THE TWIN PRIMES CONJECTURE - SOLUTIONS

Bertrand Wong, Eurotech, Singapore, Email: bwong8@singnet.com.sg

ABSTRACT

The author had published a paper on the solutions for the twin primes conjecture in an international mathematics journal in 2003. This paper, which consists of 5 parts that are each self-contained, presents strong arguments which support the validity of the twin primes conjecture.

MSC: 11-XX (Number Theory)

INTRODUCTION

In 1919, Viggo Brun (1885 - 1978) proved that the sum of the reciprocals of the twin primes converges to Brun's constant:

1/3 + 1/5 + 1/7 + 1/11 + 1/13 + 1/17 + 1/19 + …. = 1.9021605 ….

It is evident that the twin primes thin out as infinity is approached. The problem of whether there is an infinitude of twin primes is an inherently difficult one to solve, as infinity (normally symbolised by: ∞) is a difficult concept and is against common sense. It is impossible to count, calculate or live to infinity, perhaps with the exception of God. Infinity is a nebulous idea and appears to be only an abstraction devoid of any actual practical meaning. How do we quantify infinity? How big is infinity? We could either attempt to prove that the twin primes are finite, or, infinite. If the twin primes were finite, how could we prove that a particular pair of twin primes is the largest existing pair of twin primes, and, if they were infinite, how could we prove that there are always larger and larger pairs of them? It is evidently difficult to prove either, with the former appearing more difficult to prove as the odds seem against it. This paper provides proof of the latter, i.e., the infinitude of the twin primes.

PART 1

Keywords: indivisible, impossibility, new primes/twin primes

Theorem:- The twin primes are infinite.

Proof:-

Let 3, 5, 7, 11, 13, 17, 19, …….., n - 2, n be the list of consecutive primes, wherein n & n - 2 are assumed to be the largest existing twin primes pair, within the infinite list of the primes.

Let 3 x 5 x 7 x 11 x 13 x 17 x 19 = a .

Lemma: (a x …….. x n - 2 x n) - 2, &, (a x …….. x n - 2 x n) - 4 will never be divisible by any of the consecutive primes in the list: 3, 5, 7, 11, 13, 17, 19, …….., n - 2, n, whether they are prime or composite. (See appendix 1.)

This implies that:

If (a x …….. x n - 2 x n) - 2 &/V (a x …….. x n - 2 x n) - 4 are prime, then:
(a x …….. x n - 2 x n) - 2 > (a x …….. x n - 2 x n) - 4 > n > n - 2

If (a x …….. x n - 2 x n) - 2 &/V (a x …….. x n - 2 x n) - 4 are non-prime/composite, then:

(a) each prime factor, e.g., y below, of (a x …….. x n - 2 x n) - 2 > n > n - 2
(b) each prime factor, e.g., z below, of (a x …….. x n - 2 x n) - 4 > n > n - 2

(a x …….. x n - 2 x n) - 2 = prime V composite (i)

(a x …….. x n - 2 x n) - 4 = prime V composite (ii)

(i) & (ii) = twin primes, if both (i) & (ii) are prime

(i) & (ii) > n & n - 2

Let Y represent the prime factors of (a x …….. x n - 2 x n) - 2 if (a x …….. x n - 2 x n) - 2 is not prime (i.e., it is composite), each prime factor may pair up with another prime which differs from it by 2 to form twin primes. Let y = prime factor in Y .

y & y +/- 2 = twin primes, if y +/- 2 is prime

y & y +/- 2 > n & n - 2

Let Z represent the prime factors of (a x …….. x n - 2 x n) - 4 if (a x …….. x n - 2 x n) - 4 is not prime (i.e., it is composite), each prime factor may pair up with another prime which differs from it by 2 to form twin primes. Let z = prime factor in Z .

z & z +/- 2 = twin primes, if z +/- 2 is prime

z & z +/- 2 > n & n - 2

Therefore: (a x …….. x n - 2 x n) - 2 > (a x …….. x n - 2 x n) - 4 > y V y +/- 2 V z V z +/- 2 > n > n - 2

By the above, the following, which implies that n & n - 2 are the largest existing twin primes pair, is an impossibility:

n > n - 2 > (a x …….. x n - 2 x n) - 2 > (a x …….. x n - 2 x n) - 4 > y V y +/- 2 V z V z +/- 2

It is hence clear that no n & n - 2 in any list of consecutive primes can ever possibly be the largest existing twin primes pair, i.e., a largest existing twin primes pair is an impossibility, which implies that the twin primes are infinite. It is possible to find larger twin primes than n & n - 2 no matter how large n & n - 2 are to infinity, with the following formulae involving the list of consecutive primes: (a x …….. x n) - 2 & (a x …….. x n) - 4 (see appendix 1), which by the nature of their composition are capable of generating new primes/twin primes which will always be larger than n & n - 2. This is an indirect proof or proof by contradiction (reductio ad absurdum) of the infinity of the twin primes, for our assumption of n & n - 2 as the largest existing twin primes pair will be contradicted by the discovery of larger twin primes with these 2 formulae.

APPENDIX 1

Note: The (only) even prime 2 is omitted from the list of consecutive primes: 3, 5, 7, 11, 13, 17, 19, …….., n - 2, n stated in the paper, wherein n & n - 2 are assumed to be the largest existing twin primes pair.

The list of newly created primes, and, twin primes for n = 5, 7, 11, 13, 17, 19, …….. (n = 19 being the maximum limit achievable with a hand-held calculator) is as follows:-

1] For n = 5, we get the following new primes/new twin primes:

(3 x 5) - 2 = 13 (X)
(3 x 5) - 4 = 11 (Y)

2] For n = 7, we get the following new primes/new twin primes:

(3 x 5 x 7) - 2 =103 (X)
(3 x 5 x 7) - 4 = 101 (Y)

3] For n = 11, we get the following new primes/new twin primes:

(3 x 5 x 7 x 11) - 2 = 1,153 (X)
(3 x 5 x 7 x 11) - 4 = 1,151 (Y)

4] For n = 13, we get the following new prime and composite number with its prime factors:

(3 x 5 x 7 x 11 x 13) - 2 = 15,013 (X) - Prime Number
(3 x 5 x 7 x 11 x 13) - 4 = 15,011 (Y) - Composite Number (= 17 x 883, with 17 pairing with 19 to form a twin primes pair and 883 pairing with 881 to form another twin primes pair)

5] For n = 17, we get the following new primes/new twin primes:

(3 x 5 x 7 x 11 x 13 x 17) - 2 = 255,253 (X)
(3 x 5 x 7 x 11 x 13 x 17) - 4 = 255,251 (Y)

6] For n = 19, we get the following new prime and composite number with its prime factors:

(3 x 5 x 7 x 11 x 13 x 17 x 19) - 2 = 4,849,843 (X) - Prime Number
(3 x 5 x 7 x 11 x 13 x 17 x 19) - 4 = 4,849,841 (Y) - Composite Number (= 43 x 112,787, with 43 pairing with 41 to form a twin primes pair while 112,787 is a stand-alone prime)

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Results Of X & Y Above

1) X above generates 6 new primes (13; 103; 1,153; 15,013; 255,253; 4,849,843), nil composite numbers.
2) Y above generates 4 new primes (11; 101; 1,151; 255,251), 2 composite numbers (15,011 = 17 x 883; 4,849,841 = 43 x 112,787).
3) X & Y above together produce 4 pairs of new twin primes (13 & 11; 103 & 101; 1,153 & 1,151; 255,253 & 255,251).
4) The prime factors of X and Y above form 3 pairs of new twin primes with prime partners which differ from them by 2 (19 & 17; 43 & 41; 883 & 881).
5) All the new twin primes in (3) & (4) above are larger than n & n - 2, the assumed largest existing twin primes pair, which is indirect proof of the infinitude of the twin primes.

