On Quantum Gravity and the Interpretation of Quantum Mechanics

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Abstract

It is shown that classical general relativity can be obtained as the limit of fully quantum mechanical vacuum made up of a dense assembly of positive and negative Planck mass particles in equal numbers, each of them occupying a Planck length volume. The positive and negative Planck masses can be viewed as mini black and white holes. Using Heisenberg's dictionary to translate classical mechanical quantities into corresponding quantum quantities, the solution of the Boltzmann equation for this positive negative Planck mass assembly leads to the quantum mechanics of the positive and negative Planck mass particles. There then, the positive and negative Planck masses can be described by two superfluids, each one with a phonon-roton spectrum. General relativity emerges at large scales in regions with a difference between positive and negative masses, with a rippling of space-time near the Planck length. Following 'tHooft's conjecture that the solution of quantum gravity is connected to a deterministic interpretation of quantum mechanics, it is proposed that the universe is separated through an event horizon to a parallel universe, by $10^{99}cm^{-3}$ Planck length Einstein-Rosen bridges kept open by the negative Planck masses, with time and space interchanged. The observable universe carries the particle nature of matter, while the hidden parallel universe its wave nature.

1. Introduction

Recognizing that the fundamental laws of physics must be quantum laws, with the laws of classical physics limiting cases of the quantum laws, not the other way around, all attempts to obtain ultimately correct quantum laws by a quantization of the classical field theories must remain guesses, but these guesses must be governed by a number of rules. One of the rules was already set by Planck in 1899 [1]. It requires that the fundamental laws of physics should contain as the only free parameters, $\hbar$, G, and c ($\hbar$ Planck's constant, G Newton's constant and c the velocity of light), in addition to the condition that dS/dt > 0, where S is the entropy. Superstring theory in 10 dimensions, proposed as a fundamental description of nature, including quantum gravity, satisfies the first three conditions, but not the dS/dt >0 condition for the four space-time dimension of every physics laboratory.

In the context of quantum gravity, the physical quantity "mass" should be intrinsically connected to the gravitational field for which it is its source, as the quantity "electric charge" is connected to the electromagnetic field as its source. From this perspective the Higgs field appears to be "the odd man out".

It is still widely believed that the standard model (SM) excluding gravity is renormalizable, and that according to this view quantum gravity is the "odd man out". But it has been emphasized by Weinberg [2], that by including in quantum electrodynamics (QED) all the possible higher order interaction terms, not only the **j · A** term of classical electromagnetism, QED becomes non-renormalizable. The same is true for all the renormalizable field theories of the SM. Accordingly, there is much less difference between the Einstein-Hilbert Lagrangian of the general theory of relativity and the Heisenberg-Euler Lagrangian of QED. With the inclusion of the lowest perturbative terms of QED in the Heisenberg-Euler Lagrangian, both theories are nonlinear, and both Lagrangians describe quite well physical reality in the low energy limit.

Due to the impossibility of making experiments at the Planck scale, the correct fundamental law must be guessed, and that must include quantum gravity. For the correct guess one may rely on the heuristic principle that the fundamental law should be simple. This heuristic principle turned out to be successful in the entire history of physics.

A gravitational field Lagrangian containing the infinitely large number of higher order terms obtained from the curvature tensor, hardly satisfies this heuristic principle.

2. Making the Guess

For the following we define:

$$m_p = \sqrt{\hbar c / G} \approx 10^{-5} g \quad \text{Planck mass}$$

$$r_p = \sqrt{\hbar G / c^3} \approx 10^{-33} cm \quad \text{Planck length}$$

$$t_p = \sqrt{\hbar G / c^5} \approx 10^{-44} \sec \quad \text{Planck time}$$

$$F_p = c^4 / G \approx 10^{50} dyn \quad \text{Planck force}$$

we then make the assumption that the scalar curvature invariant R of the Einstein-Hilbert Lagrangian is limited by

$$R \leq \pm 1 / r_p{}^2 \tag{1}$$

This could for example be done by changing the Einstein-Hilbert Lagrangian from

$$\mathrm{L} = \sqrt{-g}R \tag{2}$$

to

$$\mathrm{L} = \frac{\sqrt{-g}R}{\left(1-r_p{}^2R\right)\left(1+r_p{}^2R\right)} \tag{3}$$

Consistent with assumption (1) we guess that the vacuum of space is filled with an equal number of positive and negative Planck masses, in the average each Planck length volume filled with one Planck mass of either sign. Inequality (1) implies that equal sign Planck masses repel each other, while Planck masses of opposite sign pass through each other, with the space curvature for both cancelling each other out. However, because for a positive-negative mass dipole, the center of mass is located at ∞, the trajectories of either particle suffer a parallel displacement in space and time following their interaction. By how much can be computed by the Planck force acting over the distance r_p. For a Planck mass it leads to acceleration $a = F_p / m_p$, lasting for the Planck time t_p, leading to the displacement and the velocity of this displacement equal to

$$\begin{gathered} \Delta q = \frac{a}{2} t_p{}^2 = \pm \frac{F_p}{2m_p} t_p{}^2 = \pm \frac{r_p}{2} \\ \Delta \dot{q} = \pm c \end{gathered} \tag{4}$$

The displacement is accompanied by a fluctuation in momentum equal to $m_p c$ for the positive mass particle, and to a fluctuation $-m_p(-c) = m_p c$ for the negative mass particle moving in the opposite direction, altogether to a fluctuation in momentum equal to

$$\Delta p = 2m_p c \tag{5}$$

whereby because of (4)

$$\Delta p \Delta q \approx \hbar \tag{6}$$

in agreement with Heisenberg's uncertainty relation, here recovered from the existence of negative masses. Rather than violating Newton's actio = reactio, the recoil of the momentum fluctuation is transmitted to all the Planck masses occupying the vacuum. A similar effect is known in condensed matter physics for a particle-hole interaction. With $\Delta E = m_p c^2$ and $\Delta t = t_p$ the momentum fluctuation is accompanied by an energy fluctuation

$$\Delta E \Delta t = \hbar \tag{7}$$

With (6) and (7) Heisenberg's uncertainty relations for momentum and energy are thus explained by the mechanical fluctuations of the positive-negative Planck mass particle fluid and it is for this reason of no surprise, that Schrödinger's equation for a Planck mass particle can be derived from the Boltzmann equation for such a fluid [3].

