The principle of least action in covariant theory of gravitation

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By variation of action integral equations for computing of metric, equations of motion of substance, as well as equations for gravitational and electromagnetic fields in covariant theory of gravitation are found. In covariant form stress-energy tensor of gravitational field, strength tensor of gravitational field and 4-current of mass are determined. The meaning of cosmological constant and its relation to components of energy density in action functional are explained. The results indicate validity of Mach's principle, assuming that gravitation effects are due to flows of gravitons. The idea substantiates that metric can be entirely determined by variables describing properties of fields.

Keywords: *action; metric; cosmological constant; stress-energy tensor of gravitational field; equations of motion; field equations; covariant theory of gravitation.*

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Covariant theory of gravitation (CTG) is one of alternative theories of gravitation in relation to general theory of relativity. The purpose of this paper is to derive equations of CTG from principle of least action. As the basis of our discussion we will use works of Einstein [1], Dirac [2], Pauli [3], Fock [4], Landau and Lifshitz [5].

We will use international system of units, basic coordinates in the form of coordinates with the contravariant indices (x^0, x^1, x^2, x^3), metric signature (+, –, –, –), metric tensor $g_{\mu\nu}$. The presence of repeated indices in formulas implies Einstein summation convention, which is a separate sum for each repeated index.

The action function

In the case of continuously distributed throughout the volume of space matter, action function for the matter in gravitational and electromagnetic fields in covariant theory of gravitation is as follows:

$$S = \int L\,dt = \int \left(k(R - 2\Lambda) - c\rho_0 - \frac{1}{c} D_\mu J^\mu + \frac{c}{16\pi\gamma} \Phi_{\mu\nu}\Phi^{\mu\nu} - \frac{1}{c} A_\mu j^\mu - \frac{c\varepsilon_0}{4} F_{\mu\nu} F^{\mu\nu} \right) \sqrt{-g}\, d\Sigma, \tag{1}$$

where L – Lagrange function or Lagrangian,

dt – differential of time in used reference frame,

k – some coefficient,

R – scalar curvature,

Λ – a constant, which characterizes the energy density of system as a whole, and therefore is a function of the system,

c – speed of light, as a measure of velocity of propagation of electromagnetic and gravitational interactions,

ρ_0 – mass of substance unit volume in reference frame in which the volume is at rest,

$D_\mu = \left(\frac{\psi}{c}, -\boldsymbol{D} \right)$ – 4-potential of gravitational field which is described by scalar potential ψ and vector potential $\boldsymbol{D}$ of the field,

J^μ – 4-vector of mass current,

γ – gravitational constant,

$\Phi_{\mu\nu} = \nabla_\mu D_\nu - \nabla_\nu D_\mu = \partial_\mu D_\nu - \partial_\nu D_\mu$ – gravitational tensor (tensor of gravitational field strength),

$\Phi^{\alpha\beta} = g^{\alpha\mu} g^{\nu\beta} \Phi_{\mu\nu}$ – definition of gravitational tensor with contravariant indices using the metric tensor $g^{\alpha\mu}$,

$A_\mu = \left(\frac{\varphi}{c}, -\boldsymbol{A} \right)$ – 4-potential of electromagnetic field, defined by scalar potential φ and vector potential $\boldsymbol{A}$ of the field,

j^μ – 4-vector of electric current density,

ε_0 – vacuum permittivity,

$F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu = \partial_\mu A_\nu - \partial_\nu A_\mu$ – electromagnetic tensor (field strength tensor),

$\sqrt{-g}\, d\Sigma = \sqrt{-g}\, c\, dt\, dx^1 dx^2 dx^3$ – invariant 4-volume, expressed through differential of time coordinate $dx^0 = c\, dt$, through product $dx^1 dx^2 dx^3$ of differentials of spatial coordinates, and through the square root $\sqrt{-g}$ of determinant g of metric tensor, taken with negative sign.

The symbol ∇_μ denotes covariant derivative with respect to coordinates (in this case the coordinates x^μ). Similarly, $\partial_\mu = \frac{\partial}{\partial x^\mu}$ is operator of partial derivative with respect to coordinates or 4-gradient.

The integrand in (1) is Lagrangian function, consisting of six terms. The first term with scalar curvature R depends on the metric tensor and its derivatives with respect to coordinates. In covariant theory of gravitation (CTG) metric is necessary to consider the impact of fundamental fields (which include electromagnetic and gravitational fields) of material bodies on results of spacetime measurements near the bodies. Effect of field on measurement results shows that under the action of field electromagnetic waves are deflected from rectilinear motion, electromagnetic clock changed its course, and measured distances changed its value. These effects can be described by introducing a curved spacetime with metric tensor $g_{\mu\nu}$ instead of flat Minkowski space with its single metric tensor $\eta_{\mu\nu}$. In CTG gravitational field is an independent physical field and the metric tensor $g_{\mu\nu}$ has the geometric meaning and is auxiliary, unlike general relativity where the metric field completely replaces gravitational field.

In CTG second term in (1) is not simply related to rest energy density of substance and its inertia with respect to applied forces. According to [6], [7], the rest mass (and density of substance at rest) is a consequence of strong gravitation and electromagnetic interactions operating at the level of elementary particles. Only the first and second terms in (1) are associated with microscopic fundamental fields, while other terms described action of macroscopic gravitational and electromagnetic fields. The division into microscopic and macroscopic fundamental fields follows from the theory of infinite nesting of matter in which at every main level of matter operates its own gravitational field. As a result, the usual gravitation is assumed as long-range component of strong gravitation.

Third term in (1) $-\frac{1}{c}D_{\mu}J^{\mu}$ is invariant with respect to different types of coordinate transformations that reflect the interaction of mass current density $J^{\mu}=\rho_0 u^{\mu}$ of arbitrary unit volume of substance with gravitational field.

According to [8], the fourth term in (1) associated with energy field, is an invariant of gravitational field does not change its form by changing the reference system. The fifth and sixth terms, for electromagnetic field, similar in structure to the third and fourth terms for gravitational field. And 4-vector of electric current density j^{μ} can be determined by charge density ρ_{0q} of substance unit volume and 4-velocity: $j^{\mu}=\rho_{0q}u^{\mu}$.

In CTG 4-potentials D_{μ} and A_{μ} with covariant indices, and 4-currents J^{μ} and j^{μ} with contravariant indices were defined in [7] and [9] as initial concepts for construction of axiomatic theory. Hence, for example, 4-vector $D^{\mu}=g^{\mu\nu}D_{\nu}$ can not be found in absence of information about metric in any frame of reference.

Variation of curvature invariant

To obtain equations for metric should be set to zero variation of action function for the case when in Lagrangian varies the metric tensor $g_{\mu\nu}$. At the same time variation of metric tensor should be zero on the borders of four-dimensional volume over which in (1) integration is performed. For full variation of the action should be the next:

$$\delta S=\delta\int\left(k(R-2\Lambda)-c\rho_0-\frac{1}{c}D_\mu\,\rho_0u^\mu+\frac{c}{16\pi\gamma}\Phi_{\mu\nu}\Phi^{\mu\nu}-\frac{1}{c}A_\mu\,\rho_{0q}u^\mu-\frac{c\varepsilon_0}{4}F_{\mu\nu}F^{\mu\nu}\right)\sqrt{-g}\,d\Sigma=0. \tag{2}$$

Let's find the variation associated with the first term in (2). Using definition of scalar curvature R through Christoffel symbols, we have as in [2]:

$$\delta S_1=\delta\int k(R-2\Lambda)\sqrt{-g}\,d\Sigma=k\delta\int R_1\sqrt{-g}\,d\Sigma+k\delta\int R_2\sqrt{-g}\,d\Sigma-2\Lambda k\int\delta\sqrt{-g}\,d\Sigma, \tag{3}$$

where $R_1=g^{\mu\nu}\left(\partial_\kappa\Gamma^\kappa_{\mu\nu}-\partial_\nu\Gamma^\kappa_{\mu\kappa}\right)$, $R_2=g^{\mu\nu}\left(\Gamma^\kappa_{\mu\nu}\Gamma^\sigma_{\kappa\sigma}-\Gamma^\kappa_{\mu\sigma}\Gamma^\sigma_{\kappa\nu}\right)$, $R=R_1+R_2$.

The expression for $R_1\sqrt{-g}$ can be obtained by differentiation by parts:

$$R_1\sqrt{-g}=\partial_\kappa\left(g^{\mu\nu}\Gamma^\kappa_{\mu\nu}\sqrt{-g}\right)-\partial_\nu\left(g^{\mu\nu}\Gamma^\kappa_{\mu\kappa}\sqrt{-g}\right)-\Gamma^\kappa_{\mu\nu}\partial_\kappa\left(g^{\mu\nu}\sqrt{-g}\right)+\Gamma^\kappa_{\mu\kappa}\partial_\nu\left(g^{\mu\nu}\sqrt{-g}\right). \tag{4}$$

The first two terms on the right-hand side of (4) are the total derivatives (divergence), and after substituting in (3), integrals of the divergence over volume under Gauss's theorem can be replaced by integrals over surface surrounding the volume on which the integration takes place. Since variation of metric tensor on the surface is zero, these terms will not contribute to the variation of action function, so that in (4) should be taken into account only the last two terms. Then we can use two relations:

$$\partial_\kappa\left(g^{\mu\nu}\sqrt{-g}\right)=\sqrt{-g}\left(g^{\mu\nu}\Gamma^\alpha_{\kappa\alpha}-g^{\nu\beta}\Gamma^\mu_{\beta\kappa}-g^{\mu\alpha}\Gamma^\nu_{\alpha\kappa}\right),\qquad \partial_\nu\left(g^{\mu\nu}\sqrt{-g}\right)=-\sqrt{-g}\,g^{\nu\beta}\Gamma^\mu_{\beta\nu}. \tag{5}$$

