# Spreading of Ultrarelativistic Wave Packet and Geometrical Optics

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*Abstract:*

The red shift of light coming to the Earth from distant objects is usually explained as a consequence of the fact that the Universe is expanding. Such an explanation implies that photons emitted by distant objects travel in the interstellar medium practically without interaction with interstellar matter and hence they can survive their long journey to the Earth. We analyze this assumption by considering wave-packet spreading for an ultrarelativistic particle. We derive a formula which shows that spreading in the direction perpendicular to the particle momentum is very important and cannot be neglected. The implications of the results are discussed.

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# 1 Introduction

The interpretation of astronomical and cosmological data is usually based on the assumption that light coming to the Earth from distant objects travels in the interstellar medium practically without interaction with interstellar matter and hence it can survive its long journey to the Earth. It is also assumed that with a very high accuracy we can describe the propagation of light from distant objects in the framework of geometrical optics. In turn, it is well-known that the approximation of geometrical optics can be obtained from semiclassical approximation in quantum theory.

The main goal of this paper is to investigate semiclassical approximation for ultrarelativistic particles and especially for photons. The main results are derived in Sec. 4. As a preparatory step, in Sec. 2 we describe well-known facts about semiclassical approximation in nonrelativistic quantum mechanics and in Sec. 3 those results are generalized for wave packets of ultrarelativistic particles in momentum space. In Sec. 5 it is shown that in classical electrodynamics the results on the validity of geometrical optics can be obtained by reformulating the results on the validity of semiclassical approximation in quantum theory Finally, Sec. 6 is a discussion.

## 2 Semiclassical approximation in quantum mechanics

In quantum theory, states of a system are represented by elements of a projective Hilbert space. The fact that a Hilbert space $H$ is projective means that if $\psi \in H$ is a state then $const\,\psi$ is the same state. The matter is that not the probability itself but only relative probabilities of different measurement outcomes have a physical meaning. In particular, normalization of states to one is only a matter of convention. In the present paper we assume this convention, i.e. we will work with states $\psi$ such that $||\psi|| = 1$ where $||...||$ is a norm. It is defined such that if $(..., ...)$ is a scalar product in $H$ then $||\psi|| = (\psi, \psi)^{1/2}$.

In quantum theory every physical quantity is described by a selfadjoint operator. Each selfadjoint operator is Hermitian i.e. satisfies the property $(\psi_2, A\psi_1) = (A\psi_2, \psi_1)$ for any states belonging to the domain of $A$. If $A$ is an operator of some quantity then the mean value of the quantity and its uncertainty in state $\psi$ are given by $\bar{A} = (\psi, A\psi)$ and $\Delta A = ||(A - \bar{A})\psi||$, respectively. The condition that a quantity corresponding to the operator $A$ is semiclassical in state $\psi$ can be defined such that $|\Delta A| \ll |\bar{A}|$. This implies that the quantity can be semiclassical only if $|\bar{A}|$ is rather large. In particular, if $\bar{A} = 0$ then the quantity cannot be semiclassical.

Let $B$ be an operator corresponding to another physical quantity and $\bar{B}$ and $\Delta B$ be the mean value and the uncertainty of this quantity, respectively. We can write $AB = \{A, B\}/2 + [A, B]/2$ where the commutator $[A, B] = AB - BA$ is anti-Hermitian and the anticommutator $\{A, B\} = AB + BA$ is Hermitian. Let $[A, B] = -iC$ and $\bar{C}$ be the mean value of the operator $C$.

A question arises whether two physical quantities corresponding to the operators $A$ and $B$ can be simultaneously semiclassical in state $\psi$. Since $||\psi_1||||\psi_2|| \geq |(\psi_1, \psi_2)|$, we have that

$$\Delta A \Delta B \geq \frac{1}{2}|(\psi, (\{A - \bar{A}, B - \bar{B}\} + [A, B])\psi)| \tag{1}$$

Since $(\psi, \{A - \bar{A}, B - \bar{B}\}\psi)$ is real and $(\psi, [A, B]\psi)$ is imaginary, we get

$$\Delta A \Delta B \geq \frac{1}{2}|\bar{C}| \tag{2}$$

This condition is known as a general uncertainty relation between two quantities. A well-known special case is that if $P$ is the $x$ component of the momentum operator and $X$ is the operator of multiplication by $x$ then $[P, X] = -i\hbar$ and $\Delta p \Delta x \geq \hbar/2$. The states where $\Delta p \Delta x = \hbar/2$ are called coherent ones. They are treated such that the momentum and the coordinate are simultaneously semiclassical in a maximal possible extent. A well-known example is that if

$$\psi(x) = \frac{1}{a\sqrt{\pi}} exp[\frac{i}{\hbar} p_0 x - \frac{1}{2a^2}(x - x_0)^2]$$

then $\bar{X} = x_0$, $\bar{P} = p_0$, $\Delta x = a/\sqrt{2}$ and $\Delta p = \hbar/(a\sqrt{2})$.

Consider first a one dimensional motion. In standard textbooks on quantum mechanics, the presentation starts with a wave function $\psi(x)$ in coordinate space since it is implicitly assumed that the meaning of space coordinates is known. Then a question arises why $P = -i\hbar d/dx$ should be treated as the momentum operator. The explanation is as follows.

Consider wave functions having the form $\psi(x) = exp(ip_0x/\hbar)a(x)$ where the amplitude $a(x)$ has a sharp maximum near $x = x_0 \in [x_1, x_2]$ such that $a(x)$ is not small only when $x \in [x_1, x_2]$. Then $\Delta x$ is of the order $x_2 - x_1$ and the condition that the coordinate is semiclassical is $\Delta x \ll |x_0|$. Since $-i\hbar d\psi(x)/dx = p_0\psi(x) - i\hbar exp(ip_0x/\hbar)da(x)/dx$, we see that $\psi(x)$ will be approximately the eigenfunction of $-i\hbar d/dx$ with the eigenvalue $p_0$ if $|p_0a(x)| \gg \hbar|da(x)/dx|$. Since $|da(x)/dx|$ is of the order of $|a(x)/\Delta x|$, we have a condition $|p_0\Delta x| \gg \hbar$. Therefore if the momentum operator is $-i\hbar d/dx$, the uncertainty of momentum $\Delta p$ is of the order of $\hbar/\Delta x$, $|p_0| \gg \Delta p$ and this implies that the momentum is also semiclassical. At the same time, $|p_0\Delta x|/2\pi\hbar$ is approximately the number of oscillations which the exponent makes on the segment $[x_1, x_2]$. Therefore the number of oscillations should be much greater than unity. In particular, semiclassical approximation cannot be valid if $\Delta x$ is very small, but on the other hand, $\Delta x$ cannot be very large since it should be much less than $x_0$. Another justification of the fact that $-i\hbar d/dx$ is the momentum operator is that in the formal limit $\hbar \to 0$ the Schroedinger equation becomes the Hamilton-Jacobi equation.

