The Hilbert book model.

in concise format.

The HBM is a simple model of fundamental physics that is strictly based on the axioms of traditional quantum logic. It uses a sequence of instances of an extension of a quaternionic separable Hilbert space that each represent a static status quo of the whole universe.

The Hilbert Book Model

In concise format

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THE HILBERT BOOK MODEL

Colophon

Written by Ir J.A.J. van Leunen
The subject of this book is a new model of physics
This is a concise format of the model

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THE HILBERT BOOK MODEL

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I also have to thank my friends and discussion partners that listened to my lengthy deliberations on this non society chitchat suitable subject and patiently tolerated that my insights changed regularly.

DETAILS

The Hilbert Book Model is the result of a research project.
That project started in 2009.

The continuing status of the project can be followed at
http://www.crypts-of-physics.eu

The author's e-print sites are:
http://vixra.org/author/Ir_J_A_J_van_Leunen .

http://vixra.org/author/Ir_J_A_J_Hans_van_Leunen .

The nice thing about laws of physics is that they repeat themselves. Otherwise they would not be noticed. The task of physicists is to notice the repetition.

Contents

Preface

This book presents a concise version of another book that also treats the Hilbert Book Model, but that in addition acts as a grab-bag of other subjects that are more or less related to the Hilbert Book Model.
Only the tale is kept as well. It is contained in part three of this book.

In fact I started the Hilbert Book Model during my studies in physics in the sixties on the Technical University of Eindhoven (TUE).

In the first two years the lectures concerned only classical physics. In the third year quantum physics was introduced. I had great difficulty in understanding why the methodology of doing physics changed drastically. So I went to the teacher, which was an old nearly retired professor and asked him:
"Why is quantum mechanics done so differently from classical mechanics?".

His answer was short. He stated":
"The reason is that quantum mechanics is based on the superposition principle".

I quickly realized that this was part of the methodology and could not be the reason of the difference in methodology. So I went back and told him my concern. He told me that he could not give me a better answer and if I wanted a useful answer I should research that myself. So, I first went to the library, but the university was quite new and only contained rather old second hand books, which they got from other institutions. Next I went to the city's book shops. I finally found a booklet from P. Mittelstaedt: (Philosophische Probleme der modernen Physik, BI Hochschultaschenbücher, Band 50, 1963) that contained a chapter on quantum logic. I concluded that this produced the answer that I was looking for. Small particles obey a kind of logic that differs from classical logic. As a result their dynamic behavior differs from the behavior of larger objects.

I searched further and encountered papers from Garret Birkhoff and John von Neumann that explained the correspondence between quantum logic and separable Hilbert spaces. That produced an acceptable answer to my question.

The lectures also told me that observables were related to eigenvalues of Hermitian operators. These eigenvalues are real numbers. However, it was clearly visible that nature has a 3+1D structure. So I tried to solve that discrepancy as well. After a few days of puzzling I discovered a new number system that had this 3+1D structure and I called them compound numbers. I went back to my professor and asked him why such compound numbers were not used in physics. Again he could not give a reasonable answer.

When I asked the same question to a much younger assistant professor he told me that these numbers were discovered more than a century earlier by William Rowan Hamilton when he was walking with his wife over a bridge in Dublin. He was so glad about his discovery that he carved the corresponding formula into the sidewall of the bridge. The inscription has faded away, but it is now molded in bronze and fixed to the same wall. The numbers are known as quaternions. So I went to the library and searched for papers on quaternions.

In those years C. Piron wrote his papers on quaternionic Hilbert spaces that completed my insight in this subject. I finalized my physics study with an internal paper on quaternionic Hilbert spaces.

The university was specialized in applied physics and not in theoretical physics. This did not stimulate me to proceed with the subject. Next, I went into a career in industry where I used my knowledge of physics in helping analyze intensified imaging and helping design night vision equipment and X-ray image intensifiers.

That put me with my nose on the notion of quanta. The image intensifiers did not show radiation. Instead they showed clouds of impinging quanta.

In those times I had not much opportunity to deliberate on that fact. However, after my retirement I started to rethink the matter. That was the instant that the Hilbert Book Model continued further.

Thus, in a few words: The Hilbert Book Model tries to explain the existence of quanta. It does that by starting from traditional quantum logic.

Part one

The fundaments

The Fundaments

Abstract

The fundaments of quantum physics are still not well established. This paper tries to find the cracks in these fundaments and suggests repair procedures. This leads to unconventional solutions and a new model of physics. The model is strictly based on the axioms of traditional quantum logic. However, in order to proceed from this point, it is necessary to extend this base, such that it also incorporates the equivalents of physical fields. This results in a model that can represent a static status quo of the whole universe. The most revolutionary introduction is the representation of dynamics by a sequence of such static models in the form of a sequence of extended separable Hilbert spaces. Together, this embodies a repair of fundaments that does not affect the building.

Logic model

The author has decided to base the Hilbert Book Model on a consistent set of axioms. It is often disputed whether a model of physics can be strictly based on a set of axioms. Still, what can be smarter than founding a model of physics on the axioms of classical logic?

Since in 1936 John von Neumann[1] wrote his introductory paper on quantum logic the scientific community knows that nature cheats with classical logic and in fact obeys traditional quantum logic.

As a consequence the Hilbert Book Model will be strictly based on the axioms of traditional quantum logic. However, this choice immediately reveals its constraints. Traditional quantum logic is not a nice playground for the mathematics that characterizes the formulation of most physical

[1] http://en.wikipedia.org/wiki/John_von_Neumann#Quantum_logics

laws. Lucky enough, von Neumann encountered the same problem and together with Garret Birkhoff[2] he detected that the set of propositions of quantum logic is lattice isomorphic with the set of closed subspaces of an infinite dimensional separable Hilbert space. Some decades later Constantin Piron[3] proved that the inner product of the Hilbert space must by defined by numbers that are taken from a division ring. Suitable division rings are the real numbers, the complex numbers and the quaternions[4]. The Hilbert Book Model takes the choice with the widest possibilities. It uses quaternionic Hilbert spaces. Quaternions play a decisive role the Hilbert Book Model. Higher dimension hyper-complex numbers may suit as eigenvalues of operators or as values of physical fields, but for the moment the HBM can do without these numbers. Instead, quaternions will be used for those purposes.

So, now we have a double model that connects logic with a flexible mathematical toolkit. But, this solution does not solve all restrictions. Neither quantum logic nor the separable Hilbert space can handle physical fields and they also cannot handle dynamics.

An extension of the basic models helps cure the first restriction. In addition it also solves another problem. The separable Hilbert space only tolerates eigenspaces of operators that contain a countable number of eigenvalues. Thus, the Hilbert space does not know continuums. A solution can be found in the Gelfand triple[5] of the Hilbert space. This sandwich contains the Hilbert space as a member and at the same time it provides operators that have a continuum as their eigenspace.

This would introduce a new problem because the continuum fits far more eigenvalues than the eigenspace of the operator in the separable Hilbert

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[2] http://en.wikipedia.org/wiki/John_von_Neumann#Lattice_theory
[3] C. Piron 1964; _Axiomatique quantique_
[4] http://en.wikipedia.org/wiki/Quaternion
[5] http://en.wikipedia.org/wiki/Gelfand_triple

space can offer. This problems makes think of a similar situation that occurs when the number N of linear equations that must be solved is larger than the number M of variables that are contained in these equations. Let the equations be available in the form:

$$\sum_{m=1}^{M} a_{mn} x_{mn} = b_n \; ; n = 1 \cdots N$$

Usually, such situations are solved by assuming that a stochastic inaccuracy exists between the values b_n in the too large result set and the actual results. The actual results would offer a consistent set of equations. These actual results make the equations interdependent. Translated to our problem, this solution comes down to linking the eigenvalues of operators in the separable Hilbert space to corresponding values in the continuum eigenspace of operators in the Gelfand triple by using a quaternionic probability amplitude distribution (QPAD) as the connection between the source and the target. As will be shown, the choice for a QPAD has significant and favorable consequences.

The attachment of QPAD's extends the separable Hilbert space and connects it in a special way to its Gelfand triple. Due to the isomorphism of the lattice structures, the quantum logic is extended in a similar way. This leads to a reformulation of quantum logic propositions that makes them incorporate stochastically inaccurate of observations instead of precise observations. The logic that is extended in this way will be called extended quantum logic. The separable Hilbert space that is extended in this way will be called extended separable Hilbert space.

The implementation of physical fields via the attachment of QPAD's to eigenvectors in the separable Hilbert space is a crucial departure from common physical methodology. Common quantum physics uses complex probability amplitude distributions (CPAD's), rather than QPAD's[6]. Quantum Field Theory[7], in the form of QED[8] or QCD[9], implements physical fields in a quite different manner.

The choice for QPAD's appears very smart. The real part of the QPAD can be interpreted as a "charge" density distribution. Similarly the imaginary part of the QPAD can be interpreted as a "current" density distribution. The squared modulus of the value of the QPAD can be interpreted as the probability of the presence of the carrier of the "charge". The "charge" can be any property of the carrier or it represents a collection of the properties of the carrier. In this way, when the wave function is represented by a QPAD the equation of motion becomes a continuity equation[10].

QPAD's use a quaternion as their parameter. Usually these parameters are values of a quaternionic distribution that uses the quaternionic number space as its parameter space. In that way the quaternionic distribution can act as a curved coordinate system.

In most cases where quaternionic distributions are used, the fact that quaternions possess two independent sign selections is ignored. The first sign selection, the conjugation, inverts the sign of all three imaginary base vectors. The second sign selection, the reflection, inverts the sign of a single imaginary base vector. The sign selections in a quaternionic distribution are all similar. Individually, the conjugation and the reflection switch the handedness of the external vector product in the product of two quaternions that are taken from the same quaternionic distribution.

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[6] http://en.wikipedia.org/wiki/Probability_amplitude
[7] http://en.wikipedia.org/wiki/Quantum_field_theory
[8] http://en.wikipedia.org/wiki/Quantum_electrodynamics
[9] http://en.wikipedia.org/wiki/Quantum_chromodynamics
[10] Also called balance equation.

For each QPAD, the mixture of conjugation and reflection produces four different sign flavors. In quantum physics these sign flavors play a crucial role. In common physics this role is hidden in alpha, beta and gamma matrices and in spinors.

The extension cured several restrictions, but one is left. Both the extended quantum logic and the extended separable Hilbert space can only represent a static status quo. The Hilbert space does not have an operator that delivers progression as an eigenvalue. Instead progression can be attached as a parameter to the whole Hilbert space including the Gelfand triple and the attached QPAD's.

Now implementing dynamics becomes a simple action. The whole Hilbert Book Model consists of an ordered sequence of sandwiches that each includes the Gelfand triple including its Hilbert space and the attached QPAD's. The progression parameter acts as page number of the book.
In the resulting Hilbert Book Model the progression is made in universe wide steps.

History

In its first years, the development of quantum physics occurred violently. Little attention was paid to a solid and consistent foundation. The development could be characterized as delving in unknown grounds. Obtaining results that would support applications was privileged before a deep understanding of the fundamentals.

The first successful results were found by Schrödinger and Heisenberg. They both used a quantization procedure that converted a common classical equation of motion into a quantum mechanical equation of motion. Schrödinger used a wave function that varied as a function of its time parameter, while operators do not depend on time. Heisenberg represented the operators by matrices and made them time dependent, while their target vectors were considered to be independent of time. This led to the distinction between the Schrödinger picture and the Heisenberg picture.

Somewhat later John von Neumann and others integrated both views in one model that was based on Hilbert spaces. Von Neumann also laid the connection of the model with quantum logic. However, that connection was ignored in later developments. Due to the restrictions that are posed by separable Hilbert spaces, the development of quantum physics moved to other types of Hilbert spaces.

After a while, it became clear that the Schrödinger picture and the Heisenberg picture represent two different views of the same situation. It appears to be unimportant were time is put as a parameter. The important thing is that the time parameter acts as a progression indicator. This observation indicates that the validity of the progression parameter covers the whole Hilbert space. With other words, the Hilbert space itself represents a static status quo.

In those days quaternions played no role. The vector spaces and functions that were used all applied complex numbers and observables were represented with self-adjoint operators. These operators are restricted to real eigenvalues.

Quaternions were discovered by the Irish mathematician Sir William Rowan Hamilton[11] in 1843. They were very popular during no more than two decades and after that they got forgotten. Only in the sixties of the twentieth century, due to the discovery of Piron that a separable Hilbert space ultimately uses quaternions for its inner product, a short upswing of quaternions occurred. But quickly thereafter they fell into oblivion again. Currently most scientists never encountered quaternions. The functionality of quaternions is taken over by complex numbers and a combination of scalars and vectors and by a combination of Clifford algebras, Grassmann algebras, Jordan algebras, alpha-, beta- and gamma-matrices and by spinors. The probability amplitude functions were taken to be complex rather than quaternionic. Except for the quaternion functionality that is hidden in the α, β, γ matrices, hardly any attention was given to the possible sign selections of quaternion imaginary base vectors and as a consequence the sign flavors of quaternionic distributions stay undetected. So, much of the typical functionality of quaternions stays obscured.

The approach taken by quantum field theory departed significantly from the earlier generated foundation of quantum physics that relied on its isomorphism with quantum logic. Both QED and QCD put the quantum scene in non-separable Hilbert spaces. Only the wave function is seen as a (complex) probability amplitude distribution. Spinors and gamma matrices are used to simulate quaternion behavior.

The influence of Lorentz transformations gives scientists the impression that space and time do not fit in a quaternion but instead in a spacetime quantity that features a Minkowski signature. Length contraction, time

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[11] http://en.wikipedia.org/wiki/William_Rowan_Hamilton

dilation and space curvature have made it improbable that progression would be seen as a universe wide parameter.

These developments cause a significant deviation between the approach that is taken in contemporary physics and the line according which Hilbert Book Model is developed.

Part two

PART TWO

The Hilbert Book Model

The Hilbert Book Model

Abstract

The extension of the separable Hilbert space by a set of QPAD's enables the interpretation of equations of movement as continuity equations. Exploring this leads to a complete set of equations that describe all elementary particles that are contained in the standard model. The equations enable the computation of the coupling factors from the configuration of the constituting fields. The properties of the particles, including the coupling factors are related to the local curvature and in this way to the notion of mass.

Role of the particle operator

The particle operator is one of the operators for which the eigenvectors are coupled to a background continuum that is related to the eigenspace of a corresponding position operator that resides in the Gelfand triple. The background continuum may be curved. It means that this background continuum is a quaternionic function of the eigenvalues of that operator. This function has the same sign flavor as its parameter space.

For each eigenvector of the particle position operator the background continuum acts as parameter space for the QPAD that connects this eigenvector with the eigenspace of the corresponding position operator that resides in the Gelfand triple.

This position operator has a canonical conjugate, which is the corresponding momentum operator. A connection of the particle eigenvector with the eigenspace of the momentum operator runs via a different QPAD. Without any curvature the two QPAD's would be each other's Fourier transform.

Hyper-complex numbers

Hyper-complex numbers form categories that are ordered with respect to their dimension. The dimension D takes the form $D = 2^n$, where n is a non-negative integer. A hyper-complex number of dimension D can be obtained from a pair of hyper-complex numbers of dimension $D - 1$ via a construction algorithm. Several construction algorithms exist. The most popular is the Cayley-Dickson construction[12]. A less known construction algorithm is the 2^n-on construction of Warren Smith[13]. This construction delivers numbers that in the higher dimensions retain better arithmetic capabilities. Up and including the octonions the two construction algorithms deliver the same numbers. The sedions differ from the 2^4-ons.

In their lower m dimensions the 2^n-ons behave similarly to the 2^m-ons. The 2^n-ons have n independent imaginary base vectors. As a consequence the 2^n-ons feature n independent sign selections.
Both construction methods ignore these sign selections. Sign selections play a crucial role in this paper.

Quaternions

A quaternion is a 1+3 dimensional hyper-complex number. It has a one dimensional real part and a three dimensional imaginary part. As a result, it can be seen as the combination of a real scalar and a three dimensional vector.

$$q = q_0 + \boldsymbol{i}q_1 + \boldsymbol{j}q_2 + \boldsymbol{k}q_3 \quad (1)$$

The quaternions form a division ring[14]. According to the Frobenius theorem[15], the only finite-dimensional division algebras over the reals are the reals themselves, the complex numbers, and the quaternions.

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[12] http://en.wikipedia.org/wiki/Cayley%E2%80%93Dickson_construction
[13] Appendix; 2^n-ons, See http://www.math.temple.edu/~wds/homepage/nce2.pdf
[14] http://en.wikipedia.org/wiki/Division_ring
[15] http://en.wikipedia.org/wiki/Frobenius_theorem_(real_division_algebras)

The coefficients $\{q_m\}$ are real numbers. Bi-quaternions exist that have complex coefficients, but these do not form a division ring.

Sign selections

The quaternions possess two independent sign selections. The conjugation $q \Leftrightarrow q^*$ inverts the sign of all imaginary base vectors. It acts isotropic.

$$q^* = q_0 - \boldsymbol{i}q_1 - \boldsymbol{j}q_2 - \boldsymbol{k}q_3 \tag{1}$$

The reflection $q \Leftrightarrow q^{1}$ inverts a single imaginary base vector and for that reason it acts anisotropic.

$$q^{1} = q_0 + \boldsymbol{i}q_1 + \boldsymbol{j}q_2 - \boldsymbol{k}q_3 \tag{2}$$

Here, the base vector $\boldsymbol{k}$ is selected arbitrarily.
These two sign selections can be mixed. They generate four sign states. Individually the conjugation and the reflection both flip the handedness of the external vector product of the imaginary part when both factors use the same sign selections.

Habits

The addition works as in all division rings, however the product of two quaternions does not commute.

Product rule

The product rule is best expressed in the form of real scalars and 3D vectors:

$$ab = a_0 b_0 - \langle \boldsymbol{a}, \boldsymbol{b} \rangle + a_0 \boldsymbol{b} + \boldsymbol{a} b_0 + \boldsymbol{a} \times \boldsymbol{b} \tag{1}$$

$$\langle \boldsymbol{a}, \boldsymbol{b} \rangle = a_1 b_1 + a_2 b_2 + a_3 b_3 \tag{2}$$

$$\boldsymbol{a} \times \boldsymbol{b} = \boldsymbol{i}(a_2 b_3 - a_3 b_2) + \boldsymbol{j}(a_3 b_1 - a_1 b_3) + \boldsymbol{k}(a_1 b_2 - a_2 b_1) \tag{3}$$

$$\boldsymbol{i}\,\boldsymbol{j} = \pm\boldsymbol{k} \tag{4}$$

The norm or modulus is defined by:

$$|a| = \sqrt{a_0 a_0 + \langle \boldsymbol{a}, \boldsymbol{a} \rangle} \tag{5}$$

Quaternionic probability amplitude distributions

A quaternionic probability amplitude distribution (QPAD)[16] is a quaternionic function. Its value can be split in a real part that can be interpreted as a charge density and an imaginary part that can be interpreted as a current density. The squared modulus of the value can be interpreted as the probability density of the presence of the carrier of the charge. The charge can be any property of the carrier or it stands for the ensemble of the properties of the carrier.

Sign flavors

The quaternions that form the values of a quaternionic distribution must all feature the same set of sign selections. This fact attaches a sign flavor to each quaternionic distribution. Quaternionic distributions come in four sign flavors[17]: $\psi^{⓪}$, $\psi^{①}$, $\psi^{②}$ and $\psi^{③}$.

We will use the symbol ψ or $\psi^{⓪}$ for the sign flavor of the quaternionic distribution that has the same sign flavor as its parameter space. The superscripts indicate the number of base vectors that changed sign.
We will use

$$\psi^{③} = \psi^{*} \tag{1}$$

And with the same symbolic:

[16] http://en.wikipedia.org/wiki/Probability_amplitude treats complex probability amplitude distributions.
[17] The notion of "sign flavor" is used because for elementary particles "flavor" already has a different meaning.

$$\psi^{①} = \psi^{1} \quad (2)$$

$$\psi^{⓪} = \psi \quad (3)$$

Often the symbols ψ and ψ^* will be used instead of the symbols $\psi^{⓪}$ and $\psi^{③}$.

QPAD multiplication

What happens when quaternions from different sign flavors will be multiplied?

1. First a reference sign flavor is selected.
2. This sign flavor is taken to be the sign flavor of the distribution that will receive the result.
3. The factors are first brought to this reference sign selection.
4. In this process nothing changes in the values of the quaternions.
5. After that the multiplication takes place.
6. The result is delivered in the reference sign flavor.

With other words the multiplication takes place with the handedness that is defined in the target distribution.

Spinors and matrices

In contemporary physics complex probability amplitude distributions (CPAD's) are used rather than QPAD's. Spinors and matrices are used to simulate QPAD behavior for CPAD's.

A spinor $[\psi]$ is a 1×4 matrix consisting of CPAD's that represent the sign flavors of a QPAD. Sometimes the spinor is represented as a 1×2 matrix.

The $\boldsymbol{\alpha}$ and β matrices influence the elements of spinor $[\psi]$.

$$\alpha_1 = \begin{bmatrix} 0 & \boldsymbol{i} \\ -\boldsymbol{i} & 0 \end{bmatrix} \quad (3)$$

$$\alpha_2 = \begin{bmatrix} 0 & \boldsymbol{j} \\ -\boldsymbol{j} & 0 \end{bmatrix} \tag{4}$$

$$\alpha_3 = \begin{bmatrix} 0 & \boldsymbol{k} \\ -\boldsymbol{k} & 0 \end{bmatrix} \tag{5}$$

$$\beta = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{6}$$

$\boldsymbol{i}, \boldsymbol{j}$ and $\boldsymbol{k}$ represent imaginary base vectors of the simulated quaternion. β represents the conjugation action for the spinor.

A relation exist between $\alpha_1, \alpha_2, \alpha_3$ and the Pauli[18] matrices $\sigma_1, \sigma_2, \sigma_3$:

$$\sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \qquad \sigma_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \qquad \sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \tag{7}$$

$$1 \mapsto I, \qquad \boldsymbol{i} \mapsto \sigma_1, \qquad \boldsymbol{j} \mapsto \sigma_2, \qquad \boldsymbol{k} \mapsto \sigma_3 \tag{8}$$

This combination is usually represented in the form of gamma matrices. In Dirac representation, the four contravariant gamma matrices are

$$\gamma^0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}, \qquad \gamma^1 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}, \tag{9}$$

$$\gamma^2 = \begin{bmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{bmatrix}, \qquad \gamma^3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

It is useful to define the product of the four gamma matrices as follows:

[18] http://en.wikipedia.org/wiki/Pauli_matrices

$$\gamma^5 = i\,\gamma^0\,\gamma^1\,\gamma^2\,\gamma^3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \tag{10}$$

The gamma matrices as specified here are appropriate for acting on Dirac spinors written in the Dirac basis; in fact, the Dirac basis is defined by these matrices. In the Dirac basis[19]:

$$\gamma^0 = \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix}, \qquad \gamma^k = \begin{bmatrix} 0 & \sigma^k \\ -\sigma^k & 0 \end{bmatrix}, \qquad \gamma^5 = \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix} \tag{11}$$

This corresponds with $\alpha_k = \gamma^k$, $\beta = \gamma^5$.
Apart from the Dirac basis, a Weyl basis exists

$$\gamma^0 = \gamma^\beta = \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix}, \qquad \gamma^k = \begin{bmatrix} 0 & \sigma^k \\ -\sigma^k & 0 \end{bmatrix}, \qquad \gamma^5 = \begin{bmatrix} -I & 0 \\ 0 & I \end{bmatrix} \tag{12}$$

The Weyl basis has the advantage that its chiral projections[20] take a simple form:

$$\psi_L = ½\,(1 - \gamma^5)[\psi] = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}[\psi] \tag{13}$$

$$\psi_R = ½\,(1 + \gamma^5)[\psi] = \begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix}[\psi] \tag{14}$$

$$[\psi^*] = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}[\psi] \tag{15}$$

Special QPAD's

We will consider a special ensemble of QPAD's $\{\,\psi_i(r, q_i)\}$.

