Is Kähler action expressible in terms of areas of minimal surfaces?

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Abstract

The general form of ansatz for preferred extremals implies that the Coulombic term in Kähler action vanishes so that it reduces to 3-dimensional surface terms in accordance with general coordinate invariance and holography. The weak form of electric-magnetic duality in turn reduces this term to Chern-Simons terms.

The strong form of General Coordinate Invariance implies effective 2-dimensionality (holding true in finite measurement resolution) so that also a strong form of holography emerges. The expectation is that Chern-Simons terms in turn reduces to 2-dimensional surface terms.

The only physically interesting possibility is that these 2-D surface terms correspond to areas for minimal surfaces defined by string world sheets and partonic 2-surfaces appearing in the solution ansatz for the preferred extremals. String world sheets would give to Kähler action an imaginary contribution having interpretation as Morse function. This contribution would be proportional to their total area and assignable with the Minkowskian regions of the space-time surface. Similar but real string world sheet contribution defining Kähler function comes from the Euclidian space-time regions and should be equal to the contribution of the partonic 2-surfaces. A natural conjecture is that the absolute values of all three areas are identical: this would realize duality between string world sheets and partonic 2-surfaces and duality between Euclidian and Minkowskian space-time regions.

Zero energy ontology combined with the TGD analog of large N_c expansion inspires an educated guess about the coefficient of the minimal surface terms and a beautiful connection with p-adic physics and with the notion of finite measurement resolution emerges. The t'Thooft coupling λ should be proportional to p-adic prime p characterizing particle. This means extremely fast convergence of the counterpart of large N_c expansion in TGD since it becomes completely analogous to the pinary expansion of the partition function in p-adic thermodynamics. Also the twistor description and its dual have a nice interpretation in terms of zero energy ontology. This duality permutes massive wormhole contacts which can have off mass shell with wormhole throats which are always massive (also for the internal lines of the generalized Feynman graphs).

Contents

1 Introduction

As I scanned of hep-th I found an interesting article by Giordano, Peschanski, and Seki [2] based on AdS/CFT correspondence. What is studied is the high energy behavior of the gluon-gluon and quark-quark scattering amplitudes of $\mathcal{N} = 4$ SUSY.

1. The proposal made earlier by Aldaya and Maldacena [1] is that gluon-gluon scattering amplitudes are proportional to the imaginary exponent of the area of a minimal surface in AdS_5 whose boundary is identified as *momentum space*. The boundary of the minimal surface would be polygon with light-like edges: this polygon and its dual are familiar from twistor approach.

2. Giordano, Peschanski, and Seki claim that quark-quark scattering amplitude for heavy quarks corresponds to the exponent of the area for a minimal surface in the *Euclidian* version of AdS_5 which is hyperbolic space (space with a constant negative curvature): it is interpreted as a counterpart of configuration space rather than momentum space and amplitudes are obtained by analytic continuation. For instance, a universal Regge behavior is obtained. For general amplitudes the exponent of the area alone is not enough since it does not depend on gluon quantum numbers and vertex operators at the edges of the boundary polygon are needed.

In the following my intention is to consider the formulation of this conjecture in quantum TGD framework. I hasten to inform that I am not a specialist in AdS/CFT and can make only general comments inspired by analogies with TGD.

2 Why Chern-Simons action should reduce to area for minimal surfaces?

The minimal surface conjectures are highly interesting from TGD point of view. The weak form of electric magnetic duality implies the reduction of Kähler action to 3-D Chern-Simons terms. Effective 2-dimensionality implied by the strong form of General Coordinate Invariance suggests a further reduction of Chern-Simons terms to 2-D terms and the areas of string world sheet and of partonic 2-surface are the only non-topological options that one can imagine. Skeptic could of course argue that the exponent of the minimal surface area results as a characterizer of the quantum state rather than vacuum functional. In the following I defend the minimal interpretation as Chern-Simons terms.

Let us look this conjecture in more detail.

1. In zero energy ontology twistor approach is very natural since all physical states are bound states of massless particles. Also virtual particles are composites of massless states. The possibility to have both signs of energy makes possible space-like momenta for wormhole contacts. Mass shell conditions at internal lines imply extremely strong constraints on the virtual momenta and both UV and IR finiteness are expected to hold true.

