

# The arithmetic of binary representations of even positive integer $2n$ and its application to the solution of the Goldbach's binary problem

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## Abstract

One of causes why Goldbach's binary problem was unsolved over a long period is that binary representations of even integer  $2n$  (BR $2n$ ) in the view of a sum of two odd primes (VSTOP) are considered separately from other BR $2n$ . By purpose of this work is research of connections between different types of BR $2n$ . For realization of this purpose by author was developed the "Arithmetic of binary representations of even positive integer  $2n$ " (ABR $2n$ ). In ABR $2n$  are defined four types BR $2n$ . As shown in ABR $2n$  all types BR $2n$  are connected with each other by relations which represent distribution of prime and composite positive integers less than  $2n$  between them. On the basis of this relations (axioms ABR $2n$ ) are deduced formulas for computation of the number of BR $2n$  (NBR $2n$ ) for each types. In ABR $2n$  also is defined and computed Average value of the number of binary sums are formed from odd prime and composite positive integers  $< 2n$  (AVNBS). Separately AVNBS for prime and AVNBS for composite positive integers. We also deduced formulas for computation of deviation NBR $2n$  from AVNBS. It was shown that if  $n$  go to infinity then NBR $2n$  go to AVNBS that permit to apply formulas for AVNBS to computation of NBR $2n$ . At the end is produced the proof of the Goldbach's binary problem with help of ABR $2n$ . For it apply method of a proof by contradiction in which we make an assumption that for any  $2n$  not exist BR $2n$  in the VSTOP then make computations at this conditions then we come to contradiction. Hence our assumption is false and for all  $2n > 2$  exist BR $2n$  in the VSTOP.

## 1 Introduction

On 7 June, 1742, the Prussian mathematician Christian Goldbach wrote a letter to Leonhard Euler in which he proposed the following conjecture: Every integer greater than 2 can be written as the sum of three primes. He considered 1 to be a prime number. In mathematics, a prime number is a natural number that has exactly two natural number divisors, which are 1 and the prime... , a convention subsequently abandoned. A modern version of Goldbach's original conjecture is: Every integer greater than 5 can be written as the sum of three primes. Euler, becoming interested in the problem, replied by noting that this conjecture is equivalent with another version: Every even integer greater than 2 can be written as the sum of two primes. Euler's version is the form in which the conjecture is usually expressed today. It is also known as the strong, even, or binary Goldbach's conjecture.

## 2 The arithmetic of binary representations of even positive integer $2n$

### 2.1 General conception

**Definition 2.1** *The binary representations of even positive integer  $2n$  in  $ABR2n$  are defined as bijective mappings :*

$$f : X \rightarrow Y \quad (1)$$

$$y = 2n - x \quad (2)$$

Where:

$$X \{x | x \in N, 1 \leq x < n\} \quad (3)$$

$$Y \{y | y \in N, n < y < 2n\} \quad (4)$$

$n$ - positive integer .

$$|X| = |Y| \quad (5)$$

**Definition 2.2** *The set of binary representations of even positive integer  $2n$  ( $SBR2n$ ) are defined as follows :*

$$SBR2n \{1 + (2n - 1) = 2n ;$$

$$2 + (2n - 2) = 2n ;$$

...

$n - 1 + (2n - (n - 1)) = 2n$   
 The last is got by represent (2)  
 in the view of  $x + y = 2n$ .

**Definition 2.3** The set  $XUY$  consist of: even positive integers ,  
 odd positive integers inclusive odd composite positive integers  
 and noncomposite positive integers (pimes and "1").

**Remark 2.1** Since  $n$  is not mapped into  $Y$  and it is mapped into itself  
 (automorphism) therefore mapping  $n \rightarrow n$  and corresponding binary rep-  
 resentation  $n + n = 2n$  do not enter into  $SBR2n$  which is formed only  
 from bijective mappings .By this reason  $n$  do not enter into  $XUY$ . But it  
 is not denoted that as a result of the exception of binary representations:  
 $2 + 2 = 4$  and  $3 + 3 = 6$  from  $SBR2n$ , 4 and 6 have not binary representa-  
 tions in the view of a sum of two noncomposite positive integers. But it is  
 not the case.In  $SBR2n$  is in existence the binary representations  $1 + 3 = 4$   
 and  $1 + 5 = 6$ . It is significant "1" with primes are related in  $ABR2n$   
 to noncomposite positive integers. At that special status of "1" in  $N$  is  
 ignored in  $ABR2n$ .

**Definition 2.4**  $s_e$  - the number of even composite positive integers into  
 $SBR2n < 2n$  .  
 $s_e$  - positive integer  $> 0 \forall 2n > 6$   $s_e = 0 \forall 2 < 2n < 8$ .

**Definition 2.5**  $s_o$  - the number of odd composite positive integers into  
 $SBR2n < 2n$  .  
 $s_o$  - positive integer  $> 0 \forall 2n > 8$   $s_o = 0 \forall 2 < 2n < 10$ .

**Definition 2.6**  $p$ - the number of odd noncomposite  
 integers (primes and "1") into  $SBR2n < 2n$  .  
 $p$  - positive integer  $> 0 \forall 2n > 2$ .

**Corollary 2.1** By reason of exclusion  $n$  from the set  $XUY$  if  $n$  is even  
 then  $n$  or  $s_e$  are decremented by two (since it is excluded  $n + n = 2n$ ):  
 $n^* = n - 2$ ;  $s_e^* = s_e - 2$ .

**Corollary 2.2** By reason of exclusion  $n$  from the set  $X U Y$  If  $n$  is odd  
 then  $n$  or  $s_o$  are decremented by two:  
 $n^* = n - 2$ ;  $s_o^* = s_o - 2$ .

**Corollary 2.3** By reason of exclusion  $n$  from the set  $X U Y$  If  $n$  is odd  
 prime then  $n$  or  $p$  are decremented by two:  
 $n^* = n - 2$ ;  $p^* = p - 2$ .

**Remark 2.2** *It needs to say that corollaries 2.1 , 2.2 , 2.3 are took into account automatically at forming SBR2n since it makes until appearance of representation in the view of  $n + n = 2n$  which throw-off . Thus if  $n$  is odd or odd prime then  $n, s_o$  automatically are decremented by two at the direct computation them into SBR2n. if  $n$  is even then  $n, s_o, p$  are not decremented since  $|Q|, |L|, |H|, s_o, p$  by definitions concern to odd composite and noncomposite integers.*

**Remark 2.3** *In the equations which include  $n$  is even  $n, s_o, p$  are not decremented (see rem 2.2 ).*

*At the computations which concern to  $s_e$   $n$  is decremented by two.*

**Remark 2.4** *In the equations which include  $n$  is odd or odd prime then it is decremented by one since  $n$  enters in the equation with value which is required the correction.*

**Proposition 2.1**  $\forall n > 1$  *always is fulfilled the condition  $|X| = |Y|$ .*

**Proof 2.1** *Taking into account that  $2n$  do not enter into the set  $XUY$  and  $n$  is excluded from the set  $XUY$  (see remark 2.1) then we have :*

$$|XUY| = 2n - 2 = 2(n - 1) \quad (6)$$

*Hence  $|XUY|$  is even for any  $n$  and thus  $\forall n > 1$  always is fulfilled the condition  $|X| = |Y|$  .  $\square$*

## 2.2 The types of binary representations even positive integer $2n$

In depend of that are  $x, y$  in the view of  $x + y = 2n$  prime or composite it can be four types binary representations of even positive integer  $2n$ :

**Definition 2.7** *There is Type "H" if  $x$ -odd prime positive integer or "1" and  $y$ - odd prime positive integer.*

$|H|$ - *the number of binary representations of Type "H".*

$|H|$ - *positive integer  $> 0 \forall 2n > 2$  see Theorem (10.1).*

**Definition 2.8** *There is Type "Q" if  $x$ -odd composite positive integer; and  $y$ -odd composite positive integer .*

$|Q|$ - *the number of binary representations of Type "Q".*

$|Q|$ - *positive integer  $> 0 \forall 2n > 22$  see corollary (9.1)*

*excepting  $2n = 26; 28; 32; 38$ ; in which  $|Q| = 0$*

*(see numerical solution  $|Q| = 0$  subsection (12.2) .*

$|Q| = 0 \forall 2 < 2n < 24.$

**Definition 2.9** *There is Type "L" if  $x$ - odd prime positive integer or "1" and  $y$ -odd composite positive integer or  $x$ -odd composite positive integer and  $y$ - odd prime positive integer.*

$|L|$  - the number of binary representations of Type "L".

$|L|$ - positive integer  $> 0 \forall 2n > 8$ .

$|L| = 0 \forall 2 < 2n < 10$ .

**Definition 2.10** *There is Type "E" if  $x$ -even positive integer and  $y$ -even positive integer .*

$|E|$ - the number of binary representations of Type "E".

$|E|$ - positive integer  $> 0 \forall 2n > 4$ .

### 2.3 The axioms of ABR2n

**Axiom 2.1** *The number of the binary representations type "H" (NBRH) is connected with The number of the binary representations type "L" (NBRL) as follows:*

$$2|H| + |L| = p \quad (7)$$

$\forall 2n > 2$ .

*The equation (7) asserts that odd noncomposite positive integers less than  $2n$  are allotted to types "H", "L" in compliance with balance (7).*

**Remark 2.5** *From equations (7 and 8) it follows :*

$$|H| = (p - |L|)/2; |Q| = (s_0 - |L|)/2 .$$

*In these equations is division by '2'. From this it can conclude that equations are correct only for  $n$  is even. But it is not the case. Below we show that in ABR2n all equations where is division by '2' It always is possibility for any  $n$ 's and  $p$ 's parity.*

**Proposition 2.2** *The equation (7) is true for any  $n$ 's and  $p$ 's parity.*

**Proof 2.2 Case 2.1** *For  $n$  is even  $p$  is even*

*By equation (7) we get:  $|H| = (p - |L|)/2$  .*

*By equation (26)  $|L| = n - p - 2|Q|$  and  $|L|$  is even*

*for  $n$  and  $p$  is even then  $(p - |L|)$  is even .*

*Hence the division by "2" is possibility then  $|H|$*

*is integer for  $n$  and  $p$  is even .*

*Thus equation (7) is true for  $n$  is even  $p$  is even.*

**Case 2.2** For  $n$  is even ;  $p$  is odd

By equation (7) we get:  $|H| = (p - |L|)/2$  .

By equation (26)  $|L| = n - p - 2|Q|$  and  $|L|$  is odd for  $n$  is even and  $p$  is odd then  $(p - |L|)$  is even .

Hence the division by "2" is possibility then  $|H|$  is integer for  $n$  is even and  $p$  is odd .

Thus equation (7) is true for  $n$  is even  $p$  is odd.

**Case 2.3** For  $n$  is odd or odd prime;  $p$  is even

By equation (7) we get:  $|H| = (p - |L|)/2$  .

By equation (27)  $|L| = n - p - 1 - 2|Q|$  and  $|L|$  is even for  $n$  is odd or odd prime and  $p$  is even then  $(p - |L|)$  is even .

Hence the division by "2" is possibility then  $|H|$  is integer for  $n$  is odd or odd prime and  $p$  is even .

