The Obscure Precession of Mercury's Perihelion

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Abstract.
The Sun's orbital motion around the Solar System barycentre contributes a small quadrupole moment to the gravitational binding energy of Mercury. This moment has until now gone undiscovered, but it actually generates 7arcsec/cy precession of Mercury's perihelion. Consequently, the residual 43arcsec/cy allocated previously to general relativity must in reality account for this new component and only 36arcsec/cy for general relativity. This means that the orbit of Mercury is grossly incompatible with the vacuum solution of GR.

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1. Introduction

The orbit of planet Mercury has been calculated by several investigators; see Clemence (1947), Brouwer & Clemence (1961), and review in Pireaux & Rozelot (2003). In their calculations, the inverse square law has been applied to set up the differential equations of motion using the measured distances and velocities between Mercury, the Sun and planets. Then the precession of the perihelion of Mercury was explained as being due to general precession in longitude, perturbation by the planets, solar oblateness, and 43arcsec/cy for general relativity.

In this paper, an obscure contribution to precession has been identified due to the actual motion of the Sun around the barycentre producing a very small quadrupole moment in the energy of Mercury. The Newtonian force exerted upon Mercury by the *moving* Sun produces a small acceleration perpendicular to the usual radial acceleration, which means that the angular momentum of Mercury fluctuates slightly. Then logically, the orbit of Mercury cannot be an ellipse around a moving Sun so its absolute potential cannot be the same as for a stationary Sun. Overall, the effect is numerically similar to solar oblateness, as if the circling solar mass is extended equatorially on average. This causes 7arcsec/cy precession of Mercury's orbit which has never before been recognised and included.

Consequently, previous investigators have calculated the orbit of Mercury with reference to a perfect ellipse, for Mercury alone around a stationary Sun, and then added 43arcsec/cy precession for GR in order to get a fit to observations. In fact this residual precession of 43arcsec/cy should account for the obscure component of 7arcsec/cy plus only 36arcsec/cy for a modified GR solution.

2. Derivation of precession due to the moving Sun

The absolute binding energy of Mercury, in the field of the Sun orbiting around the barycentre, may be calculated by using Newton's law. First, consider the *theoretical system* shown in Figure 1. Let Mercury (mass M_1) be regarded as stationary at distance (r_{1C} = 57.9x10^{6}km) from the origin C, while the Sun (mass M) travels rapidly around C at radius ($r_{SC} = 7.43\text{x}10^5$km). Then, for the Sun at distance r_1 from Mercury we can write:

$$r_1^{\ 2} = r_{1C}^{\ 2} + r_{SC}^{\ 2} - 2r_{1C}r_{SC}\cos\theta \qquad . \qquad (1a)$$

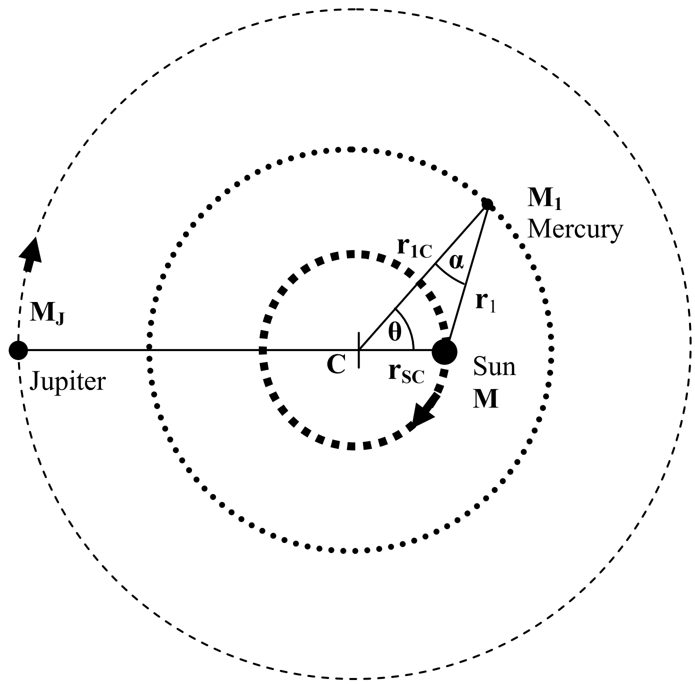

Figure 1. Schematic diagram showing Jupiter and the Sun moving around their centre of mass C. Theoretically, Mercury is considered to be stationary during one orbit of the Sun.

The *instantaneous* gravitational force exerted by the Sun on Mercury is given by the inverse square law, ($F_1 = -GMM_1 / r_1^2$), and the force directed towards C is ($F = F_1\cos\alpha$), where α is the angle between the Sun and centre C given by:

$$r_{SC}^{\ 2} = r_{1C}^{\ 2} + r_1^{\ 2} - 2r_{1C}r_1\cos\alpha \ . \qquad (1b)$$

Upon eliminating $\cos\alpha$, the force is:

$$F = F_1\cos\alpha = -\left(\frac{GMM_1}{r_1^{\ 2}}\right)\left(\frac{r_{1C} - r_{SC}\cos\theta}{r_1}\right) . \qquad (2a)$$

Now eliminate variable r_1 and get all the $\cos\theta$ terms in the numerator:

$$F \approx -\left(\frac{GMM_1}{r_{1C}{}^2}\right)\left(1-\frac{3}{2}\frac{r_{SC}{}^2}{r_{1C}{}^2}+\frac{2r_{SC}}{r_{1C}}\cos\theta+\frac{9}{2}\frac{r_{SC}{}^2}{r_{1C}{}^2}\cos^2\theta\right) \qquad (2b)$$

After averaging θ over a complete orbit of the Sun, the average force towards C becomes:

$$\tilde{F} \approx -\left(\frac{GMM_1}{r_{1C}{}^2}\right)\left[1+\frac{3}{4}\left(\frac{r_{SC}{}^2}{r_{1C}{}^2}\right)\right]. \qquad (2c)$$

This is the average force that a theoretically stationary or slow-moving Mercury responds to, even though the instantaneous force is towards the Sun according to Newton's inverse square law. The force is slightly stronger than an inverse square law for a stationary Sun at C. By integrating this force to infinity ($r_{1C} < r < \infty$), the absolute potential energy of Mercury in this theoretical system would be:

$$PE_{av} \approx -\left(\frac{GMM_1}{r_{1C}}\right)\left[1+\frac{1}{4}\left(\frac{r_{SC}{}^2}{r_{1C}{}^2}\right)\right]. \qquad (3)$$

Obviously, this is the potential energy of Mercury which would be applied for solving the equations of motion in this special system, rather than the Newtonian potential (GM/r_{1C}) for a stationary Sun. If Mercury were allowed to orbit this rapidly moving Sun, its angular momentum and kinetic energy would be slightly greater than that around a Sun stationary at C; but we are more interested in the way that this small quadrupole moment causes precession.

Strangely, most investigators exit here, rejecting this result by insisting that their calculations have proved that the absolute potential of Mercury is not affected by a moving Sun because the instantaneous gravitational force is known to be towards the Sun and independent of velocity. They accept that a ''real toroidal Sun'' would produce Eq.(3), but not a rapidly moving Sun as described herein. In rebuttal, the logical reply to this claim is that the moving Sun causes the angular momentum of Mercury to fluctuate so the orbit of Mercury cannot be an ellipse and its potential energy must be different in form. When *they* calculate the orbit of Mercury, as referred to the perfect ellipse for Mercury alone around a stationary Sun, it is therefore necessary to add 43arcsec/cy precession to get a fit to observations. If

Eq.(3) were admitted and developed as follows, then only 36arcsec/cy would be required.

Although Eq.(2c) is correct for a rapidly moving Sun, the real Sun orbits the barycentre at $7.43\text{x}10^5$km radius over 11.86 years due to Jupiter and produces a correspondingly small quadrupole moment in the long term. Mercury tries to track the Sun's motion exactly, but it has inertia and it is chasing an accelerating Sun, so some quadrupole moment is still generated. Therefore r_{SC} in these equations will be replaced by a smaller compensated value of the order ($r^{/}_{SC} \approx r_{SC}/100$), such that Eq.(2c) is modified to give the actual average acceleration of Mercury:

$$\tilde{a} \approx -\frac{GM}{{r_{1C}}^2}\left[1+\frac{3}{4}\left(\frac{r^{/}_{SC}}{r_{1C}}\right)^2\right] \quad . \tag{4}$$

After substituting ($u = 1/r_{1C}$), plus Mercury's specific angular momentum [$h \approx (GMr_{1C})^{1/2}$], then orbit theory yields a differential equation for the trajectory:

$$\frac{d^2u}{d\phi^2}+u=\frac{-\tilde{a}}{h^2u^2}=\frac{GM}{h^2}+\left(\frac{GM}{h^2}\frac{3}{4}(r^{/}_{SC})^2\right)u^2 \quad . \tag{5}$$

This type of equation has previously been solved because general relativity theory gives a similar expression for the trajectory of Mercury, (see Rindler, 2001):

$$\frac{d^2u}{d\phi^2}+u=\frac{GM}{h^2}+\left(\frac{3GM}{c^2}\right)u^2 \quad , \tag{6}$$

where the final term accounts for the 43arcsec/cy precession of Mercury's orbit. Hence, by direct comparison, we can calculate the precession to expect from the quadrupole moment in Eq.(5):

$$\delta\omega \approx \left(\frac{c}{2h}r^{/}_{SC}\right)^2 \times 43\text{arc sec/ cy} \quad . \tag{7}$$

Here, the small effect of orbit eccentricity is included in the 43arcsec/cy term.

The value of $r^{/}_{SC}$ can be found by considering Figure 2, wherein Mercury (period τ_1) is allowed to orbit around and track the Sun as it moves slowly around C with period ($\tau_{SC} = 49.2\tau_1$). Mercury's orbit is now focussed on a moving centre of mass P, at radius r_{PC} from C towards the Sun. This distance is derivable from an action principle which describes the effective action seen from Mercury orbiting around the centre of mass P. By defining *action* as (Kinetic energy x Time), it is conserved for any value of τ_1 when:

$$\tfrac{1}{2}Mv_{PC}^{2}\left(\tau_{PC}+\tau_{1}\right)=\tfrac{1}{2}Mv_{SC}^{2}\tau_{SC} \quad , \tag{8}$$

where ($\tau_{PC} = \tau_{SC}$) and ($v_{PC} / v_{SC} = r_{PC} / r_{SC}$). Here, the lower equivalent KE of the centre of mass is compensated by an extension in time of τ_1. Simplification gives:

$$r_{PC} = r_{SC}\left(\frac{\tau_{SC}}{\tau_{SC}+\tau_{1}}\right)^{1/2} \quad . \tag{9}$$