We conclude that the choice of $-i\hbar d/dx$ as the momentum operator is justified from the requirement that in semiclassical approximation this operator becomes the classical momentum. However, it is obvious that this requirement does not define the operator uniquely: any operator $\tilde{P}$ such that $\tilde{P} - P$ disappears in semiclassical limit, also can be called the momentum operator.

One might say that the choice $P = -i\hbar d/dx$ can also be justified from the following considerations. In nonrelativistic quantum mechanics we assume that the theory should be invariant under the action of the Galilei group, which is a group of transformations of Galilei space-time. The $x$ component of the momentum operator should be the generator corresponding to spatial translations along the $x$ axis and $-i\hbar d/dx$ is precisely the required operator. In this consideration one assumes that space-time has a physical meaning while, as discussed in Ref. [1] and references therein, this is not the case.

As noted in Ref. [1] and references therein, one should start not from space-time but from a symmetry algebra. Therefore in nonrelativistic quantum mechanics we should start from the Galilei algebra and consider its irreducible representations (IRs). For simplicity we again consider a one dimensional case. Let $P_x = P$ be one of representation operators in an IR of the Galilei algebra. We can implement this IR in a Hilbert space of functions $\chi(p)$ such that $\int_{-\infty}^{\infty} |\chi(p)|^2 dp < \infty$ and $P$ is the operator of multiplication by $p$, i.e. $P\chi(p) = p\chi(p)$. Then a question arises how the operator of the $x$ coordinate should be defined. In contrast to the momentum operator, the coordinate one is not defined by the representation and so it should be defined from additional assumptions. Probably a future quantum theory of measurements will make it possible to construct operators of physical quantities from the rules how these quantities should be measured. However, at present we can construct necessary operators only from rather intuitive considerations.

By analogy with the above discussion, one can say that semiclassical wave functions should be of the form $\chi(p) = exp(-ix_0p/\hbar)a(p)$ where the amplitude $a(p)$ has a sharp maximum near $p = p_0 \in [p_1, p_2]$ such that $a(p)$ is not small only when $p \in [p_1, p_2]$. Then $\Delta p$ is of the order of $p_2 - p_1$ and the condition that the momentum is semiclassical is $\Delta p \ll |p_0|$. Since $i\hbar d\chi(p)/dp = x_0\chi(p) + i\hbar exp(-ix_0p/\hbar)da(p)/dp$, we see that $\chi(p)$ will be approximately the eigenfunction of $i\hbar d/dp$ with the eigenvalue $x_0$ if $|x_0a(p)| \gg \hbar|da(p)/dp|$. Since $|da(p)/dp|$ is of the order of $|a(p)/\Delta p|$, we have a condition $|x_0\Delta p| \gg \hbar$. Therefore if the coordinate operator is $X = i\hbar d/dp$, the uncertainty of coordinate $\Delta x$ is of the order of $\hbar/\Delta p$, $|x_0| \gg \Delta x$ and this implies that the coordinate defined in such a way is also semiclassical. We can also note that $|x_0\Delta p|/2\pi\hbar$ is approximately the number of oscillations which the exponent makes on the segment $[p_1, p_2]$ and therefore the number of oscillations should be much greater than unity. It is also clear that semiclassical approximation cannot be valid if $\Delta p$ is very small, but on the other hand, $\Delta p$ cannot be very large since it should be much less than $p_0$. By analogy with the above discussion, the requirement that the operator $i\hbar d/dp$ becomes the coordinate in classical limit does not define the operator uniquely. In nonrelativistic quantum mechanics it is assumed that the coordinate is a well defined physical quantity even on quantum level and that $i\hbar d/dp$ is the most pertinent choice of the operator of this quantity.

The above results can be directly generalized to the three-dimensional case. For example, if the coordinate wave function is chosen in the form

$$\psi(\mathbf{r}) = \frac{1}{\pi^{3/4}a^{3/2}} exp[-\frac{(\mathbf{r}-\mathbf{r}_0)^2}{2a^2} + \frac{i}{\hbar}\mathbf{p}_0\mathbf{r}] \tag{3}$$

then the momentum wave function is

$$\chi(\mathbf{p}) = \int exp(-\frac{i}{\hbar}\mathbf{p}\mathbf{r})\psi(\mathbf{r})\frac{d^3\mathbf{r}}{(2\pi\hbar)^{3/2}} = \frac{a^{3/2}}{\pi^{3/4}\hbar^{3/2}} exp[-\frac{(\mathbf{p}-\mathbf{p}_0)^2a^2}{2\hbar^2} - \frac{i}{\hbar}(\mathbf{p}-\mathbf{p}_0)\mathbf{r}_0] \tag{4}$$

It is easy to verify that

$$||\psi||^2 = \int |\psi(\mathbf{r})|^2 d^3\mathbf{r} = 1, \quad ||\chi||^2 = \int |\chi(\mathbf{p})|^2 d^3\mathbf{p} = 1, \tag{5}$$

the uncertainty of each component of the coordinate operator is $a/\sqrt{2}$ and the uncertainty of each component of the momentum operator is $\hbar/(a\sqrt{2})$. Hence Eqs. (3) and (4) describe a state which is semiclassical in a maximal possible extent.