Thus equation (7) is true for  $n$  is odd or odd prime;  $p$  is even.

**Case 2.4** For  $n$  is odd or odd prime ;  $p$  is odd

By equation (7) we get:  $|H| = (p - |L|)/2$  .

By equation (27)  $|L| = n - p - 1 - 2|Q|$  and  $|L|$  is odd for  $n$  is odd or odd prime and  $p$  is odd then  $(p - |L|)$  is even .

Hence the division by "2" is possibility then  $|H|$  is integer for  $n$  is odd or odd prime and  $p$  is odd .

Thus equation (7) is true for  $n$  is odd or odd prime;  $p$  is odd.

Finally equation (7) is true for any  $n$ 's and  $p$ 's parity.  $\square$

**Axiom 2.2** The number of the binary representations Type  $Q$  (NBRQ) is connected with the number of the binary representations Type "L" (NBRL) as follows:

$$2|Q| + |L| = s_0 \quad (8)$$

$\forall 2n > 2$  .

The expression(8) asserts that odd composite positive integers less than  $2n$  are allotted to types "Q", "L" in compliance with balance (8).

**Proposition 2.3** The equation (8) is true for any  $n$ 's and  $p$ 's parity.

**Proof 2.3 Case 2.5** For  $n$  is even ;  $p$  is even

By equation (8) we get:  $|Q| = (s_0 - |L|)/2$  .

By equation (7)  $|L| = p - 2|H|$  and  $|L|$  is even for  $n$  is even and  $p$  is even .

By equation (17)  $s_0 = n - p$  is even then  $(s_0 - |L|)$  is even .

Hence the division by "2" is possibility then  $|Q|$  is integer for  $n$  is even and  $p$  is even.  
Thus equation (8) is true for  $n$  is even ;  $p$  is even.

**Case 2.6** For  $n$  is even  $p$  is odd

By equation (8) we get:  $|Q| = (s_0 - |L|)/2$  .

By equation (7)  $|L| = p - 2|H|$  and  $|L|$  is odd

for  $n$  is even ;  $p$  is odd .

By equation (17)  $s_0 = n - p$  is odd then  $(s_0 - |L|)$  is even .

Hence the division by "2" is possibility then  $|Q|$

is integer for  $n$  is even and  $p$  is odd

Thus equation (8) is true for  $n$  is even  $p$  is odd

**Case 2.7** For  $n$  is odd or odd prime ,  $p$  is even

By equation (8) we get:  $|Q| = (s_0 - |L|)/2$  .

By equation (7)  $|L| = p - 2|H|$  and  $|L|$  is even

for  $n$  is odd or odd prime  $p$  is even .

By equation (18)  $s_0 = n - p - 1$  is even then  $(s_0 - |L|)$  is even .

Hence the division by "2" is possibility then  $|Q|$

is integer for  $n$  is odd or odd prime and  $p$  is even .

Thus equation (8) is true for  $n$  is odd or odd prime ;  $p$  is even.

**Case 2.8** For  $n$  is odd or odd prime  $p$  is odd

By equation (8) we get:  $|Q| = (s_0 - |L|)/2$  .

By equation (7)  $|L| = p - 2|Q|$  and  $|L|$  is odd

for  $n$  is odd or odd prime  $p$  is odd .

By equation (18)  $s_0 = n - p - 1$  is odd then  $(s_0 - |L|)$  is even .

Hence the division by "2" is possibility then  $|Q|$

is integer for  $n$  is odd or odd prime and  $p$  is odd.

Thus equation (8) is true for  $n$  is odd or odd prime  $p$  is odd.

Finally equation (8) is true for any  $n$ 's and  $p$ 's parity.  $\square$

**Definition 2.11**  $G$ - the general number of binary representations in  $SBR_{2n}$ .

$G$  - positive integer  $\forall 2n > 2$  .

**Axiom 2.3**

$$|Q| + |L| + |H| + |E| = G \quad (9)$$

**Definition 2.12**  $F$ -the general number of binary representations with odd positive integers .

$F$  – positive integer  $\forall 2n > 2$  .

**Axiom 2.4**

$$|Q| + |L| + |H| = F \quad (10)$$

## 2.4 The computation of $G, F, S_o, p, |Q|, |L|, |H|, |E|$

### 2.4.1 The computation of $G$

**Proposition 2.4**

$$G = n - 1 \quad (11)$$

$\forall n > 1$

**Proof 2.4** The general number of elements in the set  $XUY$  by equation (6) equals  $2(n - 1)$ . Taking into account that in forming of each binary representation participate with two elements from the set  $XUY$  then we have:  $G = 1/2(2(n - 1)) = n - 1$ .  $\forall n > 1$  .  $\square$

### 2.4.2 The computation of $|E|$

**Proposition 2.5**

$$|E| = [(n - 1)/2] \quad (12)$$

$\forall n > 1$

**Proof 2.5** By definition (2.1) by elements of the set  $XUY$  are numbers of the natural scale. Half of them are even integers.

Then the number of even integers in the set  $XUY$  equals:

$$1/2|XUY| = 1/2(2(n - 1)) = n - 1 .$$

Taking into account that in forming of each binary representation participate by two elements from the set  $XUY$  we have:  $|E| = (n - 1)/2$  .

Taking into account that for  $n$  - even  $|E|$  is not integer

that breaks the status of  $|E|$  (see definition (2.10))

then  $|E| = [(n - 1)/2]$  is aliquot of  $(n - 1)/2$

then we get :  $|E| = [(n - 1)/2] \forall n > 1$   $\square$

**Proposition 2.6**

$$E = n/2 - 1 \quad (13)$$

for  $n$  is even ;  $n > 1$



**Proof 2.6** For  $n = 2i$  where  $i \in N$  by equation (12) we get:  $E = [(2i - 1)/2] = [i - 1/2] = [(i - 1) + 1/2] = i - 1$  ;  
returning to  $n$  finally we get:  $E = n/2 - 1$   $n > 1$  .  $\square$

**Proposition 2.7**

$$E = (n - 1)/2 \quad (14)$$

for  $n$  is odd or odd prime;  $n > 2$

**Proof 2.7** For  $n = 2i + 1$  where  $i \in N$  by equation (12) we get:  $E = [(2i + 1 - 1)/2] = [i] = i$  ;  
returning to  $n$  finally we get:  $E = (n - 1)/2$   $n > 2$  .  $\square$

### 2.4.3 The computation of F

**Proposition 2.8**

$$F = n/2 \quad (15)$$

for  $n$  is even  $n > 1$

**Proof 2.8** Subtracting equation (10) from equation (9) we get:  $|E| = G - F$  whence  $F = G - |E|$  . Taking into account equations (11), (12) we get:

$F = (n - 1) - [(n - 1)/2]$  for  $n = 2i$  we get:  
 $F = 2i - 1 - [(2i - 1)/2] = 2i - 1 - [i - 1/2] =$   
 $= 2i - 1 - [i - 1 + 1/2] = 2i - 1 - (i - 1) = i$   
returning to  $n$  finally we get:  $F = n/2$   $n > 1$

**Remark 2.6** Since  $n$  is even then the division by "2" is possibility. Thus  $F$  is integer and equation (15) is true for  $n$  is even.  $\square$

**Proposition 2.9**

$$F = (n - 1)/2 \quad (16)$$

for  $n$  is odd or odd prime  $n > 2$

**Proof 2.9** Subtracting equation (10) from equation (9) we get:  $|E| = G - F$  whence  $F = G - |E|$  . Taking into account equations (11), (12) we get:

$F = (n - 1) - [(n - 1)/2]$  for  $n = 2i + 1$  we get:  
 $F = 2i - i = i$  ; returning to  $n$  finally we get:  $F = (n - 1)/2$   $n > 2$  .  $\square$

**Remark 2.7** Since  $n$  is odd or odd prime then  $(n - 1)$  is even . Hence the division by "2" is possibility. Thus  $F$  is integer and equation (16) is true for  $n$  is odd or odd prime.

#### 2.4.4 The computation of $s_o$

##### Proposition 2.10

$$s_o = n - p \quad (17)$$

for  $n$  is even,  $n > 1$ .

**Proof 2.10** By definition (2.3) for computation  $s_o$  it needs to subtract from  $|XUY|$  the number of even composite positive integers  $(n - 2)$  here  $(-2)$  takes into account corollary 2.1 :  $n* = n - 2$  and also subtract the number of odd noncomposite positive integers  $p$  then we get:

$$s_o = 2(n - 1) - (n - 2) - p = n - p \text{ for } n \text{ is even } n > 1. \quad \square$$

##### Proposition 2.11

$$s_o = n - p - 1 \quad (18)$$

for  $n$  is odd or odd prime  $n > 1$ .

**Proof 2.11** Making the change equation (17) by rem 2.4 we get:  $s_o = n - p - 1$  for  $n$  is odd  $n > 1$ .  $\square$

#### 2.4.5 The computation of $p$

**Definition 2.13**  $p_1$  – the first approximation of  $p$ .

$p_2$  – the second approximation of  $p$ .

$p_3$  – the third approximation of  $p$ .

##### Proposition 2.12

$$p_1 = \text{round}(2n/\ln 2n) \quad (19)$$

$$p_2 = \text{round}(2n/\ln 2n + 2n/(\ln 2n)^2) \quad (20)$$

$$p_3 = \text{round}(2n/\ln 2n + 2n/(\ln 2n)^2 + 4n/(\ln 2n)^3) \quad (21)$$

**Proof 2.12** As everybody knows [1] that the number of the primes less than  $2n$  is expressed as follows:

$$\pi(2n) = (2n/\ln 2n) \int_0^1 (1 - (\ln y/\ln 2n) + (\ln^2 y/\ln^2 2n) + \dots) dy$$

We are limited to three of the first terms of the series.

Integrating in parts then we get:

$$\pi(2n) = (2n/\ln 2n)(1 + 1/\ln 2n + 2/\ln^2 2n)$$

Whence taking into account definition (4.1) that  $p$  is (primes + 1) then we get :

$$p = \text{round}(2n/\ln 2n + 2n/\ln^2 2n + 4n/\ln^3 2n + 1)$$

But taking into account that "2" is not odd prime and

that by def. 4.1  $p$  is the number of odd prime  $< 2n$  then  
 $p = \text{round}(2n/\ln 2n + 2n/\ln^2 2n + 4n/\ln^3 2n + 1 - 1)$  Finally we get:  $p = \text{round}(2n/\ln 2n + 2n/\ln^2 2n + 4n/\ln^3 2n)$  Whence we get:  
 $p_1 = \text{round}(2n/\ln 2n)$ ;  
 $p_2 = \text{round}(2n/\ln 2n + 2n/\ln^2 2n)$ ;  
 $p_3 = \text{round}(2n/\ln 2n + 2n/\ln^2 2n + 4n/\ln^3 2n)$  ;  $\square$

#### 2.4.6 The computation of $|Q|$

##### Proposition 2.13

$$|Q| = (n - p - |L|)/2 \quad (22)$$

for  $n$  is even  $n > 1$ .

