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Higgs α-quantized coupling constants for quarks and metastable particles

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Abstract

Higgs mass-generation particle coupling constants *g* are calculated as suggested by Lederman and Hill, who use the *top quark t* as the reference Higgs mass. Accurate α-quantized coupling constants *g* are obtained for the metastable leptons, constituent quarks, proton, B_c meson, and W and Z gauge bosons, where $\alpha = e^2/\hbar c$, and where the masses are the *inertial* Einstein masses $m = E/c^2$ of the states involved. An analogous set of inverse Higgs-like coupling constants *f* is also presented in which the *electron* serves as the reference mass, as suggested by the α-boosted mass generation structure of the experimental *f* and *g* coupling constant values.

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A key problem in elementary particle physics has been the experimental search for the Higgs boson, and the clarification of its role in generating particle masses. A step towards the solution of this problem is provided by the analysis of Lederman and Hill [1], who note that the Fermi coupling coefficient G_F in the four-point function for calculating weak-interaction decays [2] corresponds to an energy scale of 175 GeV, which matches the mass of the top quark *t*. They identify the scalar Higgs field with the field that is required for the Fermi weak interactions in a field-theory context, and they comment that the top quark mass may play a significant role in the Higgs formalism. The Higgs field couples linearly to particle masses with a coupling constant *g*, so Lederman and Hill write the Higgs electron coupling constant g_e in the form $m_e = g_e m_t$, which gives g_e = 0.0000029. However, they emphasize that in the absence of a theoretical framework for the coupling constants, the numerical value for g_e tells us "almost nothing" about the electron mass [1]. In the present paper, we introduce an α-quantized empirical framework that accurately reproduces this numerical value, where $\alpha = e^2/\hbar c \cong 1/137$.

Experimental support for the Lederman and Hill viewpoint is obtained from the mass values of the W and Z gauge bosons and top quark *t*. Due to their theoretical importance, these masses have been very accurately measured, as follows: [3]

$$m_w = 80.399 \pm 0.023\,\text{GeV}, \quad m_z = 91.1876 \pm 0.0021\,\text{GeV},$$
$$m_t = 172.9 \pm 1.1\,\text{GeV}\,.$$

The interesting result here is that the sum of the electroweak W and Z masses is 171.6 GeV, which matches the (supposedly-unrelated) *t* quark mass of 172.9 GeV to an accuracy of 0.8%. The precision of this mass equality indicates that the *mass* of the *t* quark is in some manner related to the masses of the W and Z, and hence the *t* quark mass logically plays a role in the Higgs mass formalism, as Lederman and Hill suggest [1].

A clue to the theoretical form of the Higgs coupling constants *g* can be obtained from the electroweak *lifetimes* of the metastable ($t > 10^{-21}$ sec) particles, which display an experimental quantization in powers of the fine structure constant $\alpha \cong 1/137$ that extends over many orders of magnitude [4, 5]. This lifetime α-quantization logically relates to the *mass* structures of these particles, and hence to the values of their Higgs coupling constants. We can investigate this conjecture experimentally by tracing out a mass-linked excitation chain between the electron and the top quark *t*. We have already established the *top link* of this chain, which is in the relationship between the W, Z and *t* masses. Denoting the *average mass* of the W and Z gauge bosons as the mass unit $m_{wz} \equiv (m_w + m_z)/2$, we have $m_t = 2m_{wz}$, at the 1% accuracy level.

The *second link* in the electron-to-top-quark mass chain has to span a range of masses that extends from the low-mass (below 12 GeV) well-populated region of observed particle states to the high-mass (above 80 GeV) sparsely-populated region of the W and Z bosons and *t* quark. Thus this mass link has to extend for at least an order of magnitude in particle mass values. We obtain this link in a spectacular manner by studying the Tevatron collision process that creates the W and Z particles. Counter-rotating protons and antiprotons collide together in the Fermilab Tevatron at TeV energies. The high-energy protons are relativistically flattened into pancakes that each contain three essentially free *u* and *d* quarks, and the collisions that produce W's and Z's are the rare events (1 in 10^{10}) in which the collision is a direct hit between a *u* or *d* quark in the proton and a $\overline{u}$ or $\overline{d}$ quark in the antiproton, with the colliding pair receiving most of the collision energy. Treating the *u* and *d* quarks as equal-mass *constituent-quark mass units* denoted as *ud*, we define their averaged experimental mass value as $m_{ud} = (1/3)\, m_p$. To test the conjecture of mass α quantization, we assume that high-energy tevatron collisions impart an "α-boost" factor of $1/\alpha \cong 137$ increase in mass or energy to each colliding *ud* plus $\overline{ud}$ quark mass-unit pair, thus creating a pair of high-mass ud_{GB} plus $\overline{ud}_{\mathrm{GB}}$ *gauge boson* quark mass units, which have a combined experimental mass-unit value

$$m_{ud_{GB}} + \overline{m}_{\overline{ud}_{GB}} \equiv (m_p + m_{\overline{p}})\ /\ 3\alpha \ = \ 85{,}718\ \mathrm{MeV}\ .$$

Comparing this α-boosted gauge boson quark pair mass to the experimental WZ average mass,

$$m_{wz} \equiv (m_w + m_z)/2 = 85{,}793.3\ \mathrm{MeV},$$

we see that these two masses agree to an accuracy of 0.09%. This mass equality suggests that the W and Z vector bosons are formed from (u_{GB}, d_{GB}) and ($\overline{u}_{GB}, \overline{d}_{GB}$) quark combinations. It also establishes the second link in the electron-to-top-quark mass chain—a link that is α-quantized.

The *bottom link* that is still needed is the proton-to-electron mass ratio. Proceeding as above, we note that the experimental muon-to-electron mass ratio is 3/2α, and the experimental proton-to-muon mass ratio is 9, which combine together to give

$$m_p / m_e = 27/2\alpha,\ \ (0.75\%\ \mathrm{accuracy}).$$

This bottom link thus contains a second α-quantized "α-boost" mass excitation. Joining these three electron-to-top-quark mass links together gives the calculated mass [6]

$$m_t = (18/\alpha^2)\, m_e = 172.728\,\mathrm{GeV},$$

which matches the experimental mass m_t = 172.9 GeV to an accuracy of 0.1% .The individual links in this mass excitation chain all have experimental accuracies which are at the 1% level. Thus the experimental accuracy of the excitation chain extends throughout the mass region from the 0.5 MeV electron to the 173 GeV top quark. The current experimental accuracy [3] of the top quark mass is 1.1 GeV, or 0.6%, so the calculated accuracy of 0.1% seems somewhat fortuitous.

The electron-to-top-quark mass excitation chain is diagrammed in Fig. 1, where it portrays the following mass generation sequence: (1) *electron* to *muon* to *proton*; (2) *proton* back down to *proton-quark*; (3) *proton-quark* to *gauge-boson-quark* to *gauge boson* to *top quark*. This excitation sequence disregards quantum numbers and has in common only the *total energies* of the particle states, which are proportional to the *inertial masses* of the states. The inertial mass of a particle is defined by the Einstein equation $E = mc^2$. We can denote this inertial mass as the *Einstein mass* of the particle. Thus this particle systematics can be described in either an *energy* or an *Einstein-mass* framework. Fig. 1 uses the Einstein-mass framework. In this framework, the quark inertial masses are equal to the conventional Standard Model *constituent-quark* masses [7]. The Fig. 1 excitation chain also appears as Fig. 3 of Ref. [8], where it is in the energy framework.

The challenge implicitly contained in the Lederman and Hill discussion of the Higgs coupling constant is to obtain a theoretical basis for the numerical value g_e = 0.0000029 of the electron coupling constant. The empirically-established mass generation chain displayed in Fig. 1 gives

$$g_e = \alpha^2/18 = 0.000002958,$$

which corresponds to the calculated mass value $m_t = m_e / g_e$ = 172.728 GeV for the top quark t. This mass generation chain provides a phenomenological basis for the numerical value of g_e. The calculated value for m_t is very close to the current experimental value of 172.9 GeV [3], so we employ this calculated value as the reference mass for the Lederman-Hill Higgs coupling constants g, since it brings the L-H calculations into precise agreement with a matching set of electron-based coupling constants f that we define below.

We can extend the validity of this α-quantized Higgs mass generation mechanism by applying it to the u, d, s, c, b set of quarks, the B_c meson, the μ and τ leptons, the proton, and the W and Z gauge bosons and gauge-boson quarks, which are basic excitations that represent fermion ground states.. The masses of these states are their inertial masses, which are determined from the experimental masses of the particles involved. The *u*, *d*, *s*, *c*, *b* quarks are assigned constituent-quark masses [7], whose experimental values are defined as the particle mass divided by the number of quarks and antiquarks in the particle. The *u* and *d* quarks are treated as equal-mass entities that carry the mass-unit label *ud*. Matching *quark-antiquark* binding energies are small—2-3% at energies below 2 GeV, and negligible above 6 GeV—and are not applied here [9]. *Quark-quark* binding energies are negligible. The calculated and experimental Higgs coupling constants for this set of particle states are displayed in Fig. 2, together with the experimental data used to determine inertial masses for these states.

Experimentally, the Higgs particle coupling constants g occur on three α-quantized Higgs levels: the $g \propto \alpha^2$ level, which contains just the electron and positron; the $g \propto \alpha^1$ level, which contains the leptons and hadrons; and the $g \propto \alpha^0$ level, which contains the $W^{\pm}$ and Z^{o} gauge bosons and top quark *t*. On each of these levels the lowest-mass state acts as a *unit mass*, with higher-mass metastable excitations occurring as multiples of the unit mass. On the α^2 level, the 0.511 MeV electron, the only member on this level, has the Higgs constant $g_e = \alpha^2/18$. The unit mass on the α^1 level is the 105 MeV muon, whose Higgs constant is a factor of $(3/2\alpha)$ larger than g_e, so that $g_\mu = \alpha/12$. The higher-mass τ lepton, *ud*, *s*, *c*, *b* quarks, B_c meson, and *p* proton have Higgs constants that are accurate multiples of $\alpha/12$, as displayed in Fig. 2. The unit mass on the α^0 level is the 42.86 GeV ud_{GB} gauge boson quark, whose Higgs constant is a factor of $3/\alpha$ larger than g_μ, so that $g_{ud_{GB}} = 1/4$. The WZ average-mass state and the *t* quark have Higgs constants that are accurate multiples of 1/4.

The graphical logarithmic display in Fig. 2. of the Higgs coupling constants g of these fundamental particle states reveals their excitation mechanisms in a particularly transparent manner. In particular, the three α-quantized Higgs excitation levels—$g \propto \alpha^2, \alpha^1, \alpha^0$—are clearly in evidence. The experimental data used to obtain the experimental Higgs constants are listed at the left in the figure. The agreement between the calculated and experimental Higgs values is at an accuracy level of 1%, with no free parameters used except for the use of the calculated value for the top quark reference mass. The largest deviations are for the *s* and *c* quarks, whose masses and coupling constants are deduced from the ϕ and J/ψ mesons respectively, and which require small quark-antiquark binding energy corrections [9] that are not included here.

The particle states that are displayed in Fig. 2 represent *fermion* α-boost excitations of *(1)* the electron to the muon, and *(2)* the proton *ud* quarks to the gauge boson ud_{GB} quarks. There is also a *boson* α-boost excitation of the electron to a set of *pseudoscalar* ud_{PS} pion quarks, which are required to reproduce the π, η, η', K mesons. The Higgs coupling constants *g* for these particles are multiples of the unit coupling constant $\alpha/18$. These PS mesons are discussed elsewhere [10].

The complete set of unit Higgs coupling constants *g* required for generating all of these basic Einstein-mass particle ground states is numerically as follows:

$$m_e = 0.511 \text{ MeV } (\text{J} = 1/2); \qquad g_e = \alpha^2/18;$$

$$m_{ud_{PS}} = m_e/\alpha = 70.025 \text{ MeV } (J = 0); \qquad g_{ud_{PS}} = \alpha/18;$$

$$m_\mu = 3m_e/2\alpha = 105.038 \text{ MeV } (J = 1/2); \qquad g_\mu = \alpha/12;$$

$$m_{ud_{GB}} = 9m_e/2\alpha^2 = 43{,}182 \text{ MeV } (J = 1/2); \qquad g_{ud_{GB}} = 1/4.$$

The subscripts have been defined above. The angular momentum *J*-values represent the intrinsic angular momentum each mass unit contributes to the particle excitation. The ($\alpha/18$) pseudoscalar meson mass unit ud_{PS} defined here serves as a spinless mass quantum that does not contribute angular momentum. The factor of 3/2 ratio between the ($J = 0$) 70 MeV ($\alpha/18$) mass unit and the ($J = 1/2$) 105 MeV ($\alpha/12$) mass unit can be attributed to the mathematical result that a fully relativistic spinning sphere of matter is half again as massive as its nonspinning counterpart [11].

We can obtain an alternate formulation of the Higgs coupling constants by employing the *electron* as the reference mass. This is suggested by the fact that the four basic Higgs mass units shown above are defined in terms of α as the electron mass generator, and these mass units reproduce the observed α-quantization of the Higgs coupling constants *g* in Fig. 1. We define the electron-based Higgs coupling constants by the equation $f_i = m_i/m_e$, and we use the inverse constant $\text{X} \equiv 1/\alpha \cong 137$ in the equations for the Higgs coupling constants, with X acting as a particle mass generator. In this representation, the equation for the top quark Higgs coupling constant is $f_t = 18\text{X}^2$. The four basic mass units (in MeV) are written in the X-quantized notation as

$$0.511\ (f_e = 1),\ 70.025\ (f_{ud_{PS}} = \text{X}),\ 105.038\ (f_\mu = 3\text{X}/2),\ 43{,}182\ (f_{ud_{GB}} = 9\text{X}^2/2).$$

The α-quantized Higgs coupling constants g of Fig. 1 transform into the X-quantized coupling constants f displayed in Fig. 2, where the experimental particle and quark mass values, and the calculational accuracies, are the same as in Fig. 1. Also indicated in Fig. 2 are four groupings of particle states whose masses link together in a systematic manner.

The results presented here lead to the following set of observations:

(1) The masses of leptons, quarks, hadrons, and gauge bosons combine together in a unified excitation formalism, which depends only on the total energies E, or equivalently the *inertial* masses $m = E/c^2$, of the states involved, where we can denote these inertial masses as *Einstein masses*. The mass equalities between mixed types of particle are in essence *energy* equalities.

(2) The mass/energy relationships shown here are based on experimental data, and the accuracies of their systematic excitation patterns are obtained without using adjustable parameters.

(3) The Fermi weak-interaction formalism is now commonly interpreted as a field theory, and the scalar field that it uses has an energy scale that is comparable to the mass of the top quark t. Thus the t coupling to this field is or order unity, and this coupling can be conveniently used as the reference mass for defining a set of Higgs constants $g_i = m_i/m_t$, where the scalar Fermi field serves as the scalar Higgs field.

(4) Particle mass relationships can be studied in the context of particle Higgs constants instead of particle masses or energies, which brings out the mass relationships in a transparent manner.

(5) The Higgs constants for the basic particle and quark "ground states" are α-quantized, and this empirical α-quantization is an essential feature of a mathematical formalism that ties together these fundamental ground states in a coherent and comprehensive manner.

(6) This formalism serves as the first step toward a theoretical framework that, as Lederman and Hill note [1], is required for a meaningful interpretation of the Higgs coupling constants.

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An schematic diagram of the electron (e) to top quark (t) excitation path

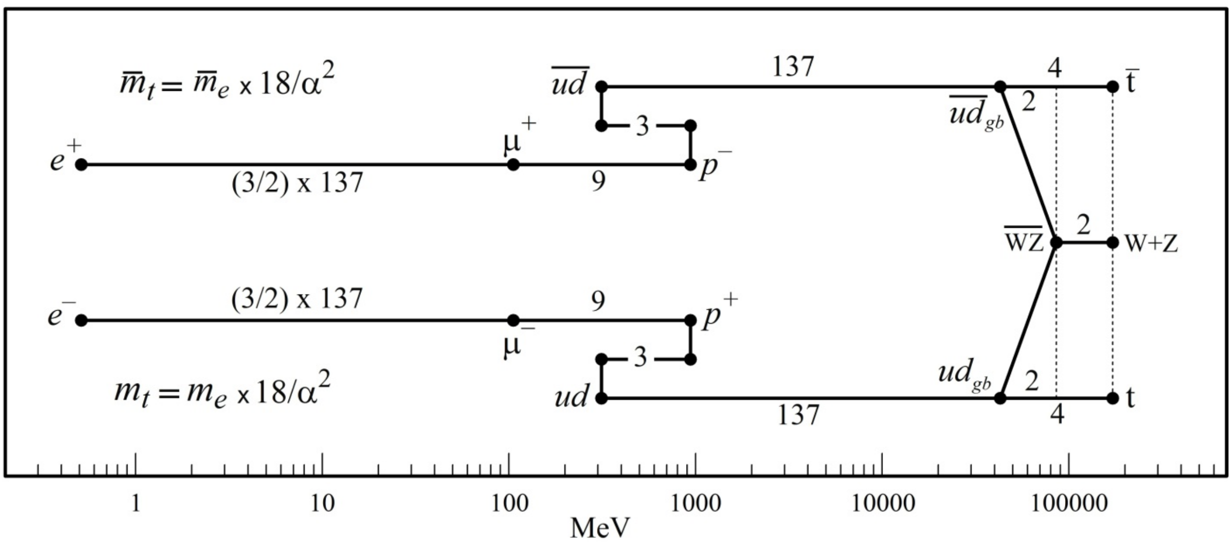

Fig. 1 TheEinstein-mass diagram for the generation of the top quark t from the electron e, showing two factor-of-137 α-boosts. The resulting Einstein mass equation, $m_t = (18/\alpha^2)m_e$, which features the *renormalized* fine structure constant α, is accurate to 0.1%. The quark states ud_p and ud_{gb} represent energy-averaged proton and gauge boson u and d quarks, respectively.

Calculated and experimental Higgs coupling constants
in the top quark representation

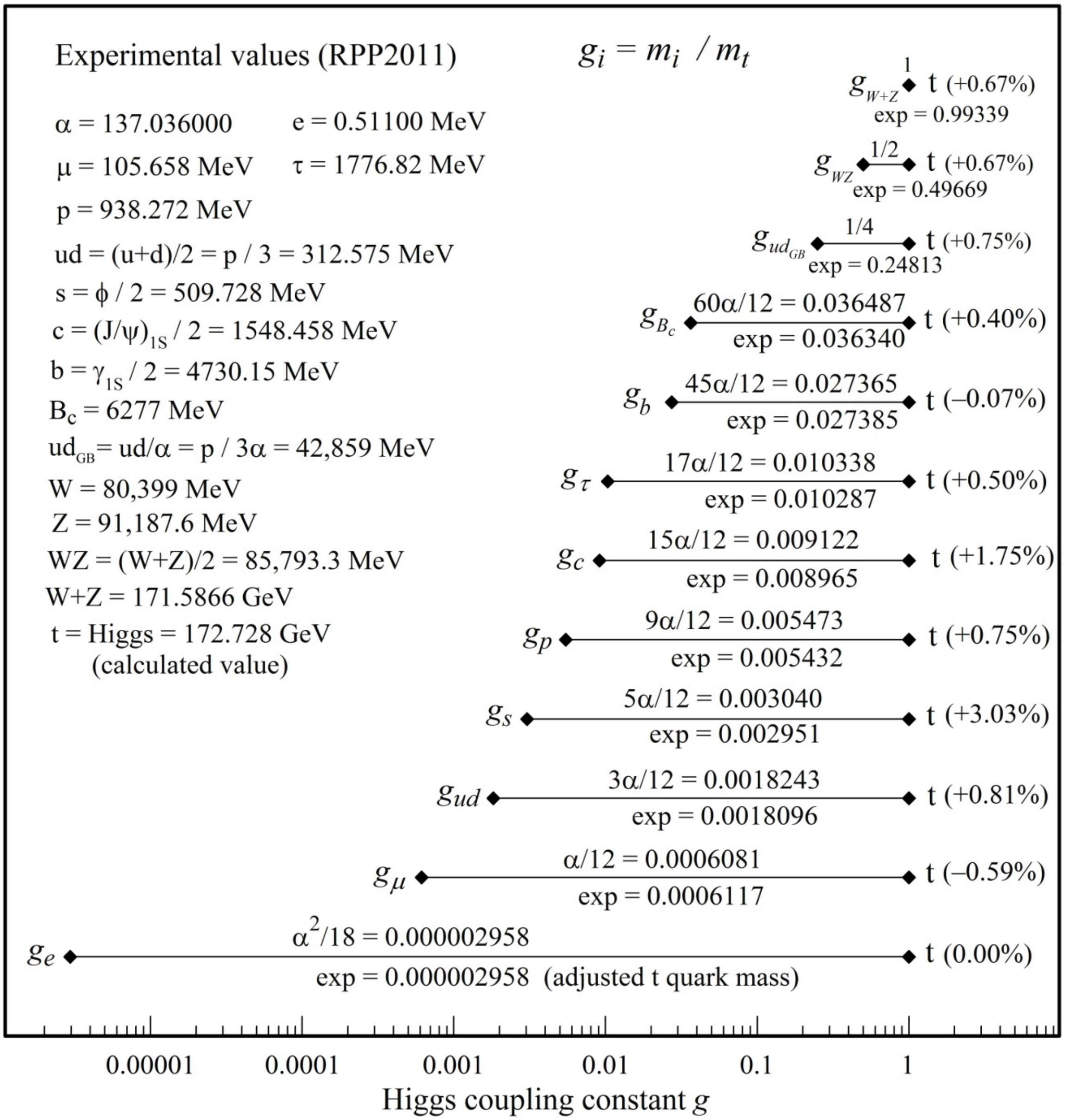

Fig. 2. Calculated and experimental Higgs coupling constants g, shown together with the accuracies of the α-quantized theoretical values (in parentheses) and the data used for the experimental values. The three α-quantized Higgs mass levels are clearly in evidence in this graphical portrayal of the Higgs coupling constants. This α-quantized coupling constant formalism contains no adjustable parameters, and its accuracy level is better than 1%, except for the s and c quarks, where low-mass binding-energy corrections [9] are needed (but are not applied).

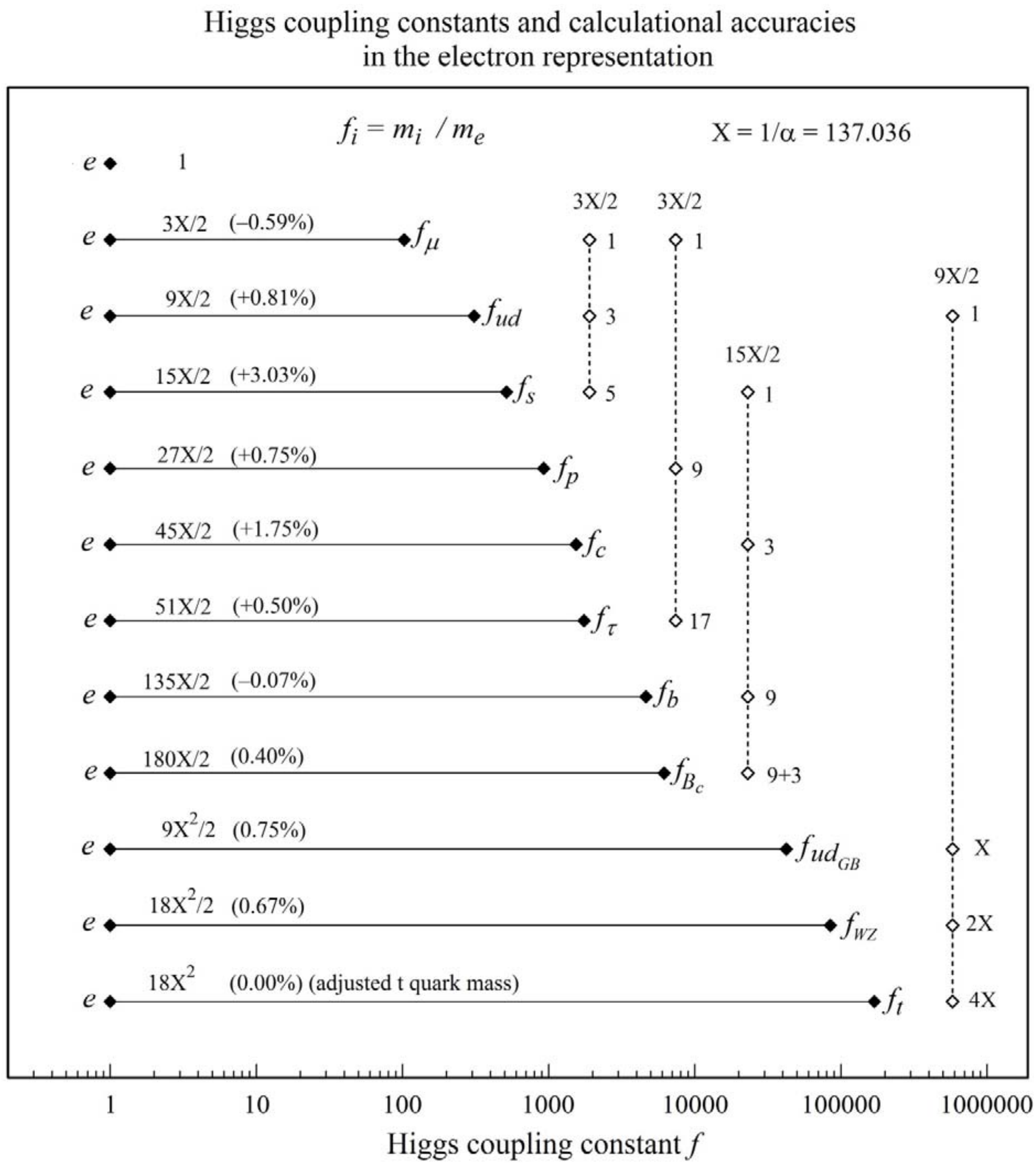

Fig. 3. The particle coupling constants of Fig. 2 converted into electron-based $X \cong 137$ Einstein mass units. Three X-quantized mass levels are clearly in evidence. Four mass-related groups of particles are delineated by vertical links, as follows: *(1)* the *ud* and *s* quarks as multiples of the muon mass; *(2)* the accurate (0.59%) linear mass spacings of the muon, proton, and tau lepton; *(3)* the successive mass triplings of the *s*, *c*, *b* quarks, and the $B_c = b\overline{c}$ meson mass; *(4)* the tevatron collision X-boost from $ud \cdot \overline{ud}$ to $ud_{GB} \cdot \overline{ud}_{GB}$ quark pairs, which then combine to form the W and Z gauge bosons and top quark *t* as a mass triad on the X^2 Higgs level.