Substituting them in the last two terms in (4) and renaming some of indexes on which there is summation, we have:

$$\begin{aligned}
&-\Gamma^\kappa_{\mu\nu}\sqrt{-g}\left(g^{\mu\nu}\Gamma^\alpha_{\kappa\alpha}-g^{\nu\beta}\Gamma^\mu_{\beta\kappa}-g^{\mu\alpha}\Gamma^\nu_{\alpha\kappa}\right)-\Gamma^\kappa_{\mu\kappa}\sqrt{-g}\,g^{\nu\beta}\Gamma^\mu_{\beta\nu}=\\
&=-\sqrt{-g}\left(g^{\mu\nu}\Gamma^\alpha_{\kappa\alpha}\Gamma^\kappa_{\mu\nu}-g^{\nu\beta}\Gamma^\mu_{\beta\kappa}\Gamma^\kappa_{\mu\nu}-g^{\mu\alpha}\Gamma^\nu_{\alpha\kappa}\Gamma^\kappa_{\mu\nu}+g^{\nu\beta}\Gamma^\mu_{\beta\nu}\Gamma^\kappa_{\mu\kappa}\right)=\\
&=-2\sqrt{-g}\left(g^{\mu\nu}\Gamma^\alpha_{\kappa\alpha}\Gamma^\kappa_{\mu\nu}-g^{\nu\beta}\Gamma^\mu_{\beta\kappa}\Gamma^\kappa_{\mu\nu}\right)=-2\sqrt{-g}\,g^{\mu\nu}\left(\Gamma^\kappa_{\mu\nu}\Gamma^\sigma_{\kappa\sigma}-\Gamma^\kappa_{\mu\sigma}\Gamma^\sigma_{\kappa\nu}\right)=-2R_2\sqrt{-g}.
\end{aligned}$$

As a result, instead of (3) can be written:

$$\delta S_1 = -k\int \delta\left(R_2\sqrt{-g}\right)d\Sigma - 2\Lambda k\int \delta\sqrt{-g}\,d\Sigma . \tag{6}$$

The first variation in (6) will be:

$$\delta\left(R_2\sqrt{-g}\right) = \delta\left(\Gamma^{\kappa}_{\mu\nu}\Gamma^{\sigma}_{\kappa\sigma}g^{\mu\nu}\sqrt{-g}\right) - \delta\left(\Gamma^{\kappa}_{\mu\sigma}\Gamma^{\sigma}_{\kappa\nu}g^{\mu\nu}\sqrt{-g}\right). \tag{7}$$

Using the relation: $\Gamma^{\sigma}_{\kappa\sigma} = \frac{1}{\sqrt{-g}}\partial_\kappa\sqrt{-g}$, differentiation by parts, and using the second relation in (5), for the first part (7) we get:

$$\begin{aligned}
\delta\left(\Gamma^{\kappa}_{\mu\nu}\Gamma^{\sigma}_{\kappa\sigma}g^{\mu\nu}\sqrt{-g}\right) &= \Gamma^{\kappa}_{\mu\nu}\delta\left(\Gamma^{\sigma}_{\kappa\sigma}g^{\mu\nu}\sqrt{-g}\right) + \Gamma^{\sigma}_{\kappa\sigma}g^{\mu\nu}\sqrt{-g}\,\delta\Gamma^{\kappa}_{\mu\nu} = \\
&= \Gamma^{\kappa}_{\mu\nu}\delta\left(g^{\mu\nu}\partial_\kappa\sqrt{-g}\right) + \Gamma^{\sigma}_{\kappa\sigma}\delta\left(g^{\mu\nu}\Gamma^{\kappa}_{\mu\nu}\sqrt{-g}\right) - \Gamma^{\sigma}_{\kappa\sigma}\Gamma^{\kappa}_{\mu\nu}\delta\left(g^{\mu\nu}\sqrt{-g}\right) = \\
&= \Gamma^{\kappa}_{\mu\nu}\delta\left(g^{\mu\nu}\partial_\kappa\sqrt{-g}\right) - \Gamma^{\sigma}_{\kappa\sigma}\delta\left(\partial_\nu\left(g^{\kappa\nu}\sqrt{-g}\right)\right) - \Gamma^{\sigma}_{\kappa\sigma}\Gamma^{\kappa}_{\mu\nu}\delta\left(g^{\mu\nu}\sqrt{-g}\right).
\end{aligned} \tag{8}$$

The expression for derivative of metric tensor has the form: $\partial_\alpha g^{\nu\beta} = -g^{\beta\kappa}\Gamma^{\nu}_{\kappa\alpha} - g^{\nu\kappa}\Gamma^{\beta}_{\kappa\alpha}$. After multiplication by $\sqrt{-g}$, taking variation and another multiplication by $\Gamma^{\alpha}_{\nu\beta}$ would be:

$$\begin{aligned}
\delta\left(\sqrt{-g}\,\partial_\alpha g^{\nu\beta}\right)\Gamma^{\alpha}_{\nu\beta} &= -\delta\left(g^{\beta\kappa}\Gamma^{\nu}_{\kappa\alpha}\sqrt{-g}\right)\Gamma^{\alpha}_{\nu\beta} - \delta\left(g^{\nu\kappa}\Gamma^{\beta}_{\kappa\alpha}\sqrt{-g}\right)\Gamma^{\alpha}_{\nu\beta} = \\
&= -2\delta\left(\Gamma^{\nu}_{\kappa\alpha}g^{\beta\kappa}\sqrt{-g}\right)\Gamma^{\alpha}_{\nu\beta}.
\end{aligned}$$

We now transform the second part of (7), using replacing of indices, operation of differentiation by parts, and previous expression:

$$\begin{aligned}
\delta\left(\Gamma^{\kappa}_{\mu\sigma}\Gamma^{\sigma}_{\kappa\nu}g^{\mu\nu}\sqrt{-g}\right) &= \delta\left(\Gamma^{\kappa}_{\mu\sigma}\Gamma^{\sigma}_{\kappa\nu}\right)g^{\mu\nu}\sqrt{-g} + \Gamma^{\kappa}_{\mu\sigma}\Gamma^{\sigma}_{\kappa\nu}\delta\left(g^{\mu\nu}\sqrt{-g}\right) = \\
&= 2\delta\left(\Gamma^{\kappa}_{\mu\sigma}\right)\Gamma^{\sigma}_{\kappa\nu}g^{\mu\nu}\sqrt{-g} + \Gamma^{\kappa}_{\mu\sigma}\Gamma^{\sigma}_{\kappa\nu}\delta\left(g^{\mu\nu}\sqrt{-g}\right) = \\
&= 2\delta\left(\Gamma^{\kappa}_{\mu\sigma}g^{\mu\nu}\sqrt{-g}\right)\Gamma^{\sigma}_{\kappa\nu} - \Gamma^{\kappa}_{\mu\sigma}\Gamma^{\sigma}_{\kappa\nu}\delta\left(g^{\mu\nu}\sqrt{-g}\right) = \\
&= -\delta\left(\sqrt{-g}\,\partial_\sigma g^{\kappa\nu}\right)\Gamma^{\sigma}_{\kappa\nu} - \Gamma^{\kappa}_{\mu\sigma}\Gamma^{\sigma}_{\kappa\nu}\delta\left(g^{\mu\nu}\sqrt{-g}\right).
\end{aligned} \tag{9}$$

Substitution of (8) and (9) in (7) yields:

$$\delta\left(R_2\sqrt{-g}\right)=\Gamma^{\kappa}_{\mu\nu}\delta\left(g^{\mu\nu}\partial_\kappa\sqrt{-g}\right)-\Gamma^{\sigma}_{\kappa\sigma}\delta\left(\partial_\nu\left(g^{\kappa\nu}\sqrt{-g}\right)\right)-\Gamma^{\sigma}_{\kappa\sigma}\Gamma^{\kappa}_{\mu\nu}\delta\left(g^{\mu\nu}\sqrt{-g}\;\right)+$$
$$+\delta\left(\sqrt{-g}\,\partial_\sigma g^{\kappa\nu}\right)\Gamma^{\sigma}_{\kappa\nu}+\Gamma^{\kappa}_{\mu\sigma}\Gamma^{\sigma}_{\kappa\nu}\delta\left(g^{\mu\nu}\sqrt{-g}\right)=$$
$$=\Gamma^{\kappa}_{\mu\nu}\delta\left(\partial_\kappa\left(g^{\mu\nu}\sqrt{-g}\right)\right)-\Gamma^{\sigma}_{\kappa\sigma}\delta\left(\partial_\nu\left(g^{\kappa\nu}\sqrt{-g}\right)\right)+\left(\Gamma^{\kappa}_{\mu\sigma}\Gamma^{\sigma}_{\kappa\nu}-\Gamma^{\sigma}_{\kappa\sigma}\Gamma^{\kappa}_{\mu\nu}\right)\delta\left(g^{\mu\nu}\sqrt{-g}\right).$$

(10)