2. The weak form of electric magnetic duality [2] implies that the exponent of Kähler action reduces to the exponent of Chern-Simons term for 3-D space-like surfaces at the ends of space-time surface inside CD and for light-like 3-surfaces. The coefficient of this term is complex since the contribution of Minkowskian regions of the space-time surface is imaginary ($\sqrt{g_4}$ is imaginary) and that of Euclidian regions (generalized Feynman diagrams) real. The Chern-Simons term from Minkowskian regions is like Morse function and that from Euclidian regions defines Kähler function and stationary phase approximation makes sense. The two contributions differ only by imaginary coefficient if Chern-Simons term contains only contributions from wormhole throats and ends of space-time at CDs. This need not be the case.

3. Electric magnetic duality [2] leads also to the conclusion that wormhole throats carrying elementary particle quantum numbers are Kähler magnetic monopoles. This forces to identify elementary particles as string like objects with ends having opposite monopole charges. Also more complex configurations are possible.

   It is not quite clear what the scale of the stringyness is. The natural first guess inspired by quantum classical correspondence is that it corresponds to the p-adic length scale of the particle characterizing its Compton length. Second possibility is that it corresponds to electroweak scale. For leptons stringyness in Compton length scale might not have any fatal implications since the second end of string contains only neutrinos neutralizing the weak isospin of the state. This kind of monopole pairs could appear even in condensed matter scales: in particular if the proposed hierarchy of Planck constants [1] is realized.

4. Strong form of General Coordinate Invariance requires effective 2-dimensionality. In given UV and IR resolutions either partonic 2-surfaces or string world sheets form a finite hierarchy of CDs inside CDs with given CD characterized by a discrete scale coming as an integer multiple of a fundamental scale (essentially CP_2 size). The string world sheets have boundaries consisting of either light-like curves in induced metric at light-like wormhole throats and space-like curves at the ends of CD whose M^4 projections are light-like. These braids intersect partonic 2-surfaces at discrete points carrying fermionic quantum numbers.

   This implies a rather concrete analogy with $AdS_5 \times S_5$ duality, which describes gluons as open strings. In zero energy ontology (ZEO) string world sheets are indeed a fundamental notion and the natural conjecture is that these surfaces are minimal surfaces whose area by quantum classical correspondence depends on the quantum numbers of the external particles. String tension in turn should depend on gauge couplings -perhaps only Kähler coupling strength- and geometric parameters like the size scale of CD and the p-adic length scale of the particle.

5. Are the minimal surfaces in question minimal surfaces of the imbedding space $M^4 \times CP_2$ or of the space-time surface X^4? All possible 2-surfaces at the boundary of CD must be allowed so that they cannot correspond to minimal surfaces in $M^4 \times CP_2$ unless one assumes that they emerge in stationary phase approximation only. The boundary conditions at the ends of CD could however be such that *any* partonic 2-surface correspond to a minimal surfaces in X^4. Same applies to string world sheets. One might even hope that these conditions combined with the weak form of electric magnetic duality fixes completely the boundary conditions at wormhole throats and space-like ends of space-time surface.

   The trace of the second fundamental form orthogonal to the string world sheet/partonic 2-surface as sub-manifold of space-time surface would vanish: this is nothing but a generalization of the geodesic motion obtained by replacing word line with a 2-D surface. It does not imply the vanishing of the trace of the second fundamental form in $M^4 \times CP_2$ having interpretation as a generalization of particle acceleration [4]. Effective 2-dimensionality would be realized if Chern-Simons terms reduce to a sum of the areas of these minimal surfaces.

These arguments suggest that scattering amplitudes are proportional to the product of exponents of 2-dimensional actions which can be either imaginary or real. Imaginary exponent would be proportional to the total area of string world sheets and the imaginary unit would come naturally from $\sqrt{g_2}$. Teal exponent proportional to the total area of partonic 2-surfaces. The coefficient of these areas would not in general be same.

The equality of the Minkowskian and Euclidian Chern-Simons terms is suggestive but not necessarily true since there could be also other Chern-Simons contributions than those assignable to wormhole throats and the ends of space-time. The equality would imply that the total area of string world sheets equals to the total area of partonic 2-surfaces suggesting strongly a duality meaning that either Euclidian or Minkowskian regions carry the needed information.

3 IR cutoff and connection with p-adic physics

In twistor approach the IR cutoff is necessary to get rid of IR divergences. Also in the AdS_5 approach the condition that the minimal surface area is finite requires an IR cutoff. The problem is that there is no natural IR cutoff. In TGD framework zero energy ontology brings in a natural IR cutoff via the finite and quantized size scale of CD guaranteeing that the minimal surfaces involved have a finite area. This implies that also particles usually regarded as massless have a small mass characterized by the size of CD. The size scale of CD would correspond to the scale parameter R assigned with the metric of AdS_5.