A well-known fact of quantum theory is that there is no operator having the meaning of the time operator. Hence a problem arises how time should be understood in quantum theory and this problem is discussed in a wide literature (see e.g. Ref. [2]). It is usually assumed that time is a classical parameter such that the dependence of the wave function on time is defined by the Hamiltonian according to the Schroedinger equation. As noted in Sec. 4, in some cases even such an interpretation of time might be problematic but in typical situations this assumption can be treated as a good approximation. In nonrelativistic quantum mechanics the Hamiltonian of the particle with the mass $m$ is $H = \mathbf{p}^2/2m$ and hence, as follows from Eq. (4), in the model discussed above the dependence of the momentum wave function on $t$ is given by

$$\chi(\mathbf{p},t) = \frac{a^{3/2}}{\pi^{3/4}\hbar^{3/2}} exp[-\frac{(\mathbf{p}-\mathbf{p}_0)^2 a^2}{2\hbar^2} - \frac{i}{\hbar}(\mathbf{p}-\mathbf{p}_0)\mathbf{r}_0 - \frac{i\mathbf{p}^2 t}{2m\hbar}] \tag{6}$$

It is easy to verify that for this state the mean value of the operator $\mathbf{p}$ and the uncertainty of each momentum component are the same for the state $\chi(\mathbf{p})$, i.e. those quantities do not change with time.

Consider now the dependence of the coordinate wave function on $t$. This dependence can be calculated by using Eq. (6) and the fact that

$$\psi(\mathbf{r},t) = \int exp(\frac{i}{\hbar}\mathbf{pr})\chi(\mathbf{p},t)\frac{d^3\mathbf{p}}{(2\pi\hbar)^{3/2}} \tag{7}$$

The result of a direct calculation is

$$\psi(\mathbf{r},t) = \frac{1}{\pi^{3/4}a^{3/2}}(1+\frac{i\hbar t}{ma^2})^{-3/2} exp[-\frac{(\mathbf{r}-\mathbf{r}_0-\mathbf{v}_0 t)^2}{2a^2(1+\frac{\hbar^2 t^2}{m^2 a^4})}(1-\frac{i\hbar t}{ma^2}) + \frac{i}{\hbar}\mathbf{p}_0\mathbf{r} - \frac{i\mathbf{p}_0^2 t}{2m\hbar}] \tag{8}$$

where $\mathbf{v}_0 = \mathbf{p}_0/m$ is the semiclassical velocity. This result shows that the semiclassical wave packet is moving along the classical trajectory $\mathbf{r}(t) = \mathbf{r}_0 + \mathbf{v}_0 t$. At the same time, it is now obvious that the uncertainty of each coordinate depends on time as $\Delta x_j(t) = \Delta x_j(0)(1+\hbar^2 t^2/m^2 a^4)^{1/2}$ $(j = 1,2,3)$ where $\Delta x_j(0) = a/\sqrt{2}$, i.e. the width of the wave packet in coordinate representation is increasing. This fact, known as the wave-packet spreading, is described in many textbooks and papers (see e.g. Ref. [3] and references therein). It shows that if a state was semiclassical in the maximal extent at $t = 0$, it will not have this property at $t > 0$ and the accuracy of semiclassical approximation will decrease with the increase of $t$. The characteristic time of spreading can be defined as $t_* = ma^2/\hbar$. For macroscopic bodies this is an extremely large quantity and hence in macroscopic physics the effect of the wave-packet spreading can be neglected.

## 3 Wave packets for relativistic particles in momentum space

The usual approach to Poincare symmetry on quantum level follows. Since classical Minkowski space is invariant under the action of the Poincare group, in quantum theory the angular momentum operators $M^{\mu\nu}$ and the four-momentum operators $P^\mu$ ($\mu, \nu = 0, 1, 2, 3; \quad M^{\mu\nu} = -M^{\nu\mu}$) should satisfy the commutation relations of the Poincare group Lie algebra

$$[P^\mu, P^\nu] = 0 \quad [P^\mu, M^{\nu\rho}] = -i(\eta^{\mu\rho}P^\nu - \eta^{\mu\nu}P^\rho)$$
$$[M^{\mu\nu}, M^{\rho\sigma}] = -i(\eta^{\mu\rho}M^{\nu\sigma} + \eta^{\nu\sigma}M^{\mu\rho} - \eta^{\mu\sigma}M^{\nu\rho} - \eta^{\nu\rho}M^{\mu\sigma}) \tag{9}$$

where $\eta^{\mu\nu}$ is the diagonal metric tensor such that $\eta^{00} = -\eta^{11} = -\eta^{22} = -\eta^{33} = 1$ and for simplicity Eqs. (9) are written in units $\hbar = c = 1$. This approach is in the spirit of the well-known Klein's Erlangen program in mathematics.

However, as we argue in Ref. [1] and references therein, quantum theory should not be based on classical space-time background and the approach should be the opposite. Each system is described by a set of independent operators. By definition, the rules how these operators commute with each other define the symmetry algebra. In particular, *by definition*, Poincare symmetry on quantum level means that the operators commute according to Eq. (9).

The next step in our construction is the definition of elementary particle. Although theory of elementary particles exists for a rather long period of time, there is no commonly accepted definition of elementary particle in this theory. In Ref. [1] and references cited therein we argue that, in the spirit of Wigner's approach to Poincare symmetry [4], a general definition, not depending on the choice of the classical background and on whether we consider a local or nonlocal theory, is that a particle is elementary if the set of its wave functions is the space of an irreducible representation (IR) of the symmetry algebra in the given theory.

There exists a wide literature describing how IRs of the Poincare algebra can be constructed. In particular, an IR can be implemented in a space of functions $\chi(\mathbf{p})$ such that the momentum operator $\mathbf{P}$ is the operator of multiplication by $\mathbf{p}$. For particles with spin, the functions $\chi(\mathbf{p})$ also depend on the spin projections but we will not write this dependence explicitly.

As follows from Eqs. (9), the operator $I_2 = E^2 - \mathbf{P}^2$, where $E = P^0$, is the Casimir operator of the second order, i.e. it is a bilinear combination of representation operators commuting with all the operators of the algebra. As follows from the well-known Schur lemma, all states belonging to an IR are the eigenvectors of $I_2$ with the same eigenvalue which is *denoted* as $m^2$. Then the energy operator is the operator of multiplication by $\pm\epsilon(\mathbf{p})$ where $\epsilon(\mathbf{p}) = (m^2 + \mathbf{p}^2)^{1/2}$. The choice of the energy sign is only a matter of convention but not a matter of principle. Indeed, the energy can be measured only if the momentum $\mathbf{p}$ is measured and then it is only a matter of convention what sign of the square root should be chosen. However, it is important that the sign should be the same for all particles. For example, if we consider a system of two particles with the same values of $m^2$ and the opposite momenta $\mathbf{p}_1$ and $\mathbf{p}_2$ such that $\mathbf{p}_1 + \mathbf{p}_2 = 0$, we cannot define the energies of the particles as $\epsilon(\mathbf{p}_1)$ and $-\epsilon(\mathbf{p}_2)$, respectively, since in that case the total four-momentum of the two-particle system will be zero what contradicts experiment.