The terms $\Gamma^{\kappa}_{\mu\nu}\delta\left(\partial_\kappa\left(g^{\mu\nu}\sqrt{-g}\right)\right)$ and $\Gamma^{\sigma}_{\kappa\sigma}\delta\left(\partial_\nu\left(g^{\kappa\nu}\sqrt{-g}\right)\right)$ in (10) can be transformed:

$$\Gamma^{\kappa}_{\mu\nu}\delta\left(\partial_\kappa\left(g^{\mu\nu}\sqrt{-g}\right)\right)=\Gamma^{\kappa}_{\mu\nu}\partial_\kappa\delta\left(g^{\mu\nu}\sqrt{-g}\right)=\partial_\kappa\left(\Gamma^{\kappa}_{\mu\nu}\delta\left(g^{\mu\nu}\sqrt{-g}\right)\right)-\left(\partial_\kappa\Gamma^{\kappa}_{\mu\nu}\right)\delta\left(g^{\mu\nu}\sqrt{-g}\right).$$
$$\Gamma^{\sigma}_{\kappa\sigma}\delta\left(\partial_\nu\left(g^{\kappa\nu}\sqrt{-g}\right)\right)=\Gamma^{\sigma}_{\kappa\sigma}\partial_\nu\delta\left(g^{\kappa\nu}\sqrt{-g}\right)=\partial_\nu\left(\Gamma^{\sigma}_{\kappa\sigma}\delta\left(g^{\kappa\nu}\sqrt{-g}\right)\right)-\left(\partial_\nu\Gamma^{\sigma}_{\kappa\sigma}\right)\delta\left(g^{\kappa\nu}\sqrt{-g}\right).$$

(11)

In equations (11) are divergences such as $\partial_\kappa\left(\Gamma^{\kappa}_{\mu\nu}\delta\left(g^{\mu\nu}\sqrt{-g}\right)\right)$, which, after substituting in (10) and then into (6) will be integrated over 4-volume and transformed into integrals over the surface, where variations are zero. With this in mind, after substituting (10) and (11) into (6) is obtained:

$$-k\int\delta\left(R_2\sqrt{-g}\right)d\Sigma=$$
$$=-k\int\left(\partial_\nu\Gamma^{\sigma}_{\mu\sigma}-\partial_\kappa\Gamma^{\kappa}_{\mu\nu}+\Gamma^{\kappa}_{\mu\sigma}\Gamma^{\sigma}_{\kappa\nu}-\Gamma^{\sigma}_{\kappa\sigma}\Gamma^{\kappa}_{\mu\nu}\right)\delta\left(g^{\mu\nu}\sqrt{-g}\right)d\Sigma=k\int R_{\mu\nu}\delta\left(g^{\mu\nu}\sqrt{-g}\right)d\Sigma. \quad (12)$$

where $R_{\mu\nu}$ is Ricci tensor.

For variations of metric tensor $g^{\mu\nu}$ and $\sqrt{-g}$ can be written:

$$\delta g^{\mu\nu}=-g^{\mu\alpha}g^{\nu\beta}\delta g_{\alpha\beta}, \qquad \delta\sqrt{-g}=\frac{\sqrt{-g}}{2}g^{\alpha\beta}\delta g_{\alpha\beta}. \quad (13)$$

Using (13) in (12), we find:

$$-k\int \delta\left(R_2\sqrt{-g}\right)d\Sigma = k\int R_{\mu\nu}\delta\left(g^{\mu\nu}\sqrt{-g}\right)d\Sigma = k\int R_{\mu\nu}\left(\sqrt{-g}\,\delta g^{\mu\nu} + g^{\mu\nu}\delta\sqrt{-g}\right)d\Sigma =$$
$$= k\int R_{\mu\nu}\left(-g^{\mu\alpha}g^{\nu\beta} + g^{\mu\nu}\frac{1}{2}g^{\alpha\beta}\right)\sqrt{-g}\,\delta g_{\alpha\beta}d\Sigma = k\int\left(-R^{\alpha\beta} + \frac{1}{2}g^{\alpha\beta}R\right)\sqrt{-g}\,\delta g_{\alpha\beta}d\Sigma.$$

With this result and expression $\delta\sqrt{-g}$ from (13), we have:

$$\delta S_1 = k\int\left(-R^{\alpha\beta} + \frac{1}{2}g^{\alpha\beta}R - \Lambda g^{\alpha\beta}\right)\sqrt{-g}\,\delta g_{\alpha\beta}d\Sigma. \tag{14}$$

Variation of invariant mass density

The second term in (2) is an invariant associated with the 4-current of mass J^μ, since we can write:

$$c\rho_0 = \sqrt{g_{\mu\nu}J^\mu J^\nu} = \sqrt{J_\nu J^\nu}\,. \tag{15}$$

4-vector J^μ can be determined through 4-velosity $u^\mu = \frac{dx^\mu}{d\tau}$, where $dx^\mu = (dx^0, dx^1, dx^2, dx^3)$ is displacement 4-vector, $d\tau$ – differential of proper time, as follows: $J^\mu = \rho_0 u^\mu = \frac{u^\mu\sqrt{g_{\alpha\beta}J^\alpha J^\beta}}{c}$, and $u_\beta u^\beta = c^2$. In elementary particle physics the values of mass and velocity of particles are secondary because only their energies E and momentums $\boldsymbol{p}$ are quantities directly found from experiments. These quantities are in 4-momentum of a particle: $p^\alpha = \left(\frac{E}{c}, \boldsymbol{p}\right) = mu^\alpha$, and the invariant mass m becomes a secondary concept, which may be found from the relation $\sqrt{p_\beta p^\beta} = mc = \frac{\sqrt{E^2 - p^2c^2}}{c}$. Accordingly, to calculate the velocity of particles in special theory of relativity is applied relation: $\boldsymbol{v} = \frac{c^2}{E}\boldsymbol{p}$.

The variation of second term in (2) with (15) has the form:

$$\delta S_2 = \int -\delta\left(\sqrt{g_{\mu\nu}J^\mu J^\nu}\sqrt{-g}\right)d\Sigma. \tag{16}$$

We define the variation in (16) with the help of (13):

$$
\begin{aligned}
&\delta\left(\sqrt{g_{\mu\nu}J^\mu J^\nu}\sqrt{-g}\right)=\sqrt{-g}\,\delta\left(\sqrt{g_{\mu\nu}J^\mu J^\nu}\right)+\sqrt{g_{\mu\nu}J^\mu J^\nu}\,\delta\sqrt{-g}=\\
&=\frac{J^\mu J^\nu\sqrt{-g}\,\delta g_{\mu\nu}}{2\sqrt{g_{\mu\nu}J^\mu J^\nu}}+\frac{g_{\mu\nu}J^\mu\sqrt{-g}\,\delta J^\nu}{\sqrt{g_{\mu\nu}J^\mu J^\nu}}+\frac{1}{2}\sqrt{g_{\mu\nu}J^\mu J^\nu}\sqrt{-g}\,g^{\alpha\beta}\,\delta g_{\alpha\beta}=\\
&=\frac{1}{2}\left(\frac{J^\alpha J^\beta}{\sqrt{g_{\mu\nu}J^\mu J^\nu}}+g^{\alpha\beta}\sqrt{g_{\mu\nu}J^\mu J^\nu}\right)\sqrt{-g}\,\delta g_{\alpha\beta}+\frac{g_{\alpha\beta}J^\alpha}{\sqrt{g_{\mu\nu}J^\mu J^\nu}}\sqrt{-g}\,\delta J^\beta.
\end{aligned}
\tag{17}
$$

In (17) is contained variation δJ^β, which according to [2], [4] can be found using displacement 4-vector ξ^μ. The displacement ξ^μ are variation of coordinates, which give variation of mass 4-current J^β:

$$
\delta J^\beta=\nabla_\sigma\left(J^\sigma\xi^\beta-J^\beta\xi^\sigma\right)=\frac{1}{\sqrt{-g}}\partial_\sigma\left[\sqrt{-g}\left(J^\sigma\xi^\beta-J^\beta\xi^\sigma\right)\right].
\tag{18}
$$

Equation (18) was obtained from the condition that mass of substance local volume in variation of coordinates remains constant despite change in density and its volume. With the help of (15) and (18) the last term in (17) can be transformed through 4-velosity u_β:

$$
\begin{aligned}
&\frac{g_{\alpha\beta}J^\alpha}{\sqrt{g_{\mu\nu}J^\mu J^\nu}}\sqrt{-g}\,\delta J^\beta=\frac{1}{c}u_\beta\,\partial_\sigma\left[\sqrt{-g}\left(J^\sigma\xi^\beta-J^\beta\xi^\sigma\right)\right]=\\
&=\frac{1}{c}\partial_\sigma\left[u_\beta\sqrt{-g}\left(J^\sigma\xi^\beta-J^\beta\xi^\sigma\right)\right]-\frac{1}{c}\partial_\sigma u_\beta\sqrt{-g}\left(J^\sigma\xi^\beta-J^\beta\xi^\sigma\right).
\end{aligned}
$$

The term with complete divergence in integration over 4-volume in the action function will not give any contribution. The remaining term in the previous equation can be transformed further:

$$
\begin{aligned}
&-\frac{1}{c}\partial_\sigma u_\beta\sqrt{-g}\left(J^\sigma\xi^\beta-J^\beta\xi^\sigma\right)=-\frac{1}{c}(\partial_\sigma u_\beta-\partial_\beta u_\sigma)J^\sigma\xi^\beta\sqrt{-g}=\\
&=-\frac{1}{c}(\nabla_\sigma u_\beta-\nabla_\beta u_\sigma)J^\sigma\xi^\beta\sqrt{-g}=-\frac{1}{c}J^\sigma\nabla_\sigma u_\beta\,\xi^\beta\sqrt{-g}\,.
\end{aligned}
\tag{19}
$$

Here was used condition $J^\sigma\nabla_\beta u_\sigma=\rho_0 u^\sigma\nabla_\beta u_\sigma=0$, since it follows from the equation $u^\sigma u_\sigma=c^2$, to which the covariant derivative ∇_β is applied.