**Proof 2.13** By equation (8) we get:  $|Q| = (s_o - |L|)/2$ . Taking into account equation (17) we get for  $n$  is even

$|Q| = (n - p - |L|)/2$ . Now we show that for  $n$  is even  $(n - p - |L|) = (n - (p + |L|))$  is even:

By equation (28)  $|L| = p - 2|H|$  whence  $|L|$  has parity of  $p$  hance  $(p + |L|)$  is even

for any  $p$ 's and  $|L|$ 's parity and if  $n$  is even then  $(n - p - |L|)$  is even then the devision by "2" is possibility

thus  $|Q|$  is integer and equation (22) is true for  $n$  is even  $n > 1$ .  $\square$

##### Proposition 2.14

$$|Q| = (n - p - 1 - |L|)/2 \quad (23)$$

for  $n$  is odd or odd prime  $n > 1$ .

**Proof 2.14** Changing equation (22) by rem (2.4) we get:  $|Q| = (n - p - 1 - |L|)/2 = (n - 1 - (p + |L|))$  for  $n$  is odd or odd prime  $n > 1$ .

Now we show that for  $n$  is odd or odd prime  $(n - p - 1 - |L|)$  is even:

By equation (28)  $|L| = p - 2|H|$ ; whence  $|L|$  has parity of  $p$  hance  $(p + |L|)$  is even for any  $p$ 's and  $|L|$ 's parity

and if  $n$  is odd or odd prime then  $(n - p - 1 - |L|)$  is even then the devision by "2" is possibility thus  $|Q|$  is integer and equation (23) is true for  $n$  is odd or odd prime  $n > 1$ .  $\square$

##### Proposition 2.15

$$|Q| = (n - 2p + 2|H|)/2 \quad (24)$$

for  $n$  is even  $n > 1$ .

**Proof 2.15** subtracting equation (7) from equation (8) we get:  $2|Q| - 2|H| = s_0 - p$  Whence we get:  $|Q| = (2|H| + s_0 - p)/2$  . Taking into account equation (17)

$|Q| = (2|H| + n - p - p)/2 = (n - 2p + 2|H|)/2$ . and for  $n$  is even  $(n - 2p + 2|H|)$  is even then the division by "2" is possibility thus  $|Q|$  is integer and equation (24) is true for  $n$  is even  $n > 1$ .  $\square$

**Proposition 2.16**

$$|Q| = (n - 2p + 2|H| - 1)/2 \quad (25)$$

for  $n$  is odd or  $n$  is odd prime  $n > 1$ .

**Proof 2.16** Changing equation (24) by rem (2.4) we get:  $|Q| = (n - 2p + 2|H| - 1)/2$  for  $n$  is odd or odd prime  $n > 1$  .

For  $n$  is odd or odd prime  $(n - 2p + 2|H| - 1)$  is even then the division by "2" is possibility thus  $|Q|$  is integer and equation (25) is true for  $n$  is odd or odd prime  $n > 1$ .  $\square$

#### 2.4.7 The computation of $|L|$

**Proposition 2.17**

$$|L| = n - p - 2|Q| \quad (26)$$

for  $n$  is even  $n > 1$

**Proof 2.17** By equation (8) we get:  $|L| = (s_0 - 2|Q|)$  Taking into account (17) we get for  $n$  is even

$|L| = (n - p - 2|Q|)$  For  $n$  is even  $n > 1$  .  $\square$

**Proposition 2.18**

$$|L| = n - p - 1 - 2|Q| \quad (27)$$

for  $n$  is odd or odd prime  $n > 1$

**Proof 2.18** Changing equation (26) by rem (2,4) we get:  $|L| = (n - p - 1 - 2|Q|)/2$  for  $n$  is odd or odd prime  $n > 1$  .  $\square$

**Proposition 2.19**

$$|L| = p - 2|H| \quad (28)$$

$\forall n > 1$

**Proof 2.19** By equation (7) and rem (2.3) we get:  $|L| = p - 2|H|$

for  $n$  is even,  $n > 1$

For  $n$  is odd or odd prime by rem (2.4) we also get:  $|L| = p - 2|H|$   $n > 1$

Thus  $|L| = p - 2|H|$   $\forall n > 1$  .  $\square$

### 2.4.8 The computation of $|H|$

#### Proposition 2.20

$$|H| = (p - |L|)/2 \quad (29)$$

$\forall n > 1$

**Proof 2.20** *By equation (7) and rem (2.3) we get:  $|H| = (p - |L|)/2$  for  $n$  is even,  $n > 1$ .*

*For  $n$  is odd or odd prime by rem (2.4) we also get:  $|H| = (p - |L|)/2$   $n > 1$ .*

*Now we show that  $\forall n$   $(p - |L|)$  is even*

*By equation (26)  $|L| = n - p - 2|Q|$  whence  $|L| + p$  is even if  $n$  is even.*

*It is possibility if  $|L|$  and  $p$  have equal parity then  $(p - |L|)$  is also even for  $n$  is even. By equation (27)  $|L| = n - p - 1 - 2|Q|$  whence  $|L| + p$  is even if  $n$  is odd or odd prime.*

*It is possibility if  $|L|$  and  $p$  have equal parity then  $(p - |L|)$  is also even for  $n$  is odd or odd prime.*

*Thus  $(p - |L|)$  is even  $\forall n$ .*

*Hence the division by "2" is possibility then  $|H|$  is integer and equation (29) is true  $\forall n > 1$ .  $\square$*

#### Proposition 2.21

$$|H| = (2p + 2|Q| - n)/2 \quad (30)$$

for  $n$  is even  $n > 1$

**Proof 2.21** *subtracting equation (7) from equation (8) we get:*

*$2|Q| - 2|H| = s_0 - p$  Whence we get:  $|H| = (2|Q| - s_0 + p)/2$ . Taking into account equation (17) we have for  $n$  is even:*

*$|H| = (2|Q| - (n - p) + p)/2 = (2p + 2|Q| - n)/2$ . Now we show that for  $n$  is even  $(2p + 2|Q| - n)$  is even*

*then the division by "2" is possibility thus  $|H|$  is integer and equation (30) is true for  $n$  is even  $n > 1$   $\square$*

#### Proposition 2.22

$$|H| = (2p + 2|Q| - n + 1)/2 \quad (31)$$

for  $n$  is odd or odd prime  $n > 1$

**Proof 2.22** subtracting equation (7) from equation (8) we get:

$2|Q| - 2|H| = s_0 - p$  Whence we get:  $|H| = (2|Q| - s_0 + p)/2$  . Taking into account (18) we have for  $n$  is odd or odd prime:

$|H| = (2|Q| - (n - p - 1) + p)/2 = (2p + 2|Q| - n + 1)/2$ . Now we show that for  $n$  is odd or odd prime

$(2p + 2|Q| - n + 1)$  is even then the division by "2" is possibility thus  $|H|$  is integer and equation (31) is true

for  $n$  is odd or odd prime  $n > 1$ .  $\square$

**Definition 2.14**  $|Q| - |H|$  - lower limit of possible range of  $|Q| - |H|$ .

$|Q| - |H|$  - positive integer  $> 0 \quad \forall 2n > 120$  .

$|Q| - |H|$  - negative integer  $< 0 \quad \forall 2 < 2n < 120$  .

excepting  $2n = 94; 96; 100; 106; 118$ ; for which  $|Q| - |H| = 0$  .

**Proposition 2.23**

$$|Q| - |H| = (n - 2p)/2 \quad (32)$$

for  $n$  is even,  $n > 1$

**Proof 2.23** subtracting equation (7) from equation (8) we get:  $2|Q| - 2|H| = s_0 - p$  Whence we get:  $|Q| - |H| = (s_0 - p)/2 =$  . Taking into account equation (17) we have for  $n$  is even:

$|Q| - |H| = (n - p - p)/2 = (n - 2p)/2$  . Since for  $n$  is even  $(n - 2p)$  is even then the division by "2"

is possibility thus  $|Q| - |H|$  is integer and equation (32) is true for  $n$  is even,  $n > 1$   $\square$

**Proposition 2.24**

$$|Q| - |H| = (n - 2p - 1)/2 \quad (33)$$

for  $n$  is odd or odd prime,  $n > 1$

**Proof 2.24** subtracting equation (7) from equation (8) we get:

$2|Q| - 2|H| = s_0 - p$  Whence we get:  $|Q| - |H| = (s_0 - p)/2 =$  . Taking into account equation (18)

we have for  $n$  is odd or odd prime :

$|Q| - |H| = (n - 1 - p - p)/2 = (n - 2p - 1)/2$  . Since for  $n$  is odd or odd prime  $(n - 2p - 1)$  is even then

the division by "2" is possibility thus  $|Q| - |H|$  is integer

and equation (33) is true for  $n$  is odd or odd prime  $n > 1$  .  $\square$

### 3 The control of ABR2n

**Remark 3.1** It needs to take into account that:  $p, s_o, |Q|, |L|, |H|, |Q|_b, |Q| - |H|$ , are defined for values  $< 2n$ .

i.e. the direct computations of them are made for all values of  $< 2n$ , but the computations by equations are made for  $n = 2n/2$ .

**Example 3.1** The computation of  $|Q|, |L|, |H|, s_o, G, F$  for  $2n = 4$

Construction SBR2n:

$$2n = 4; n = 2$$

$\circledast$  - odd noncomposite;  $\oplus$  - odd composite;  $\otimes$  - even composite.

$$\circledast 1 + 3\circledast = 4(H)$$

Data of direct computations for even  $n = 2$ , at that it is taken into account remark (2.2)

$$p = 2; s_o = 0; |Q| = 0; |L| = 0; |H| = 1; G = 1; F = 1$$

The computations of parameters of binary representations of the positive integer  $2n = 4$  with help of arithmetic stated above

at that  $p, s_o, |Q|, |L|, |H|$ , are taken from the data of direct computations.

$$n = 4/2 = 2;$$

$$G = n - 1 = 2 - 1 = 1; \text{ by equation (11)}$$

$$|E| = n/2 - 1 = 1 - 1 = 0; \text{ by equation (13)}$$

$$F = n/2 = 2/2 = 1; \text{ by equation (15)}$$

$$s_o = n - p = 2 - 2 = 0; \text{ by equation (17)}$$

$$|L| = n - p - 2|Q| = 2 - 2 - 0 = 0; \text{ by equation (26)}$$

$$|L| = p - 2|H| = 2 - 2 = 0; \text{ by equation (28)}$$

$$|H| = (2|Q| + 2p - n)/2 = (0 + 4 - 2)/2 = 1; \text{ by equation (30)}$$

$$|H| = (p - |L|)/2 = (2 - 0)/2 = 1; \text{ by equation (29)}$$

$$|Q| = (n - p - |L|)/2 = (2 - 2 - 0)/2 = 0; \text{ by equation (22)}$$

$$|Q| = (n - 2p + 2|H|)/2 = (2 - 4 + 2)/2 = 0; \text{ by equation (24)}$$

$$|Q| - |H| = 0 - 1 = -1;$$

$$|Q| - |H| = (n - 2p)/2 = (2 - 4)/2 = -1; \text{ by equation (32)}$$

$$p = 2|H| + |L| = 2 + 0 = 2; \text{ by equation (7)}$$

$$s_o = 2|Q| + |L| = 0 + 0 = 0; \text{ by equation (8)}$$

$$F = |Q| + |L| + |H| = 0 + 0 + 1 = 1; \text{ by equation (10)}$$

The compare of data of direct computations with computed values shows full coincidence of results.

