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Mathematical objects : past, present, future.

Abstract.

There are three main mathematical operations : [1] – addition, [2] - multiplication, [3] – raising to the power. Also there are three kinds of operands : A – numbers, B – vectors, C – vector spaces.

This paper shows how to define new kinds of operands : D, E, F, … and new operations : [4], [5], [6], … ; [0], [-1], [-2], [-3], … .

The study of inverse operations for new operations can open new classes of operands like it was with the operations [1], [2], [3].

Also there is a way to description of field with spin = 1/3.

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I) Operations and operands.

There are 3 main operations in mathematics now : [1] – addition, [2] - multiplication, [3] – raising to the power. Operation drives operands (such as summands, cofactors, …) into the interaction and gives the result. The result usually belongs to the same set of mathematical objects, as the operands.

There are also 3 main operands in mathematics now : A – numbers, B – vectors, C – spaces. Let us write the general formula :

Operand1 (operation) operand2 = operand3 (I .1)

There are 3*3*3*3=81 combinations of this formula. Some of them are well studied. For example :
A1 [1] A2 = A3 A1 [2] A2 = A3 A1 [3] A2 = A3
B1 [1] B2 = B3 B1 [2] B2 = A1 B1 [2] B2 = B3 A1 [2] B1 = B2
C1 [1] C2 = C3 C1 [2] C2 = C3
Some another only partly. For example : C1 [3] A1 = C2 (I . 2)
This equation is investigated only for natural A.
All those cases exampled above were studied in the Past.

Investigation of (I . 2) when A1 is rational and irrational described on the site

http://telnin.yandex.ru/ in the Part I and Part V.
This is Present.

Another cases, like : B1 [3] B2 =B3 C1 [3] B1 = C2 C1 [3] C2 =C3
belongs to the Future.

II) New operands and new operations.

1) Operands D, E, F, …

Vector is a set of numbers (projections). The addition of vectors defines by the addition of numbers (projections). The multiplication of vectors defines by the multiplication of numbers (projections).

Vector space is a set of vectors, which defines by the basis vectors. The addition of vector spaces defines by the addition of their basis vectors. The multiplication of vector spaces defines by the multiplication of basis vectors.

OBSERVATION : the set of operands of one level gives the operand of the next level. And the addition of operands of the new level defines by the addition of operands of the previous level.

Also the multiplication of the operands of the new level defines by the multiplication of the operands of the previous level.

SUPPOSITION : This must be true for operands not only A, B, C, but also for D, E, F, … . And this must be true for the operations not only [1], [2], but also for [3], [4], [5], [6], … (if it will succeed to define those new operations). And the application of [3] operation to the two operands C (vector spaces) will be defined through the application of the [3] operation to the two operands B (vectors). This reminds the process of raising one complex number into the power of another complex number.

And operand D will be the set of operands C (vector spaces). The addition of operands D would be defined by addition of operands C (vector spaces). Also the multiplication of operands D would be defined by multiplication of the operands C (vector spaces). And application of [3] operation to the two operands D will be defined by application of the [3] operation to the operands C (vector spaces).

And operand E will be the set of operands D. And so on.

2) Operations [4], [5], [6], … .

Operands and operations has deep similarity. It expresses in the connection of the mathematical objects of one level, with the mathematical objects of the next level. Here the words "mathematical objects" mean operands and operations. Every new operand is the set of the operands of the previous level. And also each new operation is the set of operations of the previous level. Let us check up that.

1 [1] 1 = 2 2 [2] 3 = 6 2 [3] 3 = 8

Let us take the set from three operations [1] with equal operands a :

a [1] a [1] a [1] a = a [2] 4

We see that this set gives the operation of the next level.
Let us take the set from three operations [2] with equal operands a :

a [2] a [2] a [2] a = a [3] 4

We see that this set of operations [2] also gives the operation of the next level.
Now we can write the universal form of connection of the operations of the level n and the level n+1 :

a [n] (a [n] (a [n] a)) =a [n+1] 4   (II . 2 .1)

Let us place n = 3 into this formula. And we derive the definition for the new operation [4] :

A [3] (a [3] (a[3]a)) = a [4] 4

And if we place n = 4 in the formula (II . 2 . 1) then we get the definition for the operation [5] through the operation [4]. By this way we can form new operations infinitly.

3) Operations [0], [-1], [-2], … .

If we inverse the (II . 2 . 1), then we get :

a [n-1] (a [n-1] (a [n-1] a)) = a [n] 4   (II . 3 .1)

Now, if we take n = 1, we will get the description of new operation [0] through the old operation [1] :

a [0] (a [0] (a [0] a)) = a [1] 4

If we take n = 0, then we will get new operation [-1] through operation [0]. And so we can define operations [-2], [-3], [-4], and others infinitely.

4) Hypothesis.

Since numbers of operations evolve like operands (from natural numbers to negative), then we can suppose that in future they can went the stagers of rational numbers, real numbers and to became by vectors. And then by vector spaces (operands C), by operands D, E, … . And we can write more general formula then (I . 1) :

Operand1 [operand2] operand3 = operand4   (II . 4 . 1)

III) Inverse operations and new operands.

Operation [1] was defined for natural operand1 and operand3. Operand4 also becomes the natural number. There is an example of this :

2 [1] 3 = 5   (III . 1)
a [1] b = c   (III . 2)

This operation has two inverse operations :

x [1] 3 = 5   x = 2 – first inverse operation
2 [1] y = 5   y = 3 – second inverse operation

Let us define symbols for **inverse** operations :

X [n] A = B	B [n,1] A = X	(III . 3)
C [n] Y = D	D [n,2] C = Y	(III . 4)

And we name operation [n] as **direct** operation.

If we demand equality for [n] and [n,1] and [n,2], then not only a and b in (III . 2) can be natural numbers, but B and A in (III . 3), and D and C in (III . 4) also may be natural numbers. But there are such pairs of natural numbers in (III . 3), (III . 4), that X and Y cannot be natural numbers. For example :

X [1] 5 = 2 X – not natural number
5 [1] Y = 2 Y – not natural number

We must define new operand : -3. So arises new class of operands – negative numbers. Natural and negative numbers are named as integer numbers. And now direct operation [1] and it's two inverse operations became equal as to if the left part of equations (III . 2), (III . 3), (III . 4) contains any two integer operands, then the right parts of those equations would also contain integer operands.

Similarly we can show that operation [2] gives inverse operations [2,1], [2,2]. And, demanding of the similar equality of all these three operations, we must arise new class of operands – rational numbers.

Further, the demanding of the equality of operations [3], [3,1], [3,2] leads to the real (rational and irrational) and imaginary numbers. Together they called by complex numbers (though they are more similar to vectors).

But what the result would be if we make this demand for the operations [4], [4,1], [4,2] ? Very likely there must be a new class of operands, more general then complex numbers. And so must be expected from [5], [6], [7], and others direct operations (and there inverse operations).

IV)Application of the raising vector spaces to rational powers to physics.

The description of the field with spin = 1/3 is given in Part IV of the site telnin@yandex.ru . There are Lagrangian, equations for field, decisions for these equations, the densities of tensor and vector of energy-momentum, the densities of spin tensor and spin vector.

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