The symmetric stress-energy tensor of substance is:

$$\phi^{\alpha\beta} = \frac{c\,J^{\alpha} J^{\beta}}{\sqrt{g_{\mu\nu} J^{\mu} J^{\nu}}} \;. \qquad (20)$$

Substituting (20) in (17) and using (19) instead of the last term in (17), we find the variation δS_2 in (16):

$$\delta S_2 = \int \left(-\frac{1}{2c}\,\phi^{\alpha\beta}\,\delta g_{\alpha\beta} - \frac{1}{2} g^{\alpha\beta} \sqrt{g_{\mu\nu} J^{\mu} J^{\nu}}\;\delta g_{\alpha\beta} + \frac{1}{c} J^{\sigma} \nabla_{\sigma} u_{\beta}\, \xi^{\beta} \right) \sqrt{-g}\, d\Sigma \,. \qquad (21)$$

Variation of Lagrangian of gravitational field and its sources

Effect of macroscopic gravitational field is in the third and fourth terms in (2), which give the following:

$$\delta S_3 = \int -\frac{1}{c}\,\delta\left(D_{\mu}\, J^{\mu} \sqrt{-g} \right) d\Sigma\,, \qquad \delta S_4 = \int \frac{c}{16\pi\gamma}\,\delta\left(\Phi_{\mu\nu} \Phi^{\mu\nu} \sqrt{-g} \right) d\Sigma \;. \qquad (22)$$

We first consider variation for δS_3 in (22), using $\delta\sqrt{-g}$ by (13) and then (18) for δJ^{μ} :

$$\delta\left(D_{\mu}\, J^{\mu} \sqrt{-g} \right) = D_{\mu}\,\delta\left(J^{\mu} \sqrt{-g} \right) + J^{\mu} \sqrt{-g}\; \delta D_{\mu} =$$
$$= D_{\mu}\, \partial_{\sigma} \left[\sqrt{-g} \left(J^{\sigma} \xi^{\mu} - J^{\mu} \xi^{\sigma} \right) \right] + \frac{1}{2} D_{\mu} J^{\mu}\, g^{\alpha\beta} \sqrt{-g}\; \delta g_{\alpha\beta} + J^{\mu} \sqrt{-g}\; \delta D_{\mu} \,.$$

Transformation of the first term:

$$D_{\mu}\, \partial_{\sigma} \left[\sqrt{-g} \left(J^{\sigma} \xi^{\mu} - J^{\mu} \xi^{\sigma} \right) \right] = \partial_{\sigma} \left[D_{\mu} \sqrt{-g} \left(J^{\sigma} \xi^{\mu} - J^{\mu} \xi^{\sigma} \right) \right] - \partial_{\sigma} D_{\mu} \sqrt{-g} \left(J^{\sigma} \xi^{\mu} - J^{\mu} \xi^{\sigma} \right).$$

Neglecting the term with total derivative, we consider the following:

$$-\partial_{\sigma} D_{\mu} \sqrt{-g} \left(J^{\sigma} \xi^{\mu} - J^{\mu} \xi^{\sigma} \right) = -(\partial_{\sigma} D_{\mu} - \partial_{\mu} D_{\sigma})\, J^{\sigma} \xi^{\mu} \sqrt{-g} = \Phi_{\mu\sigma} J^{\sigma} \xi^{\mu} \sqrt{-g} \,.$$

Substituting these results into (22), we find:

$$\delta S_3 = \int \left(-\frac{1}{c} \Phi_{\beta\sigma} J^{\sigma} \xi^{\beta} - \frac{1}{2c} D_{\mu} J^{\mu} g^{\alpha\beta} \delta g_{\alpha\beta} - \frac{1}{c} J^{\beta} \delta D_{\beta} \right) \sqrt{-g}\, d\Sigma. \qquad (23)$$

Variation for δS_4 in (22) with (13) is equal to:

$$\begin{aligned} \delta\left(\Phi_{\mu\nu} \Phi^{\mu\nu} \sqrt{-g} \right) &= \delta\left(\Phi_{\mu\nu} \Phi^{\mu\nu} \right) \sqrt{-g} + \Phi_{\mu\nu} \Phi^{\mu\nu} \delta \sqrt{-g} = \\ &= \Phi_{\mu\nu} \delta\Phi^{\mu\nu} \sqrt{-g} + \Phi^{\mu\nu} \delta\Phi_{\mu\nu} \sqrt{-g} + \frac{1}{2} \Phi_{\mu\nu} \Phi^{\mu\nu} g^{\alpha\beta} \sqrt{-g}\ \delta g_{\alpha\beta}. \end{aligned} \qquad (24)$$

Since $\Phi^{\mu\nu} = g^{\mu\alpha} g^{\beta\nu} \Phi_{\alpha\beta}$, the tensor $\Phi_{\alpha\beta}$ is antisymmetric, then using $\delta g^{\beta\nu}$ by (13), we find:

$$\begin{aligned} \Phi_{\mu\nu} \delta\Phi^{\mu\nu} \sqrt{-g} &= \Phi_{\mu\nu} \delta\left(g^{\mu\alpha} g^{\beta\nu} \Phi_{\alpha\beta} \right) \sqrt{-g} = \\ &= \Phi_{\mu\nu} \left[g^{\mu\alpha} g^{\beta\nu} \delta\Phi_{\alpha\beta} + g^{\mu\alpha} \Phi_{\alpha\beta} \delta g^{\beta\nu} + g^{\beta\nu} \Phi_{\alpha\beta} \delta g^{\mu\alpha} \right] \sqrt{-g} = \\ &= \Phi^{\alpha\beta} \delta\Phi_{\alpha\beta} \sqrt{-g} + 2\Phi_{\mu\nu} g^{\mu\alpha} \Phi_{\alpha\beta} \sqrt{-g}\, \delta g^{\beta\nu} = \\ &= \Phi^{\alpha\beta} \delta\Phi_{\alpha\beta} \sqrt{-g} - 2 g^{\nu\beta} \Phi_{\kappa\nu} \Phi^{\kappa\alpha} \sqrt{-g}\, \delta g_{\alpha\beta}. \end{aligned}$$

Substitution this expression into (24) yields:

$$\begin{aligned} \delta\left(\Phi_{\mu\nu} \Phi^{\mu\nu} \sqrt{-g} \right) &= \Phi_{\mu\nu} \delta\Phi^{\mu\nu} \sqrt{-g} + \Phi^{\mu\nu} \delta\Phi_{\mu\nu} \sqrt{-g} + \frac{1}{2} \Phi_{\mu\nu} \Phi^{\mu\nu} g^{\alpha\beta} \sqrt{-g}\ \delta g_{\alpha\beta} = \\ &= 2\Phi^{\alpha\beta} \delta\Phi_{\alpha\beta} \sqrt{-g} - 2 g^{\nu\beta} \Phi_{\kappa\nu} \Phi^{\kappa\alpha} \sqrt{-g}\, \delta g_{\alpha\beta} + \frac{1}{2} \Phi_{\mu\nu} \Phi^{\mu\nu} g^{\alpha\beta} \sqrt{-g}\ \delta g_{\alpha\beta}. \end{aligned} \qquad (25)$$

We denote $U^{\alpha\beta}$ as the stress-energy tensor of gravitational field:

$$U^{\alpha\beta} = \frac{c^2}{4\pi\gamma} \left(g^{\alpha\nu} \Phi_{\kappa\nu} \Phi^{\kappa\beta} - \frac{1}{4} g^{\alpha\beta} \Phi_{\mu\nu} \Phi^{\mu\nu} \right) = -\frac{c^2}{4\pi\gamma} \left(\Phi^{\alpha}{}_{\kappa} \Phi^{\kappa\beta} + \frac{1}{4} g^{\alpha\beta} \Phi_{\mu\nu} \Phi^{\mu\nu} \right). \qquad (26)$$

Remembering that $\Phi_{\mu\nu} = \nabla_{\mu} D_{\nu} - \nabla_{\nu} D_{\mu} = \partial_{\mu} D_{\nu} - \partial_{\nu} D_{\mu}$, using differentiation by parts, as well as equality for an antisymmetric tensor: $\partial_{\alpha}\left(\Phi^{\alpha\beta} \sqrt{-g} \right) = \sqrt{-g}\, \nabla_{\alpha} \Phi^{\alpha\beta}$, for term $2\Phi^{\alpha\beta} \delta\Phi_{\alpha\beta} \sqrt{-g}$ in (25) we have:

$$2\Phi^{\alpha\beta}\,\delta\Phi_{\alpha\beta}\sqrt{-g}=2\Phi^{\alpha\beta}\,\delta\left(\partial_\alpha D_\beta-\partial_\beta D_\alpha\right)\sqrt{-g}=2\Phi^{\alpha\beta}\left(\partial_\alpha \delta D_\beta-\partial_\beta \delta D_\alpha\right)\sqrt{-g}=$$
$$=4\Phi^{\alpha\beta}\sqrt{-g}\,\partial_\alpha \delta D_\beta=4\,\partial_\alpha\left(\Phi^{\alpha\beta}\sqrt{-g}\,\delta D_\beta\right)-4\,\partial_\alpha\left(\Phi^{\alpha\beta}\sqrt{-g}\,\right)\delta D_\beta=$$
$$=4\,\partial_\alpha\left(\Phi^{\alpha\beta}\sqrt{-g}\,\delta D_\beta\right)-4\,\nabla_\alpha\Phi^{\alpha\beta}\,\sqrt{-g}\,\delta D_\beta\,. \tag{27}$$

The term $4\partial_\alpha\left(\Phi^{\alpha\beta}\sqrt{-g}\,\delta D_\beta\right)$ in the last expression is divergence and can be neglected for variation of action function. Substituting (26) and (27) in (25), and the result in (22), we find:

$$\delta S_4=\int\left(-\frac{c}{4\pi\gamma}\nabla_\alpha\Phi^{\alpha\beta}\,\delta D_\beta-\frac{1}{2c}U^{\alpha\beta}\,\delta g_{\alpha\beta}\right)\sqrt{-g}\,d\Sigma\ . \tag{28}$$