**Example 3.2** The computation of  $|Q|, |L|, |H|, s_o, G, F$  for  $2n = 6$

.

$\odot$  - odd noncomposite;  $\oplus$  - odd composite ;  $\otimes$  - even composite .

*Construction SBR2n:*

$$2n = 6 ; n = 3$$

$$\odot 1 + 5\odot = 6(H)$$

$$\otimes 2 + 4\otimes = 6(E)$$

*Data of direct computations for odd  $n = 3$  , at that it is took into account remark (2.2)*

$$p^* = 2; s_o = 0; |Q| = 0; |L| = 0; |H| = 1; G = 2; F = 1$$

*The computations of parameters of binary representations of the positive integer  $2n = 6$  with help of arithmetic stated above*

*at that  $p, s_o, |Q|, |L|, |H|$ , are took from the data of direct computations.*

$$n = 6/2 = 3;$$

$$G = n - 1 = 3 - 1 = 2; \text{ by equation (11)}$$

$$|E| = (n - 1)/2 = 1; \text{ by equation (12)}$$

$$F = (n - 1)/2 = (3 - 1)/2 = 1; \text{ by equation (16)}$$

$$s_o = n - p - 1 = 3 - 2 - 1 = 0; \text{ by equation (18)}$$

$$|L| = n - p - 2|Q| - 1 = 3 - 2 - 0 - 1 = 0; \text{ by equation (27)}$$

$$|L| = p - 2|H| = 2 - 2 = 0; \text{ by equation (28)}$$

$$|H| = (2|Q| + 2p - n + 1)/2 = (0 + 4 - 3 + 1)/2 = 1; \text{ by equation (31)}$$

$$|H| = (p - |L|)/2 = (2 - 0)/2 = 1; \text{ by equation (29)}$$

$$|Q| = (n - p - |L| - 1)/2 = (3 - 2 - 0 - 1)/2 = 0; \text{ by equation (23)}$$

$$|Q| = (n - 2p + 2|H| - 1)/2 = (3 - 4 + 2 - 1)/2 = 0; \text{ by equation (25)}$$

$$|Q| - |H| = 0 - 1 = -1; |Q| - |H| = (n - 2p - 1)/2 = (3 - 4 - 1)/2 = -1; \text{ by equation (33)}$$

$$p = 2|H| + |L| = 2 + 0 = 2; \text{ by equation (7)}$$

$$s_o = 2|Q| + |L| = 0 + 0 = 0; \text{ by equation (8)}$$

$$F = |Q| + |L| + |H| = 0 + 0 + 1 = 1; \text{ by equation (10)}$$

*The compare of data of direct computations with computed values shows full coincidence of results.*

**Exemple 3.3** *The computation of  $|Q|, |L|, |H|, s_o, G, F$  for  $2n = 12$*

*Construction SBR2n:*

$$2n = 12 ; n = 6$$

$\odot$  - odd noncomposite;  $\oplus$  - odd composite ;  $\otimes$  - even composite .

$$\odot 1 + 11\odot = 12(H)$$

$$\otimes 2 + 10\otimes = 12(E)$$

$$\odot 3 + 9\oplus = 12(L)$$

$$\otimes 4 + 8\otimes = 12(E)$$

$$\odot 5 + 7\odot = 12(H)$$



Data of direct computations for even  $n = 2$ , at that it is took into account remark (2.2)

$$p = 5; s_o = 1; |Q| = 0; |L| = 1; |H| = 2; G = 5; F = 3$$

The computations of parameters of binary representations of the positive integer  $2n = 12$  with help of arithmetic stated above

$$n = 12/2 = 6;$$

$$G = n - 1 = 6 - 1 = 5; \text{ by equation (11)}$$

$$|E| = n/2 - 1 = 3 - 1 = 2; \text{ by equation (13)}$$

$$F = n/2 = 6/2 = 3; \text{ by equation (15)}$$

$$s_o = n - p = 6 - 5 = 1; \text{ by equation (17)}$$

$$|L| = n - p - 2|Q| = 6 - 5 - 0 = 1; \text{ by equation (26)}$$

$$|L| = p - 2|H| = 5 - 4 = 1; \text{ by equation (28)}$$

$$|H| = (2|Q| + 2p - n)/2 = (0 + 10 - 6)/2 = 2; \text{ by equation (30)}$$

$$|H| = (p - |L|)/2 = (5 - 1)/2 = 2; \text{ by equation (29)}$$

$$|Q| = (n - p - |L|)/2 = (6 - 5 - 1)/2 = 0; \text{ by equation (22)}$$

$$|Q| = (n - 2p + 2|H|)/2 = (6 - 10 + 4)/2 = 0; \text{ by equation (24)}$$

$$|Q| - |H| = 0 - 2 = -2;$$

$$|Q| - |H| = (n - 2p)/2 = (6 - 10)/2 = -2; \text{ by equation (32)}$$

$$p = 2|H| + |L| = 4 + 1 = 5; \text{ by equation (7)}$$

$$s_o = 2|Q| + |L| = 0 + 1 = 1; \text{ by equation (8)}$$

$$F = |Q| + |L| + |H| = 0 + 1 + 2 = 3; \text{ by equation (10)}$$

The compare of data of direct computations with computed values shows full coincidence of results.

**Exemple 3.4** The computation of  $|Q|$ ,  $|L|$ ,  $|H|$ ,  $s_o$ ,  $G$ ,  $F$  for  $2n = 14$

.

$\circ$  - odd noncomposite;  $\oplus$  - odd composite ;  $\otimes$  - even composite .

Construction SBR $2n$ :

$$2n = 14 ; n = 7$$

$$\circ 1 + 13\circ = 14(H)$$

$$\otimes 2 + 12\otimes = 14(E)$$

$$\circ 3 + 11\circ = 14(H)$$

$$\otimes 4 + 10\otimes = 14(E)$$

$$\circ 5 + 9\oplus = 14(L)$$

$$\otimes 6 + 8\otimes = 14(E)$$

Data of direct computations for odd  $n = 7$ , at that it is took into account remark (2.2).

$$p^* = 5; s_o = 1; |Q| = 0; |L| = 1; |H| = 2; G = 6; F = 3$$

The computations of parameters of binary representations of the positive

integer  $2n = 14$  with help of arithmetic stated above

$$n = 14/2 = 7;$$

$$G = n - 1 = 7 - 1 = 6; \text{ by equation (11)}$$

$$|E| = (n - 1)/2 = 3; \text{ by equation (12)}$$

$$F = (n - 1)/2 = (7 - 1)/2 = 3; \text{ by equation (16)}$$

$$s_o = n - p - 1 = 7 - 5 - 1 = 1; \text{ by equation (18)}$$

$$|L| = n - p - 2|Q| - 1 = 7 - 5 - 0 - 1 = 1; \text{ by equation (27)}$$

$$|L| = p - 2|H| = 5 - 4 = 1; \text{ by equation (28)}$$

$$|H| = (2|Q| + 2p - n + 1)/2 = (0 + 10 - 7 + 1)/2 = 2; \text{ by equation (31)}$$

$$|H| = (p - |L|)/2 = (5 - 1)/2 = 2; \text{ by equation (29)}$$

$$|Q| = (n - p - |L| - 1)/2 = (7 - 5 - 1 - 1)/2 = 0; \text{ by equation (23)}$$

$$|Q| = (n - 2p + 2|H| - 1)/2 = (7 - 10 + 4 - 1)/2 = 0; \text{ by equation (25)}$$

$$|Q| - |H| = 0 - 2 = -2; |Q| - |H| = (n - 2p - 1)/2 = (7 - 10 - 1)/2 = -2; \\ \text{by equation (33)}$$

$$p = 2|H| + |L| = 4 + 1 = 5; \text{ by equation (7)}$$

$$s_o = 2|Q| + |L| = 0 + 1 = 1; \text{ by equation (8)}$$

$$F = |Q| + |L| + |H| = 0 + 1 + 2 = 3; \text{ by equation (10)}$$

The compare of data of direct computations with computed values shows full coincidence of results.

**Example 3.5** The computation of  $|Q|$ ,  $|L|$ ,  $|H|$ ,  $s_o$ ,  $G$ ,  $F$  for  $2n = 132$

**Remark 3.2** The representations with  $x$  is even;  $y$  is even are excluded since

$|Q|$ ,  $|L|$ ,  $|H|$ ,  $s_o$ ,  $p$ , by definitions concern to odd composite and noncomposite integers.

$\circledast$  - odd noncomposite;  $\oplus$  - odd composite ;

Construction  $SBR2n$ :

$$2n = 132 ; n = 66$$

$$\circledast 1 + 131\circledast = 132(H)$$

$$\circledast 3 + 129\oplus = 132(L)$$

$$\circledast 5 + 127\circledast = 132(H)$$

$$\circledast 7 + 125\oplus = 132(L)$$

$$\oplus 9 + 123\oplus = 132 (Q)$$

$$\circledast 11 + 121\oplus = 132 (L)$$

$$\circledast 13 + 119\oplus = 132 (L)$$

$$\oplus 15 + 117\oplus = 132 (Q)$$

$$\circledast 17 + 115\oplus = 132 (L)$$

$$\begin{aligned}
\circlearrowleft 19 + 113\circlearrowleft &= 132 \text{ (H)} \\
\oplus 21 + 111\oplus &= 132 \text{ (Q)} \\
\circlearrowleft 23 + 109\circlearrowleft &= 132 \text{ (H)} \\
\oplus 25 + 107\oplus &= 132 \text{ (L)} \\
\oplus 27 + 105\oplus &= 132 \text{ (Q)} \\
\circlearrowleft 29 + 103\circlearrowleft &= 132 \text{ (H)} \\
\circlearrowleft 31 + 101\circlearrowleft &= 132 \text{ (H)} \\
\oplus 33 + 99\oplus &= 132 \text{ (Q)} \\
\oplus 35 + 97\oplus &= 132 \text{ (L)} \\
\circlearrowleft 37 + 95\circlearrowleft &= 132 \text{ (L)} \\
\oplus 39 + 93\oplus &= 132 \text{ (Q)} \\
\circlearrowleft 41 + 91\circlearrowleft &= 132 \text{ (L)} \\
\circlearrowleft 43 + 89\circlearrowleft &= 132 \text{ (H)} \\
\oplus 45 + 87\oplus &= 132 \text{ (Q)} \\
\circlearrowleft 47 + 85\circlearrowleft &= 132 \text{ (L)} \\
\oplus 49 + 83\oplus &= 132 \text{ (L)} \\
\oplus 51 + 81\oplus &= 132 \text{ (Q)} \\
\circlearrowleft 3 + 79\circlearrowleft &= 132 \text{ (H)} \\
\oplus 55 + 77\oplus &= 132 \text{ (Q)} \\
\oplus 57 + 75\oplus &= 132 \text{ (Q)} \\
\circlearrowleft 59 + 73\circlearrowleft &= 132 \text{ (H)} \\
\circlearrowleft 61 + 71\circlearrowleft &= 132 \text{ (H)} \\
\oplus 63 + 69\oplus &= 132 \text{ (Q)} \\
\oplus 65 + 67\oplus &= 132 \text{ (L)}
\end{aligned}$$