The notation $I_2 = m^2$ is justified by the fact that for all known particles this quantity is greater or equal than zero. Then the mass $m$ is *defined* as the square root of $m^2$ and the sign of $m$ is only a matter of convention. The usual convention is that $m \geq 0$. However, from mathematical point of view, IRs with $I_2 < 0$ are not prohibited. If the velocity operator $\mathbf{v}$ is *defined* as $\mathbf{v} = \mathbf{P}/E$ then for known particles $|\mathbf{v}| \leq 1$, i.e. $|\mathbf{v}| \leq c$ in standard units. However, for IRs with $I_2 < 0$ we will have that $|\mathbf{v}| > c$ and, at least from the point of view of mathematical construction of IRs, this case is not prohibited. The hypothetical particles with such properties are called tachyons and their possible existence is widely discussed in the literature. If the tachyon mass $m$ is also defined as the square root of $m^2$ then this quantity will be imaginary. However, this does not mean than the corresponding IRs are unphysical since all the operators of the Poincare group Lie algebra can depend only on $m^2$.

The usual choice of the Hilbert space $H$ for an IR is such that the functions belonging to $H$ are quadratically integrable over the Lorentz invariant measure $d^3\mathbf{p}/\epsilon(\mathbf{p})$. However, it is always possible to perform a unitary transformation such that the Hilbert space will be implemented as a space of functions quadratically integrable over $d^3\mathbf{p}$. For reasons which will be clear below it is convenient for us to use such a normalization.

We conclude that in relativistic quantum theory the four-momentum operators are well defined and have a clear physical meaning. In particular, it is possible to construct IRs describing the photon and other elementary particles.

Consider first a construction of the wave packet for a particle with nonzero mass. A possible way of the construction follows. We first consider the particle in its rest system, i.e. in the reference frame where the mean value of the particle momentum is zero. The wave function $\chi_0(\mathbf{p})$ in this case can be taken as in Eq. (4) with $\mathbf{p}_0 = 0$. As noted in the preceding section, such a state cannot be semiclassical. However, it is possible to obtain a semiclassical state by applying a Lorentz transformation to $\chi_0(\mathbf{p})$. As shown in a wide literature, in standard quantum theory (based on complex numbers) any IR representation of the algebra (9) by Hermitiam operators can be extended to an unitary IR of the Poincare group. One can directly verify that for a spinless particle the unitary representation operator $U(g)$ corresponding to a Lorentz transformation $g$ can be defined as

$$U(g)\chi_0(\mathbf{p}) = [\frac{\epsilon(\mathbf{p}')}{\epsilon(\mathbf{p})}]^{1/2}\chi_0(\mathbf{p}') \tag{10}$$

where $\mathbf{p}'$ is the momentum obtained from $\mathbf{p}$ by the Lorentz transformation $g^{-1}$. If $g$

is the Lorentz boost along the $z$ axis with the velocity $v$ then

$$\mathbf{p}'_\perp = \mathbf{p}_\perp, \quad p'_z = \frac{p_z - v\epsilon(\mathbf{p})}{(1-v^2)^{1/2}} \tag{11}$$

where we use the subscript $\perp$ to denote projections of vectors onto the $xy$ plane.

As follows from this expression, $exp(-\mathbf{p}'^2 a^2/2\hbar^2)$ as a function of $\mathbf{p}$ has the maximum at $\mathbf{p}_\perp = 0$, $p_z = p_{z0} = v[(m^2+\mathbf{p}_\perp^2)/(1-v^2)]^{1/2}$ and near the maximum

$$exp(-\frac{a^2\mathbf{p}'^2}{2\hbar^2}) \approx exp\{-\frac{1}{2\hbar^2}[a^2\mathbf{p}_\perp^2 + b^2(p_z - p_{z0})^2\}$$

where $b = a(1-v^2)^{1/2}$ what represents the effect of the Lorentz contraction. If $m \gg \hbar/a$ (in units where $c = 1$) then $m \gg |\mathbf{p}_\perp|$ and $p_{z0} \approx mv/(1-v^2)^{1/2}$. In this case the transformed state is semiclassical and the mean value of the momentum is exactly the classical (i.e. nonquantum) value of the momentum of a particle with mass $m$ moving along the $z$ axis with the velocity $v$. However, in the opposite case when $m \ll \hbar/a$ the transformed state is not semiclassical since the uncertainty of $p_z$ is of the same order as the mean value of $p_z$.

If the photon mass is exactly zero then the photon cannot have the rest state. However, even if the photon mass is not exactly zero, it is so small that the relation $m \ll \hbar/a$ is certainly satisfied for any realistic value of $a$. Hence a semiclassical state for the photon or a particle with a very small mass cannot be obtained by applying the Lorentz transformation to $\chi_0(\mathbf{p})$ and considering the case when $v$ is very close to unity. In this case we will describe a semiclassical state by a wave function which is a generalization of the function (4):

$$\chi(\mathbf{p},0) = \frac{ab^{1/2}}{\pi^{3/4}\hbar^{3/2}} exp[-\frac{\mathbf{p}_\perp^2 a^2}{2\hbar^2} - \frac{(p_z-p_0)^2 b^2}{2\hbar^2} - \frac{i}{\hbar}\mathbf{p}_\perp\mathbf{r}_\perp - \frac{i}{\hbar}(p_z - p_0)z] \tag{12}$$

Here we assume that the vector $\mathbf{p}_0$ is directed along the $z$ axis and its $z$ component is $p_0$. In the general case the parameters $a$ and $b$ defining the momentum distributions in the transverse and longitudinal directions, respectively, can be different. In that case the uncertainty of each transverse component of momentum is $\hbar/(a\sqrt{2})$ while the uncertainty of the $z$ component of momentum is $\hbar/(b\sqrt{2})$. In view of the above discussion one might think that, as a consequence of the Lorentz contraction, the parameter $b$ should be very small. However, the above discussion shows that the notion of the Lorentz contraction has a physical meaning only if $m \gg \hbar/a$ while for the photon the opposite relation takes place. We will see below that in typical situations the quantity $b$ is large and much greater than $a$.