Variation of Lagrangian of electromagnetic field and its sources

Variation in (2) for electromagnetic field is the same as for gravitational field in the previous section. For the fifth and sixth terms in (2) can be written:

$$\delta S_5=\int-\frac{1}{c}\,\delta\left(A_\mu\,j^\mu\sqrt{-g}\right)d\Sigma, \qquad \delta S_6=\int-\frac{c\varepsilon_0}{4}\,\delta\left(F_{\mu\nu}F^{\mu\nu}\sqrt{-g}\right)d\Sigma\,. \tag{29}$$

Replacing in (22) D_μ on A_μ, J^μ on j^μ, $\Phi_{\beta\sigma}$ on $F_{\beta\sigma}$, instead of (23) we find:

$$\delta S_5=\int\left(-\frac{1}{c}F_{\beta\sigma}\,j^\sigma\,\xi^\beta-\frac{1}{2c}A_\mu\,j^\mu\,g^{\alpha\beta}\,\delta g_{\alpha\beta}-\frac{1}{c}\,j^\beta\,\delta A_\beta\right)\sqrt{-g}\,d\Sigma. \tag{30}$$

In deriving (30) was used expression for variation of electromagnetic 4-current, similar to (18):

$$\delta j^\beta=\nabla_\sigma\left(j^\sigma\xi^\beta-j^\beta\xi^\sigma\right)=\frac{1}{\sqrt{-g}}\partial_\sigma\left[\sqrt{-g}\left(j^\sigma\xi^\beta-j^\beta\xi^\sigma\right)\right]. \tag{31}$$

The stress-energy tensor of electromagnetic field is:

$$W^{\alpha\beta}=\varepsilon_0 c^2\left(-g^{\alpha\nu}F_{\kappa\nu}F^{\kappa\beta}+\frac{1}{4}g^{\alpha\beta}F_{\mu\nu}F^{\mu\nu}\right)=\varepsilon_0 c^2\left(F^\alpha{}_\kappa F^{\kappa\beta}+\frac{1}{4}g^{\alpha\beta}F_{\mu\nu}F^{\mu\nu}\right). \tag{32}$$

With the help of this tensor variation δS_6 will be as in (28):

$$\delta S_6 = \int \left(c\varepsilon_0 \nabla_\alpha F^{\alpha\beta}\, \delta A_\beta - \frac{1}{2c} W^{\alpha\beta} \delta g_{\alpha\beta} \right) \sqrt{-g}\, d\Sigma \ . \qquad (33)$$

The equations for metric

Put it together and substitute in (2) all terms in (14), (21), (23), (28), (30) and (33) containing the variation $\delta g_{\alpha\beta}$ of metric tensor. By the arbitrariness of the variation the sum of all these terms should be zero. This gives the following:

$$k\left(-R^{\alpha\beta} + \frac{1}{2} g^{\alpha\beta} R - \Lambda g^{\alpha\beta} \right) - \frac{1}{2c}\phi^{\alpha\beta} - \frac{1}{2} g^{\alpha\beta} \sqrt{g_{\mu\nu} J^\mu J^\nu} - \frac{1}{2c} D_\mu J^\mu g^{\alpha\beta} - \frac{1}{2c} U^{\alpha\beta} -$$
$$- \frac{1}{2c} A_\mu j^\mu g^{\alpha\beta} - \frac{1}{2c} W^{\alpha\beta} = 0.$$

We rewrite this equation with $k = -\frac{c^3}{16\pi\gamma\beta}$, where β – coefficient of order 1, as equation for determination of metric tensor $g^{\alpha\beta}$ in terms of known sources of energy-momentum. Here, instead of Λ, we introduce a new constant χ, according to the relation: $\Lambda = \frac{8\pi\gamma\beta\chi}{c^4}$. As well as Λ, the constant χ determines the properties of system as a whole. This gives:

$$R^{\alpha\beta} - \frac{1}{2} g^{\alpha\beta} R = \frac{8\pi\gamma\beta}{c^4} \left(\phi^{\alpha\beta} + U^{\alpha\beta} + W^{\alpha\beta} + c\sqrt{g_{\mu\nu} J^\mu J^\nu}\, g^{\alpha\beta} + D_\mu J^\mu g^{\alpha\beta} + A_\mu j^\mu g^{\alpha\beta} - \chi g^{\alpha\beta} \right).$$

(34)

In the case of such large system, like our universe Λ has a special name – the cosmological constant. It is estimated as 10^{-52} m^{-2}. Hence the value χ is of the order $\frac{c^4 \Lambda}{8\pi\gamma\beta} \approx 5\cdot 10^{-10}$ J/m^3, having dimension of energy density. For other systems, which can roughly be considered as systems with continuously distributed throughout the volume of space matter, constants Λ and χ can have other values. This will be further discussed in section "Additional notes" at the end.

The equation (34) was obtained with condition that any variations of coordinates ξ^β and 4-potentials δD_β and δA_β in function of action (2) equal to zero, or always equal to zero the sums of all the terms with these variations. In the first case (34) is equation for the metric of system, in which were originally defined the motion of charged and gravitating substance on the known path and defined calibrated values of field potentials (with accuracy up to constants appearing in potentials). In the second case are allowed variations of coordinates (trajectories of substance) and variations of potentials due to their mutual influence on each other. However, it is assumed that each time combinations of terms in function of action defining the relationship between substance and field, including generation of fields by substance and field influence force on substance are such that they are set to zero and does not affect the function of action and the metric. The second case is the case of arbitrarily given initial distribution of substance in space and its initial velocity and initial values of the potentials, when with the laws of connection between the subsequent motion of substance and fields due to some reasons take place equation (34). Obviously, the validity of the second case requires additional evidence, or should be postulated, whereas in the former case this is not required.

Outside the substance, where gravitational and electromagnetic 4-currents J^{μ} and j^{μ} tend to zero, the contribution to the metric according to (34) is made only the stress-energy tensor of gravitational field $U^{\alpha\beta}$ (26) and the stress-energy tensor of electromagnetic field $W^{\alpha\beta}$ (32). If the metric is determined in substance, the contribution to the metric depends on all terms in (34).

Note that the right-hand side of (34) contains additional terms that in general theory of relativity are not usually considered. In particular, (34) includes all invariant scalar values from the function (1), including terms $\Phi_{\mu\nu}\Phi^{\mu\nu}$ and $F_{\mu\nu}F^{\mu\nu}$, which became components of the tensors $U^{\alpha\beta}$ and $W^{\alpha\beta}$, respectively. In general relativity, there is no stress-energy tensor of gravitational field $U^{\alpha\beta}$ in the form in which we have defined it. This follows from the fact that in general relativity, gravitational field reduces to the metric field, where the components $g_{\mu\nu}$ of metric tensor are considered as potentials describing the gravitational field. In this case, the presence $U^{\alpha\beta}$ in the right side of equation for the metric (34) would mean that gravitational field is the source itself. In the absence of substance that would lead to a vicious circle where a metric gravitational field generates itself, the field gives the metric and the metric gives the field. As contrast, in covariant theory of gravitation (CTG), the metric is only an auxiliary geometric field induced by gravitation and electromagnetic field, taken in all their forms at different scale levels of matter.

In CTG uses the metric theory of relativity [7], the essence of which is dependence of metric not only on the properties of motion of system, but also on the type of test bodies, which can be both substance particles and quanta of field. Test bodies are needed to determine the metric of system in nature, for measurement procedure of scale and time, and have different properties due to difference in equations of motion. As a result, the coefficient β in (34) may be different for different systems and should be found separately for complete definition of metric tensor. In particular, β was found in several situations, such as calculating deviation of test body's motion under the action of gravitation, and calculations of perihelion shift. Notes on the last four terms in (34) and term with constant χ will be made later in the section "Tensors of energy".

The equations of motion of substance and field

To obtain the equations of motion of substance is necessary to select in the full variation of action (2) those terms that contain variations of coordinates ξ^{β}. Due to the arbitrariness of ξ^{β} the sum of all such terms shall be equal to zero. From the sum of (21), (23) and (30) is obtained:

$$\frac{1}{c} J^{\sigma} \nabla_{\sigma} u_{\beta} - \frac{1}{c} \Phi_{\beta\sigma} J^{\sigma} - \frac{1}{c} F_{\beta\sigma} j^{\sigma} = 0 .$$

Given that $J^{\sigma} = \rho_0 u^{\sigma}$, $j^{\sigma} = \rho_{0q} u^{\sigma}$, and using operator of proper-time-derivative [7]: $u^{\sigma} \nabla_{\sigma} = \frac{D}{D\tau}$, where D denotes covariant differential, τ is proper time, the last equation can be written as:

$$\rho_0 \frac{D u_{\beta}}{D\tau} = \rho_0 a_{\beta} = \Phi_{\beta\sigma} \rho_0 u^{\sigma} + F_{\beta\sigma} \rho_{0q} u^{\sigma} , \tag{35}$$

where a_{β} – 4-acceleration with covariant index, the first term on the right is density of gravitational force, and the last term gives electromagnetic Lorentz force for charge density ρ_{0q}.

According to (35), the contribution to 4-acceleration of substance unit volume makes the gravitational acceleration $\Phi_{\beta\sigma} u^{\sigma}$ and 4-acceleration in electromagnetic field $\frac{\rho_{0q}}{\rho_0} F_{\beta\sigma} u^{\sigma}$. The physical meaning of (35) is such that it determines impact on the substance of fields when given metric tensor of the system (this means that $\delta g_{\alpha\beta} = 0$ in action function), and set field potentials ($\delta D_{\beta} = 0$, $\delta A_{\beta} = 0$ in action function).