- The $\psi_i(r, 0)$ are normalized: $\int_V |\psi_i(r,0)|^2\, dV = 1$.
- The $\psi_i(r, 0)$ must be spherically symmetric.

[19] http://en.wikipedia.org/wiki/Gamma_matrices#Dirac_basis
[20] http://en.wikipedia.org/wiki/Chirality_(physics)

- From a given minimal distance their modulus must decrease with radius r as $1/r$.

The form of the QPAD relies on Bertrand's theorem[21].

The average QPAD

The ensemble $\{ \psi_i(r,0)\}$ of the QPAD's has an average $\Psi(r,0)$

The background QPAD

The ensemble $\{ \psi_i(r,q_i)\}$ is distributed randomly over the center points $\{q_i\}$ in an affine parameter space. At a given point P in this space the superposition of all $\{ \psi_i(r,P)\}$ will be constructed.
This superposition will be renormalized and then indicated by $\Phi(r,P)$.
Thus,

$$\int_V |\Phi(r,0)|^2 \, dV = 1 \tag{1}$$

In this superposition the largest contribution comes from the $\psi_i(r,q_i)$ for which the q_i is farthest from P. Further the directions of the imaginary part are reversed with respect to the directions in the $\psi_i(r,q_i)$.
Especially at long distances, all differences are smoothed away via an averaging process.

The result is that:

$$\Phi(r,0) = \Psi^*(r,0) \tag{2}$$

We will interpret $\Phi(r,0)$ as the background QPAD.
The approach taken here, shows similarity with the approach of Denis Sciama in his paper: "On the origin of inertia"[22].

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[21] http://en.wikipedia.org/wiki/Bertrand%27s_theorem
[22] See: http://www.adsabs.harvard.edu/abs/1953MNRAS.113...34S

Differentiation and Fourier transform

A quaternionic distribution f(q) can be differentiated[23].

$$g(q) = \nabla_0 f_0(q) \mp \langle \boldsymbol{\nabla}, \boldsymbol{f}(q) \rangle \pm \nabla_0 \boldsymbol{f}(q) + \boldsymbol{\nabla} f_0(q) \pm \left(\pm \boldsymbol{\nabla} \times \boldsymbol{f}(q) \right)$$

The colored $\mp$ and $\pm$ signs refer to the influence of conjugation of $f(q)$ on quaternionic multiplication. The $\pm$sign refers to the influence of reflection of $f(q)$.

In this section, the parameter q is supposed to be taken from a non-curved parameter space. With that precondition, in Fourier space differentiation becomes multiplication with the canonical conjugate coordinate k and therefore the equivalent equation becomes:

$$\begin{aligned} \tilde{\mathrm{g}}(k) = \mathrm{k}\tilde{f}(k) \\ = \mathrm{k}_0 \tilde{f}_0(k) \mp \langle \mathbf{k}, \tilde{\boldsymbol{f}}(k) \rangle \pm \mathrm{k}_0 \tilde{\boldsymbol{f}}(k) + \mathbf{k} \tilde{f}_0(k) \\ \pm \left(\pm \mathbf{k} \times \tilde{\boldsymbol{f}}(k) \right) \end{aligned} \tag{2}$$

For the imaginary parts holds:

$$\mathbf{g}(q) = \pm \nabla_0 \boldsymbol{f}(q) + \boldsymbol{\nabla} f_0(q) \pm \left(\pm \boldsymbol{\nabla} \times \boldsymbol{f}(q) \right) \tag{3}$$

$$\tilde{\mathbf{g}}(k) = \pm \mathrm{k}_0 \tilde{\boldsymbol{f}}(k) + \mathbf{k} \tilde{f}_0(k) \pm \left(\pm \mathbf{k} \times \tilde{\boldsymbol{f}}(k) \right) \tag{4}$$

Continuity equation

When applied to a quaternionic probability amplitude distribution (QPAD), the equation for the differentiation leads to a continuity equation.

When $\rho_0(q)$ is interpreted as a charge density distribution, then the conservation of the corresponding charge is given by the continuity equation:

[23] For more details, see Appendix; Quaternionic distributions,

Total change within V = flow into V + production inside V (1)

$$\frac{d}{dt}\int_V \rho_0 \, dV = \oint_S \hat{\boldsymbol{n}}\rho_0 \frac{\boldsymbol{v}}{c} \, dS + \int_V s_0 \, dV \quad (2)$$

$$\int_V \nabla_0 \rho_0 \, dV = \int_V \langle \boldsymbol{\nabla}, \boldsymbol{\rho} \rangle \, dV + \int_V s_0 \, dV \quad (3)$$

Here $\hat{\boldsymbol{n}}$ is the normal vector pointing outward the surrounding surface S, $\boldsymbol{v}(t, \boldsymbol{q})$ is the velocity at which the charge density $\rho_0(t, \boldsymbol{q})$ enters volume V and s_0 is the source density inside V. In the above formula $\boldsymbol{\rho}$ stands for

$$\boldsymbol{\rho} = \rho_0 \boldsymbol{v}/c \quad (4)$$

It is the flux (flow per unit area and unit time) of ρ_0 .

The combination of $\rho_0(t, \boldsymbol{q})$ and $\boldsymbol{\rho}(t, \boldsymbol{q})$ is a quaternionic skew field $\rho(t, \boldsymbol{q})$ and can be seen as a probability amplitude distribution (QPAD).

$$\rho \stackrel{\text{def}}{=} \rho_0 + \boldsymbol{\rho} \quad (5)$$

$\rho(t, \boldsymbol{q})\rho^*(t, \boldsymbol{q})$ can be seen as an overall probability density distribution of the presence of the carrier of the charge. $\rho_0(t, \boldsymbol{q})$ is a charge density distribution. $\boldsymbol{\rho}(t, \boldsymbol{q})$ is the current density distribution.
The conversion from formula (2) to formula (3) uses the Gauss theorem[24].
This results in the law of charge conservation:

$$s_0(t, \boldsymbol{q}) = \nabla_0 \rho_0(t, \boldsymbol{q}) \mp \langle \boldsymbol{\nabla}, \left(\rho_0(t, \boldsymbol{q})\boldsymbol{v}(t, \boldsymbol{q}) + \boldsymbol{\nabla} \times \boldsymbol{a}(t, \boldsymbol{q})\right) \rangle \quad (6)$$

$$= \nabla_0 \rho_0(t, \boldsymbol{q}) \mp \langle \boldsymbol{\nabla}, \boldsymbol{\rho}(t, \boldsymbol{q}) + \boldsymbol{A}(t, \boldsymbol{q}) \rangle$$

$$= \nabla_0 \rho_0(t, \boldsymbol{q}) \mp \langle \boldsymbol{v}(t, \boldsymbol{q}), \boldsymbol{\nabla} \rho_0(t, \boldsymbol{q}) \rangle \mp \langle \boldsymbol{\nabla}, \boldsymbol{v}(t, \boldsymbol{q}) \rangle \, \rho_0(t, \boldsymbol{q})$$

[24] http://en.wikipedia.org/wiki/Divergence_theorem

$$\mp\langle\boldsymbol{\nabla},\boldsymbol{A}(t,\boldsymbol{q})\rangle$$

The blue colored $\pm$ indicates quaternionic sign selection through conjugation of the field $\rho(t,\boldsymbol{q})$. The field $\boldsymbol{a}(t,\boldsymbol{q})$ is an arbitrary differentiable vector function.

$$\langle\boldsymbol{\nabla},\boldsymbol{\nabla}\times\boldsymbol{a}(t,\boldsymbol{q})\rangle = 0 \tag{7}$$

$\boldsymbol{A}(t,\boldsymbol{q}) \stackrel{\text{def}}{=} \boldsymbol{\nabla}\times\boldsymbol{a}(t,\boldsymbol{q})$ is always divergence free. In the following we will neglect $\boldsymbol{A}(t,\boldsymbol{q})$.

Equation (6) represents a balance equation for charge density. What this charge actually is, will be left in the middle. It can be one of the properties of the carrier or it can represent the full ensemble of the properties of the carrier.

This only treats the real part of the full equation. The full equation runs:

$$s(t,\boldsymbol{q}) = \nabla\rho(t,\boldsymbol{q}) = s_0(t,\boldsymbol{q}) + \boldsymbol{s}(t,\boldsymbol{q}) \tag{8}$$

$$= \nabla_0\rho_0(t,\boldsymbol{q}) \mp \langle\boldsymbol{\nabla},\boldsymbol{\rho}(t,\boldsymbol{q})\rangle \pm \nabla_0\boldsymbol{\rho}(t,\boldsymbol{q}) + \boldsymbol{\nabla}\rho_0(t,\boldsymbol{q}) \pm \left(\pm\boldsymbol{\nabla}\times\boldsymbol{\rho}(t,\boldsymbol{q})\right)$$

$$= \nabla_0\rho_0(t,\boldsymbol{q}) \mp \langle\boldsymbol{v}(t,\boldsymbol{q}),\boldsymbol{\nabla}\rho_0(t,\boldsymbol{q})\rangle \mp \langle\boldsymbol{\nabla},\boldsymbol{v}(t,\boldsymbol{q})\rangle\,\rho_0(t,\boldsymbol{q})$$

$$\pm\nabla_0\boldsymbol{v}(t,\boldsymbol{q}) + \nabla_0\rho_0(t,\boldsymbol{q}) + \boldsymbol{\nabla}\rho_0(t,\boldsymbol{q})$$

$$\pm\left(\pm(\rho_0(t,\boldsymbol{q})\,\boldsymbol{\nabla}\times\boldsymbol{v}(t,\boldsymbol{q}) - \boldsymbol{v}(t,\boldsymbol{q})\times\boldsymbol{\nabla}\rho_0(t,\boldsymbol{q}))\right)$$

$$s_0(t,\boldsymbol{q}) = 2\nabla_0\rho_0(t,\boldsymbol{q}) \mp \langle\boldsymbol{v}(q),\boldsymbol{\nabla}\rho_0(t,\boldsymbol{q})\rangle \mp \langle\boldsymbol{\nabla},\boldsymbol{v}(t,\boldsymbol{q})\rangle\,\rho_0(t,\boldsymbol{q}) \tag{9}$$

$$\boldsymbol{s}(t,\boldsymbol{q}) = \pm\nabla_0\boldsymbol{v}(t,\boldsymbol{q}) \pm \boldsymbol{\nabla}\rho_0(t,\boldsymbol{q}) \tag{10}$$

$$\pm\left(\pm(\rho_0(t,\boldsymbol{q})\,\boldsymbol{\nabla}\times\boldsymbol{v}(t,\boldsymbol{q}) - \boldsymbol{v}(t,\boldsymbol{q})\times\boldsymbol{\nabla}\rho_0(t,\boldsymbol{q}))\right)$$

The red sign selection indicates a change of handedness by changing the sign of one of the imaginary base vectors. Conjugation also causes a switch of handedness. It changes the sign of all three imaginary base vectors.

Elementary particles

Elementary particles appear to obey a special kind of continuity equation. In this continuity equation the source/drain term is represented by the coupled QPAD.

Dirac equation

The best known equation of motion for elementary particles is the Dirac equation. It is written using spinors and matrices.
The Dirac equation for a free moving electron or positron is known as:

$$\nabla_0[\psi] + \boldsymbol{\nabla\alpha}[\psi] = m\beta[\psi] \tag{1}$$

This can be converted to two quaternionic equations that act on QPAD's:

$$\nabla_0\psi_R + \boldsymbol{\nabla}\psi_R = m\psi_L \tag{2}$$

$$\nabla_0\psi_L - \boldsymbol{\nabla}\psi_L = m\psi_R \tag{3}$$

In the mass term the coupling factor m couples ψ_L and ψ_R. The fact $m = 0$ decouples ψ_L and ψ_R.

$$\psi_R = \psi_L^* = \psi_0 + \boldsymbol{\psi} \tag{4}$$

In the left term $\psi_R$and ψ_L represent the wave function of the particle. In that sense term $\psi_R$and ψ_L represent each other's antiparticle.

Equations (2) and (3) are each other's quaternionic conjugate.

Reformulating these equations gives

$$\nabla\psi^{\textcircled{0}} = m\,\psi^{\textcircled{3}} \tag{5}$$

$$\nabla_0(\psi_0 + \boldsymbol{\psi}) + \boldsymbol{\nabla}(\psi_0 + \boldsymbol{\psi}) = m(\psi_0 - \boldsymbol{\psi}) \tag{6}$$

For the conjugated field holds

$$\nabla^*\psi^{\textcircled{3}} = m\,\psi^{\textcircled{0}} \tag{7}$$

$$\nabla_0(\psi_0 - \boldsymbol{\psi}) - \boldsymbol{\nabla}(\psi_0 - \boldsymbol{\psi}) = m(\psi_0 + \boldsymbol{\psi}) \tag{8}$$

This implements the reverse flip. The corresponding particle is the antiparticle.

$$\{\psi^{\textcircled{0}}, \psi^{\textcircled{3}}\} \leftrightarrow \{\psi^{\textcircled{3}}, \psi^{\textcircled{0}}\} \tag{9}$$

Both flips switch the handedness.
Summing the equations gives via

$$\boldsymbol{\nabla}\boldsymbol{\psi} = \boldsymbol{\nabla} \times \boldsymbol{\psi} - \langle \boldsymbol{\nabla}, \boldsymbol{\psi} \rangle \tag{10}$$

the result

$$\nabla_0\psi_0 - \langle \boldsymbol{\nabla}, \boldsymbol{\psi} \rangle = m\,\psi_0 \tag{11}$$

The difference gives

$$\nabla_0\boldsymbol{\psi} + \boldsymbol{\nabla}\psi_0 + \boldsymbol{\nabla} \times \boldsymbol{\psi} = -m\,\boldsymbol{\psi} \tag{12}$$

Just reversing the sign flavors does not work. The corresponding equation contains extra terms:

$$\nabla\psi^{\textcircled{3}} = \nabla_0(\psi_0 - \boldsymbol{\psi}) + \boldsymbol{\nabla}(\psi_0 - \boldsymbol{\psi}) = \nabla_0\psi_0 - \nabla_0\boldsymbol{\psi} + \boldsymbol{\nabla}\psi_0 - \boldsymbol{\nabla}\boldsymbol{\psi} \tag{13}$$

$$= (m\,\psi_0 + \langle \boldsymbol{\nabla}, \boldsymbol{\psi} \rangle) - (-m\,\boldsymbol{\psi} - \boldsymbol{\nabla}\psi_0) + \boldsymbol{\nabla}\psi_0 - (\boldsymbol{\nabla} \times \boldsymbol{\psi} - \langle \boldsymbol{\nabla}, \boldsymbol{\psi} \rangle)$$

$$= m\,\psi + 2\langle \boldsymbol{\nabla}, \boldsymbol{\psi} \rangle + 2\boldsymbol{\nabla}\psi_0$$

Thus if the reverse equation fits, then it will concern another field configuration ψ' that will not fit the original equation.
The pair $\{\psi'^{③}, \psi'^{⓪}\}$ that fits equation:

$$\nabla\psi'^{③} = m\,\psi'^{⓪} \tag{14}$$

represents a different particle than the electron $\{\psi^{⓪}, \psi^{③}\}$, which obeys equation (5). It also differs from the positron $\{\psi^{③}, \psi^{⓪}\}$, which obeys equation (7).

Where the electron couples to the background QPAD, the new particle couples to the conjugate of the background QPAD.

The coupling factor

Multiplying both sides of the equation of motion for the electron:

$$\nabla\psi^{⓪} = m\,\psi^{③} \tag{1}$$

with $\psi^{⓪}$ and then integrate over the full parameter space gives:

$$\int_V \psi^{⓪}\nabla\psi^{⓪}\,dV = m\int_V \psi^{⓪}\psi^{③}\,dV = m\int_V \left|\psi^{⓪}\right|^2 dV = m \tag{2}$$

Thus, the coupling factor m can be computed from the QPAD ψ.

The Majorana equation

The Majorana equation deviates from the Dirac equation in that is applies another sign flavor of QPAD ψ as the wave function. That other sign flavor is still coupled to the background field $\psi^{③}$.

$$\nabla\psi^{①} = m_n\, \psi^{③} \tag{1}$$

The conjugated equation defines the anti-particle.

$$\nabla^*\psi^{②} = m_n\, \psi^{⓪} \tag{2}$$

The particle is represented by the ordered pair $\{\psi^{①}, \psi^{③}\}$. The corresponding flip does not switch the handedness.
The Majorana equation is thought to hold for neutrinos, which are neutral. Equation (2) will then hold for the anti-neutrino.
The coupling coefficient m_n for the neutrino follows from:

$$\int_V \psi^{⓪}\nabla\psi^{①}\, dV = m_n \int_V \psi^{⓪}\psi^{③}\, dV = m_n \int_V \left|\psi^{⓪}\right|^2 dV = m_n \tag{3}$$

The next particle

We have exploited:

$$\nabla\psi^{⓪} = m\, \psi^{③} \tag{1}$$

and

$$\nabla\psi^{①} = m_n\, \psi^{③} \tag{2}$$

The next possibility would be:

$$\nabla\psi^{②} = m_d\, \psi^{③} \tag{3}$$

The conjugated equation is:

$$\nabla^*\psi^{①} = m_d\, \psi^{⓪} \tag{4}$$

The particle is represented by the ordered pair $\{\psi^{②}, \psi^{③}\}$. The corresponding flip does switch the handedness.
Like the electron it will have charge, but its charge will be three times lower, because only one instead of three imaginary base vectors cause the switch in handedness. Of course, this is an opportunistic interpretation, but it seems to fit when we assume that the particle is a down quark with charge equal to -⅓e.

The formula for the coupling factor m_d is:

$$\int_V \psi^{⓪} \nabla \psi^{②} \, dV = m_d \int_V \psi^{⓪} \psi^{③} \, dV = m_d \int_V \left| \psi^{⓪} \right|^2 dV = m_d \tag{5}$$

No coupling

The last possible form in which the wave function couples to the background field is:

$$\nabla \psi^{③} = m \, \psi^{③} \tag{1}$$

The formula for the coupling factor m is:

$$\int_V \psi^{⓪} \nabla \psi^{③} \, dV = m \int_V \psi^{⓪} \psi^{③} \, dV = m \int_V \left| \psi^{⓪} \right|^2 dV = m \tag{2}$$

$$\int_V \psi^{⓪} \nabla \psi^{③} \, dV = ½ \int_V \nabla |\psi|^2 \, dV = 0 \tag{3}$$

Presence does not leak. So,

$$m = 0. \tag{4}$$

The free QPAD

When for sign flavor ψ^x the coupling factor m is zero then:

$$\nabla\psi^x = 0 \tag{1}$$

$$\nabla_0\psi_0^x = \langle\boldsymbol{\nabla},\boldsymbol{\psi}^x\rangle \tag{2}$$

$$\boldsymbol{\nabla}\times\boldsymbol{\psi}^x + \boldsymbol{\nabla}\psi_0^x + \nabla_0\boldsymbol{\psi}^x = \mathbf{0} \tag{3}$$

It means that a change $\nabla_0\boldsymbol{\psi}^x$ in the speed of the current goes together with a rotation $\boldsymbol{B}$ of the current

$$\boldsymbol{B} = \boldsymbol{\nabla}\times\boldsymbol{\psi}^x \tag{4}$$

and/or a new field $\mathfrak{E}$:

$$\mathfrak{E} = -\boldsymbol{\nabla}\psi_0^x \tag{5}$$

For comparison, in the equations of Maxwell[25] the field $\boldsymbol{E}$ is defined as:

$$\boldsymbol{E} = -\boldsymbol{\nabla}\psi_0^x - \nabla_0\boldsymbol{\psi}^x = \mathfrak{E} - \nabla_0\boldsymbol{\psi}^x \tag{6}$$

In those equations $\boldsymbol{E}$ is the electric field and $\boldsymbol{B}$ is the magnetic field. However here these fields have a more general meaning.

Thus equation (3) means:

$$\boldsymbol{B} = \boldsymbol{E} \tag{7}$$

More interesting is the corollary

$$\nabla_0\boldsymbol{B} = \nabla_0\boldsymbol{E} \tag{8}$$

$$\nabla_0\boldsymbol{B} = \boldsymbol{\nabla}\times\nabla_0\boldsymbol{\psi}^x = \boldsymbol{\nabla}\times\mathfrak{E} - \boldsymbol{\nabla}\times\boldsymbol{\nabla}\times\boldsymbol{\psi}^x = -\boldsymbol{\nabla}\times\boldsymbol{\nabla}\times\boldsymbol{\psi}^x \tag{9}$$

[25] http://en.wikipedia.org/wiki/Maxwell%27s_equations#Potential_formulation

$$= \boldsymbol{\nabla}^2\boldsymbol{\psi}^x - \boldsymbol{\nabla}\langle\boldsymbol{\nabla},\boldsymbol{\psi}^x\rangle = \boldsymbol{\nabla}^2\boldsymbol{\psi}^x - \boldsymbol{\nabla}\nabla_0\psi_0^x$$

$$\nabla_0\boldsymbol{E} = -\boldsymbol{\nabla}\nabla_0\psi_0^x - \nabla_0^2\boldsymbol{\psi}^x \tag{10}$$

Thus

$$\boldsymbol{\nabla}^2\boldsymbol{\psi}^x = -\nabla_0^2\boldsymbol{\psi}^x \tag{11}$$

Or

$$\nabla^2\boldsymbol{\psi}^x = \boldsymbol{0} \tag{12}$$

Further

$$\nabla_0^2\psi_0^x = \langle\boldsymbol{\nabla},\nabla_0\boldsymbol{\psi}^x\rangle = -\langle\boldsymbol{\nabla},\boldsymbol{\nabla}\times\boldsymbol{\psi}^x\rangle - \langle\boldsymbol{\nabla},\boldsymbol{\nabla}\psi_0^x\rangle = -\boldsymbol{\nabla}^2\psi_0^x \tag{13}$$

Thus:

$$\nabla^2\psi^x = 0 \tag{14}$$

With other words a free (= not coupled) ψ^x is harmonic. This holds for all QPAD's.

Reflection

We have now exhausted all possibilities for coupling a QPAD sign flavor to the background field. We could link the analyzed particles to electrons, neutrinos, down quarks, photons and gluons. Their antiparticles fit as well. The free QPAD's are bosons and corresponds to a photon or a gluon. The others are fermions.

The ordered pair $\{\psi^x, \psi^y\}$ represents a category of elementary particle types.
For antiparticles all participating fields and the nabla operator conjugate.
Photons and gluons have a zero coupling factor. Apart from these bosons, the above treated particles appear to be fermions.
We can now try to establish the apparent rules of the game. The rules are:

- If the coupling takes place between two QPAD sign flavors with different handedness, then the corresponding particle is charged.
- The charge depends on the number and on the direction of the base vectors that differ.
- The count for each difference is ±⅓e.

The scheme does not discriminate generations of elementary particles.

No elementary particle exists that obeys the rules and features electric charge ⅔e. Such a particle may exist as a composite. Thus, according to these rules the up-quarks are not elementary particles. For that reason, they do not belong to the standard model.

The elementary particles that are not yet covered are W_+ and W_- bosons and Z bosons. We like to proceed in a similar way, but the coupling with the background QPAD is used up. Now let us try other couplings.

We already encountered one, the ordered pair of sign flavors $\{\psi^{③}, \psi^{⓪}\}$ that obeys

$$\nabla\psi^{③} = m\,\psi^{⓪} \tag{1}$$

The coupling changes the handedness, so the particle is charged. It has much in common with the positron. Still it is not the anti-particle of the electron. It might exist, but then it probably hides behind the positron.

Anisotropic coupling fields

We have explored all particles that make use of the isotropic background. These particles appear to be fermions. This means that they have half integer valued spin. What spin actually is, is not explained in this paper. Next we like to explore particles that couple to anisotropic fields. These particles will appear to be bosons. It means that they all have integer valued spin.