# 4 The photon in semiclassical approximation

Consider now a problem whether an elementary particle can be described in terms of space-time characteristics. By analogy with the discussion in Sec. 2, we can state that in relativistic quantum theory there is no operator corresponding to time and hence at best $t$ can be only a good parameter defining the evolution of the wave function according to the Schroedinger equation with the relativistic energy operator. A problem also arises whether it is possible to define a physical coordinate operator for an elementary particle. For example, in the representation where $H$ is the space of functions quadratically integrated over $d^3\mathbf{p}$ one might define the position operator by analogy with nonrelativistic quantum mechanics i.e. as $i\hbar\partial/\partial\mathbf{p}$. This is the Newton-Wigner position operator proposed in Ref. [5]. Then the coordinate wave function $\psi(\mathbf{r})$ is again defined by Eq. (3) and a question arises whether this operator has all the required properties of the physical coordinate operator.

Let us first make a few remarks about the terminology of quantum theory. The terms "wave function" and "particle-wave duality" have arisen at the beginning of quantum era in efforts to explain quantum behavior in terms of classical waves but now it is clear that no such explanation exists. The notion of wave is purely classical; it has a physical meaning only as a way of describing systems of many particles by their average characteristics. In particular, such notions as frequency and wave length can be applied only to classical waves, i.e. to systems consisting of many particles such that space-time characteristics of those systems are measured on classical level. If a particle state vector contains $exp[i(px-Et)/\hbar]$ then by analogy with the theory of classical waves one might say that the particle is a wave with the frequency $\omega = E/\hbar$ and the (de Broglie) wave length $\lambda = 2\pi\hbar/p$. However, such defined quantities $\omega$ and $\lambda$ are not real frequencies and wave lengths measured e.g. in spectroscopic experiments where only characteristics of many-particle systems are measured. In quantum theory the photon and other particles can be characterized by their energies, momenta and other quantities for which there exist well defined operators. Those quantities might be measured in collisions of those particles with other particles. The term "wave function" might be misleading since in quantum theory it defines not amplitudes of waves but only amplitudes of probabilities. So, although in our opinion the term "state vector" is more pertinent than "wave function" we will use the latter in accordance with the usual terminology.

In classical theory the notion of field, as well as that of wave, is used for describing systems of many particles by their average characteristics. For example, the electromagnetic field consists of many photons. In classical theory each photon is not described individually but the field as a whole is described by the quantities $\mathbf{E}(\mathbf{r},t)$ and $\mathbf{B}(\mathbf{r},t)$ which can be measured (in principle) by using macroscopic test bodies. In particular, the notions of electric and magnetic fields of a single photon have no physical meaning.

It has been well-known since the 1930s [5] that, when quantum mechanics is combined with relativity, there is no operator satisfying all the properties of the spatial position operator. In other words, the coordinates cannot be exactly measured even in situations when exact measurements are allowed by the non-relativistic uncertainty principle. For example, in the introductory section of the well-known textbook [6] the following arguments are given in favor of this statement. Suppose that we measure the coordinates of an electron with the mass $m$. When the uncertainty of the coordinates is of the order of $\hbar/mc$, the uncertainty of momenta is of the order of $mc$, the uncertainty of energy is of the order of $mc^2$ and hence creation of electron-positron pairs is allowed. As a consequence, it is not possible to localize the electron with the accuracy better than its Compton wave length $\hbar/mc$. Hence, for a particle with a nonzero mass the exact measurement is possible only either in the non-relativistic limit (when $c \to \infty$) or classical limit (when $\hbar \to 0$). If $m = 0$ is possible, the problem becomes even more complicated since the photon can create other photons with lesser energies. However, those arguments do not exclude a possibility that the Newton-Wigner position operator can be meaningful in semiclassical approximation.

In standard textbooks on quantum electrodynamics (see e.g. Ref. [7]) it is stated that in this theory there is no way to define a photon wave function in coordinate representation and the arguments are as follows. The electric and magnetic fields of the photon in coordinate representation are proportional to Fourier transforms of $|\mathbf{p}|^{1/2}\chi(\mathbf{p})$, not of $\chi(\mathbf{p})$. As a consequence, the quantities $\mathbf{E}(\mathbf{r})$ and $\mathbf{B}(\mathbf{r})$ are defined not by $\psi(\mathbf{r})$ but by integrals of $\psi(\mathbf{r})$ over some region. However, this argument also does not exclude the possibility that $\psi(\mathbf{r})$ can have a physical meaning in semiclassical approximation since, as noted above, the notions of the electric and magnetic fields of the single photon do not have a physical meaning.

One more argument that the Newton-Wigner position operator does not have all the required properties follows. If at $t = 0$ the function $\psi(\mathbf{r})$ has a finite carrier (i.e. $\psi(\mathbf{r}) \neq 0$ only if $\mathbf{r}$ belongs to a vicinity of some vector $\mathbf{r}_0$) and the evolution of $\psi(\mathbf{r}, t)$ is governed by the Schroedinger equation with the relativistic energy operator then it is easy to show that at any $t > 0$ the carrier of $\psi(\mathbf{r}, t)$ will belong to the whole three-dimensional space. Then at any $t > 0$ the particle can be detected at any point of the space and this contradicts the requirement that no information should be transmitted with the speed greater than $c$. A rather striking example is a photon emitted in the famous 21cm transition line between the hyperfine energy levels of the hydrogen atom. The phrase that the lifetime of this transition is of the order of $\tau = 10^7$ years implies that the width of the level is of the order of $\hbar/\tau$, i.e. experimentally the uncertainty of the photon energy is $\hbar/\tau$. Hence the uncertainty of the photon momentum is $\hbar/(c\tau)$ and with the above definition of the coordinate operators the uncertainty of the longitudinal coordinate is $c\tau$, i.e. of the order of $10^7$ light years. Then there is a nonzero probability that immediately after its creation at point A the photon can be detected at point B such that the distance between A and B is $10^7$ light years.

A problem arises how this phenomenon should be interpreted. For example, one might say that the requirement that no signal can be transmitted with the speed greater than $c$ has been obtained in Special Relativity which is a classical (i.e. nonquantum) theory which operates only with classical space-time coordinates.