The relation connecting the tensor of gravitational field $\Phi^{\mu\nu}$ with its source in the form of 4-vector J^{μ}, follows from (23) and (28) as a consequence of variation δD_{β} on gravitational 4-potential. Taking into account the antisymmetry of the tensor $\Phi^{\alpha\beta} = -\Phi^{\beta\alpha}$:

$$\nabla_{\alpha} \Phi^{\alpha\beta} = -\frac{4\pi\gamma}{c^2} J^{\beta} , \quad \text{or} \quad \nabla_{\nu} \Phi^{\mu\nu} = \frac{4\pi\gamma}{c^2} J^{\mu} . \tag{36}$$

A similar relation for electromagnetic field, from expressions for variation δA_{β} of electromagnetic 4-potential in (30) and (33) has the form:

$$\nabla_\alpha F^{\alpha\beta} = \frac{1}{c^2 \varepsilon_0} j^\beta , \quad \text{or} \quad \nabla_\nu F^{\mu\nu} = -\frac{1}{c^2 \varepsilon_0} j^\mu = -\mu_0 j^\mu , \tag{37}$$

where $\mu_0 = \frac{1}{c^2 \varepsilon_0}$ – vacuum permeability.

Equations (36) and (37), as it is evident from their receipt of variation of action function, valid in the case when the variation of coordinates of substance and the variation of metric are zero, that is $\xi^\beta = 0$, $\delta g_{\alpha\beta} = 0$. This means that if the motion of substance and metric of system are set, we can calculate how substance generates field strengths.

If we consider the definition of gravitational tensor: $\Phi_{\mu\nu} = \nabla_\mu D_\nu - \nabla_\nu D_\mu = \partial_\mu D_\nu - \partial_\nu D_\mu$, and take the covariant derivative of the tensor, followed by a cyclic permutation of indices, then the following equation is satisfied identically:

$$\nabla_\sigma \Phi_{\mu\nu} + \nabla_\nu \Phi_{\sigma\mu} + \nabla_\mu \Phi_{\nu\sigma} = 0 . \tag{38}$$

Another form of (38) is:

$$\varepsilon^{\alpha\beta\gamma\delta} \nabla_\gamma \Phi_{\alpha\beta} = 0 ,$$

where $\varepsilon^{\alpha\beta\gamma\delta}$ is the Levi-Civita symbol or totally antisymmetric unit tensor.

Equation (38) gives the gravitational field equations without sources, so that the set of equations (36) and (38) completely determines the properties of gravitational field.

For electromagnetic field we have as in (38):

$$\nabla_\sigma F_{\mu\nu} + \nabla_\nu F_{\sigma\mu} + \nabla_\mu F_{\nu\sigma} = 0 \quad \text{or} \quad \varepsilon^{\alpha\beta\gamma\delta} \nabla_\gamma F_{\alpha\beta} = 0 . \tag{39}$$

Equations (37) and (39) are the Maxwell equations, written in four-dimensional notation.

Relation (36) can be written in another way: $\nabla^\nu \Phi_{\mu\nu} = \frac{4\pi\gamma}{c^2} J_\mu$. If to take on both sides of this equation contravariant derivative ∇^μ, and use definition $\Phi_{\mu\nu} = \nabla_\mu D_\nu - \nabla_\nu D_\mu$, then due to symmetry and changing of the order of differentiation the left-hand side will vanish. This leads to the continuity equation (mass conservation), which imposes certain conditions on 4-velocity and density of substance:

$$\nabla^\mu J_\mu = \nabla_\mu J^\mu = \nabla_\mu\left(\rho_0 u^\mu\right) = 0 .$$

For conservation of electric charge, we have a similar relation:

$$\nabla^\mu j_\mu = \nabla_\mu j^\mu = \nabla_\mu\left(\rho_{0q} u^\mu\right) = 0 .$$

If to specify a condition for 4-vector of gravitational potential D_μ, or for 4-vector of electromagnetic potential A_μ, then it provides definite relation between scalar and vector potentials. The standard approach is Lorentz gauge, which gives the following conditions:

$$\nabla^\mu D_\mu = \nabla_\mu D^\mu = 0 , \qquad\qquad \nabla^\mu A_\mu = \nabla_\mu A^\mu = 0 . \qquad (40)$$

Substituting (40) into (36) and (37) and using $\Phi^{\mu\nu} = \nabla^\mu D^\nu - \nabla^\nu D^\mu$, $F^{\mu\nu} = \nabla^\mu A^\nu - \nabla^\nu A^\mu$, while the expression $\nabla_\nu \nabla^\nu = \Box$ is D'Alembert operator, we arrive to wave equations for 4-potentials in the Lorentz gauge:

$$\Box D^\mu = -\frac{4\pi\gamma}{c^2} J^\mu , \qquad\qquad \Box A^\mu = \mu_0 j^\mu .$$

About applicability of equations of motion in general case

As we mentioned above, the equation for the metric (34) holds exactly for the case when motion of substance is completely specified and field potentials are defined. However, in most cases are known only the initial state of motion and initial potentials, in later times the motion of substance begins to be determined by field and is given indirectly. How to use equation (34) in this case, and under what conditions? To answer this question, we assume that motion of substance and field potentials at a short interval of time after initial time point remains unchanged. Then we can find metric tensor from (34). After it, assuming immutability of metric and the motion of substance in the next, the second time interval, using (36) and (37) are calculated derivatives of tensors $\Phi^{\alpha\beta}$ and $F^{\alpha\beta}$ by coordinates. After integration of these derivatives may be found field strengths that are part of these tensors. Since now is known refined tensors $\Phi^{\alpha\beta}$ и $F^{\alpha\beta}$, with them on the third time interval in the equation of motion (35) can be estimated the acceleration of substance and its motion, and made the adjustment movement. On the fourth time interval can be used data about motion of substance from the third interval and about fields from the second interval in order to evaluate the change of metric. Further calculations are repeated in that order. Thus the actual motion of substance in a gravitational and electromagnetic fields, and spacetime metric can be found approximately by some iterative procedure by using the above equations for metric, motion of substance and fields.

Tensors of energy

Let's return to the equation for metric (34). It is known that covariant derivative of left-hand side of (34) is zero, which is a property of Hilbert-Einstein tensor located there. Consequently, the covariant derivative of the right-hand side of (34) must also be zero:

$$\nabla_\beta \left(\phi^{\alpha\beta} + U^{\alpha\beta} + W^{\alpha\beta} + c\sqrt{g_{\mu\nu} J^\mu J^\nu}\, g^{\alpha\beta} + D_\mu J^\mu\, g^{\alpha\beta} + A_\mu j^\mu\, g^{\alpha\beta} - \chi g^{\alpha\beta} \right) = 0 . \qquad (41)$$

Considering definition of stress-energy tensor of substance (20), relations $\sqrt{g_{\mu\nu} J^\mu J^\nu} = c\rho_0$, $J^\alpha = \rho_0 u^\alpha$, and using operator of proper-time-derivative: $u^\sigma \nabla_\sigma = \frac{D}{D\tau}$, we can write:

$$\begin{aligned} \nabla_\beta \phi^{\alpha\beta} &= \nabla_\beta \left(\frac{c J^\alpha J^\beta}{\sqrt{g_{\mu\nu} J^\mu J^\nu}} \right) = \frac{1}{\rho_0} \nabla_\beta \left(J^\alpha J^\beta \right) + J^\alpha J^\beta \nabla_\beta \left(\frac{1}{\rho_0} \right) = \\ &= u^\beta \nabla_\beta J^\alpha - u^\alpha u^\beta \nabla_\beta \rho_0 = \frac{DJ^\alpha}{D\tau} - u^\alpha \frac{D\rho_0}{D\tau} = \rho_0 \frac{Du^\alpha}{D\tau} = \rho_0 a^\alpha , \end{aligned} \qquad (42)$$

where a^α – 4-acceleration.

We now find the covariant derivative of stress-energy tensor of gravitational field. Since metric tensor under covariant differentiation behaves as a constant, using (36), we obtain from (26):

$$\begin{aligned} &\frac{4\pi\gamma}{c^2} \nabla_\beta U^{\alpha\beta} = \nabla_\beta \left(g^{\alpha\nu} \Phi_{\kappa\nu} \Phi^{\kappa\beta} - \frac{1}{4} g^{\alpha\beta} \Phi_{\mu\nu} \Phi^{\mu\nu} \right) = g^{\alpha\nu} \Phi^{\kappa\beta} \nabla_\beta \Phi_{\kappa\nu} + g^{\alpha\nu} \Phi_{\kappa\nu} \nabla_\beta \Phi^{\kappa\beta} - \\ &- \frac{1}{4} g^{\alpha\beta} \Phi^{\mu\nu} \nabla_\beta \Phi_{\mu\nu} - \frac{1}{4} g^{\alpha\beta} \Phi_{\mu\nu} \nabla_\beta \Phi^{\mu\nu} = -\Phi^{\kappa\beta} \nabla_\beta \Phi^\alpha_\kappa + g^{\alpha\nu} \Phi_{\kappa\nu} \frac{4\pi\gamma}{c^2} J^\kappa - \frac{1}{2} g^{\alpha\beta} \Phi^{\mu\nu} \nabla_\beta \Phi_{\mu\nu} . \end{aligned} \qquad (43)$$

We apply (38) to the last term in (43), with the condition that tensor $\Phi_{\alpha\beta}$ is antisymmetric:

$$-\frac{1}{2}g^{\alpha\beta}\Phi^{\mu\nu}\nabla_{\beta}\Phi_{\mu\nu}=-\frac{1}{2}g^{\alpha\beta}\Phi^{\mu\nu}\left(-\nabla_{\nu}\Phi_{\beta\mu}-\nabla_{\mu}\Phi_{\nu\beta}\right)=$$
$$=\frac{1}{2}g^{\alpha\beta}\Phi^{\mu\nu}\nabla_{\nu}\Phi_{\beta\mu}+\frac{1}{2}g^{\alpha\beta}\Phi^{\mu\nu}\nabla_{\mu}\Phi_{\nu\beta}=\frac{1}{2}\Phi^{\mu\nu}\nabla_{\nu}\Phi^{\alpha}_{\ \mu}-\frac{1}{2}g^{\alpha\beta}\Phi^{\mu\nu}\nabla_{\mu}\Phi^{\alpha}_{\ \nu}=\Phi^{\mu\nu}\nabla_{\nu}\Phi^{\alpha}_{\ \mu}.$$

