Derivation of the fine structure constant

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An alternate interpretation of Quantum Theory is given. The fine structure constant is derived. An experiment is proposed.

There are varying philosophies on quantum experimentation so let us define a process thoroughly. Two measurements must be made, A and B. The boundary condition set by A will be used to predict the state at B. The observer applies physical theory to trace a trajectory into the future and predict what the state will be at that time. Before the observer can verify the theory, sufficient time must pass that the future event occurs. Once this happens a signal from the event reaches the observer in the present and a second measurement B becomes possible.

From the present we predict into the future. In time that becomes the past. When the signal from that event reaches the observer a theory can be tested. A three-fold process.

$$\mathcal{P}resent \mapsto \mathcal{F}uture \mapsto \mathcal{P}ast \mapsto \mathcal{P}resent \tag{1}$$

Time is a real number which takes on all values between t_{min} and t_{max}. If the observer's proper time is t_0 we can write the following with certainty.

$$\begin{aligned} &\mathcal{P}ast := [t_{min}, t_0) \\ &\mathcal{P}resent := [t_0] \\ &\mathcal{F}uture := (t_0, t_{max}] \end{aligned} \tag{2}$$

In General Relativity there is no inertial frame but one is assumed and $\mathcal{L}^2$ is the vector space of this approximation. Unitary evolution in this space is characterized by orders of α. This number should be a direct prediction of a complete Quantum Theory. A finely structured theory is needed, one which does not reside in the Hilbert space $\mathcal{H}$ alone. To be precise, define a Gel'fand triple $\{\aleph, \mathcal{H}, \Omega\}$ where each set contains a Minkowski picture $\mathbf{S}$.

$$\begin{aligned} \aleph &= \{\ x^\mu_- \in \mathbf{S} \mid t_{min} \le t < t_0\} \\ \mathcal{H} &= \{\ x^\mu \in \mathbf{S} \mid t = t_0\} \\ \Omega &= \{\ x^\mu_+ \in \mathbf{S} \mid t_0 < t \le t_{max}\} \end{aligned} \tag{3}$$

The Minkowski diagram gives a clear illustration. The past and future light cones define the half spaces $\aleph$ and Ω and the hypersurface of the present is a delta function $\delta(t-t_0)$. The present is defined according to the observer so it is an axiom of this interpretation that the observer is isomorphic to the δ function.

Our task is how to reconcile calculations in $\mathcal{H}$ with the actual dynamics of Nature proceeding around us and through us in $\aleph$ and Ω. To this end define an operator $\hat{M}^3$ that is non-unitary and complimentary to the unitary evolution operator $\hat{U}$.

$$\begin{aligned} &\hat{U} : \mathcal{H} \mapsto \mathcal{H} \\ &\hat{U} := \partial_x \\ &\hat{M}^3 : \mathcal{H} \mapsto\ \Omega \mapsto \aleph \mapsto\ \mathcal{H} \\ &\hat{M} := \partial_t \end{aligned} \tag{4}$$

A particle in a square well of length L is represented by a well known state vector. A particle confined to a time box of duration D should be represented by a similar vector. The form of the state vector confined in space and time follows from the example of the 2D box.

$$\begin{aligned} |\psi; x\rangle &= \psi_n(x) := \left(\frac{n\pi x}{L}\right) \\ |\psi; t\rangle &= \psi_m(t) := \left(\frac{m\pi t}{D}\right) \\ |\psi; x, t\rangle &= \psi_{nm}(x,t) := \left[\left(\frac{n\pi x}{L}\right), \left(\frac{m\pi t}{D}\right)\right] \end{aligned} \tag{5}$$

The values of L and D should not influence the theory so let us fix the Golden Ratio in the spirit of $C = 2\pi R$. Using deparameterized functions and the identity $\Phi = \varphi^{-1}$ we probe the action of $\hat{U}$ and $\hat{M}$ on $|\psi\rangle$.

$$D = 2\varphi L \tag{6}$$

$$\begin{aligned} \hat{U}\,|\psi; x\rangle &:= \partial_x(2n\pi x) = 2n\pi \\ \hat{M}\,|\psi; t\rangle &:= \partial_t(\Phi m\pi t) = \Phi m\pi \end{aligned} \tag{7}$$

We have defined a unitary time evolution operator and a non-unitary one. Assume the correct evolution operator is the sum of a unitary part and a non-unitary part so that $\hat{\Upsilon} \equiv \hat{U} + \hat{M}^3$.

$$\langle\psi; x, t|\,\hat{\Upsilon}\,|\psi; x, t\rangle = \langle\psi|\,\hat{U}\,|\psi\rangle + \langle\psi|\,\hat{M}^3\,|\psi\rangle \tag{8}$$

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$$\langle\psi|\,\hat{M}^3\,|\psi\rangle := \int \psi^*(t)\ \delta(t)\ \partial_t^3\psi(t)\ dt \tag{9}$$

Where the inclusion of $\delta(t)$ fixes the observer at the origin. The integral over all times will trace a path through $\aleph$, $\mathcal{H}$ and Ω. To use the integrand $f(t)\delta(t)$ we must employ the familiar method from complex analysis.

$$\begin{aligned}\int_{-\infty}^{\infty}\delta(t)\ f(t)\ dt = &\int_0^{t_{max}} f(r,0)\ dt + \\ &+\int_0^{\pi} f(\infty,\phi)\ d\phi + \\ &+\int_{t_{min}}^{0} f(r,\pi)\ dt\end{aligned} \tag{10}$$

This method is an outstanding logical proxy for the process proposed in equation (1). It also telling to note that the theory of advanced electromagnetic potentials, in which future events affect the present, is one of the very few places in physics where the operator ∂_t^3 appears. It seems the three-fold interpretation is philosophically robust.

The axiom that the observer is a δ function gives meaning to normalization. This process is allowed in all versions of Quantum Theory but is poorly justified. When the observer is included in the theory as $\delta(t)$ any extraneous constants can be absorbed into the that renormalizable function. In this way states remain unchanged under the seemingly arbitrary process of normalization.

The inner product $\langle\psi|\psi\rangle$ takes place in the complex plane so the rotation in equation (10) must be through an unidentified hypercomplex plane. Using equation (3) we define a non-unitary set of basis vectors in $\mathbb{C}^3$ which will identify Dirac vectors with the state spaces $\{\aleph, \mathcal{H}, \Omega\}$.

$$\begin{aligned}|\psi\rangle\,\hat{\pi} &= \psi(x,t)\\ |\psi\rangle\,\hat{\varphi} &= \psi(x_+,t_+)\\ |\psi\rangle\,\hat{i} &= \psi(x_-,t_-)\end{aligned} \tag{11}$$

$$\|\hat{i}\| = i \qquad \|\hat{\pi}\| = 1 \qquad \|\hat{\varphi}\| = \varphi \tag{12}$$

$$\begin{aligned}\hat{\pi} &= \Phi\ \hat{\varphi} & \hat{\pi} &= -i\ \hat{i}\\ \hat{i} &= i\ \hat{\pi} & \hat{i} &= i\ \Phi\ \hat{\varphi}\\ \hat{\varphi} &= -i\varphi\ \hat{i} & \hat{\varphi} &= \varphi\ \hat{\pi}\end{aligned} \tag{13}$$

The past is defined according to the unitary vector $\hat{i}$ and the future is defined according to the non-unitary vector $\hat{\varphi}$. It follows that $|t_{max}| > |t_{min}|$ in the way that uncountable infinity is greater than $\aleph_0$. Therefore Ω is a larger space than $\aleph$ as required for a Gel'fand triple. This is the reconciliation of the limits in equation (10). Such a volume disparity will produce forward flowing time as energy densities tend to decrease.

The action of $\{\hat{i},\ \hat{\pi}, \hat{\varphi}\}$ on Dirac vectors is defined in equations (14). We quantify the forward flow of time with the hypercomplex conjugate structure defined in (15). Using $\partial_t\,|\psi\rangle := \Phi\,|\psi\rangle$ we explore the physics of the non-unitary basis in equations (16)-(18). The introduction of the hypercomplex vector $\hat{\varphi}$ brings a fractal structure to the algebra seen in equation (17) when we use the identities $\Phi^2 = \Phi + 1$ and $\Phi\hat{\varphi} = \hat{\pi}$.

$$\hat{\varphi}\,\langle\psi|\psi\rangle\,\hat{\pi} = \hat{\varphi}^*\hat{\pi}\iiint \psi^*(x_+^i)\ \psi(x^i)\ dx_+^i dx^i dt \tag{14}$$

$$\begin{aligned}\varphi^* &= \Phi\\ \Phi^* &= -i\varphi\\ \varphi^{**} &\neq \varphi\end{aligned} \tag{15}$$

$$\begin{aligned}\hat{M}^1\,|\psi\rangle\,\hat{\pi} &:= \Phi\ \ |\psi\rangle\,\hat{\varphi}\\ \hat{M}^2\,|\psi\rangle\,\hat{\pi} &:= \Phi^2\|\hat{\varphi}\|\ \ |\psi\rangle\,\hat{i}\\ \hat{M}^3\,|\psi\rangle\,\hat{\pi} &:= \Phi^3\,\|\hat{\varphi}\|\ \ \|\hat{i}\|\ \ |\psi\rangle\,\hat{\pi}\end{aligned} \tag{16}$$

$$\begin{aligned}\hat{M}^3\,|\psi\rangle\,\hat{\pi} &:= i\Phi^2\ \ |\psi\rangle\ \ \hat{\pi}\\ &:= |\psi\rangle\,i(\Phi+1)\ \hat{\pi}\\ &:= i\Phi^2\,|\psi\rangle\,\hat{\varphi} + |\psi\rangle\,\hat{i}\end{aligned} \tag{17}$$

$$\begin{aligned}\hat{M}^0\,|\psi\rangle\,\hat{\pi} &:= 1\,|\psi\rangle\,\hat{\pi}\\ \hat{M}^1\,|\psi\rangle\,\hat{\pi} &:= \Phi\ \ |\psi\rangle\ \ \hat{\varphi}\\ \hat{M}^2\,|\psi\rangle\,\hat{\pi} &:= \Phi\ \ |\psi\rangle\ \ \hat{i}\\ \hat{M}^3\,|\psi\rangle\,\hat{\pi} &:= i\Phi^2\ \ |\psi\rangle\ \ \hat{\pi}_2\\ \hat{M}^4\,|\psi\rangle\,\hat{\pi} &:= i\Phi^3\ \ |\psi\rangle\ \ \hat{\varphi}\\ \hat{M}^5\,|\psi\rangle\,\hat{\pi} &:= i\Phi^3\ \ |\psi\rangle\ \ \hat{i}\\ \hat{M}^6\,|\psi\rangle\,\hat{\pi} &:= -\Phi^4\ \ |\psi\rangle\ \ \hat{\pi}_3\\ \hat{M}^7\,|\psi\rangle\,\hat{\pi} &:= -\Phi^5\ \ |\psi\rangle\ \ \hat{\varphi}\\ \hat{M}^8\,|\psi\rangle\,\hat{\pi} &:= -\Phi^5\ \ |\psi\rangle\ \ \hat{i}\\ \hat{M}^9\,|\psi\rangle\,\hat{\pi} &:= -i\Phi^6\ \ |\psi\rangle\ \ \hat{\pi}_4\\ \hat{M}^{10}\,|\psi\rangle\,\hat{\pi} &:= -i\Phi^7\ \ |\psi\rangle\ \ \hat{\varphi}\\ \hat{M}^{11}\,|\psi\rangle\,\hat{\pi} &:= -i\Phi^7\ \ |\psi\rangle\ \ \hat{i}\\ \hat{M}^{12}\,|\psi\rangle\,\hat{\pi} &:= \Phi^8\ \ |\psi\rangle\ \ \hat{\pi}_5\end{aligned} \tag{18}$$

One rotation through the complex plane generates thirteen equations containing an eight-fold way and a five-fold symmetry. If the powers of Φ represent the Gell-Mann matrices then the generators of SU(3) are a subset of the generators $\hat{M}^i$. Odd powers of Φ appear in this sequence twice meaning the theory will support fermions. Even powers of Φ associated with the five $\hat{\pi}_i$ can support integer spin values $J = 2, 1, 0, -1, -2$.

The monotonic nature of $\hat{\varphi}$, always smaller, provides a good basis on which to construct a quantum theory of gravity where distances between particles tend to decrease. Indeed the expansion of the universe into ever larger Fibonacci cells is consistent with dark energy [1–3].

We have shown that solutions of the proposed non-unitary evolution operator exist and that the theory is consistent with the interpretation. Moving forward we calculate the eigenvalues of $\hat{\Upsilon}$.

$$\begin{aligned} \hat{\Upsilon}\,|n,m\rangle &= \upsilon_{nm}\,|n,m\rangle \\ \hat{\Upsilon}\,|n,m\rangle &:= \left(\partial_x + \partial_t^3\right)|2n\pi x, \Phi m\pi t\rangle \\ \hat{\Upsilon}\,|n,m\rangle &:= \left[2n\pi + (\Phi m\pi)^3\right]|n,m\rangle \\ \upsilon_{11} &:= 2\pi + (\Phi\pi)^3 = 137.6 \approx \alpha^{-1} \end{aligned} \tag{19}$$

The parameter characterizing each order of perturbation theory is associated with $\hat{\Upsilon}$. We have defined the flow of time as the quantum locomotion of three states through a Fibonacci structure in spacetime. Quantum statistics associated with such a trifecta have been developed by Palev and Van der Jeugt [4].

The three state spaces $\{\aleph, \mathcal{H}, \Omega\}$ mimic the structure of hadrons and compose a 9+1D space isomorphic to the domain of string theory. In its current formulation string theory takes one dimensionful parameter: the length of the string. However, it may be possible to use equation (6) to reformulate string theory without reference to dimensionful parameters. The interpretation of such a string theory will be the dynamical evolution of boundary conditions on the cosmic structure $\{\aleph, \mathcal{H}, \Omega\}$.

If variations in α can be detected by varying the delay between an event and its measurement in an experimental apparatus that will strongly support the ideas presented here.

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