As noted above, from the point of view of quantum theory the existence of tachyons is not prohibited. Note also that when two electrically charged particles exchange by a virtual photon, a typical situation is that the four-momentum of the photon is spacelike, i.e. the photon is the tachyon. On the other hand, a fully opposite explanation (pointed out to me by Alik Makarov) is as follows. We can know about the photon creation only if the photon is detected and when it was detected at point B at the moment of time $t = t_0$, this does not mean that the photon travelled from A to B with the speed greater than $c$ since the time of creation has an uncertainty of the order of $10^7$ years. After the detection, the uncertainties of the coordinates have the order of the dimensions of the detector. Note also that in this situation a description of the system (atom + electric field) by the wave function (e.g. in the Fock space) depending on a continuous parameter $t$ has no physical meaning (since roughly speaking the quantum of time in this process is of the order of $10^7$ years). If we accept this explanation then we should acknowledge that in some situations a description of evolution by a continuous classical parameter $t$ is not physical. This is in the spirit of the Heisenberg S-matrix program that in quantum theory one can describe only transitions of states from the infinite past when $t \to -\infty$ to the distant future when $t \to +\infty$.

The above discussion shows that on quantum level the physical meaning of the coordinate is not clear but at least there are reasons to think that in some cases the transverse component of the Newton-Wigner position operator has a physical meaning in semiclassical approximation (see also Sec. 5). If we also assume that in some situations time is a good approximate parameter describing the evolution according to the Schroedinger equation with the relativistic Hamiltonian then the dependence of the momentum wave function (12) on $t$ is given by

$$\chi(\mathbf{p},t) = exp(-\frac{i}{\hbar}pct)\chi(\mathbf{p},0) \tag{13}$$

where $p = |\mathbf{p}|$ and we assume that the particle is ultrarelativistic, i.e. $p \gg m$.

In view of the above discussion, the function $\psi(\mathbf{r},t)$ can be again defined by Eq. (7) where now $\chi(\mathbf{p},t)$ is defined by Eq. (13). If the variable $p_z$ in the integrand is replaced by $p_0 + p_z$ then as follows from Eqs. (7,12,13)

$$\psi(\mathbf{r},t) = \frac{ab^{1/2}exp(i\mathbf{p}_0\mathbf{r}/\hbar)}{\pi^{3/4}\hbar^{3/2}(2\pi\hbar)^{3/2}}\int exp\{-\frac{\mathbf{p}_\perp^2 a^2}{2\hbar^2} - \frac{p_z^2 b^2}{2\hbar^2} - \frac{i}{\hbar}\mathbf{pr}$$
$$-\frac{ict}{\hbar}[(p_z+p_0)^2 + \mathbf{p}_\perp^2]^{1/2}\}d^3\mathbf{p} \tag{14}$$

We now take into account the fact that in semiclassical approximation the quantity $p_0$ should be much greater than the uncertainties of the momentum in the longitudinal and transversal directions, i.e. $p_0 \gg p_z$ and $p_0 \gg |\mathbf{p}_\perp|$. Hence with a good accuracy we can expand the square root in the integrand in powers of $|\mathbf{p}|/p_0$. Taking into account the linear and quadratic terms in the square root we get

$$[(p_z + p_0)^2 + \mathbf{p}_\perp^2]^{1/2} \approx p_0 + p_z + \mathbf{p}_\perp^2/2p_0$$

Then the integral over $d^3\mathbf{p}$ can be calculated as the product of integrals over $d^2\mathbf{p}_\perp$ and $dp_z$ and the calculation is analogous to that in Eq. (8). The result of the calculation is

$$\psi(\mathbf{r},t) = [\pi^{3/4}ab^{1/2}(1+\frac{i\hbar ct}{p_0a^2})]^{-1}exp[\frac{i}{\hbar}(\mathbf{p}_0\mathbf{r} - p_0ct)]$$
$$exp[-\frac{(\mathbf{r}_\perp - \mathbf{r}_{0\perp})^2(1-\frac{i\hbar ct}{p_0a^2})}{2a^2(1+\frac{\hbar^2c^2t^2}{p_0^2a^4})} - \frac{(z-z_0-ct)^2}{2b^2}] \tag{15}$$

This result shows that the wave packet describing an ultrarelativistic particle (including a photon) is moving along the classical trajectory $z(t) = z_0 + ct$, in the longitudinal direction there is no spreading while in the transversal direction spreading is characterized by the function

$$a(t) = a(1+\frac{\hbar^2c^2t^2}{p_0^2a^4})^{1/2} \tag{16}$$

The characteristic time of spreading can be defined as $t_* = p_0a^2/\hbar c$. If $t \gg t_*$ the transversal width of the packet is $a(t) = \hbar ct/p_0a$. Hence the speed of spreading in the transversal direction is $v_* = \hbar c/p_0a$.

# 5 Geometrical optics

The relation between quantum and classical electrodynamics is well-known and is described in textbooks (see e.g. Ref. [7]). As already noted, classical electromagnetic field consists of many photons and in classical electrodynamics the photons are not described individually. Instead, classical electromagnetic field is described by field strengths which represent average characteristics of a large set of photons. For constructing the field strengths one can use the photon wave functions $\chi(\mathbf{p},t)$ or $\psi(\mathbf{r},t)$ where $E$ is replaced by $\hbar\omega$ and $\mathbf{p}$ is replaced by $\hbar\mathbf{k}$. Then the functions will not contain any dependence on $\hbar$ (note that the normalization factor $\hbar^{-3/2}$ in $\chi(\mathbf{k},t)$ will disappear since the normalization integral for $\chi(\mathbf{k},t)$ is now over $d^3\mathbf{k}$, not $d^3\mathbf{p}$). The quantities $\omega$ and $\mathbf{k}$ are now treated, respectively, as the frequency and the wave vector of the classical electromagnetic field and the functions $\chi(\mathbf{k},t)$ and $\psi(\mathbf{r},t)$ are interpreted not such that they describe probabilities for a single photon but such that they describe classical electromagnetic field and $\mathbf{E}(\mathbf{r},t)$ and $\mathbf{B}(\mathbf{r},t)$ can be constructed from these functions as described in textbooks on quantum electrodynamics (see e.g. Ref. [7]).