Substituting this into (43) gives the relation between covariant derivative of stress-energy tensor of gravitational field and 4-vector density of gravitational force:

$$\nabla_{\beta}U^{\alpha\beta}=-\Phi^{\alpha}_{\ \kappa}J^{\kappa}. \tag{44}$$

A similar expression for covariant derivative of stress-energy tensor of electromagnetic field and 4-vector density of electromagnetic force (Lorentz force density) with (32) and (39) is such:

$$\nabla_{\beta}W^{\alpha\beta}=-F^{\alpha}_{\ \kappa}j^{\kappa}. \tag{45}$$

Substituting (42), (44) and (45) into (41):

$$\nabla_{\beta}\left(\phi^{\alpha\beta}+U^{\alpha\beta}+W^{\alpha\beta}\right)=\rho_0 a^{\alpha}-\Phi^{\alpha}_{\ \kappa}J^{\kappa}-F^{\alpha}_{\ \kappa}j^{\kappa}=0. \tag{46}$$

The zero right side of (46) follows from the equations of motion of substance in gravitational and electromagnetic fields (35). Consequently, the covariant derivative in the remaining terms in (41) must also be zero:

$$\nabla_{\beta}\,g^{\alpha\beta}\left(c\sqrt{g_{\mu\nu}J^{\mu}J^{\nu}}+D_{\mu}J^{\mu}+A_{\mu}j^{\mu}-\chi\right)=\nabla^{\alpha}\left(c\sqrt{g_{\mu\nu}J^{\mu}J^{\nu}}+D_{\mu}J^{\mu}+A_{\mu}j^{\mu}-\chi\right)=0. \tag{47}$$

In brackets in (47) is a scalar quantity, in this case the covariant derivative ∇^{α} is equal to partial derivative ∂^{α} (that is 4-gradient). Relation (47) is automatically satisfied if we assume that there is a constant in brackets, which can be set equal to zero. This gives the relation:

$$c\sqrt{g_{\mu\nu}J^{\mu}J^{\nu}}+D_{\mu}J^{\mu}+A_{\mu}j^{\mu}=\chi=const. \tag{48}$$

Equality (48) is necessary to perform in (34) the limit relations for tensors at infinity, where there is no substance, no fields. As indicated in [4], at infinity the right side of (34) with energy tensors set to zero, and the spacetime becomes flat, leading to zero the left side of Hilbert-Einstein tensor. In view of (48) form of equations for metric reduces to the limit:

$$R^{\alpha\beta}-\frac{1}{2}g^{\alpha\beta}R=\frac{8\pi\gamma\beta}{c^4}\left(\phi^{\alpha\beta}+U^{\alpha\beta}+W^{\alpha\beta}\right). \tag{49}$$

Let's use (15) and reveal in (48) scalar product of 4-vectors, with the help of $D_\mu=\left(\frac{\psi}{c},-\boldsymbol{D}\right)$, $A_\mu=\left(\frac{\varphi}{c},-\boldsymbol{A}\right)$, $J^\mu=\rho_0 u^\mu$, $j^\mu=\rho_{0q}u^\mu$:

$$\rho_0 c^2+\frac{\rho_0\psi u^0}{c}-\rho_0\boldsymbol{D}\cdot\boldsymbol{u}+\frac{\rho_{0q}\varphi u^0}{c}-\rho_{0q}\boldsymbol{A}\cdot\boldsymbol{u}=\chi. \tag{50}$$

Here $\boldsymbol{u}$ is a 3-vector, which is part of 4-velocity u^μ. In no curved spacetime according to special theory of relativity $u^\mu=\left(\frac{c}{\sqrt{1-v^2/c^2}},\ \frac{\boldsymbol{V}}{\sqrt{1-v^2/c^2}}\right)$, then $u^0=\frac{c}{\sqrt{1-v^2/c^2}}$, $\boldsymbol{u}=\frac{\boldsymbol{V}}{\sqrt{1-v^2/c^2}}$, where $\boldsymbol{V}$ is velocity of motion of substance. This shows that $\frac{\rho_0\psi u^0}{c}$ is the energy density for substance in gravitational field with scalar potential ψ. The vector potential $\boldsymbol{D}$ of gravitational field is also associated with energy, but its value $-\rho_0\boldsymbol{D}\cdot\boldsymbol{u}$ may have different sign depending on direction of vector $\boldsymbol{u}$, which is proportional to speed $\boldsymbol{V}$, and direction of vector $\boldsymbol{D}$. The same picture is emerging with respect to the density of electromagnetic energy – it depends on charge density ρ_{0q}, scalar electric potential φ and vector potential $\boldsymbol{A}$ of electromagnetic field.

Now suppose that in (50) is somehow off macroscopic gravitational and electromagnetic fields and their potentials are equal to zero. In this case, the density of substance must reach a certain value ρ_0', which depends only on fundamental microscopic fields acting at the level of elementary particles. Then it will be $\chi=\rho_0' c^2$, and (49) can be rewritten as follows:

$$\rho_0 c^2=\rho_0' c^2-\frac{\rho_0\psi u^0}{c}+\rho_0\boldsymbol{D}\cdot\boldsymbol{u}-\frac{\rho_{0q}\varphi u^0}{c}+\rho_{0q}\boldsymbol{A}\cdot\boldsymbol{u}=\rho_0' c^2-\varepsilon_g-\varepsilon_e, \tag{51}$$

where ε_g and ε_e denote the energy density of substance in gravitational and electromagnetic fields, respectively.

As ε_g for substance is usually negative (due to negative gravitational potential ψ), then from (51) follows that density of substance ρ_0 in gravitational field becomes greater than density of substance ρ' in the absence of field (when the substance of a body is divided into parts and separated to infinity). The same thing can be said about mass – in gravitational field it is expected to increase due to the contribution of gravitational mass-energy of substance in the field. Thus we have obtained the result similar to those which justify us in [6] and [10], but in relation to contribution of mass-energy of field to the total mass of system of substance and field. Then we found that the mass of a spherical body grows at the expense of its field, and at constant volume, this means an increase in the effective density of the substance.

We can integrate (51) over volume of substance of a spherical uncharged body in static case, when the body is at rest and does not rotate. If the substance is infinitely slowly superimposed on the body by parts in form of thin spherical shells with the same density of substance, we can assume that in (49) $u^0 = c$, as well as:

$$\int \rho_0 c^2 dx^1 dx^2 dx^3 = mc^2, \qquad \int \rho_0' c^2 dx^1 dx^2 dx^3 = m'c^2,$$

$$\int \rho_0 \psi \, dx^1 dx^2 dx^3 = -\int \frac{\gamma \rho_0 m(r)}{r} dx^1 dx^2 dx^3 ,$$

where m – the observed mass of the body at its radius R,

m' – mass of substance of the body without taking into account the energy of gravitation,

$m(r) = \dfrac{4\pi r^3 \rho_0}{3}$ – mass inside the radius r, increasing from 0 to the radius of the body R with increasing of mass.

As a result, (51) becomes equality for the masses:

$$mc^2 = m'c^2 + \int \frac{\gamma \rho_0 m(r)}{r} dx^1 dx^2 dx^3 = m'c^2 + \frac{3\gamma m^2}{5R}, \qquad (52)$$

where γ is gravitational constant.

In (52) module of mass-energy of gravitational field is added to the mass-energy of the body. In reality, during the formation of space objects in gravitational field acts virial theorem according to which approximately half of gravitational field energy leaves the system in form of radiation, and the other half heats the substance. This reduces by half the additive to mass-energy in (52).

For the main objects of stellar level of matter contribution to (51) of energy density of substance in electromagnetic field ε_e is small compared to ε_g. In particular, for neutron stars, gravitational energy is equal to $E_\gamma = -\frac{k\gamma M_s^2}{R_s} \approx -2.4 \cdot 10^{46}$ J, here $k \approx 0.6$ in approximation of a uniform density of substance, γ – gravitational constant, $M_s = 2.7 \cdot 10^{30}$ kg, $R_s = 12$ km – mass and radius of a typical neutron star. The electromagnetic energy reaches a maximum in magnetars at magnetic pole of which magnetic field can be of order $B_m = 1.9 \cdot 10^{11}$ T. Since magnetic energy density is given in form $\frac{B^2}{2\mu_0}$, then the integral over the entire volume inside the star and beyond gives the magnitude of the magnetic energy of about 10^{41} J, which is considerably less than the modulus of gravitational energy.

A similar situation exists at the level of elementary particles, where according to theory of infinite nesting of matter [8], an analog of neutron star is nucleon. The energy of proton in its own field of strong gravitation is estimated by the formula $E_\Gamma = -\frac{k\Gamma M_p^2}{R_p}$, where M_p and R_p denote mass and radius of proton, $\Gamma = \frac{e^2}{4\pi\varepsilon_0 M_p M_e} = 1.514 \cdot 10^{29}$ m^3·kg^{-1}·s^{-2} – strong gravitational constant, e – elementary charge, ε_0 – vacuum permittivity, M_e – electron mass. The expression for electric energy of proton in the case of a uniform charge distribution is as follows: $E_e = \frac{ke^2}{4\pi\varepsilon_0 R_p}$. Consequently, for proton the ratio of module energy of strong gravitation to electrical energy equal to the ratio of proton to electron mass, and the energy of strong gravitation dominates.