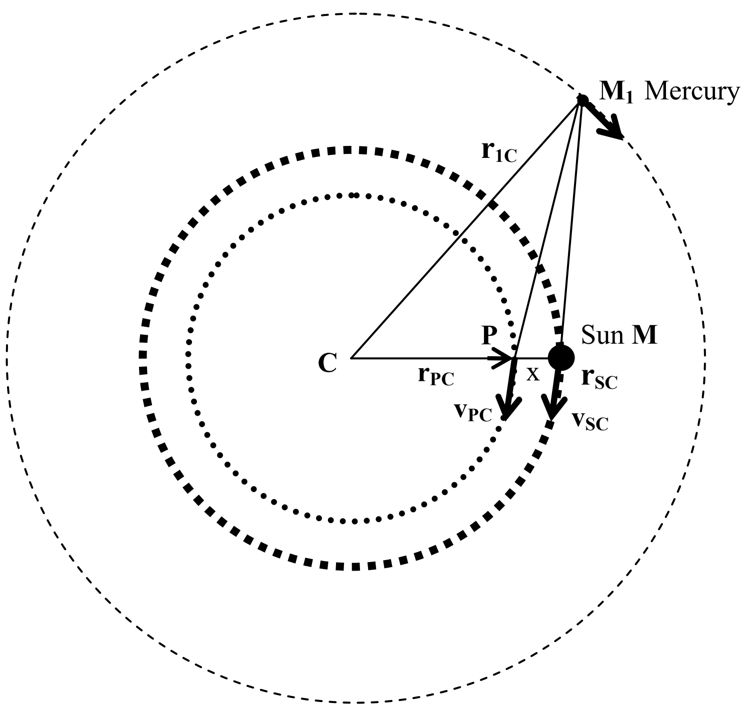

Figure 2. Schematic diagram showing the Sun moving slowly around the centre of mass C. Mercury is considered to be orbiting focal point P between C and the Sun.

From the viewpoint of an observer on Mercury, after 49.2 orbits, the Sun has moved in a circle around P of radius *x* given by:

$$x = r_{SC} - r_{PC} = r_{SC}\left[1 - \left(\frac{\tau_{SC}}{\tau_{SC} + \tau_1}\right)^{1/2}\right] \quad . \qquad (10)$$

It is this circular motion of the Sun *around P* which now determines the quadrupole moment operating on Mercury because radius *x* is equivalent to r'_{SC} in Eq.(7). Thus for (τ_1 = 88.0days), (τ_{SC} = 4331days) and ($r_{SC} = 7.43 \times 10^5$km), it evaluates to ($x \equiv r'_{SC}$ = 7433km). Substitution of this in Eq.(7), with ($h = 2.76 \times 10^{15} m^2 s^{-1}$), yields the expected precession due to the quadrupole moment:

$$\delta\omega \approx 0.162 \times 43 \approx 6.96 \text{arc sec/cy} \quad . \qquad (11)$$

This means that the 43arcsec/cy previously allocated to GR precession should be 36arcsec/cy in reality.

The absolute potential energy of Mercury in this approximately real system is finally:

$$PE_{av} \approx -\left(\frac{GMM_1}{r_{1C}}\right)\left[1 + \frac{1}{4}\left(\frac{r'^{\,2}_{SC}}{r_{1C}^{\,2}}\right)\right] \quad . \qquad (12)$$

Precession due to other planets increasing the Sun's wobble is variable because together they cause great fluctuation in r_{SC} , with a long-term average at around 8×10^5km (Landscheidt, 2007).

Precessions currently attributed to general relativity in the orbits of Venus, Earth and Icarus, will also be affected by the Sun's quadrupole moment, (Shapiro et al (1968), Lieske & Null (1969), Sitarski (1992)).

3. Conclusion

Motion of the Sun around the Solar System barycentre produces a small quadrupole moment in the gravitational binding energy of Mercury, which is estimated to generate nearly 7arcsec/cy precession of the perihelion of Mercury. This effect has been overlooked previously because the simple Newtonian potential of Mercury (GM/r) has been incorrectly employed for an orbiting/accelerating Sun as if for a stationary Sun. Therefore, this new 7arcsec/cy term and 36arcsec/cy due to general relativity are able to explain the observed 43arcsec/cy residual precession. Accordingly, the orbit of Mercury is not compatible with the vacuum solution of GR.

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References

Brouwer, D., & Clemence, G.M. 1961, "Methods of Celestial Mechanics". Academic Press, New York.

Clemence, G.M. 1947, Rev Mod Phys, 19, 361.

Landscheidt, T. 2007, http://landscheidt.wordpress.com/6000-year-ephemeris/

Lieske, H., & Null, G. 1969, Astron J, 74, 297.

Pireaux, S. and Rozelot, J.-P. 2003, Astrophys Space Science, 284, 1159.

Rindler, W. 2001, "Relativity: Special, General, and Cosmological". Oxford.

Shapiro, I. I., Ash, M.E., & Smith, W.B. 1968, Phys Rev Lett, 20, 1517.

Sitarski, G. 1992, Astron.J, 104, 1226.