1. String tension relates in AdS_5 approach to the gauge coupling g_{YM} and to the number N_c of colors by the formula

$$\lambda = g_{YM}^2 N_c = \frac{R^2}{\alpha'} . \quad (3.1)$$

$1/N_c$-expansion is in terms of $1/\sqrt{\lambda}$. The formula has an alternative form as an expression for the string tension

$$\alpha' = \frac{R^2}{\sqrt{g_{YM}^2 N_c}} . \tag{3.2}$$

The analog this formula in TGD framework suggests an connection with p-adic length scale hypothesis.

1. As already noticed, the natural counterpart for the scale R could be the discrete value of the size scale of CD. Since the symplectic group assignable to $\delta M^4_\pm \times CP_2$ (or the upper or lower boundary of CD) is the natural generalization of the gauge group, it would seem that $N_c = \infty$ holds true in the absence of cutoff. At the limit $N_c = \infty$ only planar diagrams would contribute to YM scattering amplitudes. Finite measurement resolution must make the effective value of N_c finite so that also λ would be finite. String tension would depend on both the size of CD and the effective number of symplectic colors.

2. If α' is characterized by the square of the Compton length of the particle, λ would be essentially the square of the ratio of CD size scale given by secondary p-adic lengths and of the primary p-adic length scale associated with the particle: $\lambda = g_{YM}^2\sqrt{p}$, where p is the p-adic prime characterizing the particle. Favored values of the p-adic prime correspond to primes near powers of two. The effective number of symplectic colors would be $N_c = \sqrt{p}/g_{YM}^2$ and the expansion would come in powers of $g_{YM}^2/\sqrt{p}$. For electron one would have $p = M_{127} = 2^{127-1}$ so that the expansion would converge extremely fast. Together with the amazing success of the p-adic mass calculations based on p-adic thermodynamics for the scaling generator L_0 [3] this suggests a deep connection with p-adic physics and number theoretic universality.

4 What is the interpretation of Yangian duality in TGD framework?

Minimal surfaces in both configuration space and momentum space are used in the above mentioned two articles [1, 2]. The possibility of these two descriptions must reflect the Yangian symmetry unifying the conformal symmetries of Minkowski space and momentum space in twistorial approach.

The minimal surfaces in $X^4 \subset M^4 \times CP_2$ are natural in TGD framework. Could also the minimal surfaces in momentum space have some interpretation in TGD framework? Ore more generally, what could be the interpretation of the dual descriptions provided by twistor diagrams with light-like edges and dual twistor diagrams with light-like vertices? One can imagine many interpretations but zero energy ontology suggests an especially attractive and natural interpretation of this duality as the exchange of the roles of wormhole throats carrying always on mass shell massless momenta and wormhole contacts carrying in general off-mass shell momenta and massive momenta in incoming lines.

1. For configuration space twistor diagrams vertices correspond to incoming and outgoing light-like momenta. The light-like momenta associated with the wormhole throats of the incoming and outgoing lines of generalized Feynman diagram could correspond to the light-like momenta associated with the vertices of the polygon. The internal lines defined by wormhole contacts carrying virtual off mass shell momenta would naturally correspond to to edges of the twistor diagram.

2. What about dual twistor diagrams in which light-like momenta correspond to lines? Zero energy ontology implies that virtual wormhole throats carry on mass shell massless momenta whereas incoming wormhole contacts in general carry massive particles: this guarantees the absence of IR divergences. Could one identify the momenta of internal wormhole throats as light-like momenta associated with the lines dual twistor diagrams and the incoming net momenta assignable to wormhole contacts as incoming and outgoing momenta.

Also the transition from Minkowskian to Euclidian signature by Wick rotation could have interpretation in TGD framework. Space-time surfaces decompose into Minkowskian and Euclidian regions. The latter ones represent generalized Feynman diagrams. This suggests a generalization of Wick rotation. The string world sheets in Euclidian regions would define the analogs of the minimal surfaces in Euclidian AdS_5 and the string world sheets in Minkowskian regions the analogs of Minkowskian AdS_5. The magnitudes of the areas would be identical so that they might be seen as analytical continuations of each other in some sense. Note that partonic 2-surfaces would belong to the intersection of Euclidian and Minkowskian space-time regions. This argument tells nothing about possible momentum space analog of $M^4 \times CP_2$.

Books related to TGD

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