*Data of direct computations for even  $n = 66$ , at the compare it is took into account remark (2.2).*

$$p = 32; s_o = 34; |Q| = 11; |L| = 12; |H| = 10; F = 33$$

*The computations of parameters of binary representations of the positive integer  $2n = 132$  with help of arithmetic stated above*

*at that  $p, s_o, |Q|, |L|, |H|$ , are took from the data of direct computations.*

$$n = 132/2 = 66;$$

$$G = n - 1 = 66 - 1 = 65; \text{ by equation (11)}$$

$$|E| = n/2 - 1 = 66/2 - 1 = 32; \text{ by equation (13)}$$

$$F = n/2 = 66/2 = 33; \text{ by equation (15)}$$

$$s_o = n - p = 66 - 32 = 34; \text{ by equation (17)}$$

$$|L| = n - p - 2|Q| = 66 - 32 - 22 = 12; \text{ by equation (26)}$$

$$|L| = p - 2|H| = 32 - 20 = 12; \text{ by equation (28)}$$

$$|H| = (2|Q| + 2p - n)/2 = (22 + 64 - 66)/2 = 10; \text{ by equation (30)}$$

$$|H| = (p - |L|)/2 = (32 - 12)/2 = 10; \text{ by equation (29)}$$

$$|Q| = (n - p - |L|)/2 = (66 - 32 - 12)/2 = 11; \text{ by equation (22)}$$

$$|Q| = (n - 2p + 2|H|)/2 = (66 - 64 + 20)/2 = 11; \text{ by equation (24)}$$

$$|Q| - |H| = 11 - 10 = 1; \text{ by equation (9)}$$

$$|Q| - |H| = (n - 2p)/2 = (66 - 64)/2 = 1; \text{ by equation (32)}$$

$$p = 2|H| + |L| = 20 + 12 = 32; \text{ by equation (7)}$$

$$s_o = 2|Q| + |L| = 22 + 12 = 34; \text{ by equation (8)}$$

$$F = |Q| + |L| + |H| = 11 + 12 + 10 = 33; \text{ by equation (10)}$$

The compare of data of direct computations with computed values shows full coincidence of results.

**Example 3.6** The computation of  $|Q|$ ,  $|L|$ ,  $|H|$ ,  $s_o$ ,  $G$ ,  $F$  for  $2n = 138$  .

**Remark 3.3** The representations with  $x$  is even;  $y$  is even are excluded since

$|Q| \cdot |L|, |H|, s_o, p$ , by definitions concern to odd composite and prime integers.

$\circ$  - odd noncomposite;  $\oplus$  - odd composite ;

Construction SBR2n:

$$2n = 138 ; n = 69$$

$$\circ 1 + 137 \circ = 138(H)$$

$$\circ 3 + 135 \oplus = 138(L)$$

$$\circ 5 + 133 \oplus = 138(L)$$

$$\circ 7 + 131 \circ = 138(H)$$

$$\oplus 9 + 129 \oplus = 138(Q)$$

$$\circ 11 + 127 \circ = 138(H)$$

$$\circ 13 + 125 \oplus = 138(L)$$

$$\oplus 15 + 123 \oplus = 138(Q)$$

$$\circ 17 + 121 \oplus = 138(L)$$

$$\circ 19 + 119 \oplus = 138(L)$$

$$\oplus 21 + 117 \oplus = 138(Q)$$

$$\circ 23 + 115 \oplus = 138(L)$$

$$\oplus 25 + 113 \circ = 138(L)$$

$$\oplus 27 + 111 \oplus = 138(Q)$$

$$\circ 29 + 109 \circ = 138(H)$$

$$\circ 31 + 107 \circ = 138(H)$$

$$\oplus 33 + 105 \oplus = 138(Q)$$

$$\oplus 35 + 103 \circ = 138(L)$$

$$\circ 37 + 101 \circ = 138(H)$$

$$\oplus 39 + 99 \oplus = 138(Q)$$

$$\begin{aligned}
\circlearrowleft 41 + 97\circlearrowleft &= 138(H) \\
\circlearrowleft 43 + 95\oplus &= 138(L) \\
\oplus 45 + 93\oplus &= 138(Q) \\
\circlearrowleft 47 + 91\oplus &= 138(L) \\
\oplus 49 + 89\circlearrowleft &= 138(L) \\
\oplus 51 + 87\oplus &= 138(Q) \\
\circlearrowleft 53 + 85\oplus &= 138(L) \\
\oplus 55 + 83\circlearrowleft &= 138(L) \\
\oplus 57 + 81\oplus &= 138(Q) \\
\circlearrowleft 59 + 79\circlearrowleft &= 138(H) \\
\circlearrowleft 61 + 77\oplus &= 138(L) \\
\oplus 63 + 75\oplus &= 138(Q) \\
\oplus 65 + 73\circlearrowleft &= 138(L) \\
\circlearrowleft 67 + 71\circlearrowleft &= 138(H)
\end{aligned}$$

Data of direct computations for odd  $n = 69$ , at that it is took into account remark (2.2).

$$p = 33; s_o = 35; |Q| = 10; |L| = 15; |H| = 9; F = 34$$

The computations of parameters of binary representations of the positive integer  $2n = 132$  with help of arithmetic stated above

at that  $p, s_o, |Q|, |L|, |H|$ , are took from the data of direct computations.  
 $n = 138/2 = 69$ ;

$$G = n - 1 = 69 - 1 = 68; \text{ by equation (11)}$$

$$|E| = (n - 1)/2 = 34; \text{ by equation (12)}$$

$$F = (n - 1)/2 = (69 - 1)/2 = 34; \text{ by equation (16)}$$

$$s_o = n - p - 1 = 69 - 33 - 1 = 35; \text{ by equation (18)}$$

$$|L| = n - p - 2|Q| - 1 = 69 - 33 - 20 - 1 = 15; \text{ by equation (27)}$$

$$|L| = p - 2|H| = 33 - 18 = 15; \text{ by equation (28)}$$

$$|H| = (2|Q| + 2p - n + 1)/2 = (20 + 66 - 69 + 1)/2 = 9; \text{ by equation (31)}$$

$$|H| = (p - |L|)/2 = (33 - 15)/2 = 9; \text{ by equation (29)}$$

$$|Q| = (n - p - |L| - 1)/2 = (69 - 33 - 15 - 1)/2 = 10; \text{ by equation (23)}$$

$$|Q| = (n - 2p + 2|H| - 1)/2 = (69 - 66 + 18 - 1)/2 = 10; \text{ by equation (25)}$$

$$|Q| - |H| = 10 - 9 = 1; |Q| - |H| = (n - 2p - 1)/2 = (69 - 66 - 1)/2 = 1; \text{ by equation (33)}$$

$$p = 2|H| + |L| = 18 + 15 = 33; \text{ by equation (7)}$$

$$s_o = 2|Q| + |L| = 20 + 15 = 35; \text{ by equation (8)}$$

$$F = |Q| + |L| + |H| = 10 + 15 + 9 = 34; \text{ by equation (10)}$$

The compare of data of direct computations with computed values shows full coincidence of results.

**Exemple 3.7** The computation of  $|Q|, |L|, |H|, s_o, G, F$  for  $2n = 134$

**Remark 3.4** The representations with  $x$  is even;  $y$  is even are excluded

since

$|Q| \cdot |L|, |H|, s_o, p$ , by definitions concern to odd composite and prime integers.

$\circledast$  - odd noncomposite;  $\oplus$  - odd composite ;

*Construction SBR2n::*

$$2n = 134 \quad n = 67$$

$$\circledast 1 + 133 \oplus = 134(L)$$

$$\circledast 3 + 131 \circledast = 134(H)$$

$$\circledast 5 + 129 \oplus = 134(L)$$

$$\circledast 7 + 127 \circledast = 134(H)$$

$$\oplus 9 + 125 \oplus = 134(Q)$$

$$\circledast 11 + 123 \oplus = 134(L)$$

$$\circledast 13 + 121 \oplus = 134(L)$$

$$\oplus 15 + 119 \oplus = 134(Q)$$

$$\circledast 17 + 117 \oplus = 134(L)$$

$$\circledast 19 + 115 \oplus = 134(L)$$

$$\oplus 21 + 113 \circledast = 134(L)$$

$$\circledast 23 + 111 \oplus = 134(L)$$

$$\oplus 25 + 109 \circledast = 134(L)$$

$$\oplus 27 + 107 \circledast = 134(L)$$

$$\circledast 29 + 105 \oplus = 134(L)$$

$$\circledast 31 + 103 \circledast = 134(H)$$

$$\oplus 33 + 101 \circledast = 134(L)$$

$$\oplus 35 + 99 \oplus = 134(Q)$$

$$\circledast 37 + 97 \circledast = 134(H)$$

$$\oplus 39 + 95 \oplus = 134(Q)$$

$$\circledast 41 + 93 \oplus = 134(L)$$

$$\circledast 43 + 91 \oplus = 134(L)$$

$$\oplus 45 + 89 \oplus = 134(Q)$$

$$\circledast 47 + 87 \oplus = 134(L)$$

$$\oplus 49 + 85 \oplus = 134(Q)$$

$$\oplus 51 + 83 \circledast = 134(L)$$

$$\circledast 53 + 81 \oplus = 134(L)$$

$$\oplus 55 + 79 \circledast = 134(L)$$

$$\oplus 57 + 77 \oplus = 134(Q)$$

$$\circledast 59 + 75 \oplus = 134(L)$$

$$\circledast 61 + 73 \oplus = 134(H)$$

$$\oplus 63 + 71 \circledast = 134(L)$$

$$\oplus 65 + 69 \circledast = 134(Q)$$

*Data of direct computations for odd prime  $n = 67$ , at that it is took into*

account remark (2.2)

$$p^* = 30; s_o = 36; |Q| = 8; |L| = 20; |H| = 5; F = 33$$

The computations of parameters of binary representations of the positive integer  $2n = 134$

with help of arithmetic stated above

at that  $p, s_o, |Q|, |L|, |H|$ , are took from the data of direct computations.

$$n = 134/2 = 67;$$

$$G = n - 1 = 67 - 1 = 66; \text{ by equation (11)}$$

$$|E| = (n - 1)/2 = 33; \text{ by equation (12)}$$

$$F = (n - 1)/2 = (67 - 1)/2 = 33; \text{ by equation (16)}$$

$$s_o = n - p - 1 = 67 - 30 - 1 = 36; \text{ by equation (18)}$$

$$|L| = n - p - 2|Q| - 1 = 67 - 30 - 16 - 1 = 20; \text{ by equation (27)}$$

$$|L| = p - 2|H| = 30 - 10 = 20; \text{ by equation (28)}$$

$$|H| = (2|Q| + 2p - n + 1)/2 = (16 + 60 - 67 + 1)/2 = 5; \text{ by equation (31)}$$

$$|H| = (p - |L|)/2 = (30 - 20)/2 = 5; \text{ by equation (29)} \quad |Q| = (n - p - |L| - 1)/2 = (67 - 30 - 20 - 1)/2 = 8; \text{ by equation (23)}$$

$$|Q| = (n - 2p + 2|H| - 1)/2 = (67 - 60 + 10 - 1)/2 = 8; \text{ by equation (25)}$$

$$|Q| - |H| = 8 - 5 = 3;$$

$$|Q| - |H| = (n - 2p - 1)/2 = (67 - 60 - 1)/2 = 3; \text{ by equation (33)}$$

$$p = 2|H| + |L| = 10 + 20 = 30; \text{ by equation (7)}$$

$$s_o = 2|Q| + |L| = 16 + 20 = 36; \text{ by equation (8)}$$

$$F = |Q| + |L| + |H| = 8 + 20 + 5 = 33; \text{ by equation (10)}$$

The compare of data of direct computations with computed values shows full coincidence of results.