Why It Is Impossible For Any n & n - 2 To Be The Largest Existing Twin Primes Pair

X = (3 x 5 x 7 x 11 x 13 x 17 x 19 x .... x n) - 2, and,
Y = (3 x 5 x 7 x 11 x 13 x 17 x 19 x .... x n) - 4 will never be divisible by any of the consecutive prime numbers in the list: 3, 5, 7, 11, 13, 17, 19, ..., n, whether they are prime or composite (non-prime and divisible by prime numbers or prime factors). This means that none of the consecutive prime numbers in the list: 3, 5, 7, 11, 13, 17, 19, ..., n can ever be factors of X and Y, and, X and Y must be new primes/twin primes larger than all the consecutive prime numbers in the list: 3, 5, 7, 11, 13, 17, 19, ..., n, or, if they were composite (non-prime and divisible by prime numbers or prime factors), their prime factors (and “twin prime” partners which differ from them by 2) must be larger than all the consecutive prime numbers in the list: 3, 5, 7, 11, 13, 17, 19, ..., n.

This all implies that no n & n - 2 (if n - 2 were also a prime number) in any list of consecutive prime numbers can ever possibly be the largest existing twin primes pair, since all the new primes/twin primes produced or generated by X and Y will always be larger than n & n - 2. That is, a largest existing twin primes pair is an impossibility, which implies the infinitude of the list of the primes/twin primes.

In other words, by the above, which explains why all the new primes/twin primes, which X and Y by the nature of their composition are capable of producing or generating, will always be larger than n & n - 2, no n & n - 2 in any list of consecutive prime numbers: 3, 5, 7, 11, 13, 17, 19, ..., n can ever possibly be the largest existing twin primes pair, i.e., a largest existing twin primes pair is indeed an impossibility, thus implying the infinitude of the list of the twin primes. This is a very important inference.

Regardless of how long the list of the twin primes pairs is, it is possible to find some new twin primes pairs which will always be larger than n & n - 2, our assumed largest existing twin primes pair - the largest twin primes pair in our assumed finite list of the twin primes pairs, with X and Y, which is indirect proof of the infinity of the twin primes.

APPENDIX 2

Anecdotal Evidence Of The Infinity Of The Twin Primes

<u>TOP TWIN PRIMES IN 2000, 2001, 2007 & 2009</u>

In the year 2000, 4648619711505 x $2^{60000} \pm 1$ (18,075 digits) had been the top twin primes pair which had been discovered. In the year 2001, it only ranked eighth in the list of top 20 twin primes pairs, with $318032361 \cdot 2^{107001} \pm 1$ (32,220 digits) topping the list. In the year 2007, in the list of top 20 twin primes pairs, $318032361 \cdot 2^{107001} \pm 1$ (32,220 digits) ranked eighth, while 4648619711505 x $2^{60000} \pm 1$ (18,075 digits) was nowhere to be seen; 2003663613*2^195000-1 and 2003663613*2^195000+1 (58,711 digits), which was discovered on January 15, 2007, by Eric Vautier (from France) of the Twin Prime Search (TPS) project in collaboration with PrimeGrid (BOINC platform), was at the top of the list. As at August 2009, $65516468355 \cdot 2^{333333}-1$ and $65516468355 \cdot 2^{333333}+1$ (100,355 digits) is at the top of the list of top 20 twin primes pairs, while $318032361 \cdot 2^{107001} \pm 1$ (32,220 digits) ranks $11^{th.}$, and, 2003663613*2^195000-1 and 2003663613*2^195000+1 (58,711 digits) ranks second in this list.

We can expect larger twin primes than these extremely large twin primes, much larger ones, infinitely larger ones, to be discovered in due course.

<u>LIST OF PRIMES PAIRS FOR THE FIRST 2,500 CONSECUTIVE PRIMES, 2 TO 22,307, RANKED ACCORDING TO THEIR FREQUENCIES OF APPEARANCE</u>

S. No.	Ranking	Prime Pairs	No. Of Pairs	Percentage
(1)	1	primes pair separated by 6 integers	482	19.29 %
(2)	2	primes pair separated by 4 integers	378	15.13 %
(3)	3	primes pair separated by 2 integers (t. p.)	376	15.05 %
(4)	4	primes pair separated by 12 integers	267	10.68 %
(5)	5	primes pair separated by 10 integers	255	10.20 %
(6)	6	primes pair separated by 8 integers	229	9.16 %
(7)	7	primes pair separated by 14 integers	138	5.52 %
(8)	8	primes pair separated by 18 integers	111	4.44 %
(9)	9	primes pair separated by 16 integers	80	3.20 %
(10)	10	primes pair separated by 20 integers	47	1.88 %
(11)	11	primes pair separated by 22 integers	46	1.84 %
(12)	12	primes pair separated by 30 integers	24	0.96 %
(13)	13	primes pair separated by 28 integers	19	0.76 %
(14)	14	primes pair separated by 24 integers	16	0.64 %
(15)	15	primes pair separated by 26 integers	10	0.40 %
(16)	16	primes pair separated by 34 integers	9	0.36 %
(17)	17	primes pair separated by 36 integers	5	0.20 %
(18)	18	primes pair separated by 32 integers	2	0.08 %
(19)	18	primes pair separated by 40 integers	2	0.08 %
(20)	19	primes pair separated by 42 integers	1	0.04 %
(21)	19	primes pair separated by 52 integers	1	0.04 %

Total No. Of Primes Pairs In List: 2,498

It is evident in the above list that the primes pairs separated by 6 integers, 4 integers and 2 integers (twin primes), among the 21 classifications of primes pairs separated by from 2 integers to 52 integers (primes pairs separated by 38 integers, 44 integers, 46 integers, 48 integers & 50 integers are not among them, but, they are expected to appear further down in the infinite list of the primes), are the most dominant, important. There is a long list, an infinite list, of other primes pairs, besides those shown in the above list, which also play a part as the building-blocks of the infinite list of the integers.

The list of the integers is infinite. The list of the primes is also infinite. The infinite primes are the building-blocks of the infinite integers - the infinite odd integers are all either primes or composites of primes, and, the infinite even integers, except for 2 which is a prime, are all also composites of primes. Therefore, all the primes pairs separated by the integers of various magnitudes, as described above, can never all be finite. If there is any possibility at all for any of these primes pairs to be finite, there is only the possibility that a number of these primes pairs are finite (but never all of them). However, will it have to be the primes pairs separated by 2 integers or twin primes (which are the subject of our investigation here), which are the only primes pair, or, one among a number of primes pairs, which are finite? Why question only the infinity of the primes pairs separated by 2 integers, the twin primes? Are not the infinities of the primes pairs separated by 8 integers and more, whose frequencies of appearance are lower, as compared to those of the primes pairs which are separated by 6, 4 and 2 integers respectively, in the above list of primes pairs, more questionable? Why single out only the twin primes? (There are at least 18 other primes pairs, separated by from 8 integers to 52 integers, whose respective infinities should be more suspect, as is evident from the above list of primes pairs, if any infinities should be doubted. Evidently, the primes pairs separated by 2 integers (twin primes) are not that likely to be finite.)