The cross-sign flavor equations

These equations describe the situation that a flip is made from a $\psi_i^{①}$ field to a $\psi_i^{②}$ field or vice versa. The direction i might or might not play a role.

$$\nabla\psi_i^{②} = m_{W_+}\,\psi_i^{①} \tag{1}$$

The conjugated equation is:

$$\nabla^*\psi_i^{①} = m_{W_-}\,\psi_i^{②} \tag{2}$$

Another form is

$$\nabla\psi_i^{①} = m_{W_-}\,\psi_i^{②} \tag{3}$$

The conjugated equation is:

$$\nabla^*\psi_i^{②} = m_{W_+}\,\psi_i^{①} \tag{4}$$

The sign flavor switch affects three imaginary base vectors and flips the handedness. As a consequence the particles have a full electric charge. It concerns two particles, the W^- and the W^+ bosons. These bosons carry electrical charges.
The W_+ and W_- bosons are considered to be each other's antiparticle. It is also possible that they hide between each other's antiparticle.

$$\psi_i^{②}\,\nabla\psi_i^{②} = m_{W_+}\psi_i^{②}\,\psi_i^{①} \tag{5}$$

$$\int_V \left(\psi_i^{②}\nabla\psi_i^{②}\right)\,dV = m_{W_+}\int_V \left(\psi_i^{②}\,\psi_i^{①}\right)\,dV = m_{W_+}g \tag{6}$$

$$\psi_i^{①}\,\nabla\psi_i^{①} = m_{W\_}\psi_i^{①}\,\psi_i^{②} \tag{7}$$

$$\int_V \left(\psi_i^{①}\,\nabla\psi_i^{①}\right) dV = m_{W\_}\int_V \left(\psi_i^{②}\,\psi_i^{①}\right) dV = m_{W\_}g \tag{8}$$

The Z boson

The particle that obeys:

$$\nabla\psi^{⓪} = m_Z\,\psi^{②}$$

Is a neutral boson.

$$\int_V \left(\psi_i^{①}\,\nabla\psi^{⓪}\right) dV = m_Z\int_V \left(\psi_i^{①}\,\psi_i^{②}\right) dV = m_Z\,g$$

Another possibility is:

$$\nabla\psi^{③} = m_{Z?}\,\psi^{①}$$

Again these particles may hide between each other's anti-particle.

General form

The general form of the equation for particle $\{\psi^x, \psi^y\}$ is:

$$\nabla\psi^x = m\,\psi^y \tag{1}$$

For the antiparticle:

$$\nabla^*\psi^{x*} = m\,\psi^{y*} \tag{2}$$

For all particles holds:

$$\nabla_0 \psi_0 - \langle \boldsymbol{\nabla}, \boldsymbol{\psi} \rangle = m\, \psi_0 \tag{3}$$

$$\boldsymbol{\nabla} \times \boldsymbol{\psi}^x + \boldsymbol{\nabla} \psi_0^x + \nabla_0 \boldsymbol{\psi}^x = m\, \boldsymbol{\psi}^y \tag{4}$$

$$\int_V \psi \psi^{③}\, dV = 1 \tag{5}$$

$$\int_V \psi^{①} \psi^{②}\, dV = g \tag{6}$$

The factor g is real and non-negative.
Further, the equation for coupling factor m

$$\int_V (\psi^{y*} \nabla \psi^{x})\, dV = m \int_V (\psi^{y*} \psi^{y})\, dV = m \int_V |\psi^{y}|^2\, dV \tag{7}$$

An equivalent of the Lagrangian may look like

$$\mathcal{L} \stackrel{?}{=} \psi^{y*}\, \nabla \psi^{x} - m\, \psi^{y*}\, \psi^{y} \tag{8}$$

Higher order couplings

Couplings that constitute composite particles from elementary particles or other composite particles are not treated here. It is assumed that during these couplings the constituting elementary particles keep their basic properties;

- coupling factor,
- electric charge
- and angular momentum.

The properties that characterize the coupling event are sources of secondary fields. These fields are known as physical fields.

It is thought that these secondary fields play a major role in the higher order couplings. The reason for this fact is that the mentioned coupling properties influence the curvature of the parameter space.
This fact would mean that higher order coupling is not well described by a corresponding wave equation. Instead it is better described by an equation that describes the dependence of the local curvature on the locally existing coupling properties. An equation that does something like that is the Kerr-Newman metric equation.

Curvature

Hilbert Book Model ingredients

Each page of the Hilbert Book Model consists of three quite independent ingredients.

> Ingredient 1: The quantum logic, or equivalently, its lattice isomorphic companion; the set of closed subspaces of an infinite dimensional separable Hilbert space
>
> Ingredient 2: A background coordinate system that is taken from the continuum eigenspace of an operator that resides in the Gelfand triple of the separable Hilbert space.
>
> Ingredient 3: A set of QPAD's that each couple an eigenvector of a particle position operator that resides in the Hilbert space to the background coordinate system.

Couplings between QPAD's that lead to elementary particles are characterized by three categories of properties:

- Coupling factor
- Electric charge
- Angular momentum

These properties influence the curvature that affects the third ingredient. The way that these properties influence curvature is described by metric equations, such as the Kerr-Newman metric formula.
The three ingredients have their own properties and habits. For example the QPAD's may feature a maximum speed of information transfer, while the curvature of the background coordinate system acts instantaneously on changes of the controlling properties.

Black hole features

A black hole can be considered as a geometric abnormality. Since light is the carrier of it, information can pass nor leave its skin. No distant observer can ever see that a black hole absorbs something. Still intelligent observers know that an observed BH has grown to its current size. The observer derives that information from features in the surround of the BH. However, these features must already have received enough information about the properties of the BH. Otherwise, the intelligent observer could not have derived his knowledge from those features. Thus the features got the message about the properties of the BH by a messenger that goes far faster than light. All this can be explained by the fact that the spread of the influence of the three properties; coupling factor, electric charge and angular momentum have on curvature acts instantaneously. This influence runs over the whole extent of the universe.
In comparison the transport of information runs via QPAD's and is limited by the maximum speed in that medium. These facts can be explained by the difference between the habits of the corresponding media.
For a part, the features that are described here for black holes also hold for other geometric abnormalities that have a much smaller scale.

Curvature

The coordinate system that is taken from the eigenspace of an operator that resides in the Gelfand triple is not applied directly. Instead a quaternionic distribution that uses the values of the flat coordinate system that is taken from the Gelfand triple as its parameters is used as the observed coordinate system.

Curvature can be described by the combination of a preselected coordinate system that defines location in a non-curved space and a local metric that describes the curvature in terms of that coordinate system. As is described above, the flat coordinate system is taken from the Gelfand triple.

Coordinate system

Several coordinate systems are possible. The most common coordinate systems for a non-curved three dimensional space are:

- Cartesian coordinates
- Spherical coordinates

Alternatives for spherical coordinates are:

- Schwarzschild coordinates[26]
- Kruskal-Szekeres coordinates[27]
- Lemaitre coordinates[28]
- Eddington–Finkelstein coordinates[29]

The advantage of the alternative coordinates is that they avoid unnecessary singularities. However, these alternatives are only relevant for situations in which the Schwarzschild radius plays a significant role. This is certainly the case for black holes and their environment, but it becomes irrelevant in the realm of elementary particles.

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[26] http://en.wikipedia.org/wiki/Schwarzschild_coordinates
[27] http://en.wikipedia.org/wiki/Kruskal-Szekeres_coordinates
[28] http://en.wikipedia.org/wiki/Lemaitre_coordinates
[29] http://en.wikipedia.org/wiki/Eddington%E2%80%93Finkelstein_coordinates

Metric

The best suitable local metric equation for our purposes is the Kerr-Newman metric[30]. It uses three local properties. These properties are:

- The coupling factor m
- The electric charge Q
- The angular momentum J

The angular momentum J includes the spin s.

This metric uses the sum of a category of properties that are collected within the observed sphere. However, the summation produces different centers of activity for different property categories. Thus, these centers need not be at the same location. However, for large enough selected radius r and applied to black holes or single particles, these centers coincide.

The simplest interpretation of the Kerr-Newman metric can be taken on the surface of a sphere that has a selected radius r.

The formula uses three characteristic radii. The largest characteristic radius plays the most prominent role.

This fact introduces the notion of geo-cavity.

Scales

The charge-to-mass ratio Q/M is typically larger in smaller systems[31]. For most astrophysical systems,

$$Q/M \ll 1,$$

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[30] Appendix;Metric tensor field;Local metric equation

[31] For deeper investigation, see: http://arxiv.org/abs/0802.2914

while for a Millikan oil drop,

$$Q/M \approx 10^6.$$

Going all the way down to elementary particles, the value for the electron is

$$Q/M \approx 10^{21}.$$

To achieve balance we require that Newton's gravitational force f_N has the same magnitude as Coulomb's force f_C, that is

$$|f_N| = |f_C|$$

To be more specific, let us assume that $Q = e$ where e is the elementary charge. We then adjust the mass M to the value for which the forces are balanced. This gives the Stoney mass

$$M = m_S = e \approx 2\,\mu g$$

It is only one order of magnitude lower than the Planck mass

$$m_P = \sqrt{\hbar} \approx 20\,\mu g$$

The ratio between them is given by the square root of the fine structure constant,

$$\frac{m_S}{m_P} = \sqrt{\alpha} = \sqrt{e^2/\hbar} \approx 0.1$$

Thus, in case of electric charges, the Coulomb forces are nearly in balance with the gravitational forces at the Planck scale. However, at subatomic scale this picture is disturbed by the spin.

For subatomic systems there is an additional phenomenon which comes into play. In fact, according to general relativity, the gravitational field tends to become dominated by the spin at distances of the order of the Compton wavelength. The relevant quantity which governs this behavior is the ratio S/M^2 where S is the (spin) angular momentum. For an electron,

$$S/M^2 \approx 10^{44}.$$

As a consequence, the gravitational field becomes dominated by gravitomagnetic effects in the subatomic domain. This fact has important consequences for the electromagnetic fields of spinning charged particles.

The four known gravitational and electromagnetic multi-pole moments of the electron are:

- the mass m_e,
- the spin $S_e = \hbar/2$,
- the charge e
- the magnetic moment $\mu = \frac{eS_e}{m_e}$

The spin is a gravitomagnetic dipole moment, i.e. a gravitational analogue of the magnetic dipole moment.

$$\frac{S_e}{m_e} \gg e \gg m_e$$

The corresponding Kerr-Newman field is therefore dominated by the spin in the subatomic domain. In particular, it has no event horizon and it has no ergo-region. (The ergo-region is a region of space-time located outside the event horizon of a rotating black hole where no object even if traveling at the speed of light, can remain stationary.)

An important conclusion is that gravity tends to become spin dominated in the subatomic domain.

Appendix

Logic

History of quantum logic

Around 1930 John von Neumann and Garrett Birkhoff were searching for an acceptable explanation of the results of experiments that showed that the execution of an observation of a very small object can completely destroy the validity of an earlier observation of another observable of that object. The Schrödinger equation that agreed with the dynamic behaviour of the particles already existed. Not much later Heisenberg's matrix formulation became popular as well. Quite soon the conclusion was made that something was fundamentally wrong with the logic behind the behaviour of small particles. These small objects show particle behaviour as well as wave behaviour and they show quantization effects. It was found that the distribution axiom of classical logic had to be changed. Soon it became apparent that the lattice structure of classical logic must be weakened from an ortho-complementary modular form to an ortho-complementary weakly modular lattice. The quantum logic was born. The next step was to find a useful mathematical presentation of this new logic. A historic review of what happened can be found in: "Quantum Theory: von Neumann" vs. Dirac; http://www.illc.uva.nl/~seop/entries/qt-nvd/. It includes extensions of the concept of Hilbert space and application of these concepts to quantum field theory. Another source is: http://www.quantonics.com/Foulis_On_Quantum_Logic.html.

Quantum logic

Elementary particles behave non-classical. They can present themselves either as a particle or as a wave. A measurement of the particle properties of the object destroys the information that was obtained from an earlier measurement of the wave properties of that object.
With elementary particles it becomes clear that that nature obeys a different logic than our old trusted classical logic. The difference resides in the modularity axiom. That axiom is weakened. The classical logic is congruent to an orthocomplemented modular lattice. The quantum logic

is congruent to an orthocomplemented weakly modulare lattice. Another name for that lattice is orthomodular lattice.

Lattices

A subset of the axioms of the logic characterizes it as a half ordered set. A larger subset defines it as a lattice.
A lattice is a set of elements a, b, c, ...that is closed for the connections ∩ and ∪. These connections obey:

- The set is partially ordered. With each pair of elements a, b belongs an element c, such that $a \subset c$ and $b \subset c$.
- The set is a ∩ half lattice if with each pair of elements a, b an element c exists, such that $c = a \cap b$.
- The set is a ∪ half lattice if with each pair of elements a, b an element c exists, such that $c = a \cup b$.
- The set is a lattice if it is both a ∩ half lattice and a ∪ half lattice.

The following relations hold in a lattice:

$$a \cap b = b \cap a \tag{1}$$

$$(a \cap b) \cap c = a \cap (b \cap c) \tag{2}$$

$$a \cap (a \cup b) = a \tag{3}$$

$$a \cup b = b \cup a \tag{4}$$

$$(a \cup b) \cup c = a \cup (b \cup c) \tag{5}$$

$$a \cup (a \cap b) = a \tag{6}$$

The lattice has a partial order inclusion ⊂:

$$a \subset b \Leftrightarrow a \subset b = a \quad (7)$$

A complementary lattice contains two elements n and e with each element a an complementary element a′ such that:

$$a \cap a' = n \quad (8)$$

$$a \cap n = n \quad (9)$$

$$a \cap e = a \quad (10)$$

$$a \cup a' = e \quad (11)$$

$$a \cup e = e \quad (12)$$

$$a \cup n = a \quad (13)$$

An orthocomplemented lattice contains two elements n and e and with each element a an element a'' such that:

$$a \cup a'' = e \quad (14)$$

$$a \cap a'' = n$$

$$(a'')'' = a \quad (15)$$

$$a \subset b \,\square\, b'' \subset a'' \quad (16)$$

e is the unity element; n is the null element of the lattice

A distributive lattice supports the distributive laws:

$$a \cap (b \cup c) = (a \cap b) \cup (a \cap c) \quad (17)$$

$$a \cup (b \cap c) = (a \cup b) \cap (a \cup c) \tag{18}$$

A modular lattice supports:

$$(a \cap b) \cup (a \cap c) = a \cap (b \cup (a \cap c)) \tag{19}$$

A weak modular lattice supports instead:

There exists an element d such that

$$a \subset c \Leftrightarrow (a \cup b) \cap c = a \cup (b \cap c) \cup (d \cap c) \tag{20}$$

where d obeys:

$$(a \cup b) \cap d = d \tag{21}$$

$$a \cap d = n \tag{22}$$

$$b \cap d = n \tag{23}$$

$$[(a \subset g)\, and\, (b \subset g) \Leftrightarrow d \subset g \tag{24}$$

In an atomic lattice holds

$$\exists_{p \in L} \forall_{x \in L} \{x \subset p \Rightarrow x = n\} \tag{25}$$

$$\forall_{a \in L} \forall_{x \in L} \{(a < x < a \cap p) \Rightarrow (x = a\, or\, x = a \cap p)\} \tag{26}$$

p is an atom

Both the set of propositions of quantum logic and the set of subspaces of a separable Hilbert space **H**, have the structure of an orthomodular lattice. In this respect these sets are congruent.

In Hilbert space, an atom is a pure state (a ray spanned by a single vector).

Classical logic has the structure of an orthocomplemented distributive modular and atomic lattice.
Quantum logic has the structure of an orthomodular lattice. That is an orthocomplented weakly modular and atomic lattice. The set of closed subspaces of a Hilbert space also has that structure.

Proposition

In Aristotelian logic a proposition is a particular kind of sentence, one which affirms or denies a predicate of a subject. Propositions have binary values. They are either true or they are false.

Propositions take forms like "*This is a particle or a wave*". In quantum logic "*This is a particle.*" is not a proposition.

In mathematical logic, propositions, also called "propositional formulas" or "statement forms", are statements that do not contain quantifiers. They are composed of well-formed formulas consisting entirely of atomic formulas, the five logical connectives[32], and symbols of grouping (parentheses etc.). Propositional logic is one of the few areas of mathematics that is totally solved, in the sense that it has been proven internally consistent, every theorem is true, and every true statement can be proved. Predicate logic is an extension of propositional logic, which adds variables and quantifiers.

In Hilbert space a vector is either inside or not inside a closed subspace. A proper quantum logical proposition is "*Vector |f> is inside state s*".

In Hilbert space, an atomic predicate corresponds with a subspace that is spanned be a single vector.

[32] http://en.wikipedia.org/wiki/Logical_connective

Predicates may accept attributes and quantifiers. The predicate logic is also called first order logic. A dynamic logic can handle the fact that predicates may influence each other when atomic predicates are exchanged.

Observation

In physics, particularly in quantum physics, a system **observable** is a property of the system state that can be determined by some sequence of physical operations. This paper distinguishes between measurements and observations.

- With an observation the state is considered as a linear combination of eigenvectors of the observable. An observation returns the statistical expectation value of the eigenvalue of the observable.
- A measurement transforms the observed state to one of the eigenvectors of the observable. What happens depends on the characteristics of the measuring equipment. The measurement can be seen as a combination of a transformation and an observation.

Depending on the characteristics of the measuring equipment a measurement and a clean observation can give the same result.

With this interpretation of the concept of observation it is possible to let states observe other states. A state might do a transformation before doing an observation but in general it fails the equipment to arrange that transformation. In nature observations are far more common than measurements.

Numbers

Sign selections

Four possibilities exist due to the sign selections of the quaternions. One sign selection is covered by the conjugation $a \to a^*$. This selection switches the sign of all three imaginary base vectors. The other is caused by switching the sign of a single binary base vector $a \rightarrow a^{\otimes}$. For this sign selection one of the three available base vectors is selected. When relevant, then these choices are indicated by colors (r, g or b). Both methods switch the handedness (chirality). When both sign selections combine then the superscript $a \rightarrow a^{\oplus}$ is used. This combination does not switch handedness. Also this selection is colored.

It is also possible to use the extended **quaternionic conjugation concept**:

$$a^{*} = a^{③} \tag{1}$$

$$a^{\otimes} = a^{①} \tag{2}$$

$$a^{\oplus} = a^{*\otimes} = a^{②} \tag{3}$$

The encircled number stands for the number of switched base vectors. For the single sign switch $a^{①}$, three independent direction selections are possible. We indicate these choices with r, g and b.
Similarly for the double sign switch $a^{②}$, three independent direction selections are possible. We indicate these choices also with r, g and b. This direction belongs to the non-switched direction.
Without closer description the value of $a^{①②}$ is $a^{③}$. It means that the colors are suspected to be the same.
The change from a to $a^{①}$or $a^{③}$ cause a switch of the handedness of a.

$$a^{**} = a^{③③} = \left(a^{③}\right)^{③} = a \tag{4}$$

$$a^{①①} = a^{②②} = a \tag{5}$$

$$a^{①②} = a^{②①} = a^{③}$$

The effects of the quaternionic conjugation are visible in the base numbers 1, **i**, **j**, **k**:

$$1^* = 1 \quad (6)$$

$$\mathbf{i}^{③} = -\mathbf{i}; \mathbf{j}^{③} = -\mathbf{j}; \mathbf{k}^{③} = -\mathbf{k}; \quad (7)$$

The blue colored sign selection is given by

$$\mathbf{i}^{①} = +\mathbf{i}; \mathbf{j}^{①} = +\mathbf{j}; \mathbf{k}^{①} = -\mathbf{k}; \quad (8)$$

$$\mathbf{i}^{②} = -\mathbf{i}; \mathbf{j}^{②} = -\mathbf{j}; \mathbf{k}^{②} = +\mathbf{k}; \quad (9)$$

In the blue colored sign selection, **k** follows the rules of complex conjugation. This renders its direction to a special direction.

The selected color direction is called the **longitudinal** direction. The the perpendicular directions are the transverse directions. Apart from that they are mutual perpendicular and perpendicular to the longitudinal direction, they have no preferred direction.

Sign selections and quaternionic distributions

Quaternionic distributions are supposed to obey a distribution wide sign selection. Thus, the distribution is characterized by one of the eight quaternionic sign flavors.

$$\psi^{⓪}, \psi^{①}, \psi^{①}, \psi^{①}, \psi^{②}, \psi^{②}, \psi^{②}, \text{or } \psi^{①}$$

Many of the elementary particles are characterized by an ordered pair of two field sign flavors. These fields are coupled with a coupling strength that is typical for the particle type. These particles obey a characteristic continuity equation[33].

Product rule

We use the quaternionic product rule.

[33] Hilbert field equations; Continuity equation for charges

$$ab = a_0 b_0 - \langle \boldsymbol{a}, \boldsymbol{b} \rangle + a_0 \boldsymbol{b} + \boldsymbol{a} b_0 + \boldsymbol{a} \times \boldsymbol{b} \tag{1}$$

$$\langle \boldsymbol{a}, \boldsymbol{b} \rangle = a_1 b_1 + a_2 b_2 + a_3 b_3 \tag{2}$$

$$\boldsymbol{a} \times \boldsymbol{b} = \boldsymbol{i}(a_2 b_3 - a_3 b_2) + \boldsymbol{j}(a_3 b_1 - a_1 b_3) + \boldsymbol{k}(a_1 b_2 - a_2 b_1) \tag{3}$$

Operators

The sign selections of operator $\nabla = (\nabla_0, \boldsymbol{\nabla})$ depend on the sign selections of position operator Q, which determines the sign selections for its eigenvalues $q = (q_0, \boldsymbol{q})$.

Normally we consider the sign selection for operators Q and ∇ fixed to operators $Q^{⓪}$ and $\nabla^{⓪}$. Sometimes we chose ∇^* instead of operator ∇.

Quaternionic sign selection are directly connected with the concepts of **parity** and **spin**.

For quaternionic functions symmetry reduces the differences that are produced by conjugation and anti-symmetry stresses the differences. The same holds for operators.

Matrices

Another possibility is to present sign selections by matrices[34].

$$\sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \sigma_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \quad \sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

The σ_1 matrix switches the complex fields that together form the quaternion field.

$$\begin{bmatrix} \varphi_b \\ \varphi_a \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \varphi_a \\ \varphi_b \end{bmatrix}$$

[34] http://www.vttoth.com/qt.htm

The σ_2 matrix switches the real parts and the imaginary parts of the complex fields that together form the quaternion field and it switches both fields.

$$i\begin{bmatrix}-\varphi_b\\ \varphi_a\end{bmatrix}=\begin{bmatrix}0 & -i\\ i & 0\end{bmatrix}\begin{bmatrix}\varphi_a\\ \varphi_b\end{bmatrix}$$

The σ_3 matrix switches the sign of the first complex field.

$$\begin{bmatrix}-\varphi_a\\ \varphi_b\end{bmatrix}=\begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}\varphi_a\\ \varphi_b\end{bmatrix}$$

$$\sigma_k^2=-i\sigma_1\sigma_1\sigma_1=I$$

The Pauli matrices are involutory.
The determinants[35] and traces[36] of the Pauli matrices are:

$$\det(\sigma_k)=-1$$

$$\mathrm{Tr}(\sigma_k)=0$$

$$\alpha_k=\begin{bmatrix}0 & \sigma_k\\ -\sigma_k & 0\end{bmatrix}$$

$$\alpha_1=\begin{bmatrix}0 & \boldsymbol{i}\\ -\boldsymbol{i} & 0\end{bmatrix} \quad (1)$$

$$\alpha_2=\begin{bmatrix}0 & \boldsymbol{j}\\ -\boldsymbol{j} & 0\end{bmatrix} \quad (2)$$

$$\alpha_3=\begin{bmatrix}0 & \boldsymbol{k}\\ -\boldsymbol{k} & 0\end{bmatrix} \quad (3)$$

$$\beta=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix} \quad (4)$$

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[35] http://en.wikipedia.org/wiki/Determinant
[36] http://en.wikipedia.org/wiki/Trace_of_a_matrix

The α_k matrices together select the imaginary base vectors. The β matrix exchanges the sign of all imaginary base vectors. Thus the β matrix implements the quaternionic conjugate. The conjugation also exchanges right handedness against left handedness.