## 4 The limited values of possible range of $|Q|, |L|, |H|$

**Definition 4.1**  $|H|_b$  -lower limit of possible range of  $|H|$ .

**Axiom 4.1**

$$|H|_b = 0 \tag{34}$$

**Definition 4.2**  $p(n)$  is the number of odd noncomposite integers (primes and "1")  $< n$

$$p(n) \text{ is positive integer } > 0 \quad \forall n > 1$$

**Definition 4.3**  $p - p(n)$  - the number of odd noncomposite integers in  $Y$ .

**Definition 4.4**  $|H|_c$  -upper limit of possible range of  $|H|$ .

**Proposition 4.1**

$$|H|_c = p - p(n) \quad (35)$$

**Proof 4.1** By Law of distribution of primes the number of odd noncomposite integers in  $X$  is greater than the number of odd noncomposite integers in  $Y : p(n) > p - p(n)$  .

Since the number of primes decreases with increase of  $n$ . Hence maximal number of pair of odd noncomposite integers in the set  $XUY$  equals the number of odd noncomposite integers in  $Y : p - p(n)$

then  $|H|_c = p - p(n)$  .  $\square$

**Corollary 4.1** The number of unpaired odd noncomposite positive integers in  $X$  equals :  $2p(n) - p$  and are allotted to type "L". Then  $|L| > 0 \forall 2n > 8$

**Proof 4.2** The number of unpaired odd noncomposite positive integers in  $X$  by definitions 8.4, 8.5 equals :

$$(p(n) - (p - p(n))) = 2p(n) - p$$

and are allotted to type "L".  $\square$

**Definition 4.5**  $|L|_b$  - lower limit of possible range of  $|L|$ .

**Proposition 4.2**

$$|L|_b = 2p(n) - p \quad (36)$$

**Proof 4.3** Substituting upper limit of  $|H|$  by (35) to (7) then we get: lower limit for  $|L| : |L|_b = 2p(n) - p$   $\square$

**Definition 4.6**  $|L|_c$  -upper limit of possible range of  $|L|$ .

**Proposition 4.3**

$$|L|_c = p \quad (37)$$

**Proof 4.4** Substituting lower limit of  $|H|$  by (34) to (7) then we get upper limit for  $|L| : |L|_c = p$   $\square$

**Definition 4.7**  $|Q|_b$  - lower limit of possible range of  $|Q|$ .

$|Q|_b$ - positive integer  $> 0 \forall 2n > 120$  see proposition (9.2).

$|Q|_b$  - negative integer  $< 0 \forall 2 < 2n < 120$  see proposition (9.1).

excepting  $2n = 94; 96; 100; 106; 118$ ; for which  $|Q|_b = 0$  see subsection (12.1). At that  $2n = 120$ ; ( $|Q|_b = 0$ ) is border point.



**Proposition 4.4**

$$|Q|_b = (n - 2p)/2 \quad (38)$$

$n$  is even;  $n > 1$

**Proof 4.5** Substituting upper limit of  $|L|$  by equation (37) to equation (8) then we get

lower limit for  $|Q|$ :  $|Q|_b = (S_o - p)/2$ .

Substituting  $S_o$  by equation (17), then we get:

$$|Q|_b = (n - 2p)/2 \quad n > 1 . \quad \square$$

**Proposition 4.5**

$$|Q|_b = (n - 2p - 1)/2 \quad (39)$$

$n$  is odd or odd prime ;  $n > 1$

**Proof 4.6** Substituting upper limit of  $|L|$  by equation (37) to equation (8) then we get

lower limit for  $|Q|$ :  $|Q|_b = (S_o - p)/2$  .

Substituting  $S_o$  by equation (18), finally we get:  $|Q|_b = (n - 2p - 1)/2$ ;

for  $n$  is odd or odd prime;  $n > 1$  .  $\square$

**Definition 4.8**  $|Q|_c$  - upper limit of possible range of  $|Q|$ .

**Proposition 4.6**

$$|Q|_c = (n - 2p(n))/2 \quad (40)$$

for;

**Proof 4.7** Substituting lower limit of  $|L|$  by equation (36) to equation (8) then we get

upper limit for  $|Q|$ :  $|Q|_c = (S_o - (2p(n) - p))/2$  .

Substituting  $S_o$  by equation (17) finally we get:  $|Q|_c = (n - 2p(n))/2$ ;

for  $n$  is even or  $n$  is odd prime;  $\square$

**Proposition 4.7**

$$|Q|_c = (n - 2p(n) - 1)/2 \quad (41)$$

for  $n$  is odd or odd prime;

**Proof 4.8** Substituting lower limit of  $|L|$  by equation (36) to equation (8) then we get

upper limit for  $|Q|$ :  $|Q|_c = (S_o - (2p(n) - p))/2$

Substituting  $S_o$  by equation (18) finally we get:  $|Q|_c = (n - 2p(n) - 1)/2$ ;

for  $n$  is odd or odd prime;  $\square$

**Proposition 4.8**

$$|Q| - |H| = |Q|_b \quad (42)$$

$\forall n > 1$

**Proof 4.9** By equation (32) we have:  $|Q| - |H| = (n - 2p)/2$  for  $n$  is even ; By equation (33) we have:  $|Q| - |H| = (n - 2p - 1)/2$  for  $n$  is odd or odd prime.

By equation (38) we have:  $|Q|_b = (n - 2p)/2$  for  $n$  is even. By equation (39) we have:  $|Q|_b = (n - 2p - 1)/2$  ; for  $n$  is odd or odd prime.

Whence we get:  $|Q| - |H| = |Q|_b \forall n > 1$ .  $\square$

## 5 Average value of the number of binary sums are formed from odd composite positive integers $< 2n$

**Definition 5.1**  $S$  – ordered set of odd composite positive integers  $< 2n$

$s$  – element of  $S$

$|S|_0$  – power of  $S$

$s_i$  – vary over all  $s$

$s_j$  – vary over all  $s$

**Definition 5.2**  $V \{v_k | v_k \in N, v_k = s_i + s_j\}$

is a set by elements of which are every possible binary sums of odd composite

integers  $< 2n$ . (each with all the rest )

Since  $s_i < 2n; s_j < 2n$  then  $\max v_k < 4n$ .

$|V|$  - the power of set  $V$ .

**Definition 5.3**  $W \{w | w \in N, w = 2k, 1 \leq k \leq 2n\}$

is a set of even composite positive integers  $< 4n + 2$

(inf  $W = 2$ ; sup  $W = 4n$ ).

$|W|$  – the power of set  $W$ ;  $|W| = 2n$ .

**Definition 5.4**  $|Q|_m$  - mean quantity of binary sums  $v_k = s_i + s_j$  which can be formed of odd composite positive integers  $< 2n$  and which are mapped into  $W$  by surjective mapping :  
 $f : V \Rightarrow W$

$$|Q|_m = |V|/|W| \quad (43)$$

i.e. uniform mapping regardless of real.

$|Q|_m$  - positive rational number  $> 0 \forall 2n > 4$ .

**Proposition 5.1**

$$|V| = s_0^2 \quad (44)$$

**Proof 5.1** The number of every possible binary sums in the view of  $v_k = s_i + s_j$  are formed of odd composite positive integers  $< 2n$  is equal the power of Cartesian product:  $S \times S$ . Then :

$$|V| = s_0 \cdot s_0 = s_0^2 \quad \square$$

**Proposition 5.2**

$$|Q|_m = s_0^2/2n \quad (45)$$

**Proof 5.2** By equations (43 and 44) we have :  $|Q|_m = |V|/|W| = s_0^2/2n$   
 $\square$

**Proposition 5.3**

$$|Q|_m = (n - p)^2/2n \quad (46)$$

For n is even;  $n > 1$

**Proof 5.3** Substituting  $s_0$  by equation (17) to equation (45) then we get

$$: |Q|_m = (n - p)^2/2n;$$

For n is even ;  $n > 1 \quad \square$

**Proposition 5.4**

$$|Q|_m = (n - p - 1)^2/2n \quad (47)$$

For n is odd or odd prime;  $n > 1$

**Proof 5.4** Substituting  $s_0$  by equation (18) to equation (45) then we get

$$:$$

$$|Q|_m = (n - p - 1)^2/2n;$$

For n is odd or odd prime ;  $n > 1 \quad \square$

## 6 Average value of the number of binary sums are formed from odd noncomposite positive integers $< 2n$

**Definition 6.1**  $P$  - ordered set of odd noncomposite positive integers  $< 2n$ ;

$p$  - elements of  $P$  ;

$|P| = p$  - power of  $P$  ;

$p_i$  - vary over all  $p$  ;

$p_j$  - vary over all  $p$  ;

**Definition 6.2**  $T = \{t_k | t_k \in N, t_k = p_i + p_j\}$ ;

is a set by elements of which are every possible binary sums of odd noncomposite integers  $< 2n$ . (each with all the rest )

Since  $p_i < 2n; p_j < 2n$  then  $\max t_k < 4n$ .

$|T|$  - the power of set  $T$ .

**Definition 6.3**  $|H|_m$  is mean quantity of binary sums  $p_i + p_j = 2k$  which can be formed of odd

noncomposite positive integers  $< 2n$  and which are mapped into  $W$  by surjective mapping :

$f : T \Rightarrow W$

$$|H|_m = |T|/|W| \quad (48)$$

i.e. uniform mapping regardless of real.

$|H|_m$ -positive rational number  $> 0 \quad \forall 2n > 4$ .

**Proposition 6.1**

$$|T| = p^2 \quad (49)$$

**Proof 6.1** The number of every possible binary sums

are formed of odd noncomposite positive integers  $< 2n$

in the view of  $p_i + p_j = 2k$  is equal the power of Cartesian product  $P \times P$

.

