of cycles generated through the period doubling cascade, $\overline{\delta}$ is the rate of convergence (in general, different from Feigenbaum's constant for the quadratic map) and a_n is a coefficient which becomes asymptotically independent of n, that is, $a_\infty = a$ [15]. Substituting (24) in (21) yields

$$P_j(n) = \left[M_n^{-2} \quad (g^*)_n^2 \quad (m_f^*)_n \right] \propto \overline{\delta}^{-n} \quad \text{if} \quad n >> 1 \tag{25}$$

in which $j = 1, 2, 3$ indexes the three entries of (25). Period-doubling cycles are characterized by $n = 2^p$, with $p >> 1$. The ratio of two consecutive terms in (25) is then given by

$$\boxed{\frac{P_j(p+1)}{P_j(p)} = O[\overline{\delta}^{-(2^p)}]} \tag{26}$$

Numerical results derived from (26) are displayed in Tab. 3. This table contains a side-by-side comparison of estimated versus actual mass ratios for charged leptons and quarks and a similar comparison of coupling strength ratios. Tab. 2 contains the set of known quark and gauge boson masses as well as the SM coupling strengths. All quark masses are reported at the energy scale given by the top quark mass and are averaged using reports issued by the Particle Data Group [16]. Gauge boson masses are evaluated at the EW scale and the coupling strengths at the scale set by the mass of the Z boson. The best-fit rate of convergence is $\overline{\delta} = 3.9$ which falls close to the numerical value of the Feigenbaum constant corresponding to hydrodynamic flows [13, 15, 17].

(21) and (25) imply that there is a series of terms containing massive electroweak bosons, namely

$$(M_n g_n^*)^2 = (M_{n+1} g_{n+1}^*)^2 = ... = (M_{n+q} g_{n+q}^*)^2 = ... = const. \tag{27}$$

For the first two terms of this series we obtain

$$\frac{M_Z^2}{M_W^2} = \frac{g_2^2 + e^2}{g_2^2} = 1 + \frac{\alpha_{EM}}{\alpha_2} \tag{28}$$

in which $\alpha_{EM} = e^2 / 4\pi$ is the fine-structure constant and $\alpha_2 = g_2^2 / 4\pi$ the strength of the weak interaction. The rationale for (28) lies in the fact that the charged gauge boson $W^{\pm}$ carries a superposition of weak and electromagnetic charges, whereas the neutral gauge boson Z^0 carries only the weak isospin charge. Inverting (28) and taking into account the last rows of Table 3, leads to

$$\boxed{\frac{M_W^2}{M_Z^2} = \frac{1}{1 + \frac{\alpha_{EM}}{\alpha_2}} = \frac{1}{1 + \frac{1}{\overline{\delta}}} \approx 1 - \frac{1}{\overline{\delta}} = \cos^2 \theta_W} \tag{29}$$

(29) suggests a natural explanation for the *Weinberg angle* θ_W. Likewise, we may write (27) as

$$\boxed{\frac{g_2^2}{M_W^2} = \frac{g_2^2 + e^2}{M_Z^2} = const} \tag{30a}$$

This relation offers a straightforward interpretation for both Fermi constant and the mass of the *hypothetica*l Higgs boson. Indeed, in SM we have [13]

$$\frac{g_2^2}{M_W^2} = 4\sqrt{2}G_F \tag{30b}$$

and

$$\mathrm{v}(\varphi^0) \propto \sqrt{\frac{1}{G_F\sqrt{2}}} \approx 246.22\ GeV \tag{30c}$$

where $\mathrm{v}(\varphi^0)$ denotes the vacuum expectation value for the neutral component of the "would-be" Higgs doublet.

Parameter	**Value**	**Units**
m_u	2.12	MeV
m_d	4.22	MeV
m_s	80.90	MeV
m_c	630	MeV
m_b	2847	MeV
m_t	170,800	MeV
$M_{W^{\pm}}$	80.46	GeV
M_{Z^0}	91.19	GeV
α_{EM}	1/128	-
α_W	0.0338	-
α_{QCD}	0.123	-

Tab. 2: Actual values of selected SM parameters.