The above represents anecdotal evidence that the twin primes are infinite, which is a ratification of the actual proof given earlier.

PART 2

<u>Keywords</u>: random, probability, predicted, found, induction

Theorem:- There is an infinite number of twin primes.

Numerical Evidence:-
Many believe that the twin primes are infinite. In fact, twin primes pairs could easily be found among the integers. There is evidently no region of the natural number system so remote that it lies beyond the largest twin primes pair. It is even possible to forecast the approximate number of twin primes pairs found in any region of the natural number system.

The occurrence of twin primes pairs is evidently unpredictable or random. This means that the chance of 2 numbers x and x + 2 being prime (twin primes) is somewhat similar to the chance of getting heads on 2 successive tosses of a coin. If 2 successive tosses of a coin are independent, the chance of success of obtaining heads for the 2 successive tosses of the coin is the product of the chances of success of obtaining a head for each toss of the coin. As each coin has probability ½ of coming up heads with a toss, 2 coins would have probability ½ x ½ = ¼ of coming up a pair of heads with a toss.

The prime number theorem, which had been proven, states that if n is a large number, and we select a number x at random between 0 and n, the chance that x is prime would be approximately 1/log n, the larger n is, the better would be the approximation given by 1/log n to the proportion of primes in the numbers up to n. Like 2 coins coming up heads, the chance that both x and x + 2 are prime (twin primes) would be approximately $1/(\log n)^2$. That is, there would be approximately $n/(\log n)^2$ twin primes pairs between 0 and n. As n goes to infinity, this fraction approaches infinity. This represents a quantitative version of the twin primes conjecture.

As x + 2 being prime depends on the fact that x is already prime, we should modify the estimate $n/(\log n)^2$ to $(1.32032..)n/(\log n)^2$.

The following is a comparison between the twin primes predicted by the above formula and the twin primes found, where the agreement is evidently very good:-

INTERVAL	**TWIN PRIMES**	
	PREDICTED	FOUND
100,000,000 - 100,150,000	584	601
1,000,000,000 - 1,000,150,000	461	466
10,000,000,000 - 10,000,150,000	374	389
100,000,000,000 - 100,000,150,000	309	276
1,000,000,000,000 - 1,000,000,150,000	259	276
10,000,000,000,000 - 10,000,000,150,000	221	208
100,000,000,000,000 - 100,000,000,150,000	191	186
1,000,000,000,000,000 - 1,000,000,000,150,000	166	161

All this represents numerical evidence that the twin primes are infinite as we could find more twin primes pairs whenever we look for them.

Proof:-

Lemma: According to the principle of induction in set theory, if a set of natural numbers contains 1, and if it contains n + 1 whenever it contains a number n, then it must contain every natural number, e.g., induction proves that every natural number is a product of primes.

By the above lemma, there is an infinitude of twin primes in the infinite set of the integers; twin primes are found in all the above subsets of integers, from 100,000,000 - 100,150,000 to 1,000,000,000,000,000 - 1,000,000,000,150,000, and many very much larger twin primes have also been found in the infinite set of the integers, e.g., $318032361 \cdot 2^{107001} \pm 1$ (32,220 digits), 2003663613*2^195000-1 and 2003663613*2^195000+1 (58,711 digits), and, $65516468355 \cdot 2^{333333}-1$ and $65516468355 \cdot 2^{333333}+1$ (100,355 digits), to name a few (this long list of twin primes, especially the large twin primes, are obtainable only with the help of modern computer technology) - the principle of induction implies that there must be an infinite number of twin primes, ranging from the twin primes pair, 3 & 5 (smallest twin primes pair), 5 & 7, 11 & 13, 17 & 19, ……………., $65516468355 \cdot 2^{333333}-1$ and $65516468355 \cdot 2^{333333}+1$ (100,355 digits - largest twin primes pair discovered as at August 2009), ……………. upward to infinity, in the infinite set of the integers - in other words, the twin primes conjecture must be true.

PART 3

Keywords: building-blocks, symmetry, infinity

Theorem:- The list of the twin primes pairs is infinite.

Proof:-

Lemma: According to the precepts of fractal geometry and group theory, symmetry is a very important, intrinsic part of nature. There is symmetry all around us and within us. There is evident symmetry in human bodies, the structures of viruses and bacteria, polymers and ceramic materials, the permutations of numbers, the universe and many others, even the movements of prices in financial markets, the growths of populations, the sound of music, the flow of blood through our circulatory system, the behaviour of people en masse, etc.. In other words, regularity, pattern, order, uniformity or symmetry is evident everywhere.

The reasoning here makes use of a very important idea in fractal geometry and group theory, namely, symmetry.

A prime number is an integer which is divisible only by 1 and itself, e.g., 2, 3, 7, 19, etc.. A twin primes pair are 2 primes which differ from one another by 2, e.g., 5 & 7, 11 & 13, 17 & 19, and, 29 & 31, etc.. A composite number or non-prime is a product of primes or prime factors, e.g., the composite numbers 15 is the product of 2 primes, 3 and 5 (15 = 3 x 5), and 231 is the product of 3 primes, 3, 7 and 11 (231 = 3 x 7 x 11), etc.. The integers or whole numbers are either primes or composites and are infinite.

The primes, which Euclid had proven to be infinite, are the atoms or building-blocks of the infinite integers or whole numbers, which comprise of the infinite list of the odd numbers that are all either primes or products of primes (i.e., composites), and, the infinite list of the even numbers that are all products of primes (i.e., composites, with the exception of 2 which is a prime, e.g., 6 = 2 x 3, 8 = 2 x 2 x 2 and 10 = 2 x 5, etc.). The infinite list of the integers or whole numbers may be classified as an infinite group, with various symmetries, subgroups and infinite elements, hidden within it. These various symmetries, subgroups and infinite elements, within this infinite group may be classified as follows:-

(1) Subgroup A: Infinite consecutive primes such as 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, etc. to infinity ………. , separated by 2 integers (twin primes), 4 integers, 6 integers, 8 integers, 10 integers, etc. to infinity ………. , which, incidentally, except for 2, are all odd numbers; this splitting up of the subgroup into infinite elements is shown below:

- (i) Element A1: Infinite list of all the primes pairs separated by 2 integers (twin primes) ………. . (Example: 17 & 19)
- (ii) Element A2: Infinite list of all the primes pairs separated by 4 integers/1 odd composite - single composite ………. . (Example: 79 & 83 separated by 81)
- (iii) Element A3: Infinite list of all the primes pairs separated by 6 integers/2 consecutive odd composites - twin composites ………. . (Example: 47 & 53 separated by 49 & 51)
- (iv) Element A4: Infinite list of all the primes pairs separated by 8 integers/3 consecutive odd composites - “triple” composites ………. . (Example: 359 & 367 separated by 361, 363 & 365)
- (v) Element A5: Infinite list of all the primes pairs separated by 10 integers/4 consecutive odd composites - “four-ple” composites ………. . (Example: 709 & 719 separated by 711, 713, 715 & 717)

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(2) Subgroup B: Infinite consecutive odd composites such as 9, 15, 21, 25, 27, 33, 35, 39, 45, 49, etc. to infinity ………. , of “infinite sizes” sandwiched between 2 primes; this splitting up of the subgroup into infinite elements is shown below:

- (i) Element B1: Infinite list of all “1 odd composite sandwiched between 2 primes - single composite” ………. . (Example: 9 sandwiched between the primes 7 & 11)
- (ii) Element B2: Infinite list of all “2 consecutive odd composites sandwiched between 2 primes - twin composites” ………. . (Example: 253 & 255 sandwiched between the primes 251 & 257)
- (iii) Element B3: Infinite list of all “3 consecutive odd composites sandwiched between 2 primes - “triple” composites” ………. . (Example: 685, 687 & 689 sandwiched between the primes 683 & 691)
- (iv) Element B4: Infinite list of all “4 consecutive odd composites sandwiched between 2 primes - “four-ple” composites” ………. . (Example: 2,769, 2,771, 2,773 & 2,775 sandwiched between the primes 2,767 & 2,777)
- (v) Element B5: Infinite list of all “5 consecutive odd composites sandwiched between 2 primes - “five-ple” composites” ………. . (Example: 19,291, 19,293, 19,295, 19,297 & 19,299 sandwiched between the primes 19,289 & 19,301)

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(3) <u>Subgroup C</u>: Infinite consecutive odd composites separated by 4 integers and 6 integers respectively; this splitting up of the subgroup into the 2 infinite elements is shown below:

(i) <u>Element C1</u>: Infinite list of all "2 consecutive odd composites separated by 4 integers/1 prime" ………. . (Example: 209 & 213 separated by the prime 211)
(ii) <u>Element C2</u>: Infinite list of all "2 consecutive odd composites separated by 6 integers/2 primes" ………. . (Example: 279 & 285 separated by the twin primes 281 & 283)

(4) <u>Subgroup D</u>: Infinite single primes and twin primes separating 2 consecutive odd composites; this splitting up of the subgroup into the 2 infinite elements is shown below:

(i) <u>Element D1</u>: Infinite list of all the single primes separating 2 consecutive odd composites ………. . (Example: 23 separating the 2 consecutive odd composites 21 & 25)
(ii) <u>Element D2</u>: Infinite list of all the twin primes separating 2 consecutive odd composites ………. . (Example: 11 & 13 separating the 2 consecutive odd composites 9 & 15)

(5) <u>Subgroup E</u>: Infinite consecutive even composites such as 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, etc. to infinity ………, all separated by only 2 integers; this subgroup may be classified as a single infinite element ………. . There is always 1 even number between a twin primes pair, which is separated by 2 integers, a prime and a composite which are separated by 2 integers, and, 2 composites which are separated by 2 integers. That is, the even numbers are always found in Subgroup A, Subgroup B, Subgroup C and Subgroup D above always evenly spaced out in consecutive order by 2 integers.

There is an evident symmetry in the above-mentioned infinite group, which would be broken if any of the elements within were to be finite. There are close interlinks between all the various infinite elements in all the five subgroups above, e.g., the infinity of the list of all the primes pairs each separated by 6 integers (Element A3/Subgroup A) implies the infinity of the list of all the "2 consecutive odd composites sandwiched between 2 primes - twin composites" (Element B2/Subgroup B) and vice versa, the infinity of the list of all the primes pairs each separated by 2 integers (twin primes) (Element A1/SubgroupA) implies the infinity of the list of all the "2 consecutive odd composites separated by 6 integers/2 primes" (Element C2/Subgroup C) and vice versa, the infinity of the list of all the infinite elements (A1, A2, A3, etc. to infinity) in Subgroup A above, which represents the infinity of the list of the primes which Euclid had in fact proven, implies the infinity of the list of all the "2 consecutive odd composites separated by 4 integers/1 prime" (Element C1/Subgroup C) and vice versa, the infinity of the list of all the "2 consecutive odd composites separated by 6 integers/2 primes" (Element C2/Subgroup C) implies the infinity of the list of all the twin primes separating 2 consecutive odd composites (Element D2/Subgroup D) and vice versa, the infinity of all the lists of all the infinite elements (A1, A2, A3 ……, B1, B2, B3 ……, C1 & C2, D1 & D2) in Subgroup A, Subgroup B, Subgroup C and Subgroup D above implies the infinity of the list of the consecutive even composites, i.e., 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, etc. to infinity (Subgroup E), which we know to be true in any case, and vice versa, etc. to infinity.

Subgroup A and Subgroup B above are practically "mirror" images of one another - they represent the viewing of the primes and the composites from 2 variant angles - the infinitude, or, finiteness of either implies the infinitude, or, finiteness of the other; the same applies to both Subgroup C and Subgroup D above. It is similar to the following way of viewing a glass which is partially filled: this glass could be described as "half full" or "half empty" if it is half filled, "three-quarter full" or "one-quarter empty" if it is three-quarter filled, or, "one-quarter full" or "three-quarter empty" if it is one-quarter filled, etc..

It is evident that the infinitude, or, finiteness of any one of the above-mentioned elements would imply the infinitude, or, finiteness of the other element that is interlinked with it and vice versa. All these infinite elements are evidently entangled together and complementary, being all the infinite building-blocks of the infinite integers or whole numbers. The infinity of the list of the integers or whole numbers, the primes included, in fact implies that all these various elements within it are infinite, and, vice versa, since all these various elements are closely interlinked and could not do without each other. Therefore, the breaking of the evident intrinsic symmetry of this whole infinite group, i.e., the infinite list of the integers or whole numbers, due to the finiteness of any of the elements within it, could not be possible.

We pose a very important question: Besides questioning whether the infinite list of all the primes pairs separated by 2 integers (twin primes) is really infinite, should we not also be questioning whether the following are really infinite?:

(a) Infinite lists of all the primes pairs separated respectively by 4 integers, 6 integers, 8 integers, 10 integers and sequentially larger integers to infinity (as in Subgroup A above).
(b) Infinite lists of all the respective consecutive odd composites of "infinite sizes" sandwiched between 2 primes (as in Subgroup B above).
(c) The 2 infinite lists with respectively "2 consecutive odd composites separated by 4 integers/1 prime" and "2 consecutive odd composites separated by 6 integers/2 primes" (as in Subgroup C above).

(d) The 2 infinite lists of respective single primes and twin primes separating 2 consecutive odd composites (as in Subgroup D above).
(e) Infinite list of the consecutive even composites all separated by only 2 integers (as in Subgroup E above).

Could there possibly be any symmetry-breaking in the above-mentioned infinite group whence one or more of the elements within it would be finite? In particular, could there be a possibility for the symmetry of this infinite group to be broken due to the finiteness of Element A1 (i.e., the finiteness of the twin primes) within it? Since the above-mentioned group, i.e., the list of the integers or whole numbers, is infinite, it is indeed not possible for all of these elements to be finite. And, there is no evident reason to account for why any of these elements, especially Element A1, i.e., the list of primes separated by 2 integers, or, twin primes, should be finite. In fact, all these infinite elements are like the slabs of various sizes in a building. They are all necessary for the construction of the infinite building known as the "infinite list of the integers or whole numbers" and should thus all be infinite, wherein the symmetry of the infinite group, i.e., the infinite list of the integers or whole numbers, would be preserved. The following explanation should make it clear that all these elements in the above-mentioned infinite group have to be infinite.

We here examine a hypothetical case wherein one of the elements within the infinite group is finite. Let us assume that Element A1, the list of primes separated by 2 integers, or, twin primes, in Subgroup A above, which is our target object, is finite. Let us look at the following list of consecutive integers or whole numbers where the bolded numbers are primes and the bolded, italised numbers are twin primes:-

…….. 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, **211**, 212, 213, 214, 215, 216, 217, 218, 219,

220, 221, 222, **223**, 224, 225, 226, ***227***, 228, ***229***, 230, 231, 232, **233**, 234, 235, 236, 237, 238,

239, 240, ***241***, 242, 243, 244, 245, 246, 247, 248, 249, 250, **251**, 252, 253, 254, 255, 256, **257**,

258, 259, 260, 261, 262, **263**, 264, 265, 266, 267, 268, ***269***, 270, ***271***, 272, 273, 274, 275, 276,

277, 278, 279, 280, ***281***, 282, ***283***, 284, 285, 286, 287, 288, 289, 290, 291, 292, **293**, 294, 295,

296, 297, 298, 299, 300, 301 ……..

Now, what would happen if the list of the twin primes (Element A1) were finite? Let us, e.g., assume that 269 & 271 are the largest existing twin primes pair - there are no twin primes larger than they. And let us look at the above list of consecutive integers or whole numbers now:-

…….. 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, **211**, 212, 213, 214, 215, 216, 217, 218, 219,

220, 221, 222, **223**, 224, 225, 226, ***227***, 228, ***229***, 230, 231, 232, **233**, 234, 235, 236, 237, 238,

239, 240, ***241***, 242, 243, 244, 245, 246, 247, 248, 249, 250, **251**, 252, 253, 254, 255, 256, **257**,

258, 259, 260, 261, 262, **263**, 264, 265, 266, 267, 268, ***269***, 270, ***271***, 272, 273, 274, 275, 276,

277, 278, 279, 280, , 282, , 284, 285, 286, 287, 288, 289, 290, 291, 292, **293**, 294, 295,

296, 297, 298, 299, 300, 301 ……..

Since 269 & 271 are the largest existing twin primes, the twin primes pair, 281 & 283, would not exist anymore, resulting in 2 gaps (between the integers 280 and 284) in the above list of consecutive integers or whole numbers. Thus, the earlier complete list of consecutive integers or whole numbers has become incomplete and discontinuous as a result - 2 building-blocks, 281 and 283, which are twin primes (primes separated by 2 integers), are missing from this list now - the list has now become asymmetrical, i.e., symmetry-breaking has occurred. As the list of integers or whole numbers is always complete, continuous, infinite and symmetrical, such a list with 2 missing consecutive odd numbers which are both primes (these missing consecutive odd numbers are twin primes) is an absurdity - it could never exist. Therefore, a largest existing twin primes pair, such as 269 & 271 mentioned above, is an absurdity - a largest existing twin primes pair could never exist, and, it is evident here that, of necessity, in order that the list of the integers or whole numbers would be complete, continuous, infinite and symmetrical, the list of the twin primes (primes separated by 2 integers), as well as the lists of the primes pairs separated by 4 integers, 6 integers, 8 integers, 10 integers and so on in arithmetically ascending order to infinity, have to be infinite. That is, all the elements in Subgroup A above have to be infinite.

By a similar reasoning, the same has to also apply to all the elements in Subgroup B, Subgroup C, Subgroup D and Subgroup E above.

Therefore, by the above lemma, all the elements in Subgroup A, Subgroup B, Subgroup C, Subgroup D and Subgroup E above are infinite.

It is clear that the twin primes are infinite.

PART 4

Keywords: various primes pairs, building-blocks, fraction of infinity

Theorem:- The twin primes are infinite.

Proof:-

Lemma: A fraction of infinity is also infinite. (See appendix.)

The list of the primes, which are the building-blocks of the integers, had been proven by Euclid to be infinite. The question now is whether the list of the twin primes is also infinite.

It would be appropriate to conduct an examination of the primes pairs separated by 2 integers (commonly known as twin primes) vis-a-vis the other primes pairs separated by more than 2 integers, e.g., 4 integers, 6 integers, 8 integers and more. For this purpose, we select a reasonably large list of consecutive primes, which may be regarded as a basic unit of the infinite list of the primes; we examine a compilation of such data obtained from, say, the list of the first 2,500 consecutive primes - 2 to 22,307 - which is as follows:-

<u>LIST OF PRIMES PAIRS FOR THE FIRST 2,500 CONSECUTIVE PRIMES, 2 TO 22,307, RANKED ACCORDING TO THEIR FREQUENCIES OF APPEARANCE</u>

S. No.	Ranking	Prime Pairs	No. Of Pairs	Percentage
(1)	1	primes pair separated by 6 integers	482	19.29 %
(2)	2	primes pair separated by 4 integers	378	15.13 %
(3)	3	primes pair separated by 2 integers (t. p.)	376	15.05 %
(4)	4	primes pair separated by 12 integers	267	10.68 %
(5)	5	primes pair separated by 10 integers	255	10.20 %
(6)	6	primes pair separated by 8 integers	229	9.16 %
(7)	7	primes pair separated by 14 integers	138	5.52 %
(8)	8	primes pair separated by 18 integers	111	4.44 %
(9)	9	primes pair separated by 16 integers	80	3.20 %
(10)	10	primes pair separated by 20 integers	47	1.88 %
(11)	11	primes pair separated by 22 integers	46	1.84 %
(12)	12	primes pair separated by 30 integers	24	0.96 %
(13)	13	primes pair separated by 28 integers	19	0.76 %
(14)	14	primes pair separated by 24 integers	16	0.64 %
(15)	15	primes pair separated by 26 integers	10	0.40 %
(16)	16	primes pair separated by 34 integers	9	0.36 %
(17)	17	primes pair separated by 36 integers	5	0.20 %
(18)	18	primes pair separated by 32 integers	2	0.08 %
(19)	18	primes pair separated by 40 integers	2	0.08 %
(20)	19	primes pair separated by 42 integers	1	0.04 %
(21)	19	primes pair separated by 52 integers	1	0.04 %

Total No. Of Primes Pairs In List: 2,498

It is evident in the above list that the primes pairs separated by 6 integers, 4 integers and 2 integers (twin primes), among the 21 classifications of primes pairs separated by from 2 integers to 52 integers (primes pairs separated by 38 integers, 44 integers, 46 integers, 48 integers & 50 integers are not among them, but, they are expected to appear further down in the infinite list of the primes), are the most dominant, important. There is a long list, an infinite list, of other primes pairs, besides those shown in the above list, which also play a part as the building-blocks of the infinite list of the integers. We shall prove the infinity of all these various building-blocks below.

The list of the integers is infinite. The list of the primes is also infinite. The infinite primes are the building-blocks of the infinite integers - the infinite odd integers are all either primes or composites of primes, and, the infinite even integers, except for 2 which is a prime, are all also composites of primes. Therefore, all the primes pairs separated by the integers of various magnitudes, as described above, could never all be finite. If there is any possibility at all for any of these primes pairs to be finite, there is only the possibility that a number of these primes pairs are finite (but never all of them). However, would it have to be the primes pairs separated by 2 integers or twin primes (which are the subject of our investigation here), which are the only primes pair, or, one among a number of primes pairs, which are finite? Why question only the infinity of the primes pairs separated by 2 integers, the twin primes? Are not the infinities of the primes pairs separated by 8 integers and more, whose frequencies of appearance are lower, as compared to those of the primes pairs which are separated by 6, 4 and 2 integers respectively, in the above list of primes pairs, more questionable? Why single out only the twin primes? (There are at least 18 other primes pairs, separated by from 8 integers to 52 integers, whose respective infinities should be more suspect, as is evident from the above list of primes pairs, if any infinities should be doubted. It is evident that the primes pairs separated by 2 integers (twin primes) are not that likely to be finite.)

All of the above lists of primes pairs which are respectively separated by from 2 integers to 52 integers are each respectively a fraction of the infinite list of the primes: the list of primes pairs separated by 2 integers (twin primes) is a fraction of the infinite list of the primes, the list of primes pairs separated by 4 integers is also a fraction of the infinite list of the primes, the list of primes pairs separated by 6 integers is a fraction of the infinite list of the primes too, and so on all the way down to the list of primes pairs separated by 52 integers, and beyond to infinity.

Therefore, by the above lemma, all these various lists of primes pairs separated by integers of various magnitudes are each also infinite.

<u>APPENDIX - EUCLID'S PROOF OF THE INFINITY OF THE PYTHAGOREAN TRIPLES: A FRACTION OF INFINITY IS ALSO INFINITE</u>

A Pythagorean triple is a set of 3 integers wherein 1 number squared added to another number squared equals the third number squared, e.g., 3^2 (9) + 4^2 (16) = 5^2 (25) below.

Euclid's proof of the infinity of the Pythagorean triples begins with the statement that the difference between 2 successive square numbers is always an odd number, as is evident below:

Column 1	Column 2		Column 3
1^2	1		
		}	3 (difference between 4 & 1)
2^2	4		
		}	5 (difference between 9 & 4)
3^2	9		
		}	7 (difference between 16 & 9)
4^2	16		
		}	9 (difference between 25 & 16)
5^2	25		
		}	11 (difference between 36 & 25)
6^2	36		
		}	13 (difference between 49 & 36)
7^2	49		
		}	15 (difference between 64 & 49)
8^2	64		
		}	17 (difference between 81 & 64)
9^2	81		
		}	19 (difference between 100 & 81)
10^2	100		
.	.		.
.	.		.
.	.		.

Every one of the infinity of odd numbers (in Column 3 above) could be added to a particular square number (in Column 2 above) to give another square number, e.g., 3 in Column 3 above could be added to 1 in Column 2 above to give the square number 4, 5 in Column 3 above could be added to 4 in Column 2 above to give the square number 9, 7 in Column 3 above could be added to 9 in Column 2 above to give the square number 16, and so on …… . A fraction of the infinite odd numbers in Column 3 above are themselves square numbers, e.g., the odd number 9 in Column 3 above is a square number, and, is the only square number in the list of odd numbers shown there, representing there a fraction of the infinite odd numbers which are square numbers. However, a fraction of infinity is also infinite.

Therefore, there are also an infinity of odd square numbers (in Column 3 above) which could each be added to another square number (in Column 2 above) to give another square number, i.e., there is an infinitude of Pythagorean triples.

PART 5

Keywords: prime factors, indivisible, point at infinity, new twin primes

Theorem:- The twin primes are infinite.

Numerical Evidence:-
The following algorithms would be able to generate or sieve all the twin primes in any range of odd numbers which are all larger than those in the list of known consecutive primes/twin primes; these 2 important algorithms would provide plenty of numerical evidence that the twin primes are infinite:-

Algorithm 1
We would provide an example with Items (1) to (3) from the following list of products of consecutive primes/twin primes, which should be sufficient for our purpose here:-

1) 3 x 5 = 15
2) 3 x 5 x 7 = 105
3) 3 x 5 x 7 x 11 = 1,155
4) 3 x 5 x 7 x 11 x 13 = 15, 015
5) 3 x 5 x 7 x 11 x 13 x 17 = 255,255
6) 3 x 5 x 7 x 11 x 13 x 17 x 19 = 4,849,845

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The example is as follows:-

1) For 3 x 5 = 15, we would find all the consecutive pairs of odd numbers between 5 & 15 which differ from one another by 2 and are not divisible by any of the consecutive primes/twin primes 3 & 5 in the list of consecutive primes/twin primes 3 x 5 whose product is 15.

   There is only 1 pair of odd numbers between 5 & 15 which differ from one another by 2 and are not divisible by the consecutive primes/twin primes 3 & 5 in the list of consecutive primes/twin primes 3 x 5 - they are the twin primes 11 & 13.

2) Similarly, for 3 x 5 x 7 = 105, we would find all the consecutive pairs of odd numbers between 7 & 105 which differ from one another by 2 and are not divisible by any of the consecutive primes/twin primes 3, 5 & 7 in the list of consecutive primes/twin primes 3 x 5 x 7 whose product is 105.

   The consecutive pairs of odd numbers between 7 & 105 which differ from one another by 2 and are not divisible by the consecutive primes/twin primes 3, 5 & 7 are the following consecutive twin primes:

   (a) 11 & 13
   (b) 17 & 19
   (c) 29 & 31
   (d) 41 & 43
   (e) 59 & 61
   (f) 71 & 73
   (g) 101 & 103

3) Similarly, in this final case, for 3 x 5 x 7 x 11 = 1,155, we would find all the consecutive pairs of odd numbers between 11 & 1,155 which differ from one another by 2 and are not divisible by any of the consecutive primes/twin primes 3, 5, 7 & 11 in the list of consecutive primes/twin primes 3 x 5 x 7 x 11 whose product is 1,155.

   The consecutive pairs of odd numbers between 11 & 1,155 which differ from one another by 2 and are not divisible by the consecutive primes/twin primes 3, 5, 7 & 11 are consecutive twin primes, some of which are as follows:

   (a) 17 & 19
   (b) 29 & 31
   (c) 41 & 43
   (d) 59 & 61
   (e) 71 & 73
   (f) 101 & 103
   (g) 107 & 109
   (h) 137 & 139
   (i) 149 & 151
   (j) 179 & 181
   (k) Etc. to 1,151 & 1,153

In this way, we would also be able to achieve the following:-

1) For 3 x 5 x 7 x 11 x 13 = 15,015, find all the consecutive twin primes between 13 and 15,015.
2) For 3 x 5 x 7 x 11 x 13 x 17 = 255,255, find all the consecutive twin primes between 17 and 255,255.
3) For 3 x 5 x 7 x 11 x 13 x 17 x 19 = 4,849,845, find all the consecutive twin primes between 19 and 4,849,845.

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Algorithm 2

We would, similar to Algorithm 1 above, also provide an example with Items (1) to (3) from the following list of products of consecutive primes/twin primes, which should be sufficient for our purpose here:-

1) 3 x 5 = 15
2) 3 x 5 x 7 = 105

3) 3 x 5 x 7 x 11 = 1,155
4) 3 x 5 x 7 x 11 x 13 = 15, 015
5) 3 x 5 x 7 x 11 x 13 x 17 = 255,255
6) 3 x 5 x 7 x 11 x 13 x 17 x 19 = 4,849,845

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The example is as follows:-

1) For 3 x 5 = 15, we would first find all the consecutive pairs of even numbers between 5 & 15 which differ from one another by 2 and are not divisible by any of the consecutive primes/twin primes 3 & 5 in the list of consecutive primes/twin primes 3 x 5. Then we deduct each of these consecutive pairs of even numbers which are not divisible by any of the consecutive primes/twin primes 3 & 5 from the product of these consecutive primes/twin primes 3 x 5 which is 15. The results would each be 1 pair of twin primes, 1 prime & 1 composite of primes, or, 2 composites of primes. In this way, we would be able to find all the consecutive twin primes between 5 & 15.

   There is only 1 pair of even numbers between 5 & 15 which differ from one another by 2 and are not divisible by any of the consecutive primes/twin primes 3 & 5 in the list of consecutive primes/twin primes 3 x 5 - they are the pair 2 & 4.

   The following is the result after we deduct this pair of even numbers 2 & 4 which are not divisible by any of the consecutive primes/twin primes 3 & 5 from the product of these consecutive primes/twin primes 3 x 5 which is 15:

   (a) 15 - 2 & 15 - 4: 13 & 11 (twin primes)

2) Similarly, for 3 x 5 x 7 = 105, we would first find all the consecutive pairs of even numbers between 7 & 105 which differ from one another by 2 and are not divisible by any of the consecutive primes/twin primes 3, 5 & 7 in the list of consecutive primes/twin primes 3 x 5 x 7, which are as follows:

   (a) 2 & 4
   (b) 32 & 34
   (c) 44 & 46
   (d) 62 & 64
   (e) 74 & 76
   (f) 86 & 88
   (g) 92 & 94

   Then we deduct each of these consecutive pairs of even numbers which are not divisible by any of the consecutive primes/twin primes 3, 5 & 7 from the product of these consecutive primes/twin primes 3 x 5 x 7 which is 105. The results would each be 1 pair of twin primes, 1 prime & 1 composite of primes, or, 2 composites of primes. In this way, we would be able to find all the consecutive twin primes between 7 & 105, which are as follows:

   (a) 105 - 2 & 105 - 4: 103 & 101 (twin primes)
   (b) 105 - 32 & 105 - 34: 73 & 71 (twin primes)
   (c) 105 - 44 & 105 - 46: 61 & 59 (twin primes)
   (d) 105 - 62 & 105 - 64: 43 & 41 (twin primes)
   (e) 105 - 74 & 105 - 76: 31 & 29 (twin primes)
   (f) 105 - 86 & 105 - 88: 19 & 17 (twin primes)
   (g) 105 - 92 & 105 - 94: 13 & 11 (twin primes)

3) Similarly, in this final case, for 3 x 5 x 7 x 11 = 1,155, we would first find all the consecutive pairs of even numbers between 11 & 1,155 which differ from one another by 2 and are not divisible by any of the consecutive primes/twin primes 3, 5, 7 & 11 in the list of consecutive primes/twin primes 3 x 5 x 7 x 11, some of which are as follows:

   (a) 2 & 4
   (b) 32 & 34
   (c) 62 & 64
   (d) 74 & 76
   (e) 92 & 94
   (f) 116 & 118
   (g) 122 & 124

(h) 134 & 136
(i) Etc. to 1,136 & 1,138

Next we deduct each of these consecutive pairs of even numbers which are not divisible by any of the consecutive primes/twin primes 3, 5, 7 & 11 from the product of these consecutive primes/twin primes 3 x 5 x 7 x 11 which is 1,155. The results would each be 1 pair of twin primes, 1 prime & 1 composite of primes, or, 2 composites of primes. In this way, we would be able to find all the consecutive twin primes between 11 & 1,155, some of which are as follows:

(a) 1,155 - 2 & 1,155 - 4: 1,153 & 1,151 (twin primes)
(b) 1,155 - 32 & 1,155 - 34: 1,123 (prime) & 1,121 (composite of primes which are each larger than 3, 5, 7 & 11 = 19 x 59)
(c) 1,155 - 62 & 1,155 - 64: 1,093 & 1,091 (twin primes)
(d) 1,155 - 74 & 1,155 - 76: 1,081 (composite of primes which are each larger than 3, 5, 7 & 11 = 23 x 47) & 1,079 (composite of primes which are each larger than 3, 5, 7 & 11 = 13 x 83)
(e) 1,155 - 92 & 1,155 - 94: 1,063 & 1,061 (twin primes)
(f) 1,155 - 116 & 1,155 - 118: 1,039 (prime) & 1,037 (composite of primes which are each larger than 3, 5, 7 & 11 = 17 x 61)
(g) 1,155 - 122 & 1,155 - 124: 1,033 & 1,031 (twin primes)
(h) 1,155 - 134 & 1,155 - 136: 1,021 & 1,019 (twin primes)
(i) Etc. to 1,155 - 1,136 & 1,155 - 1,138: 19 & 17 (twin primes)

In like manner, we would also be able to achieve the following:-

1) For 3 x 5 x 7 x 11 x 13 = 15,015, find all the consecutive twin primes between 13 and 15,015.
2) For 3 x 5 x 7 x 11 x 13 x 17 = 255,255, find all the consecutive twin primes between 17 and 255,255.
3) For 3 x 5 x 7 x 11 x 13 x 17 x 19 = 4,849,845, find all the consecutive twin primes between 19 and 4,849,845.

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By utilising any of the above algorithms, we would be able to find many twin primes which are all larger than those in any chosen list of consecutive primes/twin primes, i.e., we would be able to generate many larger and larger twin primes with these algorithms.

Proof:-

Lemma: According to the principle of induction in set theory, if a set of natural numbers contains 1, and if it contains n + 1 whenever it contains a number n, then it must contain every natural number, e.g., induction proves that every natural number is a product of primes.

We could have a "feel" of the infinity of the twin primes when we look at the lists of the top twin primes discovered to-date, which constitutes an argument for the twin primes' infinity; incidentally, this was more or less the kind of proof that had been used to settle the Four-Colour problem in 1976 by Wolfgang Haken and Kenneth Appel, which relied on the computing powers of powerful computers - 1,200 hours of computer time, in fact. Both Haken and Appel had studied the work of Heinrich Heesch who had claimed that the infinity of infinitely variable maps could be constructed from some finite number of finite maps and that by studying these "building-block" maps (these basic maps being the equivalent of the electron, proton and neutron, the fundamental objects from which all else could be constructed) it could be possible to get a hold on the general problem. Using the same reasoning Haken and Appel then reduced the Four-Colour problem to 1,482 "building-block" configurations, whereby proving that these 1,482 maps were four-colourable would imply that all maps would be four-colourable. Likewise, the twin primes' infinitude could be regarded as proven if a pair or more of twin primes were discovered which exceed a certain designated finite limit, a limit which is so expansive that it is indescribable, unimaginable and unnamable, such that it could be regarded as representing infinity itself - the pair of twin primes which represents this limit would be in effect a pair of googolplexes and virtually the point or focus at infinity. (Here, the designated finite limit could be treated as approximately equal to infinity.) According to the approximation theorem (20th. century), any two numbers are approximately equal - this holds whenever the range of approximation is greater than the modulus (positive value) of the difference of the numbers. In geometry, infinity is regarded as a "location", a finite entity, for instance, parallel lines could be said to intersect at a point at infinity, and, parallel planes at a line at infinity; the asymptote to a curve could be regarded as intersecting the curve at infinity. The idea of infinity as a location had been introduced by Johann Kepler, who pointed out that a parabola could be regarded as an ellipse or a hyperbola with one focus at infinity. The idea had been developed by Girard Desargues in his formulation of projective geometry, which assumed the existence of an ideal point at infinity. This equivalency link between infinity and finiteness also appears in infinite series, which are series with an unlimited number of terms. Infinite series could be either divergent or convergent. Convergence is an important feature of a series, i.e., series that are convergent play a major role in mathematics. (It is thus necessary to be able to test a series for convergence by such tests as the Cauchy convergence test, the Cauchy integral test, Abel's test and Dirichlet's test.) An infinite series which converges to a finite sum is known as a convergent series.

We could thus prove the infinitude of the twin primes by using any of the above algorithms, preferably the evidently more efficient Algorithm 1, with the help of powerful computers, to find a pair or more of googolplex twin primes beyond the designated point or focus at infinity which is also a pair of googolplex twin primes, as is described above. (As at August 2009, $65516468355 \cdot 2^{333333}-1$ and $65516468355 \cdot 2^{333333}+1$ (100,355 digits) is at the top of the list of top 20 twin primes pairs, while 2003663613*2^195000-1 and 2003663613*2^195000+1 (58,711 digits), which was the top twin primes pair in 2007, ranks second in this list. By utilising any of the above algorithms, preferably Algorithm 1, and the aid of powerful computers it would be possible to find larger and larger twin primes than these.) This would be a computer-assisted proof similar to the above-described proof of the Four-Colour problem in 1976 by Wolfgang Haken and Kenneth Appel. By the above lemma, the discovery of these larger googolplex twin primes beyond the above-said point or focus at infinity would imply that the twin primes are infinite. This would be a constructive, substantiated proof of the infinity of the twin primes. It would evidently be difficult to accept a proof of the twin primes conjecture without having to confirm or check the validity of the logic by computing a sufficiently long list of twin primes, even to the extent of looking out for counter-examples. Hence, the great importance of the above algorithms.

Also, we could have an indirect proof, a proof by contradiction, for no matter how large the twin primes we discover with any of the 2 algorithms are, we could find more twin primes with any of the 2 algorithms which would always be larger than the last pair of twin primes we discovered - all these newly found twin primes would be evidently larger than all the primes and twin primes in the list of products of consecutive primes/twin primes utilised to generate these new twin primes.

All this would constitute a well substantiated constructive proof of the infinity of the twin primes.

CONCLUSION

An array of methods has been adopted in this paper in solving the twin primes problem.

The inductive method, which is a well-established proof, is one of the methods utilised. The following lends support to this inductive proof of the infinity of the twin primes: (a) The characteristic of a mountain or infinite volume of sand is reflected in the characteristic of some grains of sand found there so that studying the characteristic of some grains of sand found there is enough for deducing the characteristic of the mountain or infinite volume of sand, to ascertain the quality of a batch of products it is only necessary to inspect some carefully selected samples from that batch of products and not everyone of the products and to carry out a population census, i.e., find out the characteristics of a population, it is only necessary to carry out a survey on some carefully selected respondents and not the whole population; in like manner, by the same principle, we just need to study a carefully selected list of integers and their associated primes and twin primes and deduce by induction whether the twin primes would always turn up, appear infinitely, in the list which is itself infinite - this act is rather like extrapolation. (For example, there are 376 pairs of twin primes (752 primes) found within the 2,500 consecutive primes from 2 to 22,307 - this means that 30.08%, which is sizeable, of the 2,500, not a small quantity, consecutive primes are twin primes. 3, 5 & 7 are the only "triple" primes found. There is no regularity in pattern in the appearance of the twin primes, except that the intervals between consecutive twin primes vary greatly by from 4 integers to 370 integers - the intervals between the consecutive twin primes increase and decrease, and, then increase and decrease again, by turns, giving rise to a graph that is characterised by many peaks, i.e., the curve is rough and nonlinear, making its description (hence, forecast of the twin primes) by differential equations practically impossible. By the principle of induction in this case we could deduce that the twin primes would be infinite.) (b) Therefore, if x is a subset of y and if x is a list of prime numbers while y is another list of prime numbers, the characteristic presence of twin primes in x suggests the characteristic presence of twin primes in y, so that if y is an infinite list of prime numbers, whence the prime numbers in it run to infinity, so do the twin primes in it. In fact, induction plays an important part in a number of the proofs.

The other argument used to prove the twin primes' infinity is the indirect (reductio ad absurdum) method, which had been used by Euclid and other mathematicians after him. Logically, 1 or 2 examples of "contradiction" should be sufficient proof of infinity, for it does not make sense to have a need for an infinite number of cases of "contradiction", as our proof would then have to be infinitely and impossibly long, an absurdity. This method of proof is "proof by implication" as a result of "contradiction" - which is a "short-cut" and smart way in proving infinity, instead of "proving infinity by counting to infinity", which is ludicrous, and, impossible. Hence, 1 or 2 cases of "contradiction" should be sufficient for implying that there would be an infinitude of twin primes, which of course also tacitly implies that there would be an infinitude of the number of cases of such "contradiction". (Euclid evidently had this logical point in mind when he formulated the indirect (reductio ad absurdum) proof of the infinity of the primes.) This method of proof had been cleverly used by a number of mathematicians, not the least by the great German mathematician, David Hilbert. For example, Hilbert had used an indirect method (the "reductio ad absurdum" proof) to prove Gordan's Theorem without having to show an actual "construction", a proof which had been accepted by his peers.

There is also the involvement of concepts from set theory, group theory, geometry, etc..

The paper describes several ways of finding or generating twin primes. Importantly, it presents 2 algorithms for generating or sieving all the twin primes in any range of odd numbers - by utilising any of these 2 algorithms, we would be able to find many twin primes which are all larger than those in any chosen list of consecutive primes, i.e., we would be able to generate many larger and larger twin primes. This is indeed significant. There is evidently some deep meaning in the ease with which the twin primes turn up, as is manifest in the above-described ways of obtaining them. It is thus evident that the twin primes are an inherent characteristic of the infinite prime numbers (as well as odd numbers), a characteristic which could be regarded as "self-similar" or "fractal". A twin primes pair is in effect any pair of odd numbers which differ from one another by 2 and are indivisible by any number except itself, the negative of itself, +1 and -1 (i.e., the pair of odd numbers are prime numbers). Any consecutive odd numbers or odd numbers that differ from one another by 2 are therefore potential prime numbers, as well as potential twin primes, and, the likelihood of them being prime is infinite (vide Euclid's proof and Dirichlet's Theorem), i.e., the primes would always be found amongst them and would be there all the way to infinity (the primes being evidently the "atoms" or building-blocks of all the whole numbers or integers, i.e., all the odd numbers and even numbers - every odd number or integer is either a prime number or composite of prime numbers (i.e., the integer has prime factors), and, every even number is the sum of two prime numbers (vide the Goldbach conjecture which, it appears, practically all mathematicians believe to be true), as well as the product of prime numbers (composite)); hence, the likelihood of them being twin primes is infinite as well (the twin primes being an inherent property of the infinite prime numbers - as well as odd numbers).

So far, there has not been any indication or confirmation that the number of twin primes is finite and the so-called largest existing pair of twin primes has not been found and confirmed (which of course would be impossible to find and confirm if the twin primes were infinite). On the other hand, practically everyone could intuit that the number of twin primes is infinite. Furthermore, there are much proofs or indications that there is an infinitude of twin primes.

We thus conclude that the list of the twin primes is infinite.

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