Then  $|T| = p \cdot p = p^2 \quad \square$

**Proposition 6.2**

$$|H|_m = p^2/2n \quad (50)$$

$\forall n > 1$

**Proof 6.2** By equations (48 and 49) we have :  $|H|_m = p^2/2n \forall n > 1$   
 $\square$

## 7 The deviation of $|Q|, |H|$ from $|Q|_m, |H|_m$

**Definition 7.1** The deviation of  $|Q|$  from  $|Q|_m$  is:

$$\begin{aligned} \Delta|Q| &= |Q|_m - |Q| \\ \Delta|Q| &> 0 \text{ if } |Q|_m > |Q| \\ \Delta|Q| &< 0 \text{ if } |Q|_m < |Q| \end{aligned}$$

**Definition 7.2** The deviation of  $|H|$  from  $|H|_m$  is:

$$\begin{aligned} \Delta|H| &= |H|_m - |H| \\ \Delta|H| &> 0 \text{ if } |H|_m > |H| ; \\ \Delta|H| &< 0 \text{ if } |H|_m < |H| \end{aligned}$$

### 7.1 Relationship between the deviations $\Delta|Q|$ and $\Delta|H|$

**Proposition 7.1**

$$\Delta Q = \Delta H + |Q|_m - |H|_m - |Q|_b \quad (51)$$

$\forall n > 1$

**Proof 7.1** By equation (42)  $|Q| - |H| = |Q|_b$ ;

By definition (7.1)  $\Delta|Q| = |Q|_m - |Q|$  ;

By definition (7.2)  $\Delta|H| = |H|_m - |H|$  ;

Whence :  $|Q| = |Q|_m - \Delta|Q|$ ;

$|H| = |H|_m - \Delta|H|$ ;

Then:

$(|Q|_m - \Delta|Q|) - (|H|_m - \Delta|H|) = |Q|_b$  ;

$|Q|_m - \Delta|Q| - |H|_m + \Delta|H| = |Q|_b$ ;

Whence:

$\Delta|Q| = \Delta|H| + |Q|_m - |H|_m - |Q|_b$  .  $\square$

### 7.2 The control of equation for $\Delta|Q|$

**Exemple 7.1 Exemple 7.2** The computation  $\Delta|Q|$  for  $n$  is even:  $n = 2, 2n = 4$  .

Data of direct computations we take from exmp 3.1

$$p = 2; s_o = 0; |Q| = 0; |L| = 0; |H| = 1; G = 1; F = 1$$

$$|Q|_m = ((n - p)^2/2n) = ((2 - 2)^2/4) = 0. \text{ by equation (46)}$$

$$|H|_m = (p^2)/2n = (2^2)/4 = 1; \text{ by equation (50)}$$

$$\Delta|Q| = |Q|_m - |Q| = 0 - 0 = 0; \text{ by definition (7.1)}$$

$$\Delta|H| = |H|_m - |H| = 1 - 1 = 0; \text{ by definition (7.2)}$$

$$|Q|_b = (n - 2p)/2 = (2 - 4)/2 = -1; \text{ by equation (38)}$$

The computation of  $\Delta|Q|$  by equation (51) .

$$\Delta|Q| = |Q|_m - |H|_m - |Q|_b + \Delta|H| = 0 - 1 + 1 - 0 = 0 .$$

The compare of  $\Delta|Q|$  computed by data of direct computations with computed value

by equation (51) shows full coincidence of results.

The computation  $\Delta|Q|$  for  $n$  is odd prime:  $n = 3; 2n = 6$  .

Data of direct computations for odd prime  $n = 3$  we take from exp 3.2

$$p^* = 2; s_o = 0; |Q| = 0; |L| = 0; |H| = 1; G = 2; F = 1$$

$$|Q|_b = |Q| - |H| = 0 - 1 = -1 . \text{ by equation (42)}$$

$$|Q|_m = ((n - p - 1)^2)/2n = ((3 - 2 - 1)^2/134) = 0. \text{ by equation (47)}$$

$$|H|_m = (p^2)/2n = (2^2)/6 = 2/3; \text{ by equation (50)}$$

$$\Delta|Q| = |Q|_m - |Q| = 0 - 8 = 0; \text{ by definition (7.1)}$$

$$\Delta|H| = |H|_m - |H| = 2/3 - 1 = -1/3; \text{ by definition (7.2)}$$

$$|Q|_b = (n - 2p - 1)/2 = (3 - 4 - 1)/2 = -1; \text{ by equation (39)}$$

The computation of  $\Delta|Q|$  by equation (51) .

$$\Delta|Q| = |Q|_m - |H|_m - |Q|_b + \Delta|H| = 0 - 2/3 + 1 - 1/3 = 0 .$$

The compare of  $\Delta|Q|$  computed by data of direct computations with computed value

by equation (51) shows full coincidence of results.

**Example 7.3** The computation  $\Delta|Q|$  for  $n$  is even:  $n = 6, 2n = 12$  .

Data of direct computations we take from exp 3.3

$$p = 5; s_o = 1; |Q| = 0; |L| = 1; |H| = 2; G = 5; F = 3$$

$$|Q|_m = ((n - p)^2/2n) = ((6 - 5)^2/12) = 1/12. \text{ by equation (46)}$$

$$|H|_m = (p^2)/2n = (5^2)/12 = 25/12; \text{ by equation (50)}$$

$$\Delta|Q| = |Q|_m - |Q| = 1/12 - 0 = 1/12; \text{ by definition (7.1)}$$

$$\Delta|H| = |H|_m - |H| = 25/12 - 2 = 1/12; \text{ by definition (7.2)}$$

$$|Q|_b = (n - 2p)/2 = (6 - 10)/2 = -2; \text{ by equation (38)}$$

The computation of  $\Delta|Q|$  by equation (51) .

$$\Delta|Q| = |Q|_m - |H|_m - |Q|_b + \Delta|H| = 1/12 - 25/12 + 2 + 1/12 = 1/12 .$$

The compare of  $\Delta|Q|$  computed by data of direct computations with computed value

by equation (51) shows full coincidence of results.

**Example 7.4** The computation  $\Delta|Q|$  for  $n$  is odd prime:  $n = 7; 2n = 14$

Data of direct computations for odd prime  $n = 67$  we take from exmp 3.4

$$p^* = 5; s_o = 1; |Q| = 0; |L| = 1; |H| = 2; G = 6; F = 3$$

$$|Q|_b = |Q| - |H| = 0 - 2 = -2 . \text{ by equation (42)}$$

$$|Q|_m = ((n - p - 1)^2)/2n = ((7 - 5 - 1)^2/14) = 1/14. \text{ by equation (47)}$$

$$|H|_m = (p^2)/2n = (5^2)/14 = 25/14; \text{ by equation (50)}$$

$$\Delta|Q| = |Q|_m - |Q| = 1/14 - 0 = 1/14; \text{ by definition(7.1)}$$

$$\Delta|H| = |H|_m - |H| = 25/14 - 2 = -3/14; \text{ by definition (7.2)}$$

$$|Q|_b = (n - 2p - 1)/2 = (7 - 10 - 1)/2 = -2 ; \text{ by equation (39)}$$

The computation of  $\Delta|Q|$  by equation (51) .

$$\Delta|Q| = |Q|_m - |H|_m - |Q|)_b + \Delta|H| = 1/14 - 25/14 + 2 - 3/14 = 1/14 .$$

The compare of  $\Delta|Q|$  computed by data of direct computations with computed value

by equation (51) shows full coincidence of results.

**Exemple 7.5** The computation  $\Delta|Q|$  for  $n$  is even:  $n = 66, 2n = 132$  .

Data of direct computations we take from exmp 3.5

$$p = 32; s_o = 34; |Q| = 11; |L| = 12; |H| = 10; F = 33$$

$$|Q|_m = ((n - p)^2/2n) = ((66 - 32)^2/132) = 8,7575757. \text{ by equation (46)}$$

$$|H|_m = (p^2)/2n = (32^2)/132 = 7,7575757; \text{ by equation (50)}$$

$$\Delta|Q| = |Q|_m - |Q| = 8,7575757 - 11 = -2,2424243; \text{ by definition (7.1)}$$

$$\Delta|H| = |H|_m - |H| = 7,7575757 - 10 = -2,2424243; \text{ by definition (7.2)}$$

$$|Q|_b = (n - 2p)/2 = (66 - 64)/2 = 1 ; \text{ by equation (38)}$$

The computation of Delta $|Q|$  by equation (51) .

$$\Delta|Q| = |Q|_m - |H|_m - |Q|)_b + \Delta|H| = 8,7575757 - 7,7575757 - 1 - 2,2424243 = -2,2424243 .$$

The compare of  $\Delta|Q|$  computed by data of direct computations with computed value

by equation (51) shows full coincidence of results.

**Exemple 7.6** The computation  $\Delta|Q|$  for  $n$  is odd :  $n = 69; 2n = 138$  .

Data of direct computations we take from exmp 3.6

$$p = 33; s_o = 35; |Q| = 10; |L| = 15; |H| = 9;$$

$$|Q|_m = ((n - p - 1)^2/2n) = ((69 - 33 - 1)^2/138) = 8,8768115. \text{ by equation (47)}$$

$$|H|_m = (p^2)/2n = (33^2)/138 = 7,8913043; \text{ by equation (50)}$$

$$\Delta|Q| = |Q|_m - |Q| = 8,8768115 - 10 = -1,123189; \text{ by definition (7.1)}$$

$$\Delta|H| = |H|_m - |H| = 7,8913043 - 9 = -1,1086957; \text{ by definition (7.2)}$$

$$|Q|_b = (n - 2p - 1)/2 = (69 - 66 - 1)/2 = 1 ; \text{ by equation (39)}$$

The computation of  $\Delta|Q|$  by equation (51) .

$$\Delta|Q| = |Q|_m - |H|_m - |Q|)_b + \Delta|H| = 8,8768115 - 7,8913043 - 1 - 1,1086957 = -1,123189 .$$

The compare of  $\Delta|Q|$  computed by data of direct computations with computed value

by equation (51) shows full coincidence of results.

**Example 7.7** The computation  $\Delta|Q|$  for  $n$  is odd prime:  $n = 67; 2n = 134$  .

Data of direct computations for odd prime  $n = 67$  we take from exmp 3.7  
 $p = 30; s_o = 36; |Q| = 8; |L| = 20; |H| = 5;$

$|Q|_b = |Q| - |H| = 8 - 5 = 3$  . by equation (42)

$|Q|_m = ((n - p - 1)^2)/2n = ((67 - 30 - 1)^2)/134 = 9,6716417$ . by equation (47)

$|H|_m = (p^2)/2n = (30^2)/134 = 6,7164179$ ; by equation (50)

$\Delta|Q| = |Q|_m - |Q| = 9,6716417 - 8 = 1,6716417$ ; by definition (7.1)

$\Delta|H| = |H|_m - |H| = 6,7164179 - 5 = 1.7164179$ ; by definition (7.2)

$|Q|_b = (n - 2p - 1)/2 = (67 - 60 - 1)/2 = 3$  ; by equation (39)

The computation of  $\Delta|Q|$  by equation (51) .

$\Delta|Q| = |Q|_m - |H|_m - |Q|_b + \Delta|H| = 2,9552238 - 3 + 1,7164179 = 1,6716417$  .

The compare of  $\Delta|Q|$  computed by data of direct computations with computed value

by equation (51) shows full coincidence of results.

Hence equation (51) is true  $\forall n > 1$  .

## 8 The relationship between $n$ and $p$

### 8.1 The relationship between $n$ and $p$ for $n < 60$

**Proposition 8.1**

$$2p > n \tag{52}$$

for  $n$  is even  $n < 60$

**Proof 8.1** From the numerical solution (see.subsection 12.1)

it follows that:

$2p > n$  for  $n$  is even  $n < 60$   $\square$

**Proposition 8.2**

$$2p + 1 > n \tag{53}$$

for  $n$  is odd or odd prime  $n < 59$



**Proof 8.2** From the numerical solution (see subsection 12.1) it follows that:

$2p + 1 > n$  for  $n$  is odd or odd prime  
 $n < 59$  .  $\square$

## 8.2 The relationship between $n$ and $p$ for $n > 60$

**Proposition 8.3**

$$n > 2p \quad (54)$$

for  $n$  is even  $n > 60$

**Proposition 8.4**

$$n > 2p + 1 \quad (55)$$

for  $n$  is odd or odd prime  $n > 59$  .