Another way of exchanging right handedness against left handedness is the exchange of the sign of one of the imaginary base vectors.

$$\begin{bmatrix} \psi_L \\ \psi_R \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \psi_R \\ \psi_L \end{bmatrix} \tag{5}$$

$$\psi^* = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \psi \tag{6}$$

The gamma matrices[37] translate directly from complex fields to fully featured quaternionic fields. In this way four sign flavors of quaternionic fields are constructed from four complex fields. This is represented by four dimensional matrices and four dimensional spinors. The equivalent of the β matrix is the γ_β matrix.

$$\begin{bmatrix} \varphi_{La} \\ \varphi_{Lb} \\ \varphi_{Ra} \\ \varphi_{Rb} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \varphi_{Ra} \\ \varphi_{Rb} \\ \varphi_{La} \\ \varphi_{Lb} \end{bmatrix} \tag{7}$$

It is false to interpret the matrices as vectors. They form a shorthand for handling spinors.

The Pauli matrix σ_1 represents the sign selection $a \rightarrow a^{\otimes}$, while the β matrix represents the sign selection $a \rightarrow a^*$. The other Pauli matrices and the α matrices implement the resulting part of the quaternion behavior for spinors.

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[37] Appendix; Gamma matrices

Construction

The Cayley-Dickson construction formula enables the generation of a quaternion from two complex numbers:

$$p = a_0 + a_1\mathbf{k} + \mathbf{i}(b_0 + b_1\mathbf{k}) \tag{1}$$

$$q = c_0 + c_1\mathbf{k} + \mathbf{i}(d_0 + d_1\mathbf{k}) \tag{2}$$

$$(a, b)\,(c, d) = (ac - db^*;\ a^*d + cb) \tag{3}$$

$$r = pq \tag{4}$$

$$r_0 = a_0c_0 - a_1c_1 - b_0d_0 - b_1d_1 \tag{5}$$

$$r_k = a_0c_1 - a_1c_0 - b_0d_1 + b_1d_0 \tag{6}$$

$$r_i = a_0d_0 + a_1d_1 + b_0c_0 - b_1c_1 \tag{7}$$

$$r_j = -a_1d_0 + a_0d_1 + b_0c_1 + b_1c_0 \tag{8}$$

Apart from the Cayley-Dickson construction the 2^n-on construction exists.[38]

Colored signs

In the following text, the consequences for the product of the sign choices of the conjugate ③ is indicated by blue color $\pm$. The extra consequence ① for the product of the choice of the handedness of the cross product is indicated by red color $\pm$. The mixed sign selection ② acts on both sign colors.

The handedness can be switched by changing the sign of all imaginary base vectors.

$$\boldsymbol{ij} = \boldsymbol{k} \ \rightarrow\ (-\boldsymbol{i})(-\boldsymbol{j}) = \boldsymbol{ij} = -\boldsymbol{k} \tag{1}$$

[38] Appendix; 2^n-on construction.

The sign selections split the ring of quaternions in four different realizations.

Warren Smith's numbers

2^n-on construction

The 2^n-ons use the following doubling formula

$$(a,b)(c,d) = (a\,c - (b\,d^*)^*, (b^*c^*)^* + (b^*(a^*((b^{-1})^*d^*)^*)^*)^*) \quad (1)$$

Up until the 16-ons the formula can be simplified to

$$(a,b)(c,d) = (a\,c - b\,d^*, c\,b + (a^*\,b^{-1})\,(b\,d)) \quad (2)$$

Up to the octonions the Cayley Dickson construction delivers the same as the 2^n-on construction. From n>3 the 2^n-ons are 'nicer'.

2^n-ons

Table of properties of the 2^n-ons. See www.math.temple.edu/~wds/homepage/nce2.ps.

Type	***name***	**Lose**
1-ons	*Reals.*	
2-ons	*Complex numbers*	$z^* = z$ (the * denotes conjugating); the ordering properties that both {z > 0, -z > 0, or z = 0} and {w > 0, z > 0 implies w + z > 0, wz > 0}.
4-ons	*Quaternions*	commutativity ab = ba; the algebraic closedness property that every univariate polynomial equation has a root.
8-ons	*Octonions*	associativity ab · c = a · bc.
16-ons	(*not Sedenions!*)	right-alternativity x · yy = xy · y; right-cancellation $x = xy \cdot y^{-1}$; flexibility x · yx = xy · x; left-linearity (b + c)a = ba + ca; anti-automorphism ab = ba, $(ab)^{-1} = b^{-1}\,a^{-1}$; left-linearity (b + c)a = ba + ca; continuity of the map x → xy;

		Moufang and Bol identities; diassociativity
32-ons		generalized-smoothness of the map x → xy; right-division properties that xa = b has (generically) a solution x, and the uniqueness of such an x; the "fundamental theorem of algebra" that every polynomial having a unique "asymptotically dominant monomial" must have a root; Trotter's formula: $\lim_{n\to\infty}\left[e^{x/n}e^{y/n}\right]^n = \lim_{n\to\infty}\left(1+\frac{x+y}{n}\right)^n = e^{x+y}$

Type	*name*	Retain
2^n-ons		Unique 2-sided multiplicative & additive identity elements 1 & 0; Norm-multiplicativity $\|xy\|^2 = \|x\|^2 \cdot \|y\|^2$; Norm-subadditivity $\|a + b\| \leq \|a\| + \|b\|$; 2-sided inverse $a^{-1} = a^*/\|a\|^2$ (a # 0); $a^{**} = a$; $(x \pm y)^* = x^* \pm y^*$; $(a^{-1})^{-1} = a$; $(a^*)^{-1} = (a^{-1})^*$; $\|a\|^2 = \|a\|^2 = a^* a$; Left-alternativity yy · x = y · yx; Left-cancellation $x = y^{-1} \cdot yx$; Right-linearity a(b + c) = ab + ac; r^{th} power-associativity $a^n a^m = a^{n+m}$; Scaling s · ab = sa · b = as · b = a · sb = a · bs = ab · s (s real); Power-distributivity $(ra^n + sa^m)b = ra^n b + sa^m b$ (r, s real); Vector product properties of the imaginary part: **ab** - re(**ab**) of the product for pure-imaginary 2^n-ons **a**,**b** regarded as $(2^n - 1)$-vectors; $\langle xa, b\rangle = \langle a, x^* b\rangle$, $\langle xa, xb\rangle = \|x\|^2 \cdot \langle a, b\rangle$ and $\langle x, y\rangle = \langle x^*, y^*\rangle$ Numerous weakened associativity, commutativity, distributivity, antiautomorphism, and Moufang and Bol properties including 9-coordinate ``niner'' versions of most of those properties; contains 2^{n-1}-ons as subalgebra.

The most important properties of 2^n-ons

If a,b,x,y are 2^n-ons, $n \geq 0$, and s and t are scalars (i.e. all coordinates are 0 except the real coordinate) then

unit: A unique 2^n-on 1 exists, with $1 \cdot x = x \cdot 1 = x$.

zero: A unique 2^n-on 0 exists, with $0 + x = x + 0 = x$ and $0 \cdot x = x \cdot 0 = 0$.

additive properties: $x+y = y+x$, $(x+y)+z = x+(y+z)$;

$-x$ exists with $x + (-x) = x - x = 0$.

norm: $|x|^2 = xx^* = x^*x$.

norm-multiplicativity: $|x|^2 \cdot |y|^2 = |x \cdot y|^2$.

scaling: $s \cdot x \cdot y = s \cdot x \cdot y = x \cdot s \cdot y = x \cdot s \cdot y = x \cdot y \cdot s$.

weak-linearity: $(x + s) \cdot y = x \cdot y + s \cdot y$ and $x \cdot (y + s) = x \cdot y + x \cdot s$.

right-linearity: $x \cdot (y + z) = x \cdot y + x \cdot z$.

inversion: If $x \neq 0$ then a unique x^{-1} exists, obeying $x^{-1} \cdot x = x \cdot x^{-1} = 1$. It is $x^{-1} = x \cdot |x|^{-2}$.

left-alternativity: $x \cdot xy = x^2 \cdot y$.

left-cancellation: $x \cdot x^{-1} \cdot y = y$.

effect on inner products: $\langle x \cdot a,b \rangle = \langle a, x^* \cdot b \rangle$, $\langle x,y \rangle = \langle x^*, y^* \rangle$, $\langle x^* \cdot a, x^{-1} \cdot b \rangle = \langle a,b \rangle$,

and $\langle x \cdot a, x \cdot b \rangle = |x|^2 \cdot \langle a,b \rangle$.

Conjugate of inverse: $(x^{-1})^* = (x^*)^{-1}$.

Near-anticommutativity of unequal basis elements: $e_k^2 = -1$ and $e_k \cdot e_l^* = -e_l \cdot e_k^*$ if $k \neq l$.

(Note: the case $k; l > 0$ shows that unequal pure-imaginary basis elements anticommute.)

Alternative basis elements: $e_k \cdot e_l \cdot e_k = e_k \cdot e_l \cdot e_k$, $e_l \cdot e_k \cdot e_k = e_l \cdot e_k \cdot e_k$, and $e_k \cdot e_k \cdot e_l = e_k \cdot e_k \cdot e_l$. (However, when $n \geq 4$ the 2^n-ons are not flexible i.e. it is not generally true that $x \cdot y \cdot x = x \cdot y \cdot x$ if x and y are 16-ons that are not basis elements. They also are not right-alternative.)

Quadratic identity: If x is a 2^n-on (over any field F with char$F \neq 2$), then $x^2 + |x|^2 = 2 \cdot x \text{ re } x$

Squares of imaginaries: If x is a 2^n-on with re $x = 0$ ("pure imaginary") then $x^2 = -|x|^2$ is nonpositive pure-real.

Powering preserves im*x* direction

Niners

Niners are 2n-ons whose coordinates with index > 8 are zero. The index starts with 0.

9-**flexibility** $xp \cdot x = x \cdot px$, $px \cdot p = p \cdot xp$.

9-**similitude unambiguity** $xp \cdot x^{-1} = x \cdot px^{-1}$, $px \cdot p^{-1} = p \cdot xp^{-1}$.

9-**right-alternativity** $xp \cdot p = x \cdot p^2$, $px \cdot x = p \cdot x^2$.

9-**right-cancellation** $xp^{-1} \cdot p = x$, $px^{-1} \cdot x = p$.

9-**effect on inner products** $\langle x, yp\rangle = \langle xp, y\rangle$, $\langle xp, yp\rangle = |p|^2\langle x, y\rangle$.

9-**left-linearity** $(x + y)p = xp + yp$, $(p + q)x = px + qx$.

9-**Jordan-identity** $xp \cdot xx = x(p \cdot xx)$, $py \cdot pp = p(y \cdot pp)$.

9-**coordinate-distributivity** $([x + y]z)_{0;...;8} = (xz + yz)_{0;...;8}$.

9-**coordinate-Jordan-identity** $[xy \cdot xx]_{0;...;8} = [x(y \cdot xx)]_{0;...;8}$.

9-**anticommutativity for orthogonal imaginary 2^n-ons**

If $\langle p, x\rangle = \text{re } p = \text{re } x = 0$ then $px = -xp$.

9-**reflection** If $|a| = 1$ and the geometric reflection operator is defined below then $-(\text{refl}[a](y))_{0;...;8} = (a \cdot y^*a)_{0;...;8}$, and $-\{\text{refl}[a](y)\}^*_{0;...;8} = (a^*y \cdot a^*)_{0;...;8}$, and

if either a or y is a niner then $-\text{refl}[a](y) = a \cdot y^*a$ and $-\text{refl}[a](y) = a^*y \cdot a^*$.

$$\text{refl}[\vec{x}](\vec{t}) \stackrel{\text{def}}{=} \vec{t} - \frac{2\langle\vec{x},\vec{t}\rangle}{|\vec{x}|^2}\vec{x} \tag{3}$$

What holds for the niners, also holds for the octonions.

Quaternionic distributions

Sign flavors

Quaternionic distributions are quaternion valued functions of a quaternionic parameter. If not otherwise stated, the quaternionic parameter space is not curved. Quaternions feature sign selections. Inside a quaternionic distribution the quaternionic sign selections of the values are all the same. Due to the four possible sign selections of quaternions, quaternionic distributions exist in four sign flavors.

Differentiation

A quaternionic distribution f(q) can be differentiated.

$$g(q) = \nabla_0 f_0(q) \mp \langle \boldsymbol{\nabla}, \boldsymbol{f}(q)\rangle \pm \nabla_0 \boldsymbol{f}(q) + \boldsymbol{\nabla} f_0(q) \pm \left(\pm \boldsymbol{\nabla} \times \boldsymbol{f}(q)\right)$$

The colored $\mp$ and $\pm$ signs refer to the influence of conjugation of $f(q)$ on quaternionic multiplication. The $\pm$sign refers to the influence of reflection of $f(q)$.

Fourier transform

In Fourier space differentiation becomes multiplication with the canonical conjugate coordinate k and therefore the equivalent equation becomes:

$$\tilde{g}(k) = \mathrm{k}\tilde{f}(k) \\ = \mathrm{k}_0\tilde{f}_0(k) \mp \langle \mathbf{k}, \tilde{\boldsymbol{f}}(k)\rangle \pm \mathrm{k}_0\tilde{\boldsymbol{f}}(k) + \mathbf{k}\tilde{f}_0(k) \\ \pm \left(\pm \mathbf{k} \times \tilde{\boldsymbol{f}}(k)\right) \tag{2}$$

For the imaginary parts holds:

$$\mathbf{g}(q) = \pm \nabla_0 \boldsymbol{f}(q) + \boldsymbol{\nabla} f_0(q) \pm \left(\pm \boldsymbol{\nabla} \times \boldsymbol{f}(q)\right) \tag{3}$$

$$\tilde{\mathbf{g}}(k) = \pm \mathrm{k}_0 \tilde{\boldsymbol{f}}(k) + \mathbf{k}\tilde{f}_0(k) \pm \left(\pm \mathbf{k} \times \tilde{\boldsymbol{f}}(k)\right) \tag{4}$$

By using[39]

$$\boldsymbol{\nabla} \times \boldsymbol{\nabla} f_0(q) = \mathbf{0} \tag{5}$$

and

$$\langle \boldsymbol{\nabla}, \boldsymbol{\nabla} \times \boldsymbol{f}(q)\rangle = 0 \tag{6}$$

It can be seen that for the static part ($\nabla_0 f(q) = 0$) holds:

$$\mathbf{g}(q) = \boldsymbol{\nabla} f_0(q) \pm \left(\pm \boldsymbol{\nabla} \times \boldsymbol{f}(q)\right) \tag{7}$$

[39] http://www.plasma.uu.se/CED/Book/EMFT_Book.pdf ;Formulas:F.104, F.105

$$\tilde{\mathbf{g}}(k) = \mathbf{k}\tilde{f}_0(k) \pm \left(\pm\mathbf{k} \times \tilde{\boldsymbol{f}}(k)\right) \tag{8}$$

Helmholtz decomposition

Formula (7) of the last paragraph leads to the Helmholtz decomposition. The Helmholtz decomposition splits the **static** vector field $\boldsymbol{F}$ in a (transversal) divergence free part $\boldsymbol{F_t}$ and a (one dimensional longitudinal) rotation free part $\boldsymbol{F_l}$.

$$\boldsymbol{F} = \boldsymbol{F_t} + \boldsymbol{F_l} = \boldsymbol{\nabla} \times \boldsymbol{f} - \boldsymbol{\nabla} f_0 \tag{1}$$

Here f_0 is a scalar field and $\boldsymbol{f}$ is a vector field. In quaternionic terms f_0 and $\boldsymbol{f}$ are the real and the imaginary part of a quaternionic field f. $\boldsymbol{F}$ is an imaginary quaternion.[40]

The significance of the terms "longitudinal"and "transversal" can be understood by computing the local three-dimensional Fourier transform of the vector field $\boldsymbol{F}$, which we call $\widetilde{\boldsymbol{F}}$. Next decompose this field, at each point $\boldsymbol{k}$, into two components, one of which points longitudinally, i.e. parallel to $\boldsymbol{k}$, the other of which points in the transverse direction, i.e. perpendicular to $\boldsymbol{k}$.

$$\widetilde{\boldsymbol{F}}(\boldsymbol{k}) = \widetilde{\boldsymbol{F}}_{\boldsymbol{l}}(\boldsymbol{k}) + \widetilde{\boldsymbol{F}}_{\boldsymbol{t}}(\boldsymbol{k}) \tag{2}$$

$$\langle \boldsymbol{k}, \widetilde{\boldsymbol{F}}_{\boldsymbol{t}}(\boldsymbol{k})\rangle = 0 \tag{3}$$

$$\boldsymbol{k} \times \widetilde{\boldsymbol{F}}_{\boldsymbol{l}}(\boldsymbol{k}) = \boldsymbol{0} \tag{4}$$

The Fourier transform converts gradient into multiplication and vice versa. Due to these properties the inverse Fourier transform gives:

$$\boldsymbol{F} = \boldsymbol{F_l} + \boldsymbol{F_t} \tag{5}$$

$$\langle \boldsymbol{\nabla}, \boldsymbol{F_t}\rangle = 0 \tag{6}$$

[40] See next paragraph

$$\boldsymbol{\nabla} \times \boldsymbol{F_l} = \mathbf{0} \tag{7}$$

So, this split indeed conforms to the Helmholtz decomposition.

This interpretation relies on idealized circumstance in which the decomposition runs along straight lines. This idealized condition is in general not provided. In normal conditions the decomposition and the interpretation via Fourier transformation only work locally and with reduced accuracy.

Continuity equation

When applied to a quaternionic probability amplitude distribution (QPAD), the equation for the differentiation leads to a continuity equation.

When $\rho_0(q)$ is interpreted as a charge density distribution, then the conservation of the corresponding charge is given by the continuity equation:

Total change within V = flow into V + production inside V (1)

$$\frac{d}{dt}\int_V \rho_0 \, dV = \oint_S \hat{\boldsymbol{n}}\rho_0 \frac{\boldsymbol{v}}{c} \, dS + \int_V s_0 \, dV \tag{2}$$

$$\int_V \nabla_0\rho_0 \, dV = \int_V \langle \boldsymbol{\nabla}, \boldsymbol{\rho} \rangle \, dV + \int_V s_0 \, dV \tag{3}$$

Here $\hat{\boldsymbol{n}}$ is the normal vector pointing outward the surrounding surface S, $\boldsymbol{v}(t, \boldsymbol{q})$ is the velocity at which the charge density $\rho_0(t, \boldsymbol{q})$ enters volume V and s_0 is the source density inside V. In the above formula $\boldsymbol{\rho}$ stands for

$$\boldsymbol{\rho} = \rho_0 \boldsymbol{v}/c \tag{4}$$

It is the flux (flow per unit area and unit time) of ρ_0 .

The combination of $\rho_0(t,\boldsymbol{q})$ and $\boldsymbol{\rho}(t,\boldsymbol{q})$ is a quaternionic skew field $\rho(t,\boldsymbol{q})$ and can be seen as a probability amplitude distribution (QPAD).

$$\rho \stackrel{\text{def}}{=} \rho_0 + \boldsymbol{\rho} \tag{5}$$

$\rho(t,\boldsymbol{q})\rho^*(t,\boldsymbol{q})$ can be seen as an overall probability density distribution of the presence of the carrier of the charge. $\rho_0(t,\boldsymbol{q})$ is a charge density distribution. $\boldsymbol{\rho}(t,\boldsymbol{q})$ is the current density distribution.
The conversion from formula (2) to formula (3) uses the Gauss theorem[41]. This results in the law of charge conservation:

$$s_0(t,\boldsymbol{q}) = \nabla_0\rho_0(t,\boldsymbol{q}) \mp \langle \boldsymbol{\nabla}, \big(\rho_0(t,\boldsymbol{q})\boldsymbol{v}(t,\boldsymbol{q}) + \boldsymbol{\nabla}\times\boldsymbol{a}(t,\boldsymbol{q})\big)\rangle \tag{6}$$

$$= \nabla_0\rho_0(t,\boldsymbol{q}) \mp \langle \boldsymbol{\nabla}, \boldsymbol{\rho}(t,\boldsymbol{q}) + \boldsymbol{A}(t,\boldsymbol{q})\rangle$$

$$= \nabla_0\rho_0(t,\boldsymbol{q}) \mp \langle \boldsymbol{v}(t,\boldsymbol{q}), \boldsymbol{\nabla}\rho_0(t,\boldsymbol{q})\rangle \mp \langle \boldsymbol{\nabla}, \boldsymbol{v}(t,\boldsymbol{q})\rangle\, \rho_0(t,\boldsymbol{q})$$

$$\mp\langle \boldsymbol{\nabla}, \boldsymbol{A}(t,\boldsymbol{q})\rangle$$

The blue colored $\pm$ indicates quaternionic sign selection through conjugation of the field $\rho(t,\boldsymbol{q})$. The field $\boldsymbol{a}(t,\boldsymbol{q})$ is an arbitrary differentiable vector function.

$$\langle \boldsymbol{\nabla}, \boldsymbol{\nabla}\times\boldsymbol{a}(t,\boldsymbol{q})\rangle = 0 \tag{7}$$

$\boldsymbol{A}(t,\boldsymbol{q}) \stackrel{\text{def}}{=} \boldsymbol{\nabla}\times\boldsymbol{a}(t,\boldsymbol{q})$ is always divergence free. In the following we will neglect $\boldsymbol{A}(t,\boldsymbol{q})$.

In Fourier space the continuity equation becomes:

$$\tilde{s}_0(t,\boldsymbol{p}) = p_0\tilde{\rho}_0(t,\boldsymbol{p}) \mp \langle \boldsymbol{p}, \tilde{\boldsymbol{\rho}}(t,\boldsymbol{p})\rangle \tag{8}$$

[41] http://en.wikipedia.org/wiki/Divergence_theorem

Equation (6) represents a balance equation for charge density. What this charge is will be left in the middle. It can be one of the properties of the carrier or it can represent the full ensemble of the properties of the carrier.

This only treats the real part of the full equation. The full equation runs:

$$s(t,\boldsymbol{q}) = \nabla\rho(t,\boldsymbol{q}) = s_0(t,\boldsymbol{q}) + \boldsymbol{s}(t,\boldsymbol{q}) \tag{9}$$

$$= \nabla_0\rho_0(t,\boldsymbol{q}) \mp \langle\boldsymbol{\nabla},\boldsymbol{\rho}(t,\boldsymbol{q})\rangle \pm \nabla_0\boldsymbol{\rho}(t,\boldsymbol{q}) + \boldsymbol{\nabla}\rho_0(t,\boldsymbol{q}) \pm \left(\pm\boldsymbol{\nabla}\times\boldsymbol{\rho}(t,\boldsymbol{q})\right)$$

$$= \nabla_0\rho_0(t,\boldsymbol{q}) \mp \langle\boldsymbol{v}(t,\boldsymbol{q}),\boldsymbol{\nabla}\rho_0(t,\boldsymbol{q})\rangle \mp \langle\boldsymbol{\nabla},\boldsymbol{v}(t,\boldsymbol{q})\rangle\,\rho_0(t,\boldsymbol{q})$$

$$\pm\nabla_0\boldsymbol{v}(t,\boldsymbol{q}) + \nabla_0\rho_0(t,\boldsymbol{q}) + \boldsymbol{\nabla}\rho_0(t,\boldsymbol{q})$$

$$\pm\left(\pm\left(\rho_0(t,\boldsymbol{q})\,\boldsymbol{\nabla}\times\boldsymbol{v}(t,\boldsymbol{q}) - \boldsymbol{v}(t,\boldsymbol{q})\times\boldsymbol{\nabla}\rho_0(t,\boldsymbol{q})\right)\right)$$

$$s_0(t,\boldsymbol{q}) = 2\nabla_0\rho_0(t,\boldsymbol{q}) \mp \langle\boldsymbol{v}(q),\boldsymbol{\nabla}\rho_0(t,\boldsymbol{q})\rangle \mp \langle\boldsymbol{\nabla},\boldsymbol{v}(t,\boldsymbol{q})\rangle\,\rho_0(t,\boldsymbol{q}) \tag{10}$$

$$\boldsymbol{s}(t,\boldsymbol{q}) = \pm\nabla_0\boldsymbol{v}(t,\boldsymbol{q}) \pm \boldsymbol{\nabla}\rho_0(t,\boldsymbol{q}) \tag{11}$$

$$\pm\left(\pm\left(\rho_0(t,\boldsymbol{q})\,\boldsymbol{\nabla}\times\boldsymbol{v}(t,\boldsymbol{q}) - \boldsymbol{v}(t,\boldsymbol{q})\times\boldsymbol{\nabla}\rho_0(t,\boldsymbol{q})\right)\right)$$

The red sign selection indicates a change of handedness by changing the sign of one of the imaginary base vectors. Conjugation also causes a switch of handedness. It changes the sign of all three imaginary base vectors.

The origin of physical fields.

The Hilbert Book Model is a simple model of physics that is strictly based on traditional quantum logic and on the lattice isomorphic model; the set of subspaces of an infinite dimensional separable Hilbert space for which the inner product is specified by using quaternions[42].

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[42] See: http://www.crypts-of-physics.eu/HilbertBookModelEssentials.pdf

This restriction results in the fact that all sets of variables are countable. At the same time most observations are taken from a continuum. As a result the set of potential observations overwhelms the set of variables[43]. The situation is comparable to the situation in which the number of equations is far larger than the number of variables that should form the result. Probably, the set of equations will appear to be inconsistent. In order to cure the situation, it is common to assume that the observations are inaccurate. The inaccuracy must be stochastic or with other words the observation result must be blurred.

Nature applies a similar solution, but instead of a simple spread function in the form of a probability density distribution, nature applies a quaternionic probability amplitude distribution (QPAD). This QPAD can be split into a real part that represents a "charge" density distribution and an imaginary part that represents a corresponding "current" density distribution. The "charge" represents the set of properties of the thing that is being observed. The parameter of the distribution represents the location at which the "charge" is observed. The squared modulus of the QPAD represents the probability density of the presence of the "charge" at the location that is specified by the parameter.

This approach transfers the dynamics of the observation into a streaming problem. The equation of motion of the "charge" becomes a continuity equation[44].

The properties of particles move according to the above principle. With each elementary particle belong one or more QPAD's that act as private fields of the particle and that determine its dynamic behavior when it moves freely. However, these fields overlap. In this way these fields and the corresponding particles interact.

A subset of the elementary particles is massless. These particles correspond to a single QPAD. That does not say that their fields cannot overlap.

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[43] A continuum has a higher cardinality than a countable set.

[44] Another name for "continuity equation" is "balance equation".

All other elementary particles are identified by an ordered pair of QPAD's that are two field sign flavors of the same base field. The coordinate system, whose values are used as field parameter, has its own field sign flavor and acts as a sign flavor reference.

Categories of fields

Two categories of fields exist.

Primary fields

The first category consists of quaternionic probability amplitude distributions (QPAD's). The QPAD's may overlap and through this superposition they may form covering fields. The QPAD's exist in four sign flavors. The same holds for the covering fields. The QPAD's may interact. When different sign flavors interact the strength of the local interaction is characterized by a coupling factor. The members of this category will be called primary fields.

Secondary fields

The second category consists of administrator fields. These fields administer the effect of interactions on the local curvature of the positioning coordinate system. For all properties that characterize a coupling of sign flavors of primary fields an administrator field exist that registers the influence of that property during interactions on the local curvature.

One of these administrator fields is the gravitation field. It administers the influence of the strength of the coupling between sign flavors of primary fields on the local curvature.

The electromagnetic fields administer the influence of the electric charge on the local curvature.

The angular momentum including the spin also influences the local curvature. Also this effect is administered.

The members of this category will be called secondary fields or administrator fields.

All these influences can be administered by using the local metric. This generates a metric tensor field.

Example potential

The influence of local properties is represented by charges. The charge carrier may contain an assembly of charges.

Spatial Harmonic functions[45] are suitable charge spread functions.
For a harmonic function $f(q)$ holds:

$$\Delta f(q) = \nabla\nabla^{①} f(q) = 0 \tag{1}$$

If there is a static spherically symmetric Gaussian charge density ϱ (r):

$$\rho(q) = \frac{Q}{\sqrt{2\pi\sigma^2}^3} \exp(-|q|^2/(2\sigma^2)) \tag{2}$$

where Q is the total charge, then the solution ϕ (r) of Poisson's equation[46],

$$\nabla^2\varphi(q) = -\frac{\rho(q)}{\varepsilon} \tag{3}$$

is given by

$$\varphi(q) = \frac{Q}{4\pi\varepsilon|q|} \operatorname{erf}\left(\frac{|q|}{\sqrt{2}\sigma}\right) \tag{4}$$

where erf(x) is the error function.

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45 http://en.wikipedia.org/wiki/Harmonic_function
46 http://en.wikipedia.org/wiki/Poisson%27s_equation

In fact the quaternionic Poisson's equation represents two separate equations:

$$(\nabla_0{}^2 - \boldsymbol{\nabla}^2)\varphi_0(q) = -\frac{\rho_0(q)}{\varepsilon} \tag{5}$$

$$(\nabla_0{}^2 - \boldsymbol{\nabla}^2)\boldsymbol{\varphi}(q) = -\frac{\boldsymbol{\rho}(q)}{\varepsilon} \tag{6}$$

Note that, for $|q|$ much greater than σ, the erf function approaches unity and the potential ϕ (r) approaches the point charge potential $\frac{Q}{4\pi\varepsilon|q|}$, as one would expect. Furthermore the erf function approaches 1 extremely quickly as its argument increases; in practice for $|q| > 3\sigma$ the relative error is smaller than one part in a thousand[47].

The definition of the quaternionic potential $\phi(q)$ is based on the convolution of a quaternionic distribution $\rho(q)$ with the real function $\varphi(q)$ See Newton potential and Bertrand's theorem in Wikipedia. The real part $\rho_0(q)$ of the distribution $\rho(q)$ can be interpreted as a charge distribution. The imaginary part $\boldsymbol{\rho}(q)$ can be interpreted as a current distribution. The convolution blurs the distribution such that the result becomes differentiable.

In configuration space holds:

$$\phi(q) = \rho(q) \circ \frac{1}{|q|}. \tag{7}$$

Reversely, according to Poisson's equation:

$$\rho(q) == -\Delta\phi(q) \tag{8}$$

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[47] http://en.wikipedia.org/wiki/Poisson's_equation#Potential_of_a_Gaussian_charge_density

The real part of $\phi(q)$ presents a scalar potential. The imaginary part presents a vector potential.

$$\phi(q) = \phi_0(q) + \boldsymbol{\phi}(q) \tag{9}$$

In the above section:

The scalar potential is a blurred charge distribution. The vector potential is a blurred current distribution. Current is moving charge.
Mass is a form of charge.

(The selected blurring function has striking resemblance with the ground state of the quantum harmonic oscillator[48]).

In Fourier space holds:

$$\tilde{\phi}(p) = \tilde{\rho}(p) \cdot \frac{1}{|p|} .= \tilde{\phi}_0(p) + \tilde{\boldsymbol{\phi}}(p) \tag{10}$$

In Fourier space the frequency spectrum of the Hilbert distribution is multiplied with the Fourier transform of the blurring function. When this falls off when the frequencies go to infinity, then as a consequence the frequency spectrum of the potential is bounded. This is valid independent of the fact that the frequency spectrum of the Hilbert distribution is unbounded.

Continuity Equations

The equation for the conservation of charge:

$$s_0(q) = \nabla_0 \rho_0(q) \mp \langle \boldsymbol{\nabla}, \boldsymbol{\rho}(q) \rangle \tag{11}$$

We can define $\mathfrak{F}$(q):

[48] Functions and fields:Functions invariant under Fourier transformation:Ladder operator:Ground state

$$\mathfrak{F}(\mathrm{q}) \stackrel{\text{def}}{=} \nabla\rho(q) \tag{12}$$

$$\mathfrak{F}_0(q) = \nabla_0\rho_0(q) \mp \langle\boldsymbol{\nabla}, \boldsymbol{\rho}(q)\rangle \tag{13}$$

$$\boldsymbol{\mathfrak{F}}(q) = \boldsymbol{\nabla}\rho_0(q) \pm \nabla_0\boldsymbol{\rho}(q) \pm \boldsymbol{\nabla} \times \boldsymbol{\rho}(q) = \boldsymbol{\mathfrak{E}}(q) + \boldsymbol{\mathfrak{B}}(q) \tag{14}$$

$$\boldsymbol{\mathfrak{E}}(q) = -\boldsymbol{\nabla}\rho_0(q) \mp \nabla_0\boldsymbol{\rho}(q) \tag{15}$$

$$\boldsymbol{\mathfrak{B}}(q) = \pm\boldsymbol{\nabla} \times \boldsymbol{\rho}(q) \tag{16}$$

The definition of $\boldsymbol{\mathfrak{B}}(q)$ and $\boldsymbol{\mathfrak{E}}(q)$ have the freedom of the gauge transform[49]

$$\boldsymbol{\rho}(q) \mapsto \boldsymbol{\rho}(q) + \boldsymbol{\nabla}\phi_0 \tag{17}$$

$$\boldsymbol{\mathfrak{E}}(q) \mapsto \boldsymbol{\mathfrak{E}}(q) - \boldsymbol{\nabla}\left(\nabla_0\phi_0(q)\right) \tag{18}$$

$$\boldsymbol{\nabla}^2\phi_0 = \nabla_0^2\phi_0 \tag{19}$$

This translates in the source free case $s_0(q) = 0$ into:

$$\nabla_0\rho_0(q) = \pm\langle\boldsymbol{\nabla}, \boldsymbol{\rho}(q)\rangle \tag{20}$$

$$\mathfrak{F}_0(q) = \nabla_0\rho_0(q) \mp \langle\boldsymbol{\nabla}, \boldsymbol{\rho}(q)\rangle = 0 \tag{21}$$

In the source divergence free case $\boldsymbol{\nabla}s_0(q) = 0$ this means:

$$\nabla_0\boldsymbol{\nabla}\rho_0(q) = \pm\boldsymbol{\nabla}\langle\boldsymbol{\nabla}, \boldsymbol{\rho}(q)\rangle \tag{22}$$

$$\nabla_0\boldsymbol{\nabla}\phi_0(q) = \pm\boldsymbol{\nabla}\langle\boldsymbol{\nabla}, \boldsymbol{\phi}(q)\rangle \tag{23}$$

[49] http://en.wikipedia.org/wiki/Gauge_fixing

$$\nabla\langle\nabla, \boldsymbol{\phi}(q)\rangle = \nabla \times \nabla \times \boldsymbol{\phi}(q) + \nabla^2 \boldsymbol{\phi}(q) \tag{24}$$

Due to the fact that there are other charges present, the divergence of the scalar potential need be in the direction of the current $\boldsymbol{\rho}(q)$, which for a spherical symmetric blur is also in the direction of the vector potential $\boldsymbol{\phi}(q)$. However, a tendency exists to minimize that difference. Thus $\nabla_0 \boldsymbol{\nabla} \boldsymbol{\phi}_0(q)$ is parallel to $\boldsymbol{\phi}(q)$. With other words:

$$\boldsymbol{\phi}(q) \times \nabla\langle\nabla, \boldsymbol{\phi}(q)\rangle = 0 \tag{25}$$

Reckoning the sign selections for the sign $\pm$ of the conjugation and the handedness $\pm$ of the cross product will provide four different sets of equations. This will provide four different Hilbert fields.

Discrete distribution

If $\rho(q)$ is discrete, such that

$$\rho(q) = \sum_i q_{E_i} \cdot \delta(q - q_i) \tag{1}$$

where q'_E is a point charge at location q′, then the contribution to the field $\boldsymbol{E}(q)$ that is generated by a point charge at location q_i is given by:

$$d\boldsymbol{E}(q) = q_{E_i} \cdot \frac{q_i - q}{|q_i - q|^3} = -q_{E_i} \cdot \nabla \cdot \frac{1}{|q_i - q|} \tag{2}$$

Differential potential equations

The gradient and curl of $\phi(q)$ are related. In configuration space holds:

$$\nabla\phi(q) = \nabla_0\phi_0(q) \mp \langle\boldsymbol{\nabla}, \boldsymbol{\phi}(q)\rangle \pm \nabla_0 \boldsymbol{\phi}(q) \pm \boldsymbol{\nabla}\phi_0(q) \pm \left(\pm\boldsymbol{\nabla} \times \boldsymbol{\phi}(q)\right) \tag{1}$$

$$\mathfrak{E}(q) \stackrel{\text{def}}{=} -\boldsymbol{\nabla}\phi_0(q) \tag{2}$$

$$\mathfrak{B}(q) \stackrel{\text{def}}{=} \boldsymbol{\nabla} \times \phi(q) \tag{3}$$

$$\mathfrak{F}(q) \stackrel{\text{def}}{=} \nabla\phi(q) = \mathfrak{F}_0(q) \mp \mathfrak{E}(q) \pm \mathfrak{B}(q) \pm \nabla_0 \boldsymbol{\phi}(q) \tag{4}$$

$$\mathfrak{F}_0(q) = \nabla_0\phi_0(q) \mp \langle\boldsymbol{\nabla}, \boldsymbol{\phi}(q)\rangle \quad (5)$$

$$\boldsymbol{\mathfrak{F}}(q) = \mp\boldsymbol{\mathfrak{E}}(q) \pm \boldsymbol{\mathfrak{B}}(q) \pm \nabla_0\boldsymbol{\phi}(q) \quad (6)$$

When the field $\phi(q)$ is split into a private field $\phi_p(q)$ and a background field $\phi_b(q)$, then $\boldsymbol{\phi_p}(q)$ corresponds to the private field of the uniform moving item. When this item accelerates, then it goes together with an extra term $\nabla_0\boldsymbol{\phi_p}(q)$. This is the reason of existence of inertia[50].

$$\langle\boldsymbol{\nabla}, \boldsymbol{\mathfrak{E}}(q)\rangle = -\boldsymbol{\nabla}^2\phi_0(q) = \rho_0(q) \quad (7)$$

$$\boldsymbol{\nabla} \times \boldsymbol{\mathfrak{E}}(q) = 0; \text{Rotation free field} \quad (8)$$

$$\langle\boldsymbol{\nabla}, \boldsymbol{\mathfrak{B}}(q)\rangle = 0; \text{Divergence free } B \text{ field} \quad (9)$$

$$\boldsymbol{\nabla} \times \boldsymbol{\mathfrak{B}}(q) = \boldsymbol{\nabla}\langle\boldsymbol{\nabla}, \boldsymbol{\phi}(q)\rangle - \boldsymbol{\nabla}^2\boldsymbol{\phi}(q) = \boldsymbol{\nabla}\langle\boldsymbol{\nabla}, \boldsymbol{\phi}(q)\rangle + \boldsymbol{\rho}(q) + {\nabla_0}^2\boldsymbol{\phi}(q) \quad (10)$$

$$\boldsymbol{\nabla} \times \boldsymbol{\mathfrak{B}}(q) = \pm\nabla_0\boldsymbol{\nabla}\phi_0(q) + \boldsymbol{\rho}(q) + {\nabla_0}^2\boldsymbol{\phi}(q) \quad (11)$$

$$= \pm\nabla_0\boldsymbol{\mathfrak{E}}(q) + \boldsymbol{\rho}(q) + {\nabla_0}^2\boldsymbol{\phi}(q)$$

Since $\nabla_0\boldsymbol{\phi}(q)$is supposed to be parallel to $\boldsymbol{\nabla}\phi_0(q)$, it is sensible to define $\boldsymbol{E}(q)$as the total field in longitudinal direction:

$$\boldsymbol{E}(q) = -\boldsymbol{\nabla}\phi_0(q) - \nabla_0\boldsymbol{\phi}(q) = \boldsymbol{\mathfrak{E}}(q) - \nabla_0\boldsymbol{\phi}(q) \quad (12)$$

And

$$\boldsymbol{B}(q) = \boldsymbol{\mathfrak{B}}(q) \quad (13)$$

With this definition:

$$\boldsymbol{\nabla} \times \boldsymbol{E}(q) = -\nabla_0\boldsymbol{B}(q) \quad (14)$$

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[50] Influence; Inertia

$$\langle \boldsymbol{\nabla}, \boldsymbol{B}(q) \rangle = 0 \tag{15}$$

$$\boldsymbol{\nabla} \times \boldsymbol{B}(q) = \boldsymbol{\rho}(q) + \nabla_0 \boldsymbol{E}(q) \tag{16}$$

In Fourier space

In Fourier space holds:

$$\widetilde{\mathfrak{F}}(p) = p_0 \tilde{\phi}_0(p) - \langle \boldsymbol{p}, \widetilde{\boldsymbol{\phi}}(p) \rangle \pm p_0 \widetilde{\boldsymbol{\phi}}(p) \pm \boldsymbol{p} \tilde{\phi}_0(p) \pm \boldsymbol{p} \times \widetilde{\boldsymbol{\phi}}(p) \tag{1}$$

$$\widetilde{\mathfrak{F}}(p) = p \tilde{\phi}(p) = \mathfrak{F}_0(p) \mp \widetilde{\boldsymbol{\mathfrak{E}}}(p) \pm \widetilde{\boldsymbol{\mathfrak{B}}}(p) \pm p_0 \widetilde{\boldsymbol{\phi}}(p) \tag{2}$$

$$\widetilde{\mathfrak{F}}_0(p) = p_0 \tilde{\phi}_0(p) - \langle \boldsymbol{p}, \widetilde{\boldsymbol{\phi}}(p) \rangle \tag{3}$$

$$\widetilde{\boldsymbol{\mathfrak{E}}}(p) = -\boldsymbol{p} \tilde{\phi}_0(p) \tag{4}$$

$$\widetilde{\boldsymbol{E}}(p) = -\boldsymbol{p} \tilde{\phi}_0(p) \pm p_0 \widetilde{\boldsymbol{\phi}}(p)$$

$$\widetilde{\boldsymbol{\mathfrak{B}}}(p) = \boldsymbol{p} \times \widetilde{\boldsymbol{\phi}}(p) \tag{5}$$

$$\widetilde{\boldsymbol{\mathfrak{F}}}(p) = \mp \widetilde{\boldsymbol{\mathfrak{E}}}(p) \pm \widetilde{\boldsymbol{\mathfrak{B}}}(p) \pm p_0 \widetilde{\boldsymbol{\phi}}(p) \tag{6}$$

$$\langle \boldsymbol{p}, \widetilde{\boldsymbol{\mathfrak{E}}}(p) \rangle = -\boldsymbol{p}^2 \tilde{\phi}_0(p) = \tilde{\rho}_0(p) \tag{7}$$

$$\boldsymbol{p} \times \widetilde{\boldsymbol{\mathfrak{E}}}(p) = 0; \text{ Rotation free field} \tag{8}$$

$$\langle \boldsymbol{p}, \widetilde{\boldsymbol{\mathfrak{B}}}(p) \rangle = 0; \text{ Divergence free } B \text{ field} \tag{9}$$

$$\boldsymbol{p} \times \widetilde{\boldsymbol{\mathfrak{B}}}(p) = \boldsymbol{p} \langle \boldsymbol{p}, \widetilde{\boldsymbol{\phi}}(q) \rangle - \boldsymbol{p}^2 \widetilde{\boldsymbol{\phi}}(q) = \boldsymbol{p} \langle \boldsymbol{p}, \widetilde{\boldsymbol{\phi}}(p) \rangle + \widetilde{\boldsymbol{\rho}}(p) \tag{10}$$

$$\boldsymbol{p} \times \widetilde{\boldsymbol{\mathfrak{B}}}(p) = \pm p_0 \boldsymbol{p} \tilde{\phi}_0(p) + \widetilde{\boldsymbol{\rho}}(p) = \pm p_0 \widetilde{\boldsymbol{\mathfrak{E}}}(p) + \widetilde{\boldsymbol{\rho}}(p) \tag{11}$$

If the distribution $\rho(q)$ is differentiable, then the same equations that hold for fields $\phi(q)$ and $\tilde{\phi}(p)$ hold for the non-blurred distributions $\rho(q)$ and $\tilde{\rho}(q)$.

Maxwell equations

First it must be noted that the above derived field equations hold for general quaternionic fields.
The resemblance with physical fields holds for electromagnetic fields as well as for gravitational fields and for any fields whose blurring function approximates

$$f(q) \approx \frac{1}{|q|}.$$

In Maxwell equations, $\boldsymbol{E}(\mathbf{r})$ is defined as:

$$\boldsymbol{E}(\boldsymbol{r},t) \equiv -\boldsymbol{\nabla}\phi_0(\boldsymbol{r},t) - \frac{\partial \boldsymbol{A}(\boldsymbol{r},t)}{\partial t} = \mathfrak{E}(\boldsymbol{r},t) - \frac{\partial \boldsymbol{A}(\boldsymbol{r},t)}{\partial t}$$

Further:

$$\langle \boldsymbol{\nabla}, \boldsymbol{E}(\boldsymbol{r},t)\rangle = -\boldsymbol{\nabla}^2\phi_0(\boldsymbol{r},t) - \frac{\partial\langle \boldsymbol{\nabla}, \boldsymbol{A}(\boldsymbol{r},t)\rangle}{\partial t}$$

$$= \frac{\rho_0(\boldsymbol{r},t)}{\varepsilon_0} - \frac{\partial\langle \boldsymbol{\nabla}, \boldsymbol{A}(\boldsymbol{r},t)\rangle}{\partial t}$$

In Maxwell equations, $\boldsymbol{B}(r)$ is defined as:

$$\boldsymbol{B}(\boldsymbol{r},t) \equiv \boldsymbol{\nabla} \times \boldsymbol{A}(\boldsymbol{r},t) = \mathfrak{B}(\boldsymbol{r},t)$$

Further:

$$\boldsymbol{\nabla} \times \boldsymbol{E}(\boldsymbol{r},t) = -\frac{\partial \boldsymbol{B}(\boldsymbol{r},t)}{\partial t}$$

$$\langle \boldsymbol{\nabla}, \boldsymbol{B}(\boldsymbol{r}, t) \rangle = 0$$

$$\boldsymbol{\nabla} \times \boldsymbol{B}(\boldsymbol{r}, t) = \mu_0(\boldsymbol{j} + \varepsilon_0 \frac{\partial \boldsymbol{E}}{\partial t})$$

Differentiable distribution

If the distribution $\rho(q)$ is differentiable, then the same equations that hold for fields $\phi(q)$ and $\tilde{\phi}(p)$ hold for the non-blurred distributions $\rho(q)$ and $\tilde{\rho}(q)$.

Using:

$$\boldsymbol{B} = \nabla \times \boldsymbol{\phi} = \boldsymbol{i}(\nabla_2\phi_\parallel - \nabla_\parallel\phi_2) + \boldsymbol{j}(\nabla_\parallel\phi_1 - \nabla_1\phi_\parallel) + \boldsymbol{k}(\nabla_1\phi_2 - \nabla_2\phi_1) \tag{1}$$

gives

$$\nabla_0\phi_\parallel(q) = \mp\nabla_\parallel\phi_0(q) \tag{2}$$

$$\nabla_0\phi_1(q) = \mp \left(\nabla_2\phi_\parallel(q) - \nabla_\parallel\phi_2(q)\right) \tag{3}$$

$$\nabla_0\phi_2(q) = \pm \left(\nabla_1\phi_\parallel(q) - \nabla_\parallel\phi_1(q)\right) \tag{4}$$

$$\nabla_0\phi_0(q) = \langle \boldsymbol{\nabla}, \boldsymbol{\phi}(q) \rangle = \nabla_\parallel\phi_\parallel(q) + \nabla_1\phi_1(q) + \nabla_2\phi_2(q) \tag{5}$$

And correspondingly in Fourier space

$$p_0\tilde{\phi}_\parallel(p) = \mp p_\parallel\tilde{\phi}_0(p) \tag{6}$$

$$p_0\tilde{\phi}_1(p) = \pm \left(p_\parallel\tilde{\phi}_2(p) - p_2\tilde{\phi}_\parallel(p)\right) \tag{7}$$

$$p_0\tilde{\phi}_2(p) = \mp \left(p_\parallel\tilde{\phi}_1(p) - p_1\tilde{\phi}_\parallel(p)\right) \tag{8}$$

$$p_0\tilde{\phi}_0(p) = \langle \boldsymbol{p}, \tilde{\boldsymbol{\phi}}(p) \rangle = p_\parallel\tilde{\phi}_\parallel(p) + p_1\tilde{\phi}_1(p) + p_2\tilde{\phi}_2(p) \tag{9}$$

Conservation laws

Flux vector

The longitudinal direction $\mathbf{k}$ of $\boldsymbol{E}(q)$ and the direction $\mathbf{i}$ of $\boldsymbol{B}(q)$ fix two mutual perpendicular directions. This generates curiosity to the significance of the direction $\mathbf{k} \times \mathbf{i}$. With other words what happens with $\boldsymbol{E}(q) \times \boldsymbol{B}(q)$.

The **flux vector** $\mathfrak{S}(q)$ is defined as:

$$\mathfrak{S}(q) \stackrel{\text{def}}{=} \boldsymbol{E}(q) \times \boldsymbol{B}(q) \tag{1}$$

Conservation of energy

Field energy density

$$\langle \boldsymbol{\nabla}, \mathfrak{S}(q) \rangle = \langle \boldsymbol{B}(q), \boldsymbol{\nabla} \times \boldsymbol{E}(q) \rangle - \langle \boldsymbol{E}(q), \boldsymbol{\nabla} \times \boldsymbol{B}(q) \rangle \tag{1}$$

$$= -\langle \boldsymbol{B}(q), \nabla_0 \boldsymbol{B}(q) \rangle - \langle \boldsymbol{E}(q), \boldsymbol{\phi}(q) \rangle - \langle \boldsymbol{E}(q), \nabla_0 \boldsymbol{E}(q) \rangle$$

$$= -½\nabla_0(\langle \boldsymbol{B}(q), \boldsymbol{B}(q) \rangle + \langle \boldsymbol{E}(q), \boldsymbol{E}(q) \rangle) - \langle \boldsymbol{E}(q), \boldsymbol{\phi}(q) \rangle$$

The **field energy density** is defined as:

$$u_{field}(q) = ½(\langle \boldsymbol{B}(q), \boldsymbol{B}(q) \rangle + \langle \boldsymbol{E}(q), \boldsymbol{E}(q) \rangle) = u_B(q) + u_E(q) \tag{2}$$

$\mathfrak{S}(q)$ can be interpreted as the **field energy current density**.
The continuity equation for field energy density is given by:

$$\nabla_0 u_{field}(q) + \langle \boldsymbol{\nabla}, \mathfrak{S}(q) \rangle = -\langle \boldsymbol{E}(q), \boldsymbol{\phi}(q) \rangle = -\phi_0(q) \langle \boldsymbol{E}(q), \boldsymbol{v}(q) \rangle \tag{3}$$

This means that $\langle \boldsymbol{E}(q), \boldsymbol{\phi}(q) \rangle$ can be interpreted as a source term.
$\phi_0(q)\boldsymbol{E}(q)$ represents **force** per unit volume.

$\phi_0(q)\langle \boldsymbol{E}(q), \boldsymbol{v}(q)\rangle$ represents **work** per unit volume, or, in other words, the power density. It is known as the Lorentz power density and is equivalent to the time rate of change of the mechanical energy density of the charged particles that form the current $\boldsymbol{\phi}(q)$**.**

$$\nabla_0 u_{field}(q) + \langle \boldsymbol{\nabla}, \mathfrak{S}(q)\rangle = -\nabla_0 u_{mechanical}(q) \tag{4}$$

$$\nabla_0 u_{mechanical} = \langle \boldsymbol{E}(q), \boldsymbol{\phi}(q)\rangle = \phi_0(q)\langle \boldsymbol{E}(q), \boldsymbol{v}(q)\rangle \tag{5}$$

$$\nabla_0 \left(u_{field}(q) + u_{mechanical}(q)\right) = -\langle \boldsymbol{\nabla}, \mathfrak{S}(q)\rangle \tag{6}$$

Total change within V = flow into V + production inside V (7)

$$u(q) = u_{field}(q) + u_{mechanical}(q) = u_B(q) + u_E(q) + u_{mechanical}(q) \tag{8}$$

$$U = U_{field} + U_{mechanical} = U_B + U_E + U_{mechanical} = \int_V u\, dV \tag{9}$$

$$\frac{d}{dt}\int_V u\, dV = \oint_S \langle \widehat{\boldsymbol{n}}, \mathfrak{S}\rangle dS + \int_V s_0\, dV \tag{10}$$

Here the source s_0 is zero.

How to interprete $U_{mechanical}$

$U_{mechanical}$ is the energy of the private field (wave function) of the involved particle(s).

Conservation of linear momentum

Field linear momentum

$\mathfrak{S}(q)$ can also be interpreted as the **field linear momentum density**. The time rate change of the field linear momentum density is:

$$\nabla_0 \mathfrak{S}(q) = \boldsymbol{g}_{field}(q) = \nabla_0\, \boldsymbol{E}(q) \times \boldsymbol{B}(q) + \boldsymbol{E}(q) \times \nabla_0 \boldsymbol{B}(q) \tag{1}$$

$$= \left(\boldsymbol{\nabla} \times \boldsymbol{B}(q) - \boldsymbol{\rho}(q)\right) \times \boldsymbol{B}(q) - \boldsymbol{E}(q) \times \boldsymbol{\nabla} \times \boldsymbol{E}(q) \tag{2}$$

$$\boldsymbol{G}(\boldsymbol{E}) = \boldsymbol{E} \times (\boldsymbol{\nabla} \times \boldsymbol{E}) = \langle \boldsymbol{\nabla E}, \boldsymbol{E} \rangle - \langle \boldsymbol{E}, \boldsymbol{E} \rangle = ½\boldsymbol{\nabla}\langle \boldsymbol{E}, \boldsymbol{E} \rangle - \langle \boldsymbol{E}, \boldsymbol{E} \rangle \tag{3}$$

$$= -\boldsymbol{\nabla}(\mathbf{EE}) + ½\boldsymbol{\nabla}\langle \boldsymbol{E}, \boldsymbol{E} \rangle + \langle \boldsymbol{\nabla}, \boldsymbol{E} \rangle \boldsymbol{E}$$

$$= -\boldsymbol{\nabla}(\mathbf{EE} + ½\mathbf{1_3}\langle \boldsymbol{E}, \boldsymbol{E} \rangle) + \langle \boldsymbol{\nabla}, \boldsymbol{E} \rangle \boldsymbol{E}$$

$$\boldsymbol{G}(\boldsymbol{B}) = \boldsymbol{B} \times (\boldsymbol{\nabla} \times \boldsymbol{B}) = -\boldsymbol{\nabla}(\mathbf{BB} + ½\mathbf{1_3}\langle \boldsymbol{B}, \boldsymbol{B} \rangle) + \langle \boldsymbol{\nabla}, \boldsymbol{B} \rangle \boldsymbol{B} \tag{4}$$

$$\boldsymbol{H}(\boldsymbol{B}) = -\boldsymbol{\nabla}(\mathbf{BB} + ½\mathbf{1_3}\langle \boldsymbol{B}, \boldsymbol{B} \rangle) \tag{5}$$

$$\nabla_0 \mathfrak{S}(q) = \boldsymbol{G}(\boldsymbol{B}) + \boldsymbol{G}(\boldsymbol{E}) - \boldsymbol{\rho}(q) \times \boldsymbol{B}(q) \tag{6}$$

$$= \boldsymbol{H}(\boldsymbol{E}) + \boldsymbol{H}(\boldsymbol{B}) - \boldsymbol{\rho}(q) \times \boldsymbol{B}(q) + \langle \boldsymbol{\nabla}, \boldsymbol{B} \rangle \boldsymbol{B} + \langle \boldsymbol{\nabla}, \boldsymbol{E} \rangle \boldsymbol{E}$$

$$= \boldsymbol{H}(\boldsymbol{E}) + \boldsymbol{H}(\boldsymbol{B}) - \boldsymbol{\rho}(q) \times \boldsymbol{B}(q) - \rho_0(q)\, \boldsymbol{E}(q)$$

$$= \boldsymbol{H}(\boldsymbol{E}) + \boldsymbol{H}(\boldsymbol{B}) - \boldsymbol{f}(q) = \boldsymbol{\mathcal{T}}(q) - \boldsymbol{f}(q)$$

$\mathcal{T}$(q) is the linear momentum flux tensor.
The linear momentum of the field contained in volume V surrounded by surface S is:

$$\boldsymbol{P}_{field} = \int_V \boldsymbol{g}_{field}\, dV = \int_V \rho_0 \boldsymbol{\phi}\, dV + \int_V \langle \nabla \boldsymbol{\phi}, \boldsymbol{E} \rangle\, dV + \oint_S \langle \widehat{\boldsymbol{n}}, \boldsymbol{EA} \rangle dS \tag{7}$$

$$\boldsymbol{f}(q) = \boldsymbol{\rho}(q) \times \boldsymbol{B}(q) + \rho_0(q)\, \boldsymbol{E}(q) \tag{8}$$

Physically, $\boldsymbol{f}(q)$ is the Lorentz force density. It equals the time rate change of the mechanical linear momentum density $\boldsymbol{g}_{mechanical}$.

$$\boldsymbol{g}_{mechanical}(q) = \rho_{0m}(q)\boldsymbol{v}(q) \tag{9}$$

The force acted upon a single particle that is contained in a volume V is:

$$\boldsymbol{F} = \int_V \boldsymbol{f}\, dV = \int_V (\boldsymbol{\rho} \times \boldsymbol{B} + \rho_0\, \boldsymbol{E})\, dV \tag{10}$$

Brought together this gives:

$$\nabla_0 \left(\boldsymbol{g}_{field}(q) + \boldsymbol{g}_{mechanical}(q) \right) = -\langle \boldsymbol{\nabla}, \boldsymbol{\mathcal{T}}(q) \rangle \tag{11}$$

This is the continuity equation for linear momentum.
The component $\mathcal{T}_{ij}$ is the linear momentum in the i-th direction that passes a surface element in the j-th direction per unit time, per unit area.

$$\text{Total change within } V = \text{flow into } V + \text{production inside } V \tag{12}$$

$$\boldsymbol{g}(q) = \boldsymbol{g}_{field}(q) + \boldsymbol{g}_{mechanical}(q) \tag{13}$$

$$\boldsymbol{P} = \boldsymbol{P}_{field} + \boldsymbol{P}_{mechanical} = \int\limits_V \boldsymbol{g}\, dV \tag{14}$$

$$\frac{d}{dt} \int\limits_V \boldsymbol{g}\, dV = \oint_S \langle \widehat{\boldsymbol{n}}, \boldsymbol{\mathcal{T}} \rangle dS + \int\limits_V \boldsymbol{s}_{\boldsymbol{g}}\, dV \tag{15}$$

Here the source $\boldsymbol{s}_g = 0$.

Conservation of angular momentum

Field angular momentum

The angular momentum relates to the linear momentum.

$$\boldsymbol{h}(\boldsymbol{q}_c) = (\boldsymbol{q} - \boldsymbol{q}_c) \times \boldsymbol{g}(q) \tag{1}$$

$$\boldsymbol{h}_{field}(\boldsymbol{q}_c) = (\boldsymbol{q} - \boldsymbol{q}_c) \times \boldsymbol{g}_{field}(q) \tag{2}$$

$$\boldsymbol{h}_{mechanical}(q) = (\boldsymbol{q} - \boldsymbol{q}_c) \times \boldsymbol{g}_{mechanical}(q) \tag{3}$$

$$\boldsymbol{\mathcal{K}}(\boldsymbol{q}_c) = (\boldsymbol{q} - \boldsymbol{q}_c) \times \boldsymbol{\mathcal{T}}(q) \tag{4}$$

This enables the balance equation for angular momentum:

$$\nabla_0 \left(\boldsymbol{h}_{field}(\boldsymbol{q}_c) + \boldsymbol{h}_{mechanical}(\boldsymbol{q}_c) \right) = -\langle \boldsymbol{\nabla}, \boldsymbol{\mathcal{K}}(\boldsymbol{q}_c) \rangle \tag{5}$$

Total change within V = flow into V + production inside V

$$\boldsymbol{J} = \boldsymbol{J}_{field} + \boldsymbol{J}_{mechanical} = \int_V \boldsymbol{h}\, dV \tag{6}$$

$$\frac{d}{dt} \int_V \boldsymbol{h}\, dV = \oint_S \langle \widehat{\boldsymbol{n}}, \boldsymbol{\mathcal{K}} \rangle dS + \int_V \boldsymbol{s_h}\, dV \tag{7}$$

Here the source $\boldsymbol{s}_h = 0$.

For a localized charge density contained within a volume V holds for the mechanical torsion:

$$\tau(\boldsymbol{q}_c) = \int_V (\boldsymbol{q}' - \boldsymbol{q}_c) \times \boldsymbol{f}(q') dV \tag{8}$$

$$= \int_V (\boldsymbol{q}' - \boldsymbol{q}_c) \times \left(\rho_0(q') \boldsymbol{E}(q') + \boldsymbol{j}(q') \times \boldsymbol{B}(q') \right) dV$$

$$= Q(\boldsymbol{q} - \boldsymbol{q}_c) \times (\boldsymbol{E}(q) + \boldsymbol{v}(q) \times \boldsymbol{B}(q))$$

$$\boldsymbol{J}_{field}(\boldsymbol{q}_c) = \boldsymbol{J}_{field}(\boldsymbol{0}) + \boldsymbol{q}_c \times \boldsymbol{P}(q) \tag{9}$$

Using

$$\langle \boldsymbol{\nabla} \boldsymbol{a}, \boldsymbol{b} \rangle = \boldsymbol{n}_\nu \frac{\partial a_\mu}{\partial q_\nu} b_\mu \tag{10}$$

$$\langle \boldsymbol{b}, \boldsymbol{\nabla a} \rangle = \boldsymbol{n}_\mu \frac{\partial a_\mu}{\partial q_\nu} b_\mu \tag{11}$$

holds

$$\boldsymbol{J}_{field}(\boldsymbol{0}) = \int_V \boldsymbol{q}' \times \mathfrak{S}(q') dV = \int_V \boldsymbol{q}' \times \boldsymbol{E}(q') \times \boldsymbol{\nabla} \times \boldsymbol{\phi}(q')\, dV \tag{12}$$

$$= \int_V (\boldsymbol{q}' \times \langle (\boldsymbol{\nabla\phi}), \boldsymbol{E} \rangle - \langle \boldsymbol{q}' \times \boldsymbol{E}, (\boldsymbol{\nabla\phi}) \rangle)\, dV$$

$$= \int_V \boldsymbol{q}' \times \langle (\boldsymbol{\nabla\phi}), \boldsymbol{E} \rangle dV$$

$$+ \int_V \boldsymbol{E} \times \boldsymbol{\phi}\, dV - \int_V \langle \boldsymbol{\nabla}, \boldsymbol{E q}' \times \boldsymbol{\phi} \rangle dV$$

$$+ \int_V (\boldsymbol{q}' \times \boldsymbol{\phi}) \langle \boldsymbol{\nabla}, \boldsymbol{E} \rangle dV$$

Spin

Define the non-local spin term, *which does not depend on* $\boldsymbol{q}'$ as:

$$\boldsymbol{\Sigma}_{field} = \int_V \boldsymbol{E}(q) \times \boldsymbol{\phi}(q) dV \tag{13}$$

Notice

$$\boldsymbol{\phi}(q) \times \boldsymbol{\nabla}\phi_0(q) = \phi_0 \boldsymbol{\nabla} \times \boldsymbol{\phi}(q) + \boldsymbol{\nabla} \times \big(\phi_0(q) \boldsymbol{\phi}(q)\big)$$

And

$$\boldsymbol{L}_{field}(\boldsymbol{0}) = \int_V \boldsymbol{q}' \times \langle (\boldsymbol{\nabla\phi}), \boldsymbol{E} \rangle dV + \int_V \boldsymbol{q}' \times \rho_0 \boldsymbol{\phi} dV \tag{14}$$

Using Gauss:

$$\int_V \langle \boldsymbol{\nabla}, \boldsymbol{a} \rangle dV = \oint_S \langle \widehat{\boldsymbol{n}}, \boldsymbol{a} \rangle dS \tag{15}$$

And

$$\rho_0 = \langle \boldsymbol{\nabla}, \boldsymbol{E} \rangle \tag{16}$$

Leads to:

$$\boldsymbol{J}_{field}(\boldsymbol{0}) = \boldsymbol{\Sigma}_{field} + \boldsymbol{L}_{field}(\boldsymbol{0}) + \oint_S \langle \widehat{\boldsymbol{n}}, \boldsymbol{E}\boldsymbol{q}' \times \boldsymbol{\phi} \rangle dS \tag{17}$$

Spin discussion

The spin term is defined by:

$$\boldsymbol{\Sigma}_{field} = \int_V \boldsymbol{E}(q) \times \boldsymbol{\phi}(q) dV \tag{1}$$

In free space the charge density ρ_0 vanishes and the scalar potential ϕ_0 shows no variance. Only the vector potential $\boldsymbol{\phi}$ may vary with q_0. Thus:

$$\boldsymbol{E} = \boldsymbol{\nabla}\phi_0 - \nabla_0 \boldsymbol{\phi} \approx -\nabla_0 \boldsymbol{\phi} \tag{2}$$

$$\boldsymbol{\Sigma}_{field} \approx \int_V (\nabla_0 \boldsymbol{\phi}(q)) \times \boldsymbol{\phi}(q) dV \tag{3}$$

Depending on the selected field $\boldsymbol{\Sigma}_{field}$ has two versions that differ in their sign. These versions can be combined in a single operator:

$$\boldsymbol{\Sigma}_{field} = \begin{bmatrix} \boldsymbol{\Sigma}^{+}{}_{field} \\ \boldsymbol{\Sigma}^{-}{}_{field} \end{bmatrix} \tag{4}$$

If $\frac{\boldsymbol{\phi}(q)}{|\boldsymbol{\phi}(q)|}$ can be interpreted as tantrix (q_0)) and $\frac{\nabla_0\boldsymbol{\phi}(q)}{|\nabla_0\boldsymbol{\phi}(q)|}$ can be interpreted as the principle normal $\boldsymbol{N}(q_0)$, then $\frac{(\nabla_0\boldsymbol{\phi}(q))\times\boldsymbol{\phi}(q)}{|(\nabla_0\boldsymbol{\phi}(q))\times\boldsymbol{\phi}(q)|}$ can be interpreted as the binormal $\boldsymbol{B}(q_0)$.
From these quantities the curvature and the torsion[51] can be derived.

$$\begin{bmatrix}\dot{\boldsymbol{T}}(t)\\ \dot{\boldsymbol{N}}(t)\\ \dot{\boldsymbol{B}}(t)\end{bmatrix} = \begin{bmatrix}0 & \kappa(\mathrm{t}) & 0\\ -\kappa(\mathrm{t}) & 0 & \tau(\mathrm{t})\\ 0 & -\tau(\mathrm{t}) & 0\end{bmatrix}\begin{bmatrix}\boldsymbol{T}(t)\\ \boldsymbol{N}(t)\\ \boldsymbol{B}(t)\end{bmatrix} \tag{5}$$

Metric tensor field

The metric tensor is an example of a tensor field. This means that relative to a locally non-affected coordinate system[52] on the manifold, a metric tensor takes on the form of a symmetric matrix whose entries transform covariantly under changes to the coordinate system. Thus a metric tensor is a covariant symmetric tensor[53]. From the coordinate-independent point of view, a metric tensor is defined to be a non-degenerate symmetric bilinear form[54] on each tangent space that varies smoothly from point to point.

Curved path

In a Riemannian manifold[55] M with metric tensor[56] g, the length of a continuously differentiable curve $\gamma: [a, b] \to M$ is defined by

$$L(\gamma) = \int_a^b \sqrt{g_{\gamma(t)(\dot{\gamma}(t),\dot{\gamma}(t))}}\, dt \tag{1}$$

The distance $d(p, q)$ between two points p and q of M is defined as the infimum[57] of the length taken over all continuous, piecewise continuously

[51] Path characteristics
[52] http://en.wikipedia.org/wiki/Local_coordinate_system
[53] http://en.wikipedia.org/wiki/Symmetric_tensor
[54] http://en.wikipedia.org/wiki/Symmetric_bilinear_form
[55] http://en.wikipedia.org/wiki/Riemannian_manifold
[56] http://en.wikipedia.org/wiki/Metric_tensor

differentiable curves $\gamma: [a, b] \to M$ such that $\gamma(a) = p$ and $\gamma(b) = q$. With this definition of distance, geodesics in a Riemannian manifold are then the locally distance-minimizing paths, in the above sense.
The minimizing curves of L in a small enough open set[58] of M can be obtained by techniques of calculus of variations[59]. Typically, one introduces the following action[60] or energy functional[61]

$$E(\gamma) = \tfrac{1}{2} \int_a^b g_{\gamma(t)(\dot{\gamma}(t),\dot{\gamma}(t))}\, dt \tag{2}$$

It is then enough to minimize the functional E, owing to the Cauchy–Schwarz inequality[62]

$$L(\gamma)^2 \leq 2(b-a)\, E(\gamma) \tag{3}$$

with equality if and only if $|d\gamma/dt|$ is constant.
The Euler–Lagrange[63] equations of motion for the functional E are then given in local coordinates by

$$\frac{d^2x^\lambda}{dt^2} + \Gamma^\lambda_{\mu\nu} \cdot \frac{dx^\mu}{dt} \cdot \frac{dx^\nu}{dt} = 0 \tag{4}$$

where $\Gamma^\lambda_{\mu\nu}$ are the Christoffel symbols[64] of the metric. This is the **geodesic equation**.

57 http://en.wikipedia.org/wiki/Infimum
58 http://en.wikipedia.org/wiki/Open_set
59 http://en.wikipedia.org/wiki/Calculus_of_variations
60 http://en.wikipedia.org/wiki/Action_(physics)
61 http://en.wikipedia.org/wiki/Energy_functional
62 http://en.wikipedia.org/wiki/Cauchy%E2%80%93Schwarz_inequality
63 Appendix; Derivation of the one dimensional Euler Langrange equation
64 Equations of motion; Path through field; Christoffel symbols

Calculus of variations

Techniques of the classical calculus of variations[65] can be applied to examine the energy functional E. The first variation[66] of energy is defined in local coordinates by

$$\delta E(\gamma)(\varphi) = \left.\frac{\partial}{\partial t}\right|_{t=0} E(\gamma + t\,\varphi) \tag{1}$$

The critical points[67] of the first variation are precisely the geodesics. The second variation is defined by

$$\delta^2 E(\gamma)(\varphi,\psi) = \left.\frac{\partial^2}{\partial t^2}\right|_{t=0} E(\gamma + t\,\varphi + s\psi) \tag{2}$$

In an appropriate sense, zeros of the second variation along a geodesic γ arise along Jacobi fields[68]. Jacobi fields are thus regarded as variations through geodesics.

By applying variational techniques from classical mechanics[69], one can also regard geodesics as Hamiltonian flows[70]. They are solutions of the associated Hamilton–Jacobi equations[71], with (pseudo-)Riemannian metric taken as Hamiltonian[72].

Affine geometry

A **geodesic** on a smooth manifold M with an affine connection[73] ∇ is defined as a curve $\boldsymbol{\gamma}(t)$ such that parallel transport[74] along the curve preserves the tangent vector to the curve, so

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65 http://en.wikipedia.org/wiki/Calculus_of_variations
66 http://en.wikipedia.org/wiki/First_variation
67 http://en.wikipedia.org/wiki/Critical_point_(mathematics)
68 http://en.wikipedia.org/wiki/Jacobi_field
69 http://en.wikipedia.org/wiki/Classical_mechanics
70 http://en.wikipedia.org/wiki/Geodesics_as_Hamiltonian_flows
71 http://en.wikipedia.org/wiki/Hamilton%E2%80%93Jacobi_equation
72 http://en.wikipedia.org/wiki/Hamiltonian_mechanics
73 http://en.wikipedia.org/wiki/Affine_connection

$$\nabla_{\dot{\gamma}}\dot{\boldsymbol{\gamma}}(t) = 0 \qquad (1)$$

at each point along the curve, where $\dot{\boldsymbol{\gamma}}$ is the derivative with respect to t. More precisely, in order to define the covariant derivative of $\dot{\boldsymbol{\gamma}}$ it is necessary first to extend $\dot{\boldsymbol{\gamma}}$ to a continuously differentiable imaginary Hilbert field in an open set[75]. However, the resulting value of the equation is independent of the choice of extension.
Using local coordinates[76] on M, we can write the **geodesic equation** (using the summation convention[77]) as

$$\frac{d^2x^\lambda}{dt^2} + \Gamma^\lambda_{\mu\nu} \cdot \frac{dx^\mu}{dt} \cdot \frac{dx^\nu}{dt} = 0 \qquad (2)$$

where $x^\mu(t)$ are the coordinates of the curve $\boldsymbol{\gamma}(t)$ and $\Gamma^\lambda_{\mu\nu}$ are the Christoffel symbols[78] of the connection ∇. This is just an ordinary differential equation for the coordinates. It has a unique solution, given an initial position and an initial velocity.
From the point of view of classical mechanics, geodesics can be thought of as trajectories of free particles in a manifold. Indeed, the equation $\nabla_{\dot{\gamma}}\dot{\boldsymbol{\gamma}}(t) = 0$ means that the acceleration of the curve has no components in the direction of the surface (and therefore it is perpendicular to the tangent plane of the surface at each point of the curve). So, the motion is completely determined by the bending of the surface. This is also the idea of the general relativity where particles move on geodesics and the bending is caused by the gravity.

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[74] http://en.wikipedia.org/wiki/Parallel_transport
[75] http://en.wikipedia.org/wiki/Open_set
[76] http://en.wikipedia.org/wiki/Local_coordinates
[77] http://en.wikipedia.org/wiki/Summation_convention
[78] http://en.wikipedia.org/wiki/Christoffel_symbol

Christoffel symbols

If x^i, $i = 1,2,\ldots,n$, is a local coordinate system on a manifold M, then the tangent vectors

$$e_\mu = \frac{\partial}{\partial x_\mu}, \qquad \mu = 1, 2, \ldots, n \tag{1}$$

define a basis of the tangent space of M at each point. The Christoffel symbols $\Gamma^\lambda_{\mu\nu}$ are defined as the unique coefficients such that the equation

$$\nabla_\mu e_\nu = \Gamma^\lambda_{\mu\nu} \cdot e_\lambda \tag{2}$$

holds, where ∇_μ is the Levi-Civita connection[79] on M taken in the coordinate direction e_μ.

The Christoffel symbols can be derived from the vanishing of the covariant derivative of the metric tensor g_{ik}:

$$0 = \nabla_\lambda g_{\mu\nu} = \frac{\partial g_{\mu\nu}}{\partial x_\lambda} - g_{\eta\mu} \cdot \Gamma^\eta_{\mu\lambda} - g_{\mu\eta} \cdot \Gamma^\eta_{\nu\lambda} \tag{3}$$

By permuting the indices, and re-summing, one can solve explicitly for the Christoffel symbols as a function of the metric tensor:

$$\Gamma^\mu_{\nu\lambda} = ½ \cdot g^{\mu\nu} \cdot \left(\frac{\partial g_{\eta\nu}}{\partial x^\lambda} + \frac{\partial g_{\eta\lambda}}{\partial x^\nu} - \frac{\partial g_{\nu\lambda}}{\partial x^\eta}\right) \tag{4}$$

where the matrix $(g^{\mu\nu})$ is an inverse of the matrix $(g_{\mu\nu})$, defined as (using the Kronecker delta, and Einstein notation for summation)

$$g^{\lambda\mu} \cdot g_{\mu\nu} = \delta^\lambda_\nu \tag{5}$$

[79] http://en.wikipedia.org/wiki/Levi-Civita_connection

Although the Christoffel symbols are written in the same notation as tensors with index notation, they are **not** tensors, since they do not transform like tensors under a change of coordinates.
Under a change of variable from (x^1, …., x^n) to (y^1, …., y^n), vectors transform as

$$\frac{\partial}{\partial y^i} = \frac{\partial x^k}{\partial y^i} \cdot \frac{\partial}{\partial x^k} \tag{6}$$

and so

$$\Gamma_{ij}^{k} = \frac{\partial x^p}{\partial y^i} \cdot \frac{\partial x^q}{\partial y^j} \cdot \Gamma_{pq}^{r} \cdot \frac{\partial y^k}{\partial x^r} + \frac{\partial y^k}{\partial x^m} \cdot \frac{\partial^2 x^m}{\partial y^i \partial y^j} \tag{7}$$

where the underline denotes the Christoffel symbols in the y coordinate frame. Note that the Christoffel symbol does **not** transform as a tensor, but rather as an object in the jet bundle.
At each point, there exist coordinate systems in which the Christoffel symbols vanish at the point. These are called (geodesic) normal coordinates, and are often used in Riemannian geometry.
The Christoffel symbols are most typically defined in a coordinate basis, which is the convention followed here. However, the Christoffel symbols can also be defined in an arbitrary basis of tangent vectors e_μ by

$$\nabla_{e_\mu} e_\nu = \Gamma_{\mu\nu}^{\lambda} \cdot e_\lambda \tag{8}$$

Local metric equation

The local metric equation relates the local value of the metric tensor field to the influence of the properties of the local particles on the local curvature.

In order to do this it requires a non-affected coordinate system and a way to qualify the influence that the local value of the particle properties have on the resulting curved coordinate system.

For example the Kerr Newman metric equation uses the per category summed property values of the local coupling factors, the electric charges of the local particles and the angular momenta of the local particles in order to relate these to the local curvature[80].

Kerr-Newman metric equation

The Kerr–Newman metric equation describes the geometry of spacetime in the vicinity of a rotating mass M with charge Q. The formula for this metric depends upon what coordinates or coordinate conditions are selected.

It uses three local properties. These properties are:

- The coupling factor m
- The electric charge Q
- The angular momentum J

The angular momentum J includes the spin s.

In most cases, the simplest interpretation of the Kerr-Newman metric can be taken on the surface of a sphere that has a selected radius r. This metric uses the sum of a category of properties that are collected within the observed sphere. However, the summation produces different centers of activity for different property categories. Thus, these centers need not be at the same location. However, for large enough selected radius r and applied to black holes or single particles, these centers coincide.
The formula uses three characteristic radii, whose prominence usually differs with the content of the investigated sphere.

[80] See next part.

The metric uses a non-curved coordinate system to start with. Several coordinate systems can be used. The most common coordinate systems for a non-curved three dimensional space are:

- Cartesian coordinates
- Spherical coordinates

Alternatives for spherical coordinates are:

- Schwarzschild coordinates[81]
- Kruskal-Szekeres coordinates[82]
- Lemaitre coordinates[83]
- Eddington–Finkelstein coordinates[84]

The advantage of the alternative coordinates is that they avoid unnecessary singularities.

Spherical coordinates

The line element dτ in spherical coordinates is given by:

$$c^2\,d\tau^2 = -\left(\frac{dr^2}{\Delta} + d\theta^2\right)\rho^2 + (c\,dt - \alpha\ \sin^2(\theta)\ d\phi)^2\,\frac{\Delta}{\rho^2} \tag{1}$$

$$-\left((r^2+\alpha^2)\,d\phi - \alpha\,c\,dt\right)^2\,\frac{\sin^2(\theta)}{\rho^2}$$

where the coordinates r, θ and ϕ are the parameters of the standard spherical coordinate system. The length-scales α, ρ and Δ have been introduced for brevity.

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[81] http://en.wikipedia.org/wiki/Schwarzschild_coordinates
[82] http://en.wikipedia.org/wiki/Kruskal-Szekeres_coordinates
[83] http://en.wikipedia.org/wiki/Lemaitre_coordinates
[84] http://en.wikipedia.org/wiki/Eddington%E2%80%93Finkelstein_coordinates

$$\alpha = \frac{J}{M\,c} \quad (2)$$

$$\rho^2 = r^2 + \alpha^2 \cos^2(\theta) \quad (3$$

$$\Delta = r^2 - r_s\,r + \alpha^2 + r_Q^2 \quad (4)$$

r_s is the Schwarzschild radius[85] (in meters) of the massive body, which is related to its mass M by

$$r_s = \frac{2GM}{c^2} \quad (5)$$

where G is the gravitational constant[86]. In case of a single encapsulated elementary particle, M stands for the coupling constant m.

Compare this with the Planck length, $l_{Pl} = \sqrt{\hbar G/c^3}$
The Schwarzschild radius is radius of a spherical geo-cavity with mass M. The escape speed from the surface of this geo-cavity equals the speed of light. Once a stellar remnant collapses within this radius, light cannot escape and the object is no longer visible. It is a characteristic radius associated with every quantity of mass.

r_Q is a length-scale corresponding to the electric charge Q of the mass

$$r_Q^2 = \frac{Q^2 G}{4\pi\varepsilon_0 c^4} \quad (6)$$

where $\frac{1}{4\pi\varepsilon_0}$ is Coulomb's force constant[87].

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85 http://en.wikipedia.org/wiki/Schwarzschild_radius
86 http://en.wikipedia.org/wiki/Gravitational_constant
87 http://en.wikipedia.org/wiki/Coulomb%27s_law

Cartesian coordinates

The Kerr–Newman metric can be expressed in "Kerr–Schild" form, using a particular set of Cartesian coordinates

$$g_{\mu\nu} = \eta_{\mu\nu} + f\, k_\mu\, k_\nu \tag{1}$$

$$f = \frac{G\, r^2}{r^4 + a^2\, z^2}\,[2\, M\, r - Q^2] \tag{2}$$

$$k_x = \frac{r\, x + a\, y}{r^2 + a^2} \tag{3}$$

$$k_y = \frac{r\, y - a\, x}{r^2 + a^2} \tag{4}$$

$$k_0 = 1 \tag{5}$$

Notice that $\boldsymbol{k}$ is a unit vector. Here M is the constant mass of the spinning object, Q is the constant charge of the spinning object, η is the Minkowski tensor, and a is a constant rotational parameter of the spinning object. It is understood that the vector $\boldsymbol{a}$ is directed along the positive z-axis. The quantity r is not the radius, but rather is implicitly defined like this:

$$1 = \frac{x^2 + y^2}{r^2 + a^2} + \frac{z^2}{r^2} \tag{6}$$

Notice that the quantity r becomes the usual radius $R = \sqrt{x^2 + y^2 + z^2}$ when the rotational parameter a approaches zero. In this form of solution, units are selected so that the speed of light is unity ($c = 1$).

In order to provide a complete solution of the Einstein–Maxwell Equations, the Kerr–Newman solution not only includes a formula for the metric tensor, but also a formula for the electromagnetic potential:

$$A_\mu = \frac{Q\, r^3}{r^4 + a^2\, z^2}\, k_\mu \tag{7}$$

At large distances from the source (R>>a), these equations reduce to the Reissner-Nordstrom metric[88] with:

$$A_\mu = \left(-\phi, A_x, A_y, A_z\right) \tag{8}$$

The static electric and magnetic fields are derived from the vector potential and the scalar potential like this:

$$\boldsymbol{E} = -\boldsymbol{\nabla}\phi \tag{9}$$

$$\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A} \tag{10}$$

Schwarzschild metric

Schwarzschild coordinates

Specifying a metric tensor[89] is part of the definition of any Lorentzian manifold[90]. The simplest way to define this tensor is to define it in compatible local coordinate charts and verify that the same tensor is defined on the overlaps of the domains of the charts. In this article, we will only attempt to define the metric tensor in the domain of a single chart.

In a Schwarzschild chart[91] (on a static spherically symmetric spacetime), the line element ds takes the form

$$\mathrm{ds}^2 = -\left(\mathrm{f(r)}\right)^2 \mathrm{dt} + \left(\mathrm{g(r)}\right)^2 \mathrm{dr} + \mathrm{r}^2(\mathrm{d}\theta^2 + \sin^2(\theta)\,\mathrm{d}\phi^2) \tag{1}$$

$$-\infty < \mathrm{t} < \infty,\ \mathrm{r}_0 < \mathrm{r} < \mathrm{r}_1, 0 < \theta < \pi, -\pi < \phi < \pi$$

In the Schwarzschild chart, the surfaces $t = t_0, r = r_0$ appear as round spheres (when we plot loci in polar spherical fashion), and from the form

[88] http://en.wikipedia.org/wiki/Reissner%E2%80%93Nordstr%C3%B6m_metric
[89] http://en.wikipedia.org/wiki/Metric_tensor
[90] http://en.wikipedia.org/wiki/Lorentzian_manifold
[91] http://casa.colorado.edu/~ajsh/schwp.html

of the line element, we see that the metric restricted to any of these surfaces is

$$d\sigma = r_0^2(d\theta^2 + \sin^2(\theta)\, d\phi^2), \qquad 0 < \theta < \pi, -\pi < \phi < \pi \tag{2}$$

That is, these nested coordinate spheres do in fact represent geometric spheres with
surface area

$$A = 4\pi r_0^2 \tag{3}$$

And Gaussian curvature

$$K = 1/r_0^2$$

That is, they are geometric round spheres. Moreover, the angular coordinates θ, ϕ are exactly the usual polar spherical angular coordinates: θ is sometimes called the colatitude and ϕ is usually called the longitude. This is essentially the defining geometric feature of the Schwarzschild chart.

With respect to the Schwarzschild chart, the Lie algebra of Killing vector fields is generated by the time-like irrotational Killing vector field $\boldsymbol{\partial_t}$ and three space-like Killing vector fields

$\boldsymbol{\partial_\phi}, \sin(\phi)\, \boldsymbol{\partial_\theta} + \cot(\theta)\cos(\phi)\ \boldsymbol{\partial_\phi}, \cos(\phi)\ \boldsymbol{\partial_\theta} - \cot(\theta)\sin(\phi)\ \boldsymbol{\partial_\phi}$

Here, saying that $\boldsymbol{\partial_t}$ is irrotational means that the vorticity tensor of the corresponding time-like congruence vanishes; thus, this Killing vector field is hyper-surface orthogonal. The fact that our spacetime admits an irrotational time-like Killing vector field is in fact the defining characteristic of a static spacetime. One immediate consequence is that the constant time coordinate surfaces $t = t_0$ form a family of (isometric) spatial hyper-slices. (This is not true for example in the Boyer-Lindquist chart for the exterior region of the Kerr vacuum, where the time-like coordinate vector is not hyper-surface orthogonal.)

It may help to add that the four Killing fields given above, considered as abstract vector fields on our Lorentzian manifold, give the truest expression of both the symmetries of a static spherically symmetric spacetime, while the particular trigonometric form which they take in our chart is the truest expression of the meaning of the term Schwarzschild chart. In particular, the three spatial Killing vector fields have exactly the same form as the three non-translational Killing vector fields in a spherically symmetric chart on E3; that is, they exhibit the notion of arbitrary Euclidean rotation about the origin or spherical symmetry.
However, note well: in general, the Schwarzschild radial coordinate does not accurately represent radial distances, i.e. distances taken along the space-like geodesic congruence which arise as the integral curves of ∂r. Rather, to find a suitable notion of 'spatial distance' between two of our nested spheres, we should integrate $g(r)dr$ along some coordinate ray from the origin:

$$\Delta\rho = \int_{r_1}^{r_2} g(r)dr \tag{4}$$

Similarly, we can regard each sphere as the locus of a spherical cloud of idealized observers, who must (in general) use rocket engines to accelerate radially outward in order to maintain their position. These are static observers, and they have world lines of form $r = r_0, \theta = \theta_0, \phi = \phi_0$, which of course have the form of vertical coordinate lines in the Schwarzschild chart.
In order to compute the proper time interval between two events on the world line of one of these observers, we must integrate $f(r)dt$ along the appropriate coordinate line:

$$\Delta\tau = \int_{t_1}^{t_2} f(r)dt \tag{5}$$

Schwarzschild metric

In Schwarzschild coordinates[92], the Schwarzschild metric has the form:

$$c^2\,d\tau^2 = \left(1 - \frac{r_s}{r}\right) c^2\,dt^2 - \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2(\theta)\,d\phi^2) \tag{6}$$

---

[92] http://en.wikipedia.org/wiki/Schwarzschild_coordinates

where:

- τ is the proper time (time measured by a clock moving with the particle) in seconds,
- c is the speed of light in meters per second,
- t is the time coordinate (measured by a stationary clock at infinity) in seconds,
- r is the radial coordinate (circumference of a circle centered on the star divided by 2π) in meters,
- θ is the colatitude (angle from North) in radians,
- φ is the longitude in radians, and
- r_s is the Schwarzschild radius (in meters) of the massive body.

Lemaître coordinates

In Schwarzschild coordinates the Schwarzschild metric has a singularity. Georges Lemaître was the first to show that this is not a real physical singularity but simply a manifestation of the fact that the static Schwarzschild coordinates cannot be realized with material bodies inside the gravitational radius[93]. Indeed inside the gravitational radius everything falls towards the center and it is impossible for a physical body to keep a constant radius.

A transformation of the Schwarzschild coordinate system from $\{t, r\}$ to the new coordinates $\{\tau, \rho\}$,

$$d\tau = dt + \frac{\sqrt{r_s/r}}{\left(1 - \frac{r_s}{r}\right)}\, dr \tag{1}$$

$$d\rho = dt + \frac{\sqrt{r/r_s}}{\left(1 - \frac{r_s}{r}\right)}\, dr \tag{2}$$

leads to the Lemaître coordinate expression of the metric,

$$ds^2 = d\tau^2 - \frac{r_s}{r} d\rho^2 - r^2(d\theta^2 + \sin^2(\theta)\, d\phi^2) \tag{3}$$

[93] http://en.wikipedia.org/wiki/Lemaitre_coordinates

Where

$$r = r_s^{1/3} \left[\frac{3\,(\rho - \tau)}{2} \right]^{2/3} \tag{4}$$

In Lemaître coordinates there is no singularity at the gravitational radius, which instead corresponds to the point $\frac{3\,(\rho - \tau)}{2} = r_s$. However, there remains a genuine gravitational singularity at the centrum, where $\rho - \tau = 0$, which cannot be removed by a coordinate change.

The Lemaître coordinate system is synchronous, that is, the global time coordinate of the metric defines the proper time of co-moving observers. The radially falling bodies reach the gravitational radius and the center within finite proper time.

Along the trajectory of a radial light ray,

$$dr = \left(\pm 1 - \sqrt{r_s / r} \right) d\tau \tag{5}$$

therefore no signal can escape from inside the Schwarzschild radius, where always $dr < 0$ and the light rays emitted radially inwards and outwards both end up at the origin.

The action along the live path

The integrated action S_{ab} is performed over a distance along the action trail or equivalently over a period of coordination time

$$S_{ab} = -\int_a^b m \cdot c^2 \cdot ds + matter\ terms \tag{1}$$

$$= -\int_{\tau_a}^{\tau_b} m \cdot c^2 \cdot \sqrt{1 - \left(\frac{v}{c}\right)^2} \cdot d\tau + matter\ terms$$

$$= \int_{\tau_a}^{\tau_b} \mathcal{L} \cdot d\tau$$

m is the mass of the considered item.
v is the speed in Q space.
$\mathcal{L}$ is the Lagrangian.

The first line of this formula can be considered as an integral along the trail in coordinate space or equivalently over the trail in Hilbert space. The next lines concern integrals over the corresponding path in observed space combined with coordinate time. It must be noticed that these spaces have different signature.

$$\mathcal{L} = -m \cdot c^2 \cdot \frac{ds}{d\tau} + \text{matter terms} \tag{2}$$

In general relativity, the first term generalizes (includes) both the classical kinetic energy and interaction with the Newtonian gravitational potential. It becomes:

$$m \cdot c^2 \cdot \frac{ds}{d\tau} = -m \cdot c \cdot \sqrt{g_{\alpha\beta} \cdot \dot{q}_\alpha \cdot \dot{q}_\beta} \tag{3}$$

$g_{\alpha\beta}$ is the rank 2 symmetric metric tensor which is also the gravitational potential. Notice that a factor of c has been absorbed into the square root. The matter terms in the Lagrangian $\mathcal{L}$ differ from those in the integrated action S_{ab}.

$$S_{ab\_matter} = -\int_a^b e \cdot A_\gamma \cdot dq^\gamma + \text{other matter terms} \tag{4}$$

The matter term in the Lagrangian due to the presence of an electromagnetic field is given by:

$$\mathcal{L} = -m \cdot c^2 \cdot \frac{ds}{d\tau} + e \cdot \dot{q}^\gamma \cdot A_\gamma + \text{other matter terms} \tag{5}$$

A_γ is the electromagnetic 4-vector potential.

Black hole

Classical black hole

According to classical mechanics the no-hair theorem[94] states that, once a black hole achieves a stable condition after formation, it has only three independent physical properties:

- mass,
- charge, and
- angular momentum.

The surface gravity[95] κ may be calculated directly from Newton's Law of Gravitation[96], which gives the formula

$$\kappa = \frac{\mathrm{Gm}}{\mathrm{r}^2} \tag{2}$$

[94] http://en.wikipedia.org/wiki/No-hair_theorem
[95] http://en.wikipedia.org/wiki/Surface_gravity
[96] http://en.wikipedia.org/wiki/Newton%27s_Law_of_Gravitation

where m is the mass of the object, r is its radius, and G is the gravitational constant[97]. If we let $\rho = m/V$ denote the mean density of the object, we can also write this as

$$\kappa = \frac{4\pi}{3} G\rho r \tag{3}$$

For fixed mean density ρ, the surface gravity κ is proportional to the radius r. (4)

Sciama[98] relates G to the potential that is raised by the community of particles. For fixed mean density ρ this is shown by

$$\Phi = -\int_V \frac{\rho}{r} dV = -\rho \int_V \frac{dV}{r} = \rho 2\pi R^2 \tag{5}$$

$$G \approx \frac{-c^2}{\Phi} = \frac{-c^2}{\rho 2\pi R^2} \tag{6}$$

Here R is the current radius of the universe.

Simple black hole

The Schwarzschild radius r_S for a non-rotating spherical black hole is

$$r_S = \frac{2Gm}{c^2} \tag{1}$$

General black hole

More generally holds

[97] http://en.wikipedia.org/wiki/Gravitational_constant

[98] Influence;Inertia

$$dM = \frac{\kappa}{8\pi} dA + \Omega dJ + \phi dQ$$

where

- M is the mass/energy,
- A is the horizon area,
- Ω is the angular velocity,
- J is the angular momentum,
- ϕ is the electrostatic potential,
- κ is the surface gravity,
- Q is the electric charge.

For a stationary black hole, the horizon has constant surface gravity. It is not possible to form a black hole with surface gravity. $\kappa = 0$.

Quantum black hole

When quantum mechanical effects are taken into account, one finds that black holes emit thermal radiation (Hawking radiation) at temperature

$$T_H = \frac{\kappa}{2\pi}$$

A quantum black hole is characterized by an entropy S and an area A. The entropy of a black hole is given by the equation:

$$S = \frac{c^3 kA}{4\hbar G}$$

The Bekenstein-Hawking Entropy of three-dimensional black holes exactly saturates the bound

$$S = \frac{kA_P}{4}$$

where A_P is the two-dimensional area of the black hole's event horizon in units of the Planck area,

$$A_P = l_p^2 = \frac{\hbar G}{c^3}.$$

In the Hilbert book model this equals the number of granules that covers the horizon of the black hole.
The horizon of the black hole is an event horizon because information cannot pass this horizon. (Near the horizon the speed of light goes to zero.)

Holographic principle

The holographic principle[99] states that the entropy contained in a closed surface in space equals the entropy of a black hole that has absorbed everything that is contained in this surface.
In the Hilbert book model it means that if the surface is considered as a sparsely covered horizon, then that sparse horizon contains as many granules as the densely covered horizon of the corresponding black hole.
It also means that the maximum entropy that can be contained inside a surface corresponds to a dense coverage with granules of that surface.
In the Hilbert book model, any dense or sparse horizon reflects via its contained entropy the number of granules that are contained in the corresponding volume.

We might extend this picture by stating that the number of granules in a volume corresponds with the entropy in the volume. In the Hilbert book model the number of granules corresponds to the number of Hilbert vectors that are attached to a QPAD. It also corresponds to the number of anchor points of the primary physical fields.

The eigenvectors of the strand operator correspond to quantum logical propositions that represent physical particles. These propositions have a

[99] http://en.wikipedia.org/wiki/Holographic_principle

binary yes/no value. In the extended model these propositions get extra content via the attached QPAD's.

Chandrasekhar limit

The Chandrasekhar limit[100] is an upper bound on the mass of a stable white dwarf star:

$$M_{limit} = \frac{\omega_3^0 \sqrt{3\pi}}{2} \left(\frac{\hbar c}{G}\right)^{3/2} \frac{1}{(\mu_e m_H)^2}$$

where:

- $\hbar$ is the reduced Planck constant
- c is the speed of light
- G is the gravitational constant
- μ_e is the average molecular weight per electron, which depends upon the chemical composition of the star.
- m_H is the mass of the hydrogen atom.
- $\omega_3^0 \approx 2.018236$ is a constant connected with the solution to the Lane-Emden equation.

Approximately:

$$M_{limit} \propto \frac{M_P^3}{m_H^2}.$$

Where

$$M_P = \sqrt{\hbar c/G}$$ is the Planck mass

[100] http://en.wikipedia.org/wiki/Chandrasekhar_limit

Birth of the universe

The unit sphere of the separable Hilbert space **Ḥ** is an affine space. All unit size eigenvectors end in this sphere.

The eigenvectors of the strand operator are exceptional. They are surrounded by a QPAD *that installs the tendency to keep these vectors together*. The parameter of these distributions is taken from a background coordinate system. This means that also the eigenvectors of the strand operator possess a position in this background coordinate system. The background coordinate system is formed by the eigenspace of an operator that houses in the Gelfand triple **Ħ** of the Hilbert space **Ḥ**. The coupling between the eigenvectors of the strand operator and the eigenspace of the operator in the rigged Hilbert space that provides the background coordinate system is not precise. It is stochastic and of the order of the Planck-length. That is why the granules have this size.

The eigenvectors of the strand operator all touch a granule. The relation with quantum logic means that the Hilbert vector stands for a proposition that has a yes/no value. In case of the Hilbert vectors that are attached to the granules the yes/no value represents group membership. Thus each granule represents a bit of information.

For the eigenvectors vectors of the strand operator a densest packaging exists. It means that in that condition the QPAD's have shrunk to their smallest possible location difference.

Assumption 1: *In that condition, due to the properties of the QPAD's, the mutual tension works asymmetrically*.

This asymmetry means that in a surface that is formed by a set of densely packed granules the tension on one side is stronger than the surface tension at the other side. As a consequence the final configuration of a densest packaging becomes an empty bubble.

In the starting condition all eigenvectors of the strand operator are densely packed in one assembly.

Assumption 2: *After that moment the packaging density relaxes*.

The number of granules does not change. Thus, during this spreading the total entropy does not change.

The package may fall apart in several separated subassemblies and a large series of single or more loosely packed granules. For the single and the more loosely packed granules the corresponding QPAD's fold out. The densely packed subassemblies take again a bubble shape.

This process may occur instantly or gradually, but most probably it will be done in a sequence of these two possibilities.

First occurs a sudden change of scale between the strand operator in the separable Hilbert space **Ң** and the GPS operator that delivers the background coordinate system and that resides in the rigged Hilbert space **Ħ**. It is possible that originally the bubble covered the whole of the unit sphere of the Hilbert space **Ң,** or it may just cover a finite dimensional subspace of **Ң**. This means that the bubble contains an infinite or a finite amount of granules, which suddenly get diffused in a much larger space. That space is affine like the unit sphere of the Hilbert space **Ң**. The diffusion takes place at every occupied location in the background coordinate system.

This kind of universe has no spatial origin or it must be the center of the outer horizon. With the aid of the background coordinate system, it will be possible to indicate a center of that universe. Each item in this universe has its own private information horizon. This horizon is set by the reach of the light that has been travelling since the birth of the universe. As long as this light does not reach the outer horizon that sub-universe looks isotropic. A multitude of such sub-universes exist that need not overlap.

However, they all look at their border at an image of part of the start horizon. Such, sub-universes obey the cosmological principle[101].

In the next phase the further expansion occurs gradually. Because the QPAD's that are attached to the granules install a tendency for the granules to stay together, a different motor must be present behind this expansion. This motor can be found in the fact that with increasing radius the number of pulling granules grows faster than the decrease of the forces that are executed by the fields of these granules that is caused by the increasing distance. In an affine space this is always and everywhere true. This effect is also the source of inertia.

Due to local attraction, loosely packed and single granules may reassemble in bubble shaped subassemblies. These subassemblies are known as black holes. Single granules and small aggregates of granules are known as elementary particles, nuclei or atoms.

Much larger aggregates may be formed as well but these are not densely packed. Elementary particles can be categorized according to the configuration of their private fields. The private fields determine whether the particle is matter, with other words whether it has mass or not.

Inside the bubble the fact that the granule represents matter is not recognizable. It is only recognizable when the attached QPAD gets the chance to unfold. That condition is true when the granule is not part of a densely packed subassembly.

The requirements for the birth of the universe are:

1. The existence of a strand operator
2. The existence of QPAD's that install the tendency to keep these eigenvectors of the strand operator together

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101 http://en.wikipedia.org/wiki/Cosmological_principle

3. When the large numbers of eigenvectors are densely packed, then the assembly forms a bubble, because due to the properties of the QPAD's, the mutual tension works asymmetrically
4. In advance the eigenvectors of the strand operator are densely packed in one bubble.
5. A non-zero probability exists that the package density will be relaxed and the package falls apart. This may happen in a two stage process
    a. A sudden reduction of scale occurs
    b. Next a force that pulls the granules further away from each other exists

In the first episode of the universe the sudden scale change took place. This ripped the original bubble apart. Next a gradual further expansion took place.

The granules that move freely can at the utmost take one space step at every progression step. When the ratio of the space step and the progression step is fixed, then this determines a maximum speed of granules. A certain type of granules takes a space step at every progression step. That type transports information at the maximum possible speed.

When the path of these information transmitting particles is a straight line, then after a while, the other types of granules no longer get messages from the birth episode of the universe. But this need not be the case.

Since the messenger has a finite speed, it brings information from the past. First of all the speedy messenger and the slow addressee may have started from different locations. Further, due to curvature of space the path of the speedy messenger may take much longer than the duration of the much straighter path that the much slower addressee has taken. The information about the past that is included in the message might be close to the episode in which the granules were combined in one large bubble.

Thus despite the fact that most of the information that is generated during the birth of the universe is long gone, still some of that information may reach particles long after the instance of birth. When this information is interpreted it gives the impression of a metric expansion of the universe[102].

[102] http://en.wikipedia.org/wiki/Metric_expansion_of_space

Part three

PART THREE

A Tall Quantum Tale

A Tall Quantum Tale

I state you a proposition
and that proposition indicates
how the world works

Story

Prelude

A group of elderly Magi sit in a circle and discuss what happens around them. That is not much. The youngest of them gets bored and starts considering their discussion. The chat appears regulated, because if they start from a false proposition they will be able to draw any inference, whether true or not true, and then the conversation ends only in balderdash ad infinitum.

After some time, he has collected the rules. These rules prevent the conversations from getting out of control. He proposes these rules to his companion discussers. They are very pleased. From this moment on, every conversation runs fluently. The inventor writes his finding in a book and calls that book "Logic".

However, in their environment still little occurs that is worth a proper discussion. Since the talks no longer get out of control, most of the time passes in silence. The inventor feels bored again and therefore he tries to invent something else. He realizes that if he changes the rules in his book a little, then as a result, the discussions could be become much more interesting. He writes a new book that contains the changed rules. Next he changes the forest that exists in their neighbourhood in order to reflect the discussion rules.

After finishing this book and the forest, the situation has completely changed. Continuously, things appear in the forest around them that keep their conversations for ever alive. The writer calls the second book "Quantum Logic" and he renames his first book "Classical Logic". The toolkit that he uses to create the new structure of the forest also has a name. It is called "Mathematics".


M

S


The encounter

An old, very experienced senior meets a young curious guy, which is full of questions about the things that he has observed during his trip through his world. The youngster asks the elder whether he can ask him a few of his most urging questions. The senior reacts positively by nicking shortly. However, already the first question of the studious guy startles him:

S: Mister, can you explain me how the world works?

The elder thinks a while very deeply and comes then with his answer:

M: That would be a hell of a job, but I can at least give it a try. Please, sit down on that stone, because this will take some time.

The lad sits down and looks expectantly to his narrator. The old man takes a breath and starts:

M: This can be done in the form of a tale. It could be done better in the form of a truck load of formulas, but I doubt that you would understand these formulas. Do you accept that I pack the story in a tale?

S: Well I like a tale much better than a truck load of formulas. I probably would not understand one of them. So please start with your tale.

The elder takes a breath and starts his tale.

M: The world is governed by a book of laws. It must conform to these laws. There is no punishment in not following the laws, but the world cannot do anything else then operate according to the rules that are written in the book of laws.

S: Where is that book and how is it called?

M: It is in the possession of the governor of Hilbert's bush. The book's name is "The rules of quantum logic".

S: What is in that book?

M: The book contains a small set of rules that regulate what the relations are between propositions that can be made about things that live in our world.

S: What things?

M: Well, anything that has an identity and that stores the condition it is in. Let us call such a thing an item or a particle and let us use the name state for the condition it is in. Mostly the concerned things are very small. However, these things can be very large.

S: What is different with that logic? I know only one kind of logic.

M: You know the kind of logic that humans base their reasoning on. They use the rules of logic in their discussions when they start with truth and want to stay with truth. Nature uses a kind of logic that has a much richer structure. However, in that logic only one rule is different.

S: How many rules contains the book and what do these rules mean?

M: The book contains somewhat more than twenty rules and they specify the structure of the relations between the allowable propositions.

S: There are not much rules in the book! How can that book rule the world?

M: You are right about this, but these rules are very powerful.

S: Please explain that.

M: Well, the structure of the propositions is reflected in the structure of Hilbert's bush. Hilbert's bush is a huge and dense forest and is connected to our world. Via these connections Hilbert's bush controls how the world works.

S: Thus, if I visit Hilbert's bush, then I can see how the world works?

M: No, if you visit Hilbert's bush, then you can see how the world is controlled.

S: How, can I visit Hilbert's bush?

M: Well, you can join me on a virtual trip to Hilbert's bush. I will be your guide.

S: Fine. How does Hilbert's bush look?

The man describes a very strange environment. The chap follows the old man in his mind and shows astonished. However, in advance his guide warned that he would present a tale. So, he must belief what the man tells.

M: It is like a huge forest of poles. All poles have the same length and the feet of all poles are hooked at the same point in the centre of the bush. In this way the poles form an enormous sphere.

S: Where do these poles stand for?

M: The poles are the axes of a multidimensional cube that has an enormous dimension. First think of a three dimensional cube. Take a corner of it and take the three axes at that corner. You can identify the position of all points in the cube by three positions on rulers that are taken along the three axes.
Now, as in an umbrella, fold these axes together, such that they form a small bundle. Next add a large amount of axes to that bundle. Give every axis a unique label in the form of one or more numbers. Add a ruler to each of these axes. You can still define the position of each point in the multidimensional cube by stating the corresponding positions on the rulers. Next increase the number of dimensions until it reaches infinity. The axes now form a dense ball and they all are numbered with a unique label. Finally unfold in your imagination the "umbrella" again until all axes are again perpendicular to each other. You can start counting the dimensions of the cube, but you will never finish counting.

S: Thus the poles are a plain set of axes.

M: Yes, but the space between the perpendicular axes can also be filled with poles. In this way several sets of mutually perpendicular axis poles can be found.

S:What is the function of these axis poles?

M: The axis poles have colours. Some axis poles are green poles. Together they form a base in which the position of all other poles can be expressed. Another set of axis poles are red. Also they form a base. Some of the poles are silver white. They are not necessarily axis poles. The silver white poles appear in bundles.

S: That is a strange kind of forest!

M: Indeed, but it is not the only thing that is strange about Hilbert's bush. Let me tell more about the silver white poles. The bundles of white poles represent and at the same time control the items in our world.

S: How is that arranged?

M: The items in our world are reflections of the bundles of white poles in Hilbert bush. What happens to the bundles will happen to the items.

The student tries to imagine the strange situation. Apparently two worlds exist. One in which he lives and one from where his live is controlled. He visualizes the forest in his brain.

S: What is the function of the green and red poles?

M: At their top these other poles contain a data store in the form of a label. The data stores of the green poles contain position data. They are a kind of kilometre indications that you find along our roads. Instead of a single number the stores contain all three coordinates. It works like a kind of primitive GPS system.

S: With some trouble I can understand what you paint for me.

M: The data stores of the red poles contain speed data, or better said momentum data. In this way a bundle of silver white poles can determine the current position and the momentum of the moves of its pupil in the real world.

S: Why are there two types of data poles?

M: The governor arranged it that way. In this way the bundle cannot determine both types of data at the same time. It is another detail of how the governor models our world. The stores of the poles contain the values of the properties of the type observation to which the pole belongs. Mathematicians call these values eigenvalues and the corresponding poles eigenvectors. With this trick the governor leaves us uncertain about our exact condition.

S: What are mathematicians?

M: Mathematicians are scientists that amongst other things study the mechanisms, which determine the structure and behaviour of Hilbert's forest. The creator of the forest used mathematics to give it its functionality.

S: Can white poles read data?

M: No, in fact a shepherd that takes care of the silver white bundle does that. The forest is very dense. So, the shepherd can walk on top of these poles and guard his herd of sheep. From now on, I will call the silver white poles the shepherd's sheep.

S: How does the shepherd read the data?

M: The shepherd must turn to the data pole in order to read its data. If he is close to a green pole, then he is rather far from a red pole. In fact he

may be at nearly the same distance from a series of red poles. He will usually read the nearest data pole. The same holds when the shepherd looks at other colours. Thus, the governor plays a strange trick with our world.
For the insiders: This is the source for the existence of Heisenberg's uncertainty principle. It is the cause of the quantum behaviour of small particles.

S: I must say, that is a strange situation!

M: Yes, let me proceed. It will become even much stranger.

S: Please, go on.

M: The shepherd drives his sheep through Hilbert's bush. He does that guided by the smells that he receives from other silver white bundles. The smells are mixtures of perfumes that are attractive and perfumes that are repellent. The shepherd reacts on these smells.

S: What is causing these smells?

M: These smells are caused by the properties of the sheep. They hang as a blurring mist around each white pole, thus around each individual sheep. The sheep may also move inside the scope of the herd. That movement may also be caused by the influence of the emitted smells.

S: How does the shepherd keep his sheep together?

M: Well, that happens in a particular way. The bush is so dense, that it is impossible to let the poles move. Instead at each of his steps the shepherd redefines the poles that belong to his herd. These poles turn silver white. The poles that get outside of the herd obtain their original green or red colour. The smells create a tendency to minimize action of the cheap. Further there exists another mechanism, which is called inertia.

S: What is inertia?

M: The smells invoke a sticky resistance of the system of all herds against change. Inertia represents the combined influence of all other herds. The most distant herds together form the largest part of the set of herds. So, they have the largest effect. The influence of each individual herd decreases with distance. However, the number of herds increases faster with distance. The difference between the distant herds averages away. As a consequence the distant herds form a uniform background influence.

S: What is the effect of inertia on a herd?

M: Locally the inertia produces an enormous smell pressure. A smooth uniform movement does not disturb this potential. When the herd accelerates it stirs the perfumes and in this way the inertia produces a smell that goes together with this movement.

S: I understand now how position is treated. What about time?

M: The shepherd owns a simple clock. That clock counts his steps. His steps are all the same size. When he drives his sheep around, he follows a track in Hilbert's bush. All shepherds take their steps in synchrony. In facts at each of their steps the complete forest is redefined. In this process the smells act as a guide. They store the current condition of the forest and these represent the preconditions for the new version of the forest. You can say that the smells represent potential versions of the forest. This includes potential versions of sheep. These potential sheep are virtual sheep.

S: So, compared to space, time is handled quite differently.

M: You understand it quickly and perfectly! You understand it better than the physicists of the last few centuries. Most of them were wrong with this subject. They think that time and space belong in one inseparable observable characteristic.

S: How many of these herds exist?

M: As many as there are particles in our world. So, there exist an enormous number of herds, but they are still countable. They can all be identified. All shepherds take their own track through Hilbert's bush.

S: That must make Hilbert's bush very large!

M: It is. Let me proceed. It must be obvious now that the herds influence each other's movements via their smells.

The lad reflects and pictures the forest in his mind as an enormous sphere. On top of that sphere a large number of shepherds push their own herd of silver white lights forward on curving tracks that are determined by the smells that other herds produce. At each of the shepherd's steps Hilbert's forest is reconfigured. The old man must have a strange image of the world. Nonetheless, he must have his reasons.

S: So, the shepherds play a crucial role!

M: Yes, they manipulate their own herd. However, the smells of their sheep influence for other shepherds the observation of the position and momentum of other herds.

S: How do the smells influence that observation?

M: They give the data that are transmitted in the smell an extra turn. It means that other shepherds do not get a proper impression of the position and momentum data that are sent by other herds.

S: Is there a good reason for this confusing behaviour?

M: No, there is no reason. It is just a built in habit of all sheep. On the other hand, the governor established that habit when he designed mathematics. He designed mathematics such, that Hilbert's bush and its inhabitants behave according to the rules in his book.

S: What is the consequence of this strange behaviour?

M: The consequence is that the particles in the world get the wrong impression of the position and momentum of other items. For them it appears that there exists a maximum speed. And these items think that they live in a curved space.
For the insiders: This is the source of the existence of relativity as it was discovered, but not explained by Einstein.

S: Do they think that?

M: For them, it is the truth!

S: So, I live in a curved space and for me there exists a maximum speed.

M: That is right. You properly understand how the world is controlled. As long as you do not interpret that maximum speed as the limit set by your local police officer.

S: What happens inside a herd?

M: The sheep inside a well-shaped herd perform rhythmic movements. You could say that they are dancing. Physicists call it harmonic movements. These dances occur under the control of the shepherd. He considers them as his own possession.

S: What do you mean with a well-shaped herd?

M: A well-formed herd represents in our world a well-formed object, such as an atom.

S: Why is everything set up in such a strange way?

M: The governor of Hilbert's bush is very intelligent, but also very lazy. He does not want to create many rules, so that he does not have to write much in his law book. That is why he invented Hilbert's bush. He builds the consequences of all his rules into the structure and the dynamics of Hilbert's bush. That structure is in principle very simple. The same holds for the dynamics. In this way he does not have to take care on how the world evolves. However, this leaves an enormous freedom for what happens in the world that is controlled by Hilbert's bush. That on itself results in an enormous complexity of the world we live in. That renders the governor very, very smart and very, very lazy.

S: How did Hilbert's bush get its name?

M: Hilbert was the first human that discovered the governor's bush. So people give it his name.

S: Can everybody visit Hilbert's bush?

M: In principle yes. Everybody that possesses sufficient imagination can visit Hilbert's bush. There exist two guides. A mister Schrödinger tells the story as we did. He tells the story as if the bundle of silver white poles moves through the bush of green and red poles. The other guide, mister Heisenberg tells the story as if the bundle of white poles is stationary and the bush of green and red poles moves around. For the world it does not matter what moves. It only senses the relative motion.

S: How did intelligent creatures like us enter that world?

M: The governor installed a tendency to reduce complexity by means of modularization into his forest. When more compatible modules become available it becomes easier to construct more capable modules and more capable items from these modules. Given enough time, more and more capable items are created, which finally result in intelligent creatures. Scientists call this process evolution. It is a chaotic process, but it possesses a powerful tendency.

S: Uch. Can I tell this to my friends?

M: Yes, you can. And if you have learned to read formulas and work with them you can come back and I will tell you the same story in a cart load of formulas.

S: Thanks. I will come back when I am grown up. Can I still ask a final question?

M: You are a sauce-box, but you are smart. Go ahead.

S: What are you going to do after this?

M: I will visit a very old and very wise scientist, called Mendel. He claims that he has a cohesive explanation for all smells that shepherds react to.

S: Why is that important?

M: If his claim is right, then he has found the Holy Grail of physics.

S: Gosh!

After this the boy departs. Later he will become a good physicist.

Interpretation

The book of laws contains a number of axioms that define the structure of traditional quantum logic as an orthomodular lattice.

Hilbert's bush stands for an infinite dimensional separable Hilbert space that is defined over the number field of the quaternions. The set of the closed subspaces of the Hilbert space has the same lattice structure as traditional quantum logic.

The green poles represent an orthonormal base consisting of eigenvectors of the normal operator Q. This operator represents an observable quantity, which indicates the location of the item in space.

The red poles represent an orthonormal base consisting of eigenvectors of the normal operator P. This operator is the canonical conjugate of Q and represents an observable quantity, which indicates the momentum of the item.

The bundle of silver white poles and the herd of sheep represent a closed subspace of the Hilbert space that on its turn represents a particular quantum logical statement. This statement concerns a particle or a wave packet in our surroundings. Q describes the thing as a particle. P describes the thing as a wave packet.

The shepherd represents a complicated operator U_t that pushes the subspace, which is represented by his herd, around in the Hilbert space. The operator U_t may be seen as a trail of infinitesimal unitary operators. It is a function of the trail progression parameter t. The progression parameter differs from our common notion of time, which is the coordinate time.

Traditional quantum logic defines only the stationary structure of what happens in Hilbert’s bush. The dynamics are introduced by the shepherds that react on the smells.

The smells correspond to physical fields. The fields transport information about the conserved quantities that characterize the movements of the item and its elements. Each type of preserved quantity has its own field type. The operators U_t react on these fields. Inertia shows how these operators reflect the actions of the fields. Any acceleration of the item goes together with a reconfiguration of the fields.

The operator U_t transforms the observation operators Q and P into respectively

$$Q_t = U_t^{-1} \cdot Q \cdot U_t$$

and

$$P_t = U_t^{-1} \cdot P \cdot U_t$$

.This distorts the correct observation and ensures that the observer experiences a speed maximum and a curved space.

The eigenvalues of Q and P and the trail progression parameter t characterize the space-time in our live space. As already indicated t is not the same as our common coordinate time.

De eigenfunctions of U_t control the (harmonic) internal movements of the particles.

The sheep represent the elements/properties of the particle.

The effect of modularization is treated in http://www.crypts-of-physics.eu/ThereExistsATendencyInNatureToReduceComplexity.pdf.; part four of this book

HvL


The Philosopher Tempted.
PLATO
DANTE
Nietzsche
MARX
HEGEL
KANT
ZENO
&
the
Alchemists
SUNLIGHT Falling on a tree of Jewels
MEDITATIONS

INDEX

The Hilbert Book Model
Author: Hans van Leunen

This book starts from the axioms of traditional quantum logic and extends this model such that it incorporates physical fields as well as dynamics.
It uses the isomorphism between the set of propositions of traditional quantum logic and the set of closed subspaces of an infinite dimensional separable Hilbert space that uses quaternions in order to specify its inner products.
The book finds solutions for the anomalies that are raised by the countability of the eigenspaces of normal quaternionic operators. It also takes the consequence of the observation that all information about nature becomes available in the form of clouds of information carrying quanta.
The book unifies all fields, such that except for the curvature field, all fields including the wave functions are considered as QPAD's. The curvature field is derived from the curvature of the superposition of all these primary fields. The curvature follows from the decomposition of this covering field in rotation free and divergence free parts.
In order to implement dynamics, the developed model applies a sequence of extended quantum logics or equivalently a sequence of extended separable Hilbert spaces. Each of the members of the sequence represents a static status quo of the universe. This leads to a new model of physics:

The Hilbert Book Model

Apart from this main subject the book contains a series of related papers.

http://vixra.org/author/Ir_J_A_J_van_Leunen
http://vixra.org/author/Ir_J_A_J_Hans_van_Leunen

http://www.crypts-of-physics.eu

The crypt is under a friend's house in Nesle, France

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