The Boltzmann equation is given by [4]

$$\frac{\partial f}{\partial t}+\mathbf{v}\cdot\frac{\partial f}{\partial \mathbf{r}}+\mathbf{a}\cdot\frac{\partial f}{\partial \mathbf{v}}=\int \mathrm{v}_{rel}\left(f^{'}f^{'}_{1}-ff_{1}\right)d\sigma d\mathbf{v}_{1} \tag{8}$$

where f is the distribution function of the colliding particles, $f^{'}$, $f^{'}_{1}$ before and f, f_1 after the collision, with $f^{'}_{1}$ and f_1 the distribution functions of the particles which by colliding with those particles belonging to $f^{'}$ and f change the distribution function from $f^{'}$ to f. The magnitude of the relative collision velocity is $\mathrm{v}_{rel}=c$, and the collision cross section is σ. The particle number density is $n(\mathbf{r},t)=\int f(\mathbf{v},\mathbf{r},t)d\mathbf{v}$ and the average velocity $V=\int \mathbf{v}f(\mathbf{v},\mathbf{r},t)d\mathbf{v}/n(\mathbf{r},t)$. The acceleration is $\mathbf{a}=\mp(1/m_p)\nabla U$, where $U(\mathbf{r})$ is the potential of a force.

The Boltzmann equation for the distribution function $f_{\pm}$ of the positive and negative Planck mass particle is then

$$\frac{\partial f_{\pm}}{\partial t}+\mathrm{v}_{\pm}\cdot\frac{\partial f_{\pm}}{\partial \mathbf{r}}\mp\frac{1}{m_p}\frac{\partial U}{\partial \mathbf{r}}\cdot\frac{\partial f_{\pm}}{\partial \mathbf{v}_{\pm}}=4cr_p^{2}\int\left(f_{\pm}^{'}f_{\mp}^{'}-f_{\pm}f_{\mp}\right)d\mathbf{v}_{\pm} \tag{9}$$

where we have set $\sigma=\left(2r_p\right)^2=4r_p^{\,2}$ and $\mathrm{v}_{rel}=c$. U describes the average potential of all Planck mass particles on one Planck mass particle. The constraint keeping constant the average number density of all Planck mass particle leads to a pressure which has to be included in the potential U. It can be viewed as a potential holding together the positive and negative Planck mass particles, which otherwise would fly apart. The effective interaction between the positive and negative Planck mass particles is separated into the short range part entering the collision integral and the long range average potential part included in the potential U.

Because of (4) one has

$$f^{'}_{\pm}(\mathbf{r})=f_{\pm}\left(\mathbf{r}\pm\mathbf{r}_p/2\right) \tag{10}$$

where one has to average over all possible displacements and velocities of the “Zitterbewegung”. With the distribution function $f^{'}$ before the collision set equal to the displaced distribution function f, the direction of the “Zitterbewegung” velocity is in the opposite direction of the displacement vector $\mathbf{r}_p/2$. With (10) the integrand in the collision integral becomes

$$f_{\pm}^{'}f_{\mp}^{'}-f_{\pm}f_{\mp}=f_{\pm}\left(\mathbf{r}\pm\frac{\mathbf{r}_p}{2}\right)f_{\mp}\left(\mathbf{r}\mp\frac{\mathbf{r}_p}{2}\right)-f_{\pm}(\mathbf{r})\,f_{\mp}(\mathbf{r}) \tag{11}$$

Expanding $f_{\pm}\left(\mathbf{r}\pm\frac{\mathbf{r}_p}{2}\right)$ and $f_{\mp}\left(\mathbf{r}\mp\frac{\mathbf{r}_p}{2}\right)$ into a Taylor series

$$f_\pm\left(\mathbf{r}\pm\frac{\mathbf{r}_p}{2}\right)=f_\pm\pm\frac{\mathbf{r}_p}{2}\cdot\frac{\partial f_\pm}{\partial\mathbf{r}}+\frac{\mathbf{r}_p{}^2}{8}\cdot\frac{\partial^2 f_\pm}{\partial\mathbf{r}^2}+\cdots$$
$$f_\mp\left(\mathbf{r}\mp\frac{\mathbf{r}_p}{2}\right)=f_\mp\mp\frac{\mathbf{r}_p}{2}\cdot\frac{\partial f_\mp}{\partial\mathbf{r}}+\frac{\mathbf{r}_p{}^2}{8}\cdot\frac{\partial^2 f_\mp}{\partial\mathbf{r}^2}+\cdots \tag{12}$$

one finds up to second order that

$$\begin{aligned} f_\pm{}' f_\mp{}' - f_\pm f &\cong \pm\frac{\mathbf{r}_p}{2}\cdot\left(f_\mp\frac{\partial f_\pm}{\partial\mathbf{r}}-f_\pm\frac{\partial f_\mp}{\partial\mathbf{r}}\right)\\ &-\frac{\mathbf{r}_p{}^2}{4}\frac{\partial f_\pm}{\partial\mathbf{r}}\cdot\frac{\partial f_\mp}{\partial\mathbf{r}}+\frac{\mathbf{r}_p{}^2}{8}\left(f_\mp\frac{\partial^2 f_\pm}{\partial\mathbf{r}^2}+f_\pm\frac{\partial^2 f_\mp}{\partial\mathbf{r}^2}\right)\end{aligned} \tag{13}$$

with higher order terms suppressed by the Planck length. Because approximately

$f_\mp(\mathbf{v}_\mp,\mathbf{r},t)\cong f_\pm(\mathbf{v}_\pm,\mathbf{r},t)$, one has

$$\begin{aligned} f_\pm{}' f_\mp{}' - f_\pm f_\mp &\cong -\frac{\mathbf{r}_p{}^2}{4}\left(\frac{\partial f_\pm}{\partial\mathbf{r}}\right)^2+\frac{\mathbf{r}_p{}^2}{4}f_\pm\frac{\partial^2 f_\pm}{\partial\mathbf{r}^2}=\left(\frac{\mathbf{r}_p}{2}\right)^2 f_\pm{}^2\frac{\partial^2\log f_\pm}{\partial\mathbf{r}^2}\\ &\cong\left(\frac{\mathbf{r}_p}{2}\right)^2 f_\pm f_\mp\frac{\partial^2\log f_\pm}{\partial\mathbf{r}^2}\end{aligned} \tag{14}$$

To obtain the net displacement over a sphere with a volume to surface ratio $(r_p/2)^3/(r_p/2)^2=r_p/2$, (14) must be multiplied by the operator $(1/2)\mathbf{r}_p\cdot\partial/\partial\mathbf{r}$, and to obtain the net value in velocity space it must in addition be multiplied by the operator $\mathbf{c}\cdot\partial/\partial\mathbf{r}$, with the vector $\mathbf{c}$ in opposite direction to $\mathbf{r}_p$.

Integrating the r.h.s. of (9) over $d\mathbf{v}_\mp$, and setting $\int f_\mp d\mathbf{v}_\mp\cong 1/2r_p{}^3$, the number density of one Planck mass species in the undistributed configuration of the Planck mass particles filling space, one then has

$$\frac{\partial f_\pm}{\partial t}+\mathbf{v}_\pm\cdot\frac{\partial f_\pm}{\partial\mathbf{r}}\mp\frac{1}{m_p}\frac{\partial U}{\partial\mathbf{r}}\cdot\frac{\partial f_\pm}{\partial\mathbf{v}_\pm}=-\frac{c^2\mathbf{r}^2{}_p}{4}\frac{\partial^2}{\partial\mathbf{v}_\pm\partial r}\left(f_\pm\frac{\partial^2\log f_\pm}{\partial\mathbf{r}^2}\right) \tag{15}$$

For an approximate solution of (15) one computes its zeroth and first moment. The zeroth moment is obtained by integrating (15) over $d\mathbf{v}_\pm$, with the result that

$$\frac{\partial n_\pm}{\partial t}+\frac{\partial(n_\pm V_\pm)}{\partial\mathbf{r}}=0 \tag{16}$$

which is the continuity equation for the macroscopic quantities $n_\pm$ and $\mathbf{V}_\pm$. The first moment is obtained by multiplying (15) with $\mathbf{v}_\pm$ and integrating over $d\mathbf{v}_\pm$. Because the logarithmic dependence can with sufficient be written accuracy as $\partial^2 \log f_\pm / \partial \mathbf{r}^2 \approx \partial^2 \log n_\pm / \partial \mathbf{r}^2$, one finds

$$\frac{\partial\left(n_\pm \mathbf{V}_\pm\right)}{\partial t}+\frac{\partial\left(n_\pm \mathbf{V}_\pm \cdot \mathbf{V}_\pm\right)}{\partial \mathbf{r}}=\mp\frac{n_\pm}{m_p}\frac{\partial U}{\partial \mathbf{r}}+\frac{c^2\mathbf{r}^2{}_p}{4}\frac{\partial}{\partial \mathbf{r}}\left(n_\pm \frac{\partial^2 \log n_\pm}{\partial \mathbf{r}^2}\right) \tag{17}$$

With the help of (16) this can be written as

$$\frac{\partial \mathbf{V}_\pm}{\partial t}+\mathbf{V}_\pm \cdot \frac{\partial \mathbf{V}_\pm}{\partial \mathbf{r}}=\mp\frac{1}{m_p}\frac{\partial U}{\partial \mathbf{r}}+\frac{\hbar^2}{4{m_p}^2 n_\pm}\frac{\partial}{\partial \mathbf{r}}\left(n_\pm \frac{\partial^2 \log n_\pm}{\partial \mathbf{r}^2}\right) \tag{18}$$

for which one can also write

$$\frac{\partial \mathbf{V}_\pm}{\partial t}+\mathbf{V}_\pm \cdot \frac{\partial \mathbf{V}_\pm}{\partial \mathbf{r}}=\mp\frac{1}{m_p}\frac{\partial U}{\partial \mathbf{r}}+\frac{\hbar^2}{2{m_p}^2}\frac{\partial}{\partial \mathbf{r}}\left(\frac{1}{\sqrt{n_\pm}}\frac{\partial^2 \sqrt{n_\pm}}{\partial \mathbf{r}^2}\right) \tag{19}$$

The equivalence of (16) and (19) with the one-body Schrödinger equation for a positive or negative Planck mass can now be established by Madelung's transformation [5].

$$\begin{aligned} n_\pm &= {\psi_\pm}^* \psi_\pm \\ n_\pm V_\pm &= \mp \frac{i\hbar}{2m_p}\left[{\psi_\pm}^* \nabla \psi_\pm - \psi_\pm \nabla {\psi_\pm}^*\right] \end{aligned} \tag{20}$$

transforming Schrödinger's equation of a Planck mass $\pm m_p$

$$i\hbar \frac{\partial \psi_\pm}{\partial t} = \mp \frac{i\hbar}{2m_p}\nabla^2 \psi_\pm + U(\underline{r})\psi_\pm \tag{21}$$

into

$$\begin{aligned} &\frac{\partial n_\pm}{\partial t}+\frac{\partial\left(n_\pm \mathbf{V}_\pm\right)}{\partial \mathbf{r}}=0 \\ &\frac{\partial \mathbf{V}_\pm}{\partial t}+\mathbf{V}_\pm \cdot \frac{\partial \mathbf{V}_\pm}{\partial \mathbf{r}}=\mp\frac{1}{m_p}\frac{\partial}{\partial \mathbf{r}}\left[U+Q_\pm\right] \end{aligned} \tag{22}$$

where

$$Q_\pm = \mp \frac{\hbar^2}{2m_p}\frac{1}{\sqrt{n_\pm}}\frac{\partial^2 \sqrt{n_\pm}}{\partial \mathbf{r}^2} \tag{23}$$

is the so called quantum potential.

The importance of this result is that quantum mechanics is shown to have its cause in the existence of negative masses at the Planck scale. Another important result is that quantum mechanics becomes invalid for masses $m > m_p$. For masses large compared to m_p it has to be replaced by Newtonian mechanics , because the negative Planck masses of the vacuum cannot exert an appreciable "Zitterbewegung" on a mass $m >> m_p$. The uncertainty in quantum mechanics is not seen here due to a fundamental noncausal structure, but rather the consequence of the principal inability to make measurements for distances and times smaller than r_p and t_p, not permitting to calculate the otherwise deterministic outcome of the collisions between Planck mass particles which would require the knowledge obtained from such measurements.

The Schrödinger equation (21) is in line with Heisenberg's dictionary for translating classical into quantum mechanics, here obtained from the assumption of negative masses in the classical mechanical Boltzmann equation. The change of the function $f(\mathbf{r},t)$ in phase space, obtained by solving the Boltzmann equation, involves the scattering of a positive with a negative mass particle. In quantum mechanics the scattering of two positive mass particles can be described by the S-matrix. But since here the scattering of a positive with a negative Planck mass particle involves the non-Euclidean space-time geometry of a "black hole" interacting with a "white hole", the multivalued topology of this geometry may in a very fundamental way be at the root of the strange feature of quantum mechanics.

3. Quantum Mechanics of the Planck Mass Plasma

Having established quantum mechanics for a single Planck mass particle within a dense assembly of positive and negative Planck mass particle, a quantum mechanical description of the many body problems for all the Planck mass particles can be given. It is achieved 1) by setting the potential U in (21) equal to

$$U = 2\hbar c r_p^{\,2}\left[\psi^*_+\psi_+ - \psi_-^*\psi_-\right] \tag{24}$$

2) by replacing the field functions $\psi_\pm, \psi^*_\pm$ with the operators $\psi_\pm, \psi_\pm^{\;\dagger}$ obeying the canonical commutation relations

$$\left[\psi_\pm(r), \psi^\dagger_\pm(r')\right] = \delta\left(r - r'\right); \quad \left[\psi_\pm(r), \psi_\pm(r')\right] = \left[\psi^\dagger_\pm(r), \psi^\dagger_\pm(r')\right] = 0 \tag{25}$$

whereby (21) becomes the operator field equation

$$i\hbar\frac{\partial\psi_\pm}{\partial t} = \mp\frac{i\hbar}{2m_p}\nabla^2\psi_\pm \pm 2\hbar c r_p^{\,2}\left(\psi^\dagger_\pm\psi_\pm - \psi^\dagger_\mp\psi_\mp\right)\psi_\pm \tag{26}$$

We justify it as follows: An undisturbed dense assembly of Planck mass particles, each particle occupying the volume $r_p^{\,3}$, has the expectation value $<\psi_\pm^*\psi_\pm> = 1/2r_p^{\,3}$ whereby $2\hbar c r_p^{\,2} <\psi_\pm^*\psi_\pm> = m_p c^2$, implying an average potential energy $\pm m_p c^2$ for the positive and negative Planck mass particles within the assembly of all Planck masses, consistent with the value of the potential $F_p r_p = m_p c^2$ of the Planck for acting over the distance r_p. The interaction term between the positive and negative Planck mass fluid results from the constraint demanding that the number density of Planck mass particles shall (in the average) be equal to $1/r_p^{\,3}$. The rules of quantum mechanics for one Planck mass consistent with the Schrödinger equation, imply the one-particle commutation rule $[p,q] = \hbar/i$, which for a many-particle system of Planck mass particles leads to the canonical commutation relation (25) for the operator field equation (26) describing the many Planck mass particle system.

Equation (26) has the form of a nonrelativistic nonlinear Heisenberg equation, similar to Heisenberg's nonlinear spinor field equation proposed by him as a model of elementary particles. The two values for the chiralty of the zero rest mass spinors in his equation are replaced by the two signs for the Planck mass m_p in the kinetic energy term of (26). The limiting mass, conjectured by Heisenberg to separate the Hilbert space I, containing states of positive norm, from those of Hilbert space II having those of negative norm, becomes the Planck mass. But in contrast to Heisenberg's relativistic spinor equation, eq. (26) is nonrelativistic.

4. Hartree and the Hartree-Fock Approximation

To obtain solutions of the nonlinear quantized field equation (26), it appears that suitable nonperturbative approximation methods must be used. But perturbation theory contradicts the spirit of the theory, because before perturbation theory can be applied, a spectrum of elementary particles should be derived nonperturbatively. Fortunately, this is possible for a nonrelativistic theory. The most simple nonperturbative method which can be used to obtain approximate solutions of (26) is the self-consistent Hartree approximation.

In the Hartree approximation, one sets the expectation value of the product of three field operators equal to the product of their expectation values

$$\begin{aligned} &< \psi_{\pm}{}^{\dagger}\psi_{\pm}\psi_{\pm} > \cong \varphi_{\pm}{}^{*}\varphi^{2}{}_{\pm} \\ &< \psi_{\mp}{}^{\dagger}\psi_{\mp}\psi_{\pm} > \cong \varphi_{\mp}{}^{*}\varphi_{\mp}\varphi_{\pm} \end{aligned} \tag{27}$$

where $< \psi_{\pm} > = \varphi_{\pm}$, $< \psi_{\pm}{}^{\dagger} > = \varphi_{\pm}{}^{*}$. Taking the expectation value of (26), one obtains in this approximation

$$i\hbar \frac{\partial \varphi_{\pm}}{\partial t} = \mp \frac{\hbar^2}{2m_p} \nabla^2 \varphi_{\pm} \pm 2\hbar c r_p{}^2 \left(\varphi^{*}{}_{\pm}\varphi_{\pm} - \varphi^{*}{}_{\mp}\varphi_{\mp} \right) \varphi_{\pm} \tag{28}$$

which is a classical field equation.

However, if the temperature of the Planck mass plasma is close to absolute zero, each component is superfluid and should be described by a completely symmetric wave function. Under these circumstances, the Hartree approximation has to be replaced by the more accurate Hartree-Fock approximation, taking into account the exchange interactions neglected in the Hartree approximation. In the Hartree-Fock approximation one has to consider the symmetric wave function of two indentical Planck masses

$$\psi(1,2) = \frac{1}{\sqrt{2}} \left[\varphi_1(\mathbf{r})\varphi_2(\mathbf{r}') + \varphi_1(\mathbf{r}')\varphi_2(\mathbf{r}) \right] \tag{29}$$

There the expectation value for a delta-function-type contact interaction between the identical Planck mass particles is

$$< \psi(1,2) \,|\, \delta(\mathbf{r} - \mathbf{r}') \,|\, \psi(1,2) > = 2\varphi_1{}^2(\mathbf{r})\varphi_2{}^2(\mathbf{r}) \tag{30}$$

with the direct and exchange integrals making an equal contribution. One therefore has to put instead of (27)

$$< \psi_\pm^\dagger \psi_\pm \psi_\pm > \cong 2\varphi_\pm^* \varphi^2_\pm$$
$$< \psi_\mp^\dagger \psi_\mp \psi_\pm > \cong \varphi_\mp^* \varphi_\mp \varphi_\pm \tag{31}$$

In this approximation, one obtains from (26)

$$i\hbar \frac{\partial \varphi_\pm}{\partial t} = \mp \frac{\hbar^2}{2m_p} \nabla^2 \varphi_\pm \pm 2\hbar c r_p{}^2 \left(2\varphi^*{}_\pm \varphi_\pm - \varphi^*{}_\mp \varphi_\mp \right) \varphi_\pm \tag{32}$$

In the Hartree-Fock approximation the twice as large interaction between indentical Planck masses results from the completely symmetric wave function of the superfluid state.

5. Hydrodynamic formulation

In its ground state the Planck mass plasma is superfluid and has a phonon-roton spectrum for both the positive and negative mass components. Above the ground state it has a large number of quasiparticles, made up from stable vortex solutions. This can best be seen by the hydrodynamic formulation, for the Hartree and Hartree-Fock equations, in which equations (28) and (29), have the form of the nonlinear Schrödinger equations, obtained by the Madelung transformation [4]

$$n_\pm = \varphi^*{}_\pm \varphi_\pm$$
$$n_\pm \mathrm{v}_\pm = \mp \frac{i\hbar}{2m_p} \left[\varphi^*{}_\pm \nabla \varphi_\pm - \varphi_\pm \nabla \varphi^*{}_\pm \right] \tag{30}$$

they become

$$\frac{\partial \mathbf{v}_\pm}{\partial t} + \left(\mathbf{v}_\pm \cdot \nabla \right) \mathbf{v}_\pm = -\frac{1}{m_p} \nabla \left(U_\pm + Q_\pm \right)$$
$$\frac{\partial n_\pm}{\partial t} + \nabla \cdot \left(n_\pm \mathbf{v}_\pm \right) = 0 \tag{31}$$

In (31) $U_\pm$ is called the ordinary and $Q_\pm$ the quantum potential.
For the Hartree approximation one finds

$$U_\pm = 2m_p c^2 r^3{}_p \left(n_\pm - n_\mp \right) \tag{33}$$

and for the Hartree-Fock approximation

$$U_\pm = 2m_p c^2 r^3{}_p \left(2n_\pm - n_\mp \right) \tag{34}$$

The quantum potential in both cases is (somewhat different definition than in (23))

$$Q_\pm = -\frac{\hbar^2}{2m_p} \frac{\nabla^2 \sqrt{n_\pm}}{n_\pm} \tag{35}$$

The connection between (30) and (28) or (32) for the Hartree or the Hartree-Fock approximation is given by

$$\varphi_\pm = A_\pm e^{iS_\pm}, A_\pm > 0, \qquad 0 \le S_\pm \le 2\pi$$
$$n_\pm = A_\pm{}^2, \qquad \underline{\mathbf{v}}_\pm = \pm \frac{\hbar}{m_p} \operatorname{grad} S_\pm \tag{36}$$

showing that $\operatorname{curl} \mathbf{v}_\pm = 0$. The uniqueness of $S_\pm$ requires that

$$\oint \mathbf{v}_\pm \cdot d\mathbf{r} = 0 \tag{37}$$

but the uniqueness of $\varphi_\pm$ only requires that

$$\oint \mathbf{v}_\pm \cdot d\mathbf{r} = \pm nh/m_p \quad \mathrm{n} = 0,1,2\ldots \tag{38}$$

Implying multiply quantized vortices. From (38) one obtains the Helmholtz theorem for the quantized vortices

$$\frac{d}{dt}\oint \mathbf{v}_\pm \cdot d\mathbf{r} = 0 \tag{39}$$

With the vorticity vector $\boldsymbol{\omega}_\pm = \nabla \times \mathbf{v}_\pm$ and the vector identity $\left(\mathbf{v}_\pm \cdot \nabla\right)\mathbf{v}_\pm = \nabla\left(\mathbf{v}_\pm{}^2/2\right) - \mathbf{v}_\pm \mathrm{x} \mathit{curl}\mathbf{v}_\pm$ one finds for the vortex field both in the Hartree and Hartree-Fock approximation:

$$\frac{\partial \boldsymbol{\omega}_\pm}{\partial t} = \mathit{curl}\left(\mathbf{v}_\pm \mathrm{x} \boldsymbol{\omega}_\pm\right) \tag{40}$$

By order of magnitude the core radius of a vortex is obtained by equating the quantum potential $Q_\pm$ with $U_\pm$ one finds that the core radius is about equal to r_p, with the fluid velocity reached at this radius equal to c. Therefore, the velocity distribution in a singly quantized line vortex, expressed in cylindrical coordinates is

$$\begin{aligned} \left|\mathbf{v}_\pm\right| = \mathrm{v}_\phi &= c\left(r_p/r\right), \qquad && r > r_p \\ &= 0, && r < r_p \end{aligned} \tag{41}$$

From the stable vortex solutions a large number of possible quasiparticles emerge, from which Dirac spinor particles are composed of positive and negative masses [6]. Without the assumption of negative masses, models of a superfluid vacuum have also been proposed by Volovik [7], along the line of theories made for superfluid helium [8]. Electromagnetic and gravitational waves can be explained as two kinds of vortex waves in a lattice of vortex rings [9,10].

6. Towards a solution of the hierarchy problem

The positive-negative Planck mass model, which made it possible to derive quantum mechanics from the Boltzmann equation, also leads to a solution of the hierarchy problem: That the Fermi weak interaction constant is by 32 orders of magnitude larger than Newton's constant, contradicting the idea of "naturalness". And also why is the Higgs mass so much smaller than the Planck mass. To solve the hierarchy problem the following ideas have been proposed: Extra dimensions, brane worlds and

supersymmetry. Of these proposals only supersymmetry is not extravagant, but supersymmetry has so far not been observed.
From the experimentally established phonon-roton spectrum in superfluid helium, one obtains for the roton energy a value about 0.16 times the Debye energy. Replacing the Debye energy with the Planck energy $m_p c^2$, the mass of the rotons in the Planck mass plasma should be $m_r \approx 0.16 m_p$. In the two-component superfluid Planck mass plasma, there are positive and negative mass rotons $m_r^\pm \simeq \pm 0.16 m_p$.

We now consider the gravitational interaction of a positive mass roton with a negative mass roton separated by the distance r. For this (positive-negative) mass dipole the energy of the gravitational field is positive and with $m_r^- = -m_r^+$ given by

$$E = -\frac{G m_r^- m_r^+}{r} = \frac{G\left|m_r^\pm\right|^2}{r} \tag{42}$$

According to the mass-energy equivalence, this field has the mass

$$m = \frac{E}{c^2} = \frac{G\left|m_r^\pm\right|^2}{c^2 r} \tag{43}$$

A second equation is given by the uncertainty relation

$$\left|m_r^\pm\right| rc \simeq \hbar \tag{44}$$

Eliminating r from (2) and (3), one obtains

$$m = \frac{G\left|m_r^\pm\right|^3}{c\hbar} \tag{45}$$

or because of $G m_p^2 = \hbar c$,

$$\frac{m}{m_p} = \left(\frac{\left|m_r^\pm\right|}{m_p}\right)^3 \tag{46}$$

With $m_r^\pm \simeq \pm 0.16 m_p$, one finds that $m/m_p \simeq 4\times 10^{-3}$ or $mc^2 = 10^{16} GeV$, about equal to the GUT scale. If the positive gravitational field mass is added to the positive mass of the mass dipole, one obtains a pole-dipole mass configuration from which one can derive the Dirac equation. It is the small residual mass m of the gravitational field which is the mass of a Dirac particle.

While without the mass of the gravitational field a mass dipole would lead to self-acceleration, a pole-dipole configuration leads to a helical motion reaching the velocity of light. It is from this configuration that one can derive the Dirac equation. We therefore call this configuration a spinor roton, and suggest that the non-baryonic cold dark matter is made up of it.

Quasiparticles below the roton energy scale follow from resonant excitations of vortex rings. As shown by J.J. Thomson [11] a vortex ring with a vortex core radius r_o and a ring radius R, has a resonance frequency

$$\omega_r \approx c r_o / R^2 \tag{47}$$

The superfluid Planck mass plasma has quantized line vortices with the core radius $r_o = r_p$. A quantized ring vortex thus leads to quasiparticles with the energy

$$\hbar\omega_r \approx m_p c^2 \left(r_p / R\right) \tag{48}$$

A lattice of line vortices is stable if the ratio of the vortex core radius r_o to the distance of separation l between two line vortices is equal to [12]

$$r_o / l = 3.4\times10^{-3} \tag{49}$$

Setting $l = 2R$ and $r_o = r_p$, one has

$$R / r_p \approx 147 \tag{50}$$

No comparable stability calculation has been made for a three-dimensional lattice of vortex rings, but one can make a guess. The instability in a lattice of line vortices arises from the fluid velocity of one vortex on the adjacent vortex. For a ring vortex the velocity at the distance R / r_p is larger by the factor $\log\left(8R / r_p\right)$[13] .

A better value for R / r_p can therefore be obtained by solving the equation

$$R / r_p \approx 147 \log(8R / r_p) \tag{51}$$

With the result

$$R / r_p \approx 1360 \tag{52}$$

With this value one obtains for a positive and negative mass vortex ring quasiparticles with the energy

$$m^{\pm}c^2 = \pm m_p c^2 \left(r_p / R\right)^2 \approx \pm 1.32\times10^{12}\,\text{GeV} \tag{53}$$

setting $m^{\pm}$ equal to $m_r^{\pm}$ and inserting it into (46) one obtains

$$\frac{m}{m_p} = \left(\frac{r_p}{R}\right)^6 \approx 1.6\times10^{-19} \tag{54}$$

resulting in $m \approx 386\,\text{MeV}$, within the baryon energy scale of the standard model.

In arriving at these results we used the Newtonian approximation to compute the gravitational interaction energy of a positive mass with a negative mass. Including the lowest corrections from quantum gravity for the potential between energy between two masses m_1 and m_2, one has [14]

$$V(r) = -\frac{Gm_1 m_2}{r}\left(1+3\frac{G(m_1+m_2)}{2rc^2}+\frac{41}{10\pi}\frac{r_p^2}{r^2}\right) \tag{55}$$

For $m_2 = -m_1 = m$, this simplifies to

$$V(r) = \frac{Gm^2}{r}\left(1+\frac{41}{10\pi}\frac{r_p^2}{r^2}\right) \tag{56}$$

Even for rotons, where $r \approx 10r_p$, does quantum gravity only lead to a small correction. The situation resembles Bohr's theory of the hydrogen atom, where corrections from QED to the Coulomb potential of classical electrodynamics can be neglected.

It was shown by Redington [15], the Planck mass plasma hypothesis implies a "literal rippling" of space-time near the Planck length, but this is not enough to formulate a theory of quantum gravity.

7. Quantum gravity and the interpretation of quantum mechanics

According to 'tHooft the unsolved problem of quantum gravity might be intimately connected to the interpretation of quantum mechanics [16]. He asks: " Will the Copenhagen interpretation survive the 21st Century? " He does not take a stand against Born's statistical meaning of the wave function, but against its agnostic part which claims that "we will never be able to determine what actually happened during a physical experiment, and it is asserted that a deterministic theory is impossible." And he believes that a deterministic interpretation of quantum mechanics must come from a solution of quantum gravity as a dissipative deterministic system.

In the positive-negative Planck mass plasma the positive Planck mass can be interpreted as Planck mass black holes, and the negative Planck masses as Planck mass white holes. In this configuration our universe is connected to a second invisible parallel universe by a Planck length texture of 10^{99} cm^{-3} Einstein-Rosen bridges, with the negative masses keeping open the bridges. Taken as small Schwarzschild black holes the positive mass Planck mass particles have an event horizon at $r = r_p$, where upon crossing the event horizon from $r > r_p$ to $r < r_p$, the roles of space and time are interchanged.

With the negative mass Planck mass particles taking the role of small white holes, the vacuum becomes a texture of $r_p^{-3} \cong 10^{99}\mathrm{cm}^{-3}$ Einstein-Rosen bridges to a world where space and time are interchanged, with the negative mass Planck mass particles keeping the bridges open for a Planck time. This separates reality into an upper world where space and time appear as in our daily experience, and a lower world, parallel to the upper world, where the rules of space and time are interchanged, but with both communicating by a myriad of Einstein-Rosen bridges. We propose the hypothesis that the upper world is the world of what we perceive as particles, and the lower world as what we think are probability waves. We now can also understand why the solution of the Boltzmann equation for this positive-negative Planck mass particle plasma leads to the Schrödinger equation as a shadow of the upper particle world cast on the lower wave world.

Introducing the Planck length as a smallest length leads to the paradox that a smallest length is in violation of the special theory of relativity. In the proposed hypothesis that the vacuum of space is in reality a texture of Planck mass black and white holes, this paradox is eliminated by replacing the special theory of relativity with the general theory of relativity at this length, where this length rather is the extension of an event horizon.

With this division, into two different universes connected to each other by a texture of Einstein-Rosen bridges, the outcome of an EPR experiment becomes understandable. While in our universe the two particles have the same time in the moment an observation is made but are separated by a large distance, in the invisible parallel universe, both particles remain at the same position, with one of the particles getting younger, which means it is moving back in time. Accordingly, the wave-particle duality results from the topology of the two spaces connected by a large number of Einstein-Rosen (wormhole) bridges, one representing particles in time, which is the space of everyday experience, and one representing waves in time, with one space and three time coordinates.

The EPR experiment and the double slit experiment, the wave-particle dualism in the upper particle and lower wave world is sketched in Fig. 1 and Fig. 2.

8. Conclusion

The idea that quantum mechanics might be connected with the problem of quantum gravity and that it has a deterministic interpretation, first seems to show up in the derivation of quantum mechanics from the Boltzmann equation from the hypothesis that the vacuum is made up from a medium composed of positive and negative Planck mass particles. It is furthermore shown that this hypothesis can lead to a resolution of the hierarchy problem and to a spectrum of quasiparticles resembling the particles of the standard model.

Ultimately, a general relativistic treatment of the positive-negative Planck mass plasma, leads to the possibility to explain the strange long-range nonlocal quantum correlations by a vacuum texture of many Einstein-Rosen bridges to a hidden parallel universe with an “exchanged” space-time metric.

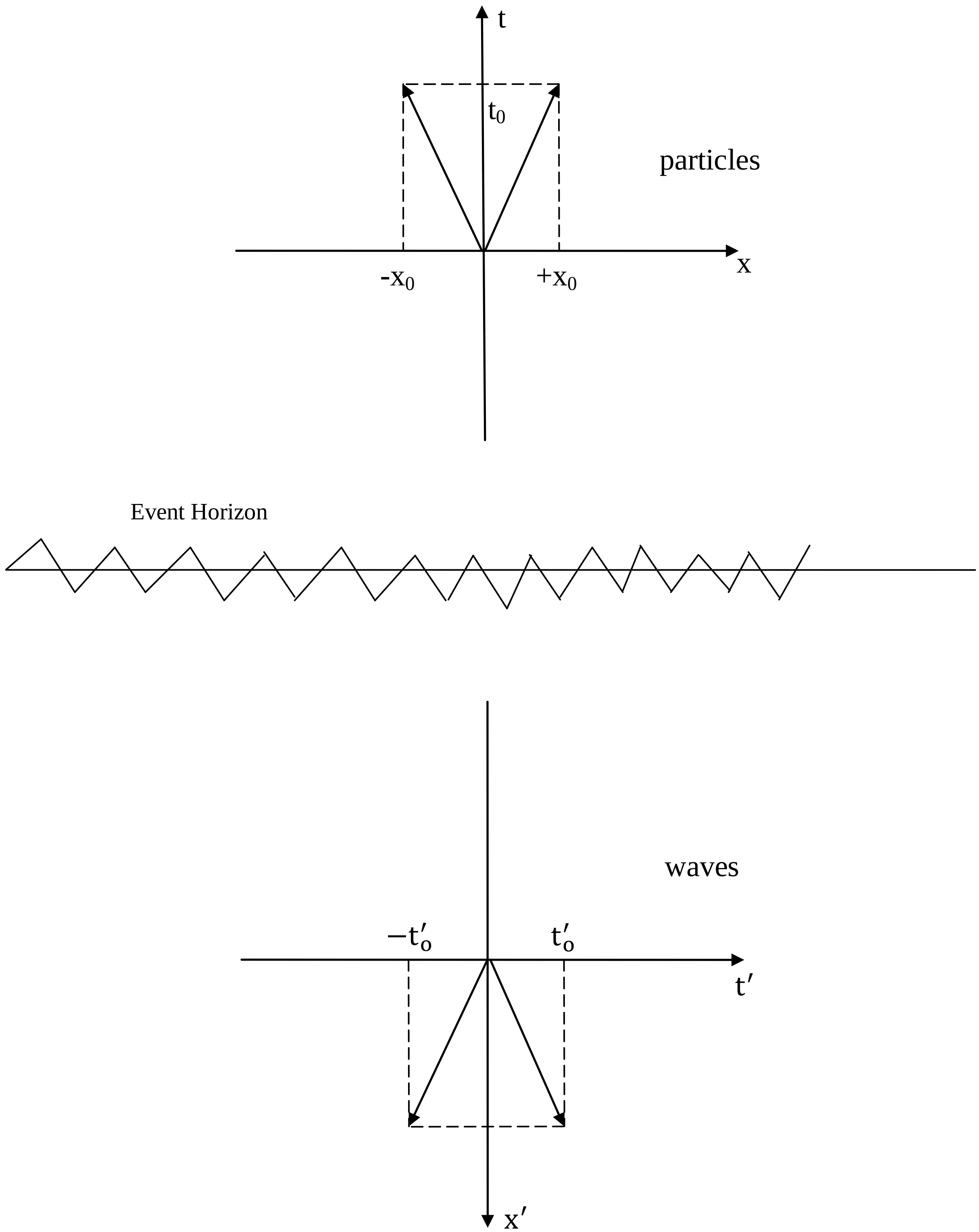

Fig. 1

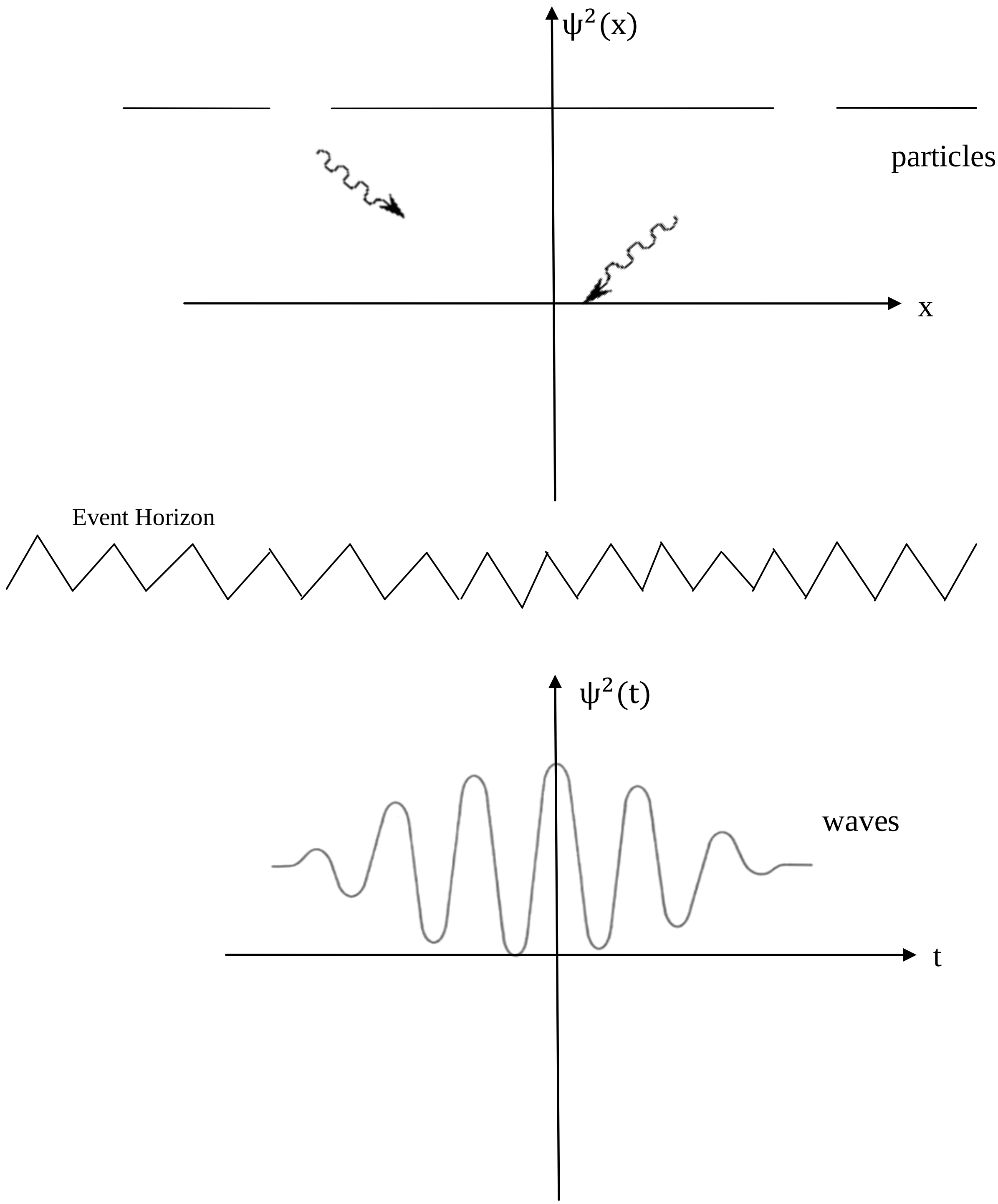

Fig. 2

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