Parameter ratio	Behavior	Actual	Predicted
m_u / m_c	$\bar{\delta}^{-4}$	3.365×10^{-3}	4.323×10^{-3}
m_c / m_t	$\bar{\delta}^{-4}$	3.689×10^{-3}	4.323×10^{-3}
m_d / m_s	$\bar{\delta}^{-2}$	0.052	0.066
m_s / m_b	$\bar{\delta}^{-2}$	0.028	0.066
m_e / m_μ	$\bar{\delta}^{-4}$	4.745×10^{-3}	4.323×10^{-3}
m_μ / m_τ	$\bar{\delta}^{-2}$	0.061	0.066
M_W / M_Z	$(1-\frac{1}{\bar{\bar{\delta}}})^{1/2}$	0.8823	0.8623
$(\alpha_{EM} / \alpha_W)^2$	$\bar{\delta}^{-2}$	0.053	0.066
$(\alpha_{EM} / \alpha_{QCD})^2$	$\bar{\delta}^{-4}$	4.034×10^{-3}	4.323×10^{-3}

Tab 3: Actual versus predicted ratios of SM parameters (except neutrinos)

A similar comparison may be drawn on *neutrinos*. Since neutrino oscillation experiments are only sensitive to neutrino mass squared differences and not to the absolute neutrino mass scale (m_ν^0), they can only supply lower limits for two of the neutrino masses, that is,

$(m_{ATM}^2)^{1/2} \approx 5\times10^{-2}$ eV and $(m_{SOL}^2)^{1/2} \approx 1\times10^{-2}$ eV [27]. As a result, it is more relevant to consider experimentally constrained bounds on m_ν^0 reported from beta decay, neutrinoless double beta decay as well as from cosmological observations [26]:

- Beta decay: electron neutrino mass is $m_{\nu_e} < 0.2$ to 2 eV.
- Neutrinoless double beta decay: effective electron neutrino mass is $|m_{ee}| < 0.1$ eV.
- Cosmological measurements: sum of neutrino masses is $\sum_i m_{\nu_i} < 0.17$ eV at 95% confidence level.

Based on these inputs, it makes sense to set the upper (U) and lower (L) limit values for the absolute neutrino mass scale as $(m_\nu^0)_U = 2$ eV and $(m_\nu^0)_L = 0.1$ eV. According to Tab. 3, charged lepton masses ratios scale as $\overline{\delta}^{-2}$ and $\overline{\delta}^{-4}$, which suggests that m_ν^0 should naturally follow a $\overline{\delta}^{-8}$ or $\overline{\delta}^{-16}$ pattern . Table 4 displays a side-by-side comparison on the mass ratio m_ν^0 / m_e for $(m_\nu^0)_U$ and $(m_\nu^0)_L$, respectively, and shows that numerical predictions line up fairly well with current observations.

Parameter ratio	**Behavior**	**Actual**	**Predicted**
$\mathrm{m}_\nu^0 / \mathrm{m}_e$	$\overline{\delta}^{-8}$	$< 2\times10^{-7}$[...] $< 4\times10^{-6}$[...]	1.87×10^{-5}

m_ν^0 / m_e	$\overline{\delta}^{-16}$	$< 2\times10^{-7}$[...] $< 4\times10^{-6}$[...]	3.5×10^{-10}

Tab. 4: Actual vs. predicted ratios of neutrino mass scales.

7. A natural solution for the hierarchy problem

It is known that the technique of renormalization in QFT is conceived as a two-step program: regularization and subtraction. One first controls the divergence present in momentum integrals by inserting a suitable "regulator", and then brings in a set of "counter-terms" to cancel out the divergence. Momentum integrals in QFT have the generic form

$$I = \int_0^\infty d^4q\, F(q) \tag{31}$$

Two regularization techniques are frequently employed to manage (31), namely "momentum cutoff" and "dimensional regularization". When the momentum cutoff scheme is applied for regularization in the UV region, the upper limit of (31) is replaced by a finite cutoff Λ,

$$I \to I_\Lambda = \int_0^\Lambda d^4q\, F(q) \tag{32}$$

Explicit calculation of the convergent integral (32) amounts to a sum of three polynomial terms

$$I_\Lambda = A(\Lambda) + B + C(1/\Lambda) \tag{33}$$

Dimensional regularization proceeds instead by shifting the momentum integral (33) from a four-dimensional space to a continuous D - dimensional space

$$I \to I_D = \int_0^\infty d^Dq\, F(q) \tag{34}$$

Introducing the dimensional parameter $\varepsilon = 4 - D$ leads to

$$I_D \to I_\varepsilon = A'(\varepsilon) + B' + C'(1/\varepsilon) \tag{35}$$

In general, Λ and ε are not independent regulators and relate to each other via the approximate connection (13)

$$\varepsilon = 4 - D = \frac{1}{\log(\Lambda^2/\mu_0^2)} \tag{36}$$

where $\mu_0 < \Lambda$ stands for an arbitrary but non-vanishing reference scale.

A similar technique can be used to regularize field theory in the IR limit whereby Γ is taken to represent the lowest bound scale. A strictly positive ε on less than four dimensions ($D < 4$) requires taking the reciprocal of the logarithm in (36) to comply with $\mu_0 > \Gamma$. The infrared version of (36) accordingly reads:

$$\varepsilon' = 4 - D = \frac{1}{\log(\mu_0^2/\Gamma^2)} \tag{37}$$

We next proceed with the following assumptions

7.1) The deep IR cutoff of field theory is set by the cosmological constant scale

$$\Gamma = (\Lambda_{cc})^{1/4} \tag{38}$$

where Λ_{cc} represents the cosmological constant.

7.2) The deep UV cutoff of field theory is set by the Planck scale:

$$\Lambda_{UV} = \Lambda_{Pl} \tag{39}$$

Combining 7.1) and 7.2) implies that, as the EW scale is approached from above or below, (36) and (37) naturally converge to a common value. Taking $\mu_0 = \mu_{EW}$ and replacing in (36) and (37) yields

$$\boxed{\frac{\mu_{EW}}{\Gamma} = \frac{\Lambda_{Pl}}{\mu_{EW}} \rightarrow (\Lambda_{cc})^{1/4} = \frac{\mu_{EW}^2}{\Lambda_{Pl}}} \tag{40}$$

Direct substitution of Planck's mass $\Lambda_{Pl} \approx 10^{19}$ GeV and $\mu_{EW} = O(G_F^{-1/2}) \approx 300$ GeV leads to

$$(\Lambda_{cc})^{1/4} \approx 10^{-5}\ \text{eV} \tag{41}$$

This result falls in line with the upper limit set by the cosmological value of vacuum energy density [ ], that is,

$$\boxed{(\Lambda_{cc})^{1/4} = O(\rho_{\text{v}}) < 10^{-3}\ \text{eV}} \tag{42}$$

Several conclusions may be drawn from (40), namely,

a) Asymptotic approach to four-dimensional space-time explains the existence of the deep IR cutoff (Λ_{cc}) and deep UV cutoff (Λ_{Pl}). Stated differently, *fractal space-time* description supplied by the condition $\varepsilon > 0$ and $\varepsilon' > 0$ appears to be linked to these natural bounds [20].

b) Fixing two out of the three scales involved in (40) automatically determines the third one.

c) The gauge hierarchy problem, cosmological constant problem and EWSB appear to be deeply interconnected.

d) The derivation presented here stands in sharp contrast with sophisticated approaches to the hierarchy problem based on supersymmetry, Technicolor, extra-dimensions, anthropic arguments, fine-tuning or gauge unification near the Planck scale.

8. Preserving unitarity in di-boson scattering without the Higgs

As it is known, unitarity provides a strong argument for the existence of the EW Higgs scalar [30]. Without it, elastic scattering of longitudinally polarized W bosons diverges at the tree level and violates unitarity. In what follows we briefly review this argument. Let P denote the magnitude of the three-momentum in the center-of-mass frame and let "c" represent the cosine of the angle between the initial and final boson states. The scattering amplitudes describing the high energy limit $P \gg M_W$ are represented by [30]

$$M_{1s} = \frac{g^2 P^4}{M_W^4}(1-c)(3+c) + \frac{g^2 P^2}{2M_W^2}(9+7c-4c^2) + O(P^0)$$

$$M_{1t} = -4\frac{g^2 P^4}{M_W^4}c - 9\frac{g^2 P^2}{M_W^2}c + O(P^0)$$

$$M_{1u} = -\frac{g^2 P^4}{M_W^4}(3-6c-c^2) - 2\frac{g^2 P^2}{M_W^2}(2-3c-c^2) + O(P^0) \tag{43}$$

$$\sum M_1 = \frac{g^2 P^2}{2M_W^2}(1+c) + O(P^0)$$

Here, s,t,u denote Mandelstam variables of the scattering process and P^0 is the total energy measured in the center-of-mass frame. It is apparent that the amplitude diverges as $P \to \infty$ and unitarity is violated. Similar argument holds for amplitudes computed in the $SU(2)\times U(1)$ theory where the mediating gauge boson is either a Z boson or a photon.

A natural question is then: *How can unitarity be restored in our Higgs-less model*?

To properly answer this question, we recall that the LGW program infers the desired IR behavior of Φ from a systematic study of its scaling behavior in the UV regime corresponding to $\mu \to \Lambda$. In constructing the LGW model we had tacitly taken the IR limit $t \to \infty$, where t parameterizes continuous scaling of cutoff Λ. Stated differently, the limit of infinitesimal scaling is described by

$$\Lambda \to \Lambda' = \frac{\Lambda}{b}, \quad b = 1 + dt \quad (dt << 1) \tag{44}$$

and both scale-invariant regime and the WF fixed point of RG flow (17) are approached as $t \to \infty$ [19]. It is apparent that (17) is formally equivalent to differential equations describing temporal behavior of generic dynamical systems from an initial time corresponding to the UV limit to a final time that defines the IR limit.

Next, we recall that a basic hypothesis of equilibrium critical behavior is that the outcome of (17) is *independent* of initial conditions corresponding to $dt <<$ 1. However, in the early stages of scaling regime the evolution of the flow is not yet stationary and correlation functions cannot be considered *translational invariant*. In particular, since initial conditions break translation invariance with respect to the choice of t, the two-point correlation functions will explicitly depend on two independent variables namely t and $t' \neq t$ [27]. This situation is a hallmark of non-equilibrium dynamics and leads to the concept on *non-locality*. Moreover, non-equilibrium field theory restores unitarity of scattering processes and blurs the distinction between locality and non-locality. As explained in [28], these arguments imply that a correct treatment of Higgs-less models in the high-energy limit $P \gg M_W$ can no longer be formulated using perturbative QFT and requires instead the framework of *fractional dynamics*. An attractive feature of this framework is that it supplies a natural mechanism for breaking of discrete symmetries (P and CP), as well as a plausible source for anomalous behavior near or beyond the EW scale [28, 29].

9. Open questions

- Are there additional generations of gauge bosons and fermions or is there a stability limit of RG trajectories constraining the number of these flavors? [31].

- Can flavor mixing and the absence of flavor changing neutral currents be explained using mixing of RG trajectories near transition to chaos?

- Can all electroweak precision observables (including Peskin-Takeuchi parameters) be correctly recovered from our model?.

- Can all decay channels be understood as the result of *chaotic mixing* and *diffusion* of RG trajectories on strange attractors?.

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