**Proof 8.3** We need to prove that:  $n > 2p$  for  $n$  is even,  $n > 60$  .  
 $n > 2p + 1$  for  $n$  is odd or odd prime,  $n > 59$  .

For it we must find a dependence of differences :

$f_1 = n - 2p$ ; and  $f_2 = n - (2p + 1)$ ; from  $n$

Substituting for  $p$  its the second order approximation by equation (20) we get:

$$f_1 = n - 2(2n/\ln 2n + 2n/\ln^2 2n) f_2 = n - (2(2n/\ln 2n + 2n/\ln^2 2n) + 1) .$$

$$f_1 = n - 4n/\ln 2n - 4n/\ln^2 2n f_2 = n - 4n/\ln 2n - 2n/\ln^2 2n - 1 .$$

Next we computation the derivatives :

$$(f_1)' = (\ln^4 2n - 4\ln^3 2n + 8\ln 2n)/\ln^4 2n .$$

$$(f_2)' = (\ln^4 2n - 4\ln^3 2n + 8\ln 2n)/\ln^4 2n .$$

Whence  $(f_1)' > 0$  ;  $(f_2)' > 0$  ;  $\forall 2n > 2$  .

Then  $f_1, f_2$  increase  $\forall 2n > 2$  .

Next we compute the points of intersection of  $f_1, f_2$  with abscissa axis.

For it we need to test of fulfillment of conditions:

$$(f_1(2n) = 0 ; f_1(2n + 2) > 0) \text{ and } (f_2(2n) = 0; f_2(2n + 4) > 0).$$

As follows from numerical solution (see subsubsection 12.1).

The points of intersection are:

$2n = 120$  for  $n$  is even; and  $2n = 118$  for  $n$  is odd or odd prime

Since conditions are fulfilled only for them.

Hence  $f_1 > 0$  for  $n$  is even,  $n > 60$  .

And  $f_2 > 0$  for  $n$  is odd or odd prime  $n > 59$

Thus  $n > 2p$ ; for  $n$  is even ,  $n > 60$ .

And  $n > 2p + 1$  ; for  $n$  is odd or odd prime,  $n > 59$  .  $\square$

### 8.2.1 The control of relationship $n > 2p$ ; $n > 2p + 1$

**Exemple 8.1 Remark 8.1**  $p^*$  is decremental value of  $p$  by corollary 2.3

From data direct computations exmp 3.5; examp 3.6 ; examp 3.7 we have :

$n = 66$  ,  $p = 32$  ;  $n = 67$ ,  $p^* = 30$  ;  $n = 69$ ,  $p = 33$  ;

By proposition (8.3)  $66 > 64$ ; .

By proposition (8.4)  $67 > 60 + 1$  ;  $69 > 66 + 1$  . Hence propositions (8.3 and 8.4) are correct for  $n$  stated above.

## 8.3 The relationship between $n$ and $p \forall n$

**Definition 8.1** The correspondence  $p$  to finite set of  $2n$  it is when for several  $2n$  it takes at the same  $p$  at computations by equations ABR $2n$ .

**Proposition 8.5** For each  $p$  correspond to finite set of  $2n$  included between neighboring values of  $p$  .

**Proof 8.4** Let for any  $2n_1$  exists  $p < 2n_1$  and for  $2n_k$  exists  $p + 1 < 2n_k$  then  $p$  correspond to set of  $2n$

$2n_1 \dots 2n_k$  This set is restrictedly. Hence For each  $p$  correspond to finite set

of  $2n$  included between neighboring values of  $p$ .  $\square$

### 8.3.1 The examples of the set of $n$ included between neighboring values of $p$ .

**Exemple 8.2** Let we have the set XUY 1,2,3,4,5,6,7,8,9,10,11,12,13,14,15, U 17,18,19,20,21,22,23,24,25,26,27,28,29,30,31

$n=16$  is excluded by rem (2.1). Corollaries (2.1 and 2.2 and 2.3)change following way:  $s_e^* = s_e - 1$ ;  $s_o^* = s_o - 1$  :  $p^* = p - 1$

since in this case is excluded one  $n$ .

We can see that :

$p = 2$  correspond  $2n = 4$

$p = 3$  correspond  $2n = 6$

$p = 4$  correspond  $2n = 8,10$

$p = 5$  correspond  $2n = 12$

$p = 6$  correspond  $2n = 14,16$

$p = 7$  correspond  $2n = 18$

$p = 8$  correspond  $2n = 20,22$

$p = 9$  correspond  $2n = 24,26,28$

$p = 10$  correspond  $2n = 30$

We control the following correspondences:  $p = 4$  correspond  $2n = 8, 10$  subsets:  $XUY 1, 2, 3U5, 6, 7$   $n = 4$  is excluded by rem.(2.1);

$XUY 1, 2, 3, 4U6, 7, 8, 9$   $n = 5$  is excluded.

$p = 6$  correspond  $2n = 14, 16$  subsets:  $XUY 1, 2, 3, 4, 5, 6U8, 9, 10, 11, 12, 13$   $n = 7$  is excluded ;

$XUY 1, 2, 3, 4, 5, 6, 7U9, 10, 11, 12, 13, 14, 15$   $n = 8$  is excluded

$p = 8$  correspond  $2n = 20, 22$  subsets:  $XUY 1, 2, 3, 4, 5, 6, 7, 8, 9U11, 12, 13, 14, 15, 16, 17, 18, 19$   $n = 10$  is excluded ;

$XUY 1, 2, 3, 4, 5, 6, 7, 9, 10, U12, 13, 14, 15, 16, 17, 18, 19, 20, 21$   $n = 11$  is excluded

$p = 9$  correspond  $2n = 24, 26, 28$  subsets:  $XUY 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11U13, 14, 15, 16, 17, 18, 19, 20, 21$   $n = 12$  is excluded ;

$XUY 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12U14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25$   $n = 13$  is excluded

$XUY 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13U15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27$   $n = 14$  is excluded

By equation (17) we get :

for  $2n - 8$ ;  $n = 4$  ;  $p = 4$  .

$s_o = n - p = 4 - 4 = 0$ . By direct computation for  $2n = 8$ ,  $s_o = 0$ .

for  $2n - 10$ ;  $n = 5$  ;  $p^* = 3$  (corrolary (2.3) with real changes).

By equation (18) we get :

$s_o = n - p - 1 = 5 - 3 - 1 = 1$  By direct computation for  $2n = 10$ ,  $s_o = 1$ .

for  $2n - 14$ ;  $n = 7$  ;  $p^* = 5$  (corrolary (2.3) with real changes) .

By equation (18) we get :

$s_o = n - p - 1 = 7 - 5 - 1 = 1$  By direct computation for  $2n = 14$ ,  $s_o = 1$ .

for  $2n - 16$ ;  $n = 8$  ;  $p = 6$  .

By equation (17) we get :

$s_o = n - p = 8 - 6 = 2$  By direct computation for  $2n = 16$ ,  $s_o = 2$ .

for  $2n - 20$ ;  $n = 10$  ;  $p = 8$  .

By equation (17) we get :

$s_o = n - p = 10 - 8 = 2$  By direct computation for  $2n = 20$ ,  $s_o = 2$ .

for  $2n - 22$ ;  $n = 11$ ;  $p^* = 7$  (corrolary (2.3) with real changes) .

By equation (18) we get :

$s_o = n - p - 1 = 11 - 7 - 1 = 3$  By direct computation for  $2n = 22$ ,  $s_o = 3$ .

for  $2n - 24$ ;  $n = 12$  ;  $p = 9$  .

By equation (17) we get :

$s_o = n - p = 12 - 9 = 3$  By direct computation for  $2n = 24$ ,  $s_o = 3$ .

for  $2n - 26$ ;  $n = 13$ ;  $p^* = 8$  (corrolary (2.3) with real changes) .

By equation (18) we get :

$s_o = n - p - 1 = 13 - 8 - 1 = 4$  By direct computation for  $2n = 26$ ,  $s_o = 4$ .

for  $2n - 28$ ;  $n = 14$  ;  $p = 9$  .

By equation (17) we get :

$s_o = n - p = 14 - 9 = 5$  By direct computation for  $2n = 28$ ,  $s_o = 5$ .  
 Thus for several  $2n$  it takes at the same  $p$  at computations by equations  
 $ABR2n$  and these computations are correct.  
 Hence for each  $p$  correspond to finite set of  $2n$  included between neighbor-  
 ing values of  $p$

## 9 The properties of $|Q|_b$

**Proposition 9.1** In the range of  $2 < 2n < 120$   $|Q|_b$  is negative integer  
 excepting  $2n = 94, 96, 100, 118$ .

**Proof 9.1** By proof (4.5)  $|Q|_b$  is integer for  $n$  is even .  
 By proof (4.6)  $|Q|_b$  is integer for  $n$  is odd or odd prime .  
 Then  $|Q|_b$  is integer  $\forall n > 1$  .  
 Since by propositions (8.1 and 8.2 )  $2p > n$  and  $2p + 1 > n$   
 then by equation (38)  $|Q|_b < 0$  for  $n$  is even  $< 60$  .  
 And by equation (39)  $|Q|_b < 0$  for  $n$  is odd or odd prime  $< 60$ .  
 By direct computation with help of numerical solution of  $|Q|_b$  (see subsec-  
 tion 12.1)  
 we find  $2n$  for which  $|Q|_b = 0$ .  
 Thus In the range of  $2 < 2n < 120$   $|Q|_b$  is negative integer  
 excepting  $2n = 94, 96, 100, 118$ .  $\square$

**Proposition 9.2**  $|Q|_b$  is positive integer  $\forall 2n > 120$

**Proof 9.2** By proof (4.5)  $|Q|_b$  is integer for  $n$  is even .  
 By proof (4.6)  $|Q|_b$  is integer for  $n$  is odd or odd prime .  
 Then  $|Q|_b$  is integer  $\forall n > 1$  .  
 Since by propositions (8.3 and 8.4)  $2p < n$  and  $2p + 1 < n$   
 then by equation (38)  $|Q|_b > 0$  for  $n$  is even  $> 60$  .  
 And by equation (39)  $|Q|_b > 0$  for  $n$  is odd or odd prime  $> 60$ .  
 Thus  $|Q|_b$  is positive integer  $\forall 2n > 120$   $\square$

**Corollary 9.1** There is no less than  $|Q|_b$  of representations of the  
 type "Q"  $\forall 2n > 120$  .

**Remark 9.1** The following proposition explains the cause by which  $|Q| =$   
 $0$  in the range of  $2 < 2n < 120$  .

**Proposition 9.3** In the range of  $2 < 2n < 120$  if fulfilled the condition  
 $|H| = ||Q|_b|$  then  $|Q| = 0$  .

**Proof 9.3** By equation (42) and proposition (9.1) — we have for  $2 < 2n < