

# Fine structure constant and square root of Planck momentum

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The natural constants  $G$ ,  $h$ ,  $e$  and  $m_e$  are commonly used but are themselves difficult to measure experimentally with a high precision. Defining the Planck Ampere in terms of the square root of Planck momentum, referred to here as Quintessence momentum, and by assigning a formula for the electron as a magnetic monopole in terms of  $e$  and  $c$ , a formula for the Rydberg constant can be derived.  $G$ ,  $h$ ,  $e$  and  $m_e$  can then each be written in terms of more precise constants; the speed of light  $c$  (fixed value), the Rydberg constant (12 digit precision) and alpha, the fine structure constant (10 digit precision).

## 1 Introduction

Some things never change. Physicists call them the constants of nature. Such quantities as the velocity of light,  $c$ , Newton's constant of gravitation,  $G$ , and the mass of the electron,  $m_e$ , are assumed to be the same at all places and times in the universe. They form the scaffolding around which the theories of physics are erected, and they define the fabric of our universe. Physics has progressed by making ever more accurate measurements of their values. And yet, remarkably, no one has ever successfully predicted or explained any of the constants. Physicists have no idea why they take the special numerical values that they do. In SI units,  $c$  is 299,792,458;  $G$  is 6.673e-11; and  $m_e$  is 9.10938188e-31 -numbers that follow no discernible pattern. The only thread running through the values is that if many of them were even slightly different, complex atomic structures such as living beings would not be possible. The desire to explain the constants has been one of the driving forces behind efforts to develop a complete unified description of nature, or "theory of everything." Physicists have hoped that such a theory would show that each of the constants of nature could have only one logically possible value. It would reveal an underlying order to the seeming arbitrariness of nature. [1]

## 2 Quintessence momentum

I have assigned the letter  $Q$  to represent the square root of Planck momentum. [2]

$$Q = 1.019\ 113\ 4112\dots \text{ units} = \sqrt{\frac{kg.m}{s}} \quad (1)$$

the integer constants become;

$$\text{Planck momentum} = 2.\pi.Q^2, \text{ units} = \frac{kg.m}{s} \quad (2)$$

$$m_p = \frac{2.\pi.Q^2}{c}, \text{ units} = kg \quad (3)$$

$$G = \frac{l_p.c^3}{2.\pi.Q^2}, \text{ units} = \frac{m^3}{kg.s^2} \quad (4)$$

$$h = 2.\pi.Q^2.2.\pi.l_p, \text{ units} = \frac{kg.m^2}{s} \quad (5)$$

$$\hbar = 2.\pi.Q^2.l_p \quad (6)$$

## 3 Ampere

A formula for a Planck Ampere is proposed.

$$A = \frac{8.c^3}{\pi.\alpha.Q^3}, \text{ units} = \frac{m^2}{kg.s^2.\sqrt{(kg.m/s)}} \quad (7)$$

Planck time

$$t_p = \frac{2.\pi.l_p}{c} \quad (8)$$

And as elementary charge  $e=A.s$

$$e = \frac{16.l_p.c^2}{\alpha.Q^3}, \text{ units} = \frac{m^2}{kg.s.\sqrt{(kg.m/s)}} \quad (9)$$

## 4 Magnetic (electric) constant

In a vacuum, the force per meter of length between the two infinite straight parallel conductors carrying a current of 1 A and spaced apart by 1 m, is exactly  $2.10^{-7}N/m$  [3]

$$\mu_0 = 4.\pi.10^{-7} N/A^2$$

Planck force  $F_p$  [4]

$$F_p = \frac{E_p}{l_p} = \frac{2.\pi.Q^2.c}{l_p} \quad (10)$$

The electric force is weaker than the strong force by a factor of alpha.

$$F_{electric} = \frac{F_p}{\alpha} \quad (11)$$

Using Amperes force law and from eqn.7

$$\mu_e = \frac{F_{electric}}{A^2} \quad (12)$$

$$\mu_e = \pi \cdot \mu_0 \quad (13)$$

Giving

$$\mu_0 = \frac{\pi^2 \cdot \alpha \cdot Q^8}{32 \cdot l_p \cdot c^5} \quad (14)$$

$$\epsilon_0 = \frac{32 \cdot l_p \cdot c^3}{\pi^2 \cdot \alpha \cdot Q^8} \quad (15)$$

$$k_e = \frac{\pi \cdot \alpha \cdot Q^8}{128 \cdot l_p \cdot c^3} \quad (16)$$

## 5 Planck length $l_p$

$l_p$  in terms of  $Q$ ,  $\alpha$ ,  $c$ . The magnetic constant  $\mu_0$  has a fixed value. From eqn.14

$$l_p = \frac{\pi^2 \cdot \alpha \cdot Q^8}{27 \cdot \mu_0 \cdot c^5} \quad (17)$$

$$\mu_0 = 4 \cdot \pi \cdot 10^{-7} \text{ N/A}^2$$

$$l_p = \frac{5^7 \cdot \pi \cdot \alpha \cdot Q^8}{c^5} \quad (18)$$

## 6 Electron as magnetic monopole

$m_e$  in terms of  $m_p$ ,  $t_p$ ,  $\alpha$ ,  $e$ ,  $c$ . [6]

The ampere-meter is the SI unit for pole strength (the product of charge and velocity) in a magnet ( $A \cdot m = e \cdot c$ ). A Magnetic monopole [5] is a hypothetical particle that is a magnet with only 1 pole. A dimensionless geometrical formula for the electron is proposed that is a derivative of a magnetic monopole  $\sigma_e$ . Planck mass =  $m_p$ , electron mass =  $m_e$ .

$$m_e = 2 \cdot m_p \cdot t_x \cdot \sigma_e^3 \quad (19)$$

where...

$$\sigma_e = \frac{2 \cdot \pi^2}{3 \cdot \alpha^2 \cdot e_x \cdot c_x} \quad (20)$$

nb. the conversion of Planck time  $t_p$ , elementary charge  $e$  and speed of light  $c$  to 1s, 1C, 1m/s requires dimensionless numbers which are numerically equivalent ( $t_x, e_x, c_x$ ).

$$\frac{t_p}{t_x} = \frac{5.3912 \dots e^{-44} s}{5.3912 \dots e^{-44}} = 1s$$

$$\frac{e}{e_x} = \frac{1.6021764 \dots e^{-19} C}{1.6021764 \dots e^{-19}} = 1C$$

$$\frac{c}{c_x} = \frac{299792458 m/s}{299792458} = 1m/s$$

## 7 Reduced formulas

Replacing  $l_p$  with eqn.18, the natural constants can be reduced to  $Q$ ,  $\alpha$ ,  $c$

$$h = \frac{2^2 \cdot 5^7 \cdot \pi^3 \cdot \alpha \cdot Q^{10}}{c^5} \quad (21)$$

$$e = \frac{2^4 \cdot 5^7 \cdot \pi \cdot Q^5}{c^3} \quad (22)$$

$$m_e = m_p \cdot \frac{\pi^4}{2^8 \cdot 3^3 \cdot 5^{14} \cdot \alpha^5 \cdot Q_x^7} \quad (23)$$

The Rydberg constant  $R_\infty$ , which incorporates the other constants, is the most accurately measured fundamental physical constant with a precision to 12 digits.

$$R_\infty = \frac{m_e \cdot e^4 \cdot \mu_0^2 \cdot c^3}{8 \cdot h^3} \quad (24)$$

$$R_\infty = \frac{\pi^2 \cdot c^5}{2^{10} \cdot 3^3 \cdot 5^{21} \cdot \alpha^8 \cdot Q^{15}} \quad (25)$$

von Klitzing constant  $R_K = h/e^2$

$$R_K = \frac{\pi \cdot \alpha \cdot c}{5000000} \quad (26)$$

Magnetic flux quantum  $\sigma_0 = h/2e$

$$\sigma_0 = \frac{\pi^2 \cdot \alpha \cdot Q^5}{8 \cdot c^2} \quad (27)$$

electron gyromagnetic ratio  $\gamma_e = g_e \cdot \mu_B / \hbar$

$$\gamma_e = \frac{2^{11} \cdot 3^3 \cdot 5^{21} \cdot Q^{10} \cdot \alpha^5 \cdot 1.00115965218076}{\pi^4 \cdot c^2} \quad (28)$$

nb. 1.00115965218076 = electron magnetic moment [7]

## 8 Fine structure constant

We can define  $Q$  using  $R_\infty$ . The numerical values for the natural constants can then be determined using  $\alpha$ ,  $c$ ,  $R_\infty$ . As  $c$  has a fixed value and as the Rydberg constant has a precision several magnitudes greater than the other natural constants, we can use the experimental values for the natural constants to suggest solutions for alpha.

$Q$  in terms of the Rydberg constant, from eqn.25

$$Q^{15} = \frac{\pi^2 \cdot c^5}{2^{10} \cdot 3^3 \cdot 5^{21} \cdot \alpha^8 \cdot R_\infty} \quad (29)$$

CODATA 2010 values

$$\begin{aligned} \alpha &= 137.035\,999\,074(44) \text{ [9]} \\ R_\infty &= 10\,973\,731.568\,539(55) \text{ [7]} \\ h &= 6.626\,069\,57(29) \text{ e} - 34 \text{ [8]} \\ l_p &= 1.616\,199(97) \text{ e} - 35 \text{ [10]} \\ e &= 1.602\,176\,565(35) \text{ e} - 19 \text{ [11]} \\ m_e &= 9.109\,382\,91(40) \text{ e} - 31 \text{ [12]} \\ G &= 6.673\,84(80) \text{ e} - 11 \text{ [14]} \\ \sigma_0 &= 2.067\,833\,758(46) \text{ e} - 15 \text{ [15]} \\ R_K &= 25\,812.807\,4434(84) \text{ [16]} \\ \gamma_e &= 1.760\,859\,708(39) \text{ e}11 \text{ [17]} \end{aligned}$$

Using

$$\begin{aligned} R_\infty &= 10\,973\,731.568\,539 \\ c &= 299792458 \end{aligned}$$

and the mean CODATA values as reference, we have the following solutions for alpha.

$$\begin{aligned} h- &> 137.035\,997\,435 \\ e- &> 137.035\,997\,416 \\ m_e- &> 137.035\,995\,287 \\ \sigma_0- &> 137.035\,996\,427 \\ R_K- &> 137.035\,999\,074 \\ \gamma_e- &> 137.036\,012\,180 \\ G- &> 137.027\,382\,183 \\ l_p- &> 137.031\,783\,789 \end{aligned}$$

We may note that the von Klitzing constant is equivalent to, or derived from, the CODATA alpha value.

$$R_K- > 137.035\,999\,074(44).$$

Using

$$\begin{aligned} \alpha &= 137.035\,999\,074 \\ R_\infty &= 10\,973\,731.568\,539 \\ c &= 299792458 \end{aligned}$$

gives

$$\begin{aligned} h &= 6.626\,069\,148 \text{ e} - 34 \\ l_p &= 1.616\,036\,603 \text{ e} - 35 \\ e &= 1.602\,176\,513 \text{ e} - 19 \end{aligned}$$

$$\begin{aligned} m_e &= 9.109\,382\,323 \text{ e} - 31 \\ G &= 6.672\,497\,199 \text{ e} - 11 \\ \sigma_0 &= 2.067\,833\,691 \text{ e} - 15 \\ \gamma_e &= 1.760\,859\,764 \text{ e}11 \\ Q &= 1.019\,113\,411\,247 \\ \mu_0 &= 4.\pi/10000000 \end{aligned}$$

Refer to online calculator at [www.planckmomentum.com](http://www.planckmomentum.com)

## 9 Summary

The CODATA values for the listed constants are influenced by the CODATA value for alpha and so are of limited use as an independent verification.

The proposed formula for the electron as a symmetrical magnetic monopole was used to formulate the Rydberg constant and so may be justified, both geometrically and numerically. Although the electron is predicted to be slightly aspheric, with a distortion characterized by the electric dipole moment, experimental results indicate that the electron is spherical. [20]

Quintessence momentum was proposed as the link between mass and charge; with the formulas for the natural constants describing geometrical shapes in terms of  $Q$ ,  $c$  and  $\alpha$ .

## 10 Note: Reference formulas

These formulas are cross referenced with common formulas

$$\begin{aligned} \alpha &= \frac{2.h}{\mu_0 \cdot e^2 \cdot c} \\ 2 \cdot 2.\pi \cdot Q^2 \cdot 2.\pi \cdot l_p &= \frac{32.l_p \cdot c^5}{\pi^2 \cdot \alpha \cdot Q^8} \frac{\alpha^2 \cdot Q^6}{256.l_p^2 \cdot c^4} \frac{1}{c} \\ \alpha &= \alpha \end{aligned} \quad (30)$$

$$\begin{aligned} c &= \frac{1}{\sqrt{\mu_0 \cdot \epsilon_0}} \\ \mu_0 \cdot \epsilon_0 &= \frac{\pi^2 \cdot \alpha \cdot Q^8}{32.l_p \cdot c^5} \frac{32.l_p \cdot c^3}{\pi^2 \cdot \alpha \cdot Q^8} = \frac{1}{c^2} \\ c &= c \end{aligned} \quad (31)$$

$$R_\infty = \frac{m_e \cdot e^4 \cdot \mu_0^2 \cdot c^3}{8.h^3}$$

$$m_e = \frac{65536.l_p^4 \cdot c^8}{\alpha^4 \cdot Q^{12}} \frac{\pi^4 \cdot \alpha^2 \cdot Q^{16}}{1024.l_p^2 \cdot c^{10}} c^3 \frac{1}{8} \frac{1}{8.\pi^3 \cdot Q^6 \cdot 8.\pi^3 \cdot l_p^3}$$

$$R_{\infty} = \frac{m_e}{4\pi.l_p.\alpha^2.m_P} \quad (32)$$

$$E_n = -\frac{2.\pi^2.k_e^2.m_e.e^4}{h^2.n^2}$$

$$2.\pi^2 \frac{\pi^2.\alpha^2.Q^{16}}{16384.l_p^2.c^6} m_e \frac{65536.l_p^4.c^8}{\alpha^4.Q^{12}} \frac{1}{4.\pi^2.Q^4.4.\pi^2.l_p^2}$$

$$E_n = -\frac{m_e.c^2}{2.\alpha^2.n^2} \quad (33)$$

$$q_p = \sqrt{4.\pi.\epsilon_0.\hbar.c}$$

$$q_p = \sqrt{4.\pi \frac{32.l_p.c^3}{\pi^2.\alpha.Q^8} 2.\pi.Q^2.l_p c} = \sqrt{\alpha}.e \quad (34)$$

$$r_e = \frac{e^2}{4.\pi.\epsilon_0.m_e.c^2}$$

$$r_e = \frac{256.l_p^2.c^4}{\alpha^2.Q^6} \frac{1}{4.\pi} \frac{\pi^2.\alpha.Q^8}{32.l_p.c^3} \frac{1}{m_e.c^2} = \frac{l_p.m_P}{\alpha.m_e} \quad (35)$$

$$m_e = \frac{B^2.r^2.e}{2.V}$$

$$V_p = \frac{E_p}{e}$$

$$\frac{B^2.r^2.e^2}{E_p} = \frac{\pi^2.\alpha^2.Q^{10}}{64.l_p^4.c^4} l_p^2 \frac{256.l_p^2.c^4}{\alpha^2.Q^6} \frac{1}{2.\pi.Q^2.c}$$

$$\frac{B^2.r^2.e^2}{E_p} = m_P \quad (36)$$

11. <http://physics.nist.gov/cgi-bin/cuu/Value?e>

12. <http://physics.nist.gov/cgi-bin/cuu/Value?me>

13. <http://physics.nist.gov/cgi-bin/cuu/Value?mu0>

14. <http://physics.nist.gov/cgi-bin/cuu/Value?bg>

15. <http://physics.nist.gov/cgi-bin/cuu/Value?flxquhs2e>

16. <http://physics.nist.gov/cgi-bin/cuu/Value?rk—search+for=von+Klitzing+constant>

17. <http://physics.nist.gov/cgi-bin/cuu/Value?gammae>

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2. Plato's Cave (2003), Malcolm Macleod <http://www.platoscode.com>
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4. <http://en.wikipedia.org/wiki/Planck-force>
5. [en.wikipedia.org/wiki/Magnetic-monopole](http://en.wikipedia.org/wiki/Magnetic-monopole)
6. Plato's Cave (2007 rev), Malcolm Macleod <http://www.platoscode.com>
7. <http://physics.nist.gov/cgi-bin/cuu/Value?ryd>
8. <http://physics.nist.gov/cgi-bin/cuu/Value?ha>
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10. <http://physics.nist.gov/cgi-bin/cuu/Value?plkl>