The global dominance of gravitational forces over electromagnetic forces leads to possibility of formation of elementary particles of substance, massive bodies and other objects found in space. For the observable universe cosmological constant Λ is estimated by 10^{-52} m^{-2}, and constant χ reaches $\chi \approx 5 \cdot 10^{-10}$ J/m^3. We believe that $\chi = \rho_0' c^2$ characterizes the visible universe as a whole, setting the rest energy density of substance distributed in space, without taking into account the energy fields. We further assume that gravitational fields are a consequence of flows of gravitons, which are produced by tiny particles of all the substance that exists in universe. The more substance in the universe, the greater density of the substance and more flux density of gravitons. Then (50) supports the idea of Einstein that inertia of a body must increase near other gravitational masses [11], which is in turn the development of Mach's principle on impact of distant masses to acceleration bodies.

Additional notes

In our view, the equation of motion of substance (35) should look a little different:

$$\frac{DJ_\beta}{D\tau}=\frac{D(\rho_0 u_\beta)}{D\tau}=\Phi_{\beta\sigma}\rho_0 u^\sigma+F_{\beta\sigma}\rho_{0q}u^\sigma . \qquad (53)$$

In (53) density of substance ρ_0 has been included under the sign of total derivative with respect to proper time. This allows us to describe cases where the substance density changes and thus creates an additional acceleration of substance. Meanwhile, equation (35) was obtained from the variation of coordinates described in [2] and [4], at a constant mass in the variation. This led to the fact that (35) differs from (53), as in case $\rho_0 = const$, and therefore ρ_0 can be outside of the total differential.

It is interesting that we can choose the stress-energy tensor of substance such that its covariant derivative just gives the rate of change of mass 4-current. This tensor has an unusual form in terms of indices, but formal covariant derivative gives the desired result. Instead of (20) we write: $\phi^{\alpha\beta}=\frac{2cJ^\alpha J^\beta}{\sqrt{g_{\alpha\beta}J^\alpha J^\beta}}$. Taking into account the continuity equation $\nabla_\beta J^\beta=0$ we have:

$$\nabla_\beta\phi^{\alpha\beta}=\nabla_\beta\left(\frac{2cJ^\alpha J^\beta}{\sqrt{g_{\alpha\beta}J^\alpha J^\beta}}\right)=\frac{2cJ^\beta\nabla_\beta J^\alpha}{\sqrt{g_{\alpha\beta}J^\alpha J^\beta}}+2cJ^\alpha J^\beta\nabla_\beta\left(\frac{1}{\sqrt{g_{\alpha\beta}J^\alpha J^\beta}}\right)=$$
$$=\frac{2cJ^\beta\nabla_\beta J^\alpha}{\sqrt{g_{\alpha\beta}J^\alpha J^\beta}}-\frac{cJ^\alpha J^\beta}{(g_{\alpha\beta}J^\alpha J^\beta)^{1,5}}\nabla_\beta\left(g_{\alpha\beta}J^\alpha J^\beta\right)=\frac{cJ^\beta\nabla_\beta J^\alpha}{\sqrt{g_{\alpha\beta}J^\alpha J^\beta}}=u^\beta\nabla_\beta J^\alpha=\frac{DJ^\alpha}{D\tau}.$$

In fact, we found the tensor $\phi^{\alpha\beta}=\frac{2cJ^\alpha J^\beta}{\sqrt{g_{\alpha\beta}J^\alpha J^\beta}}$ in (17). The reason that we have not used it, is that instead of the exact form of variation δJ^β as a function of variations in coordinates, in derivation of equations of motion was used simplified form (18). This leads to (35), but not to (53).

From variation (48) implies that the total variation of mass 4-current δJ^β associated with variation of metric tensor $\delta g_{\mu\nu}$, variation of electromagnetic 4-current δj^μ and variations of 4-potentials:

$$\delta\left(c\sqrt{g_{\mu\nu}J^\mu J^\nu}+D_\mu J^\mu+A_\mu j^\mu-\chi\right)=0, \qquad (54)$$

$$\frac{1}{2}u^\mu J^\nu\delta g_{\mu\nu}+u_\nu\delta J^\nu+D_\mu\delta J^\mu+J^\mu\delta D_\mu+A_\mu\delta j^\mu+j^\mu\delta A_\mu=0 .$$

The relationship of variations in (54) is connected through variation of coordinates ξ^{β} in such a way that mass of any local volume does not change during variation. However, a case is possible when the mass-energy is converted into radiation energy, or density of substance changes due to inflow or outflow of mass. Then, some presented in this paper results will require a corresponding change.

Let's now substitute (48) into (2), and with equations $\sqrt{g_{\mu\nu} J^{\mu} J^{\nu}} = c\rho_0$, $-2k\Lambda - \frac{\chi}{c} = 0$, we obtain:

$$\delta S = \delta \int \left(kR + \frac{c}{16\pi\gamma} \Phi_{\mu\nu} \Phi^{\mu\nu} - \frac{c\varepsilon_0}{4} F_{\mu\nu} F^{\mu\nu} \right) \sqrt{-g}\, d\Sigma = 0 .$$

Following the implementation of variation in this equation we arrive to the equations for metric (49), but without the tensor $\phi^{\alpha\beta}$:

$$R^{\alpha\beta} - \frac{1}{2} g^{\alpha\beta} R = \frac{8\pi\gamma\beta}{c^4} \left(U^{\alpha\beta} + W^{\alpha\beta} \right). \qquad (55)$$

The metric which is obtained in (55) is what it should be outside of substance, and this metric is directly dependent on magnitude of existing fields and indirectly from distribution of substance in this field.

In CTG there is the concept of geodesic line, which coincides with the expression of general theory of relativity, but only for field quanta. The equation of motion (35), taking into account $D\tau = d\tau$, can be written as:

$$\frac{D}{d\tau}\left(\frac{dx_{\beta}}{d\tau} \right) = \frac{Ddx_{\beta}}{d\tau d\tau} = \Phi_{\beta\sigma} \frac{dx^{\sigma}}{d\tau} + \frac{\rho_{0q}}{\rho_0} F_{\beta\sigma} \frac{dx^{\sigma}}{d\tau} . \qquad (56)$$

For field quanta $ds = cd\tau = 0$. Multiplying (56) by $d\tau d\tau$, we obtain the vanishing of the right-hand side: $Ddx_{\beta} = 0$. Dividing this by the square of differential of time coordinate λ, which is a measure time along the trajectory of photon, and recalling definition of operator of proper-time-derivative, we obtain the equation of geodesic in covariant indices:

$$\frac{Ddx_\beta}{d\lambda d\lambda}=\frac{D}{d\lambda}\left(\frac{dx_\beta}{d\lambda}\right)=\frac{dx^\sigma}{d\lambda}\nabla_\sigma\left(\frac{dx_\beta}{d\lambda}\right)=\frac{dx^\sigma}{d\lambda}\partial_\sigma\left(\frac{dx_\beta}{d\lambda}\right)-\Gamma^\gamma_{\sigma\beta}\frac{dx^\sigma}{d\lambda}\frac{dx_\gamma}{d\lambda}= \\ =\frac{d}{d\lambda}\left(\frac{dx_\beta}{d\lambda}\right)-\Gamma^\gamma_{\sigma\beta}\frac{dx^\sigma}{d\lambda}\frac{dx_\gamma}{d\lambda}=0. \quad (57)$$

While the field quanta are distributed outside of substance in a given field (gravitational and electromagnetic), their movement is in accordance with the equation of motion (57), and the space-time metric is determined from (55). What may change as the field quanta pass through substance? If the substance is rare and does not interact with photons, the quanta move between the particles of substance. Then again, in principle, should be valid equation (55) for the metric with the amendment that is now necessary to take into account stress-energy tensor of field of strong gravitation acting at the level of elementary particles. This new tensor should look as a supplement to the stress-energy tensor $U^{\alpha\beta}$ of normal gravitation (26), replacing the constant of gravitation γ by constant of strong gravitation Γ, and with some factor of proportionality. Instead of using this new tensor, in practice, it is replaced by stress-energy tensor of substance $\phi^{\alpha\beta}$, and one say that "substance alters the spacetime metric inside, and affects the metric beyond its borders".

This leads to equation for the metric in the form of (49). But another interpretation is possible – always a field affects the metric, whereas the role of substance is reduced only to creation of field. In this case is necessary to impose the condition on the properties of test objects with which we study metric and find metric tensor components – these test bodies must interact with substance at a distance and only through the fields, without mechanical contact, randomly changing the movement.

The above interpretation of relationship between substance, metric and field is difficult in general relativity, in which gravitation is hiding in shadow of geometrical metric field and losing the physical essence. Metric gravitational field (metric of spacetime) in general relativity depends on substance and electromagnetic field and is fully determined by them. But how substance changes physically metric field even far away from itself? What is the relationship mechanism between substance and field? All this remains a mystery.

In covariant theory of gravitation as the basic idea of generating gravitational field Fatio-Le Sage's theory of gravitation is considered, allowing in the same way to describe strong gravitation at the level of elementary particles and usual macroscopic gravitation [12], as well as electromagnetic interaction between bodies [7]. Quanta of gravitation, which are formed by relativistic objects at the lower levels of matter presumably in the form of electromagnetic radiation and neutrinos, become gravitons for objects of higher levels of matter and give there gravitational interaction. The gradients of energy density flux of gravitons may be regarded as gravitational field strengths. Then gravitational potential is difference between the energy density flux of gravitons near or inside bodies, and the energy density flux of gravitons at infinity in the absence of bodies. That flows of gravitons are responsible for deviation of test particles and field quanta near massive bodies. In this picture field quanta of lower levels of matter generate macroscopic fields and form macroscopic metric, and substance (regarded as a collection of objects at different levels of matter, distinguished by their characteristic sizes and masses) interacts with the field quanta and itself generates them.

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