# Do We Need Dark Energy to Explain the Cosmological Acceleration?

Felix M. Lev

*Artwork Conversion Software Inc., 1201 Morningside Drive, Manhattan Beach, CA 90266, USA (Email: felixlev314@gmail.com)*

**Abstract**

We argue that the phenomenon of the cosmological acceleration can be easily and naturally explained from first principles of quantum theory without involving empty spacetime background, dark energy and other artificial notions.

The discovery of the cosmological acceleration (see e.g., [1, 2]) has ignited a vast discussion on how this phenomenon should be interpreted. The results of the observations are usually represented in terms of the parameter, which is called the cosmological constant (CC) and denoted by $\Lambda$. The meaning of this quantity will be discussed below. According to Refs. [3, 4], the observational data on the value of $\Lambda$ define it with the accuracy better than 5%. Therefore the possibilities that $\Lambda = 0$ or $\Lambda < 0$ are practically excluded.

Before discussing the CC problem, we first consider whether the gravitational constant $G$ can be treated as a fundamental constant. The Newton law of gravity says that the force of attraction between two bodies is proportional to their masses, inversely proportional to the square of distance between them and the coefficient of proportionality is called the gravitational constant. Numerous experimental data show that the Newton law works with a very high accuracy. However, this only means that $G$ is a good *phenomenological* parameter. At the level of the Newton law we cannot prove that $G$ is the exact constant which does not change with time, does not depend on masses, distances etc.

From this point of view, in General Relativity (GR) the situation is analogous. Here we have two different quantities which have different dimensions: the stress energy tensor of matter $T_{\mu\nu}$ and the Ricci tensor $R_{\mu\nu}$ describing the curvature of space-time. We will not discuss the meaning of those quantities and only note that the Einstein equations read

$$R_{\mu\nu} = (8\pi G/c^4)T_{\mu\nu} \quad (\mu, \nu = 0, 1, 2, 3) \tag{1}$$

Therefore $G$ is the *phenomenological* coefficient of proportionality between those quantities. In the formalism of GR, $G$ can be only a constant but GR cannot calculate it or give a *theoretical* explanation why this value should be as it is.

GR is a classical (i.e. non-quantum) theory based on the minimum action principle. The Lagrangian of GR should be invariant under coordinate transformations and the simplest way to satisfy this requirement is a choice when it is proportional to the so called tensor of scalar curvature $R_c$. In this case the Newton gravitational law is recovered in the nonrelativistic approximation and the theory is successful in explaining several well known phenomena. However, the argument that this choice is simple and agrees with the data, cannot be treated as a fundamental requirement. Another reason for choosing the linear case is that here equations of motions are of the second order while in quadratic, cubic cases etc. they will be of higher orders. However, this reason also cannot be treated as fundamental. It has been argued in the literature that GR is a low energy approximation of a theory where equations of motion contain higher order derivatives. In particular, a rather popular approach is when the Lagrangian contains a function $f(R_c)$ which should be defined from additional considerations. In that case the constant $G$ in the Lagrangian is not the same as the standard gravitational constant. It is believed that the nature of gravity will be understood in the future quantum theory of gravity but efforts to construct this theory has not been successful yet. Therefore the above remarks show that there are no solid reasons to treat $G$ as an exact fundamental constant.

Special Relativity works with Minkowski space, which is also called the space of *events*. The points of the space are defined by time $t$ and coordinates $(x, y, z)$. For example, when we consider the motion of a body, the first event is that at the moment of time $t_1$ the coordinates of the body were $(x_1, y_1, z_1)$; the second event is that at the moment of time $t_2$ the coordinates were $(x_2, y_2, z_2)$ etc. One can define intervals between different events. It is very important to note that Minkowski space has a physical meaning only as *a space of events for real bodies*. In particular, the notion of empty space has no physical meaning since it contradicts the physical principle that a definition of a physical quantity is a description of how this quantity should be measured. In particular, one can discuss how coordinates of *real bodies* can be measured but there is no way to measure coordinates of the empty space which exist only in our imagination.

Physicists consider others spaces of events, for example de Sitter (dS) space. Below we discuss a relation between Minkowski and dS spaces but first we consider a simpler example. Consider the surface of the Earth and assume that this surface is a two-dimensional sphere $S^2$ in the three-dimensional space. If $R_E$ is the radius of the Earth and $(x, y, z)$ are the coordinates of the three-dimensional space then the surface of the Earth contains only points whose coordinates satisfy the requirement $x^2+y^2+z^2 = R_E^2$. Since we have one restriction on three coordinates, $S^2$ is indeed two-dimensional. For example, instead of characterizing the points of $S^2$ by three coordinates with one restriction, we can characterize them by polar coordinates $(\theta, \varphi)$. With such a description we use only dimensionless quantities (angles) and there is no need to know the value of $R_E$ and the units in which the quantities $(x, y, z)$ are measured.

Suppose that the coordinates of the North pole of the Earth in the three-dimensional space are $(0, 0, R_E)$ and consider a vicinity of the North pole consisting of points on the Earth surface such that $z$ is very close to $R_E$ and the values of $|x|$ and $|y|$ are much less than $R_E$. If a man lives in this vicinity and does not go to areas where $|x|$ and $|y|$ are comparable to $R_E$, he does not feel the curvature of the Earth surface and thinks that it is flat. He can describe points in this vicinity not only by $(\theta, \varphi)$ but also by $(x, y)$. In the latter case the units for $(x, y)$ depend on the units in which $R_E$ is measured.

dS space is a set of points characterizing by five coordinates $(t, x, y, z, u)$ which satisfy the restriction $t^2 - x^2 - y^2 - z^2 - u^2 = -R^2$ where $R$ is some parameter and we assume that time $t$ has the same dimension as the spacial coordinates $(x, y, z, u)$. Therefore dS space is four-dimensional. By analogy with the example with the Earth, one can describe points on dS space by four dimensionless analogs of angular coordinates and those coordinates do not depend on $R$ at all. Consider a vicinity of the North pole of dS space assuming that the pole has the coordinates $(0, 0, 0, 0, R)$. If we consider only such points of dS space that $u$ is close to $R$ and all the values of $(t, x, y, z)$ are much less than $R$ then in this vicinity, geometry is very close to that of Minkowski space. The dimension of the quantities $(t, x, y, z)$ in this vicinity depends on the dimension in which $R$ is measured. For historical reasons, the curvature of dS space is discussed not in terms of $R$ but in terms of the CC $\Lambda = 3/R^2$. Then the experimental results [1-3] say that $R$ is of order $10^{26}m$. One might say that the greater the value of $R$ is, the bigger vicinity of the North pole is similar to Minkowski space. This discussion shows that in dS theory $\Lambda$ is not present at all; it appears only when one wishes to parametrize dS space by dimensionful coordinates. Hence the question of why $\Lambda$ is as it is, is not fundamental since the answer is: because we want to measure distances in meters.

When the Lagrangian is linear in $R_c$, the most general Einstein equations are not (1) but

$$R_{\mu\nu} + \Lambda g_{\mu\nu} = (8\pi G/c^4)T_{\mu\nu} \tag{2}$$

where $g_{\mu\nu}$ is the space-time metric. As follows from this expression, in GR the curvature and the metric depend on the presence of matter. In the formal limit, when matter disappears, solutions of Eq. (2) are Minkowski space when $\Lambda = 0$, dS space when $\Lambda > 0$ and anti de Sitter (AdS) space when $\Lambda < 0$. In this connection the following extremely important question arises. As discussed above, these spaces have a physical meaning only as *spaces of events for real bodies*. At the same time, in GR those spaces arise as solutions of the Einstein equations when matter is absent. In other words, those spaces arise only as empty spaces. Of course, in mathematics one can consider different spaces without thinking about the physical meaning of empty space. But in physics the notion of empty space has no meaning. We believe that these remarks show that the formal limit of GR when matter disappears is unphysical.

The history of the appearance of $\Lambda$ in Eq. (2) is well known. Einstein originally wrote his equations without $\Lambda$, i.e. in the form (1). Then, as it has been shown by Fridman, the Universe is not stationary. For this reason Einstein introduced $\Lambda$ in his equations. However, when Hubble discovered that the Universe was not stationary, Einstein said that introducing $\Lambda$ was the greatest blunder of his life. In textbooks on gravity written before 1998 (when the cosmological acceleration was discovered) it is often claimed that $\Lambda$ is not needed since its presence contradics the philosophy of GR: matter creates curvature of space-time, so in the absence of matter space-time should be flat (i.e. Minkowski) while empty dS space is not flat.

As noted above, such a philosophy has no physical meaning since the notion of empty space is unphysical. That's why the discovery of the fact that $\Lambda \neq 0$ has ignited many discussions. The most popular approach is as follows. One can move the term with $\Lambda$ in Eq. (2) from the left-hand side to the right-hand one:

$$R_{\mu\nu} = (8\pi G/c^4)T_{\mu\nu} - \Lambda g_{\mu\nu} \tag{3}$$

Then the term with $\Lambda$ is treated as the stress-energy tensor of a hidden matter which is called dark energy: $(8\pi G/c^4)T_{\mu\nu}^{DE} = -\Lambda g_{\mu\nu}$. With the observed value of $\Lambda$ this dark energy contains approximately 75% of the energy of the Universe. In this approach $G$ is treated as a fundamental constant and one might try to express $\Lambda$ in terms of $G$. The existing quantum theory of gravity cannot perform this calculation unambiguously since the theory contains strong divergences. With a reasonable cutoff parameter, the result for $\Lambda$ is such that in units where $\hbar = c = 1$, $G\Lambda$ is of order unity. This result is expected from dimensional considerations since in these units, the dimension of $G$ is $length^2$ while the dimension of $\Lambda$ is $1/length^2$. However, such value of $\Lambda$ is greater than the observed one by 122 orders of magnitude. This problem is called the CC problem or dark energy problem.

Several authors criticized this approach from the following considerations. GR without the contribution of $\Lambda$ has been confirmed with a good accuracy in experiments in the Solar system. If $\Lambda$ is as small as it has been observed then it can have a significant effect only at cosmological distances while for experiments in the Solar system the role of such a small value is negligible. The authors of Ref. [5] entitled "Why All These Prejudices Against a Constant?", note that even in a special case $f(R_c) = R_c$, the most general form of the Einstein equations is as in Eq. (2) and so it is not clear why we should think that only a special case (1) is allowed. If we accept the theory containing a constant $G$ which cannot be calculated and is taken from the outside then why can't we accept a theory containing two independent (*phenomenological*) constants?

In our approach one can easily and naturally explain the cosmological acceleration proceeding only from basic principles of quantum theory without involving space-time background, dark energy etc. In our Refs. [6, 7] it has been explained in detail that symmetry on quantum level is defined by commutation relations between the operators belonging to the symmetry algebra. In particular, *by definition*, relativistic (Poincare) symmetry on quantum level means that the four-momentum $P^\mu$ operator and the operators of Lorentz angular momenta $M^{\mu\nu}$ ($\mu, \nu = 0, 1, 2, 3, \quad M^{\mu\nu} = -M^{\nu\mu}$)

satisfy the commutation relations

$$[P^\mu, P^\nu] = 0 \quad [P^\mu, M^{\nu\rho}] = -i(\eta^{\mu\rho} P^\nu - \eta^{\mu\nu} P^\rho)$$
$$[M^{\mu\nu}, M^{\rho\sigma}] = -i(\eta^{\mu\rho} M^{\nu\sigma} + \eta^{\nu\sigma} M^{\mu\rho} - \eta^{\mu\sigma} M^{\nu\rho} - \eta^{\nu\rho} M^{\mu\sigma}) \tag{4}$$

where $\eta^{\mu\nu}$ is the diagonal metric tensor such that $\eta^{00} = -\eta^{11} = -\eta^{22} = -\eta^{33} = 1$. dS or AdS symmetries on quantum level *are defined* such that that the operators describing a quantum system under consideration, satisfy the commutation relations

$$[M^{ab}, M^{cd}] = -i(\eta^{ac} M^{bd} + \eta^{bd} M^{ac} - \eta^{ad} M^{bc} - \eta^{bc} M^{ad}) \tag{5}$$

where $a, b = 0, 1, 2, 3, 4$, $M^{ab} = -M^{ba}$ and $\eta^{ab}$ is the diagonal metric tensor such that $\eta^{00} = -\eta^{11} = -\eta^{22} = -\eta^{33} = 1$, $\eta^{44} = \mp 1$ for the dS and AdS cases, respectively.

It is usually said that Eqs. (4) and (5) are written in units $c = \hbar = 1$. However, as we note in Ref. [6], in relativistic quantum theory the quantities $c$ and $\hbar$ *should not* be present at all; they arise only if one wishes to express physical quantities in this theory in terms of standard units $(kg, m, sec)$. Therefore in the case of dS and AdS symmetries, all the operators $M^{ab}$ are dimensionless while in the case of Poincare symmetry only the operators of the Lorentz algebra are dimensionless while the momentum operators have the dimension $1/length$. Eq. (4) is a special case of Eq. (5) obtained as follows. If $R$ is a parameter with the dimension *length* and the operators $P^\mu$ are *defined* as $P^\mu = M^{4\mu}/R$ then in the formal limit $R \to \infty$ one gets Eq. (4) from Eq. (5). This contraction procedure is well known. Hence from the point of view of symmetry on quantum level, dS and AdS symmetries are more natural and general than Poincare symmetry. It is also clear that on quantum level dS and AdS theories can be constructed without parameters having the dimension of length. Such parameters may be used if one wishes to interpret the results in classical approximation or in Poincare limit but they are not fundamental. In particular, neither $\Lambda$ nor $G$ can be fundamental.

The next step in our construction is a definition of an elementary particle. This problem has a long history. In Refs. [6, 7] we argue that a general definition, not depending on the choice of the classical background and on whether we consider a local or nonlocal theory, is that a particle is elementary if the set of its wave functions is the space of an irreducible representation (IR) of the symmetry algebra in the given theory. This is in the spirit of Wigner's approach to Poincare symmetry. The construction of IRs is needed not only for describing elementary particles but even for describing the motion of a macroscopic body as a whole. For example, when we consider the interaction between two macroscopic bodies such that the distance between them is much greater than their sizes, it suffices to describe each body as a whole by using the IR with the corresponding mass.

The above notions are fully sufficient to describe systems of *free* bodies in Poincare, dS or AdS theories. In particular, we don't need Minkowski, dS or AdS spaces at all.

A standard quantum-mechanical calculation, which is described in detail in our Ref. [6], shows that in the approximation when classical mechanics works with a good accuracy (so called semiclassical approximation in quantum theory), the relative acceleration $\mathbf{a}$ of two *free* bodies is zero only in Poincare-invariant theory while in dS and AdS cases, $\mathbf{a} = \Lambda c^2\mathbf{r}/3$ where $\mathbf{r}$ is the vector of the relative distance between the particles. The result is not zero (as one would expect from the first Newton law) but in the formal limit $\Lambda \to 0$ the first Newton law is recovered since $\mathbf{a}$ becomes zero. The result depends on $\Lambda$ only because we wish to express the acceleration in terms of standard time and coordinates (see the above discussion about the relation between Minkowski and dS spaces). From the formal point of view, the result is the same as in GR on dS or AdS spaces, where $\Lambda > 0$ in the dS case and $\Lambda < 0$ in the AdS one. However, our result has been obtained by using only standard quantum-mechanical notions while dS or AdS spaces, their metric, connection etc. have not been involved at all. In view of this approach, the data of Refs. [1-3] that $\Lambda > 0$ should be interpreted not such that the spacetime background is dS space but that the dS algebra is more pertinent than the Poincare or AdS ones. As shown in our papers (see e.g. Ref. [7] and references therein), this opens a radically new approach to gravity where the quantity $G$ is not taken from the outside but (in principle) can be calculated. We believe that our result is a strong indication that the results of GR can be recovered from semiclassical approximation in quantum theory without using spacetime background and differential geometry at all.

The above discussion shows that the phenomenon of cosmological acceleration can be naturally explained from first principles of quantum theory without involving spacetime background, dark energy or other artificial notions.

**Acknowledgements:** The author is grateful to Volodya Netchitailo for reading the manuscript and important remarks.

# References

[1] S. Perlmutter, G. Aldering, G. Goldhaber *et al.* Astrophys. J. **517**, 565-586 (1999).

[2] A. Melchiorri, P.A.R. Ade, P. de Bernardis *et al.* Astrophys. J. **536**, L63-L66 (2000).

[3] D.N. Spergel, R. Bean, O. Dore *et. al.* Astrophys. J. Suppl. **170**, 377-408 (2007).

[4] K. Nakamura and Particle Data Group. J. Phys. **G37**, 075021 (2010).

[5] E. Bianchi and C. Rovelli. arXiv:1002.3966v3.

[6] F. Lev. Symmetry, Special Issue: Quantum Symmetry. **2(4)**, 1945-1980 (2010).

[7] F. Lev. arXiv:1110.0240; viXra:1110.0003.