As noted in the preceding section, some authors (see e.g. Ref. [7]) state that the function $\psi(\mathbf{r},t)$ cannot be interpreted as the photon wave function in coordinate representation since for each value of $\mathbf{r}$, $\mathbf{E}(\mathbf{r},t)$ and $\mathbf{B}(\mathbf{r},t)$ depend not only on $\psi(\mathbf{r},t)$ but on the values of the function $\psi$ in some vicinity of $\mathbf{r}$. This vicinity has dimensions of the order of the wave length. Hence, for example for visible light, where the wave length is of the order of hundreds of nanometers, the quantity $\mathbf{r}$ can be treated as a good coordinate in semiclassical approximation. Another argument in favor of this statement is that in classical electrodynamics the quantities $\mathbf{E}(\mathbf{r},t)$ and $\mathbf{B}(\mathbf{r},t)$ for the free field should satisfy the wave equation $\partial^2\mathbf{E}/c^2\partial t^2 = \Delta\mathbf{E}$ and analogously for $\mathbf{B}(\mathbf{r},t)$. Hence if $\mathbf{E}(\mathbf{r},t)$ and $\mathbf{B}(\mathbf{r},t)$ are constructed from $\psi(\mathbf{r},t)$ as described in textbooks (see e.g. Ref. [7]), they will satisfy the wave equation since, as follows from Eqs. (7,12,13), $\psi(\mathbf{r},t)$ also satisfies this equation.

The approximation of geometrical optics can be formulated in full analogy with semiclassical approximation in quantum theory. This approximation implies that if $\mathbf{k}_0$ and $\mathbf{r}_0$ are the average values of the wave vector and the spatial radius vector for a wave packet describing the electromagnetic wave then the uncertainties $\Delta k$ and $\Delta r$, which are the average values of $|\mathbf{k}-\mathbf{k}_0|$ and $|\mathbf{r}-\mathbf{r}_0|$, respectively, should satisfy the requirements $\Delta k \ll |\mathbf{k}_0|$ and $\Delta r \ll |\mathbf{r}_0|$. Analogously, in full analogy with the derivation of Eq. (2), one can show that for each $j=1,2,3$ the uncertainties of the corresponding projections of the vectors $\mathbf{k}$ and $\mathbf{r}$ satisfy the requirement $\Delta k_j \Delta r_j \geq 1/2$ (see e.g. Ref. [8]). In particular, an electromagnetic wave satisfies the approximation of geometrical optics in the greatest possible extent if $\Delta k \Delta r$ is of the order of unity.

In view of this discussion, the results of the preceding section can be fully applied to spreading of wave packet describing the classical electromagnetic wave. In particular, the parameters of spreading can be characterized by the function $a(t)$ (see Eq. (16)) and the quantities $t_*$ and $v_*$ (see the end of the preceding section) which in terms of the wave length $\lambda = 2\pi c/\omega_0$ can be written as

$$a(t) = a\Big(1+\frac{\lambda^2 c^2 t^2}{4\pi^2 a^4}\Big)^{1/2}, \quad t_* = \frac{2\pi a^2}{\lambda c}, \quad v_* = \frac{\lambda c}{2\pi a} \tag{17}$$

The quantity $N_{||} = b/\lambda$ shows how many oscillations the oscillating exponent in Eq. (15) makes in the region where the wave function or the amplitude of the classical wave is significantly different from zero. As noted in Sec. 2, for the validity of semiclassical approximation this quantity should be very large. In nonrelativistic quantum mechanics $a$ and $b$ are of the same order and hence the same can be said about the quantity $N = a/\lambda$. As noted above, in the case of the photon we don't know the relation between $a$ and $b$. In terms of the quantity $N$ we can rewrite the expressions for $t_*$ and $v_*$ in Eq. (17) as

$$t_* = 2\pi N^2 T, \quad v_* = \frac{c}{2\pi N} \tag{18}$$

Hence the accuracy of semiclassical approximation (or the approximation of geometrical optics in classical electrodynamics) increases with the increace of $N$.

# 6 Discussion

In plain language, the problem discussed in this paper can be formulated as follows. Is the fact that we can see distant stars and even planets compatible with the another known fact that the wave function of the photon which has managed to survive its long journey to the Earth was the subject of the wave-packet spreading? For understanding this problem we should first answer the question of what can be said about the characteristics of photons coming to the Earth from distance objects.

Typical conclusions based on numerous experiments with light coming to the Earth from the Sun are as follows. We know that with a good accuracy this light can be described in the framework of geometrical optics i.e. one can approximately treat the light as a collection of particles moving along classical trajectories. Since we know that the photons came from the Sun then with a good accuracy we know the direction of their momenta and since we know the distribution of wave lengths then (in the approximation described in Sec. 5) we know the distribution of photon energies.

The next question is what we know about the width of the coordinate photon wave functions in the direction perpendicular to the photon momentum in the approximation when the coordinate operators are defined as in Sec. 4. Suppose that a wide beam of light falls on a screen which is perpendicular to the direction of light. Suppose that the total area of the screen is $S$ but the surface contains slits with the total area $S_1$. We are interested in the question of what part of the light will pass the screen. One might think that the obvious answer is that the part equals $S_1/S$. This answer follows from the picture that the light consists of many photons moving along geometrical trajectories and hence only the $S_1/S$ part of the photons will pass the surface. Numerous experiments show that deviations from the above answer begin to manifest in interference experiments where dimensions of slits and distances between them have the order of tens or hundreds of microns or even less. Hence one can conclude that the width of the photon wave functions cannot be of the order of say centimeters or meters since in that case deviations from the $S_1/S$ law would be visible if the slits and the distances between them would have the corresponding dimensions but this does not happen.

Consider, for example, the Lyman transition $2P \rightarrow 1S$ in the hydrogen atom on the Sun. In this case the energy of the photon is $E = 10.2eV$, its wave length is $\lambda = 121.6nm$, the lifetime is $\tau = 1.6 \cdot 10^{-9}s$ and the period of the wave is $T \approx 4 \cdot 10^{-16}s$. Since the lifetime is rather small, one might think that a description of the process by a continuous parameter $t$ is a good approximation (see the discussion in Sec. 4). Hence the phrase that the lifetime is $\tau$ can be interpreted such that the uncertainty of the energy is $\hbar/\tau$, the uncertainty of the longitudinal momentum is

$\hbar/c\tau$ and $b$ is of the order of $c\tau \approx 0.48m$ or greater. In view of the above discussion, the estimation $a \approx b$ seems to be very favorable since one might expect that the value of $a$ is much less than $0.48m$. With this estimation $N = a/\lambda \approx 4 \cdot 10^6$. So the value of N is rather large and in view of Eq. (18) one might think that the effect of spreading is not important. However, this is not the case since, as follows from Eq. (18), $t_* \approx 0.04s$. Since the distance between the Sun and the Earth is approximately $t = 8$ light minutes and this time is much greater than $t_*$, the width of the wave packet when it arrives to the Earth is $v_* t \approx 5760m$. It is obvious from the above discussion that such a value of the width is unrealistically large. On the other hand, if we assume that the initial value of $a$ is of the order of several wave lengths then the value of $N$ is much less and the width of the wave packet coming to the Earth is much greater.

Consider now a photon which was created in the same reaction but on Sirius which is the brightest star on our sky. Since the distance to Sirius is 8.6 light years, an analogous estimation shows that even in the favorable scenario the width of the wave packet coming to the Earth from Sirius will be approximately equal to $3 \cdot 10^6 km$ but in less favorable situations the width will be much greater.

A standard understanding of light coming to the Earth from the Sun is such that the major part of the light comes not from transitions between atomic levels but from processes which can be approximately described as a black body radiation and in that case the spectrum of the radiation is approximately continuous. In that case we cannot estimate the quantity $a$ as above. However, even if we take for $a$ a very favorable value of the same order as above we obviously will come to the same conclusion that the width of the wave packet will be unreasonably high.

One might say that the expression for $v_*$ in Eqs. (17) and (18) resembles a well-known phenomenon of diffraction: if a wave encounters an obstacle having a dimension $d$ it begins to diverge and the angle of divergence is of the order of $\lambda/d$. However, the phenomenon of wave-packet spreading implies that the width of the wave packet in the transversal direction is growing even when the packet propagates in empty space. This phenomenon takes place only for wave function in coordinate representation while the distribution of momenta remains unchanged. As a consequence, even when at some moment of time the packet was maximally semiclassical (i.e. for each component of the coordinate and momentum the product of their uncertainties is of the order of $\hbar$), this property is not conserved with time since uncertainties of coordinates in the transversal direction become greater. The problem of the wave-packet spreading in the ultrarelativistic case has been discussed in a wide literature but typically the authors consider cases when the law of dispersion $\omega(k)$ changes as a result of propagation in a medium, when there are obstacles etc. On the other hand, the propagation of light in the empty space in the approximation of geometrical optics is sometimes characterized such that light behaves as particles, for examples as bullets. However, the phenomenon of the wave-packet spreading takes place for bullets too. In this sense the difference between bullets and photons is only quantitative: since bullets have large masses and dimensions, the characteristic time of spreading for them is extremely large (see the end of Sec. 2) while for photons this time is much smaller.

In view of the above discussion a problem arises why we can see separate stars at all. Indeed, if the width of a wave packet is growing as discussed in the above examples then we would see not separate stars but only a continuous background of light coming to us from many stars. A possible explanation might be such that the interaction of light with the interstellar medium cannot be neglected.

On quantum level a process of propagation of photons in the medium is rather complicated because several mechanisms of propagation should be taken into account. For example, a possible process is such that a photon can be absorbed by an atom and reemitted in approximately the same direction. This process makes it clear why the speed of light in the medium is less than $c$: because the atom which absorbed the photon is in an excited state for some time before reemitting the photon. However, this process is also important from the following point of view: even if the coordinate photon wave function had a large width before absorption, as a consequence of the phenomenon known as the collapse of the wave function, the wave function of the emitted photon will have in general much smaller dimensions since after detection the width is defined only by parameters of the corresponding detector. If the photon encounters many atoms on its way, this process does not allow the photon wave function to spread significantly. Analogous remarks can be made about other processes, for example about rescattering of photons on large groups of atoms, rescattering on elementary particles if they are present in the medium etc.

In 1678 Huygens proposed a principle (later developed by Fresnel) that every point reached by a light wave can be treated as if it is an imagined source of a secondary wave. If the above qualitative picture of light propagation is realistic then the Hyugens principle can be understood such that the source of secondary waves is not imagined but real.

The above qualitative picture of light propagation in the interstellar medium seems also reasonable in view of hypotheses that the density of the interstellar medium is much greater than usually believed. Among the most popular scenarios are dark energy, dark matter etc. As shown in our papers (see e.g. Refs. [9, 1] and references therein), the phenomenon of the cosmological acceleration can be easily and naturally explained from first principles of quantum theory without involving dark energy, empty space-background and other artificial notions. On the other hand, the other scenarios seem to be more realistic and one might expect that they will be intensively investigated.

Consider now the 21 cm radio emission line discussed above. Here $\tau \approx 10^7$ light years, $\lambda \approx 21.1cm$ and $T \approx 7 \cdot 10^{-10}s$. In the scenario when the quantity $b$ can be estimated as $c\tau \approx 10^{23}m$ (see the above discussion) and $a$ has the same order as $b$, we get that $N \approx 5 \cdot 10^{23}$ and $t_* \approx 3 \cdot 10^{31}$ years. Hence in this scenario the effect of the wave-packet spreading is negligible. However, if we assume that the quantity

$a$ is of the order of several wave lengths then the situation will be fully different.

If our qualitative picture is realistic then the interpretation of several well-known facts in astronomy and cosmology should be reconsidered. For example, a part of the red shift of light coming to us from distant objects can be a consequence of the fact that we observe not photons emitted by stars many years ago but photons which reached the Earth as a result of many rescatterings and reemissions. It is reasonable to expect that energies of such photons are less than energies of photons originally emitted by stars and the greater the distance to a star is, the greater is the energy loss. There exists a vast literature where the authors argue that the phenomenon of the red shift can be explained not only by the Doppler effect. However, to the best of our knowledge, the effect of wave-packet spreading has not been considered in this literature.

Another phenomenon which might be reconsidered is as follows. As already noted, if a photon reaches the Earth without interaction with the interstellar medium, then its wave function is spread in a great extent and this photons cannot be described in the framework of geometrical optics. It is well-known that some low energy photons (e.g. those emitted in the 21cm transition line) can propagate in the interstellar space practically without interaction with the interstellar medium. Hence when we observe photons coming to the Earth in this part of spectrum, we can see not separate objects but almost a continuous background of photons coming from many objects. In this scenario it is possible that at least a part of photons which are believed to belong to the relic radiation, in fact have been emitted by existing stars since their spectrum also will be isotropic in a great extents.

The above discussion shows that the wave-packet spreading has important implications in astronomy and cosmology and should be taken into account for explaining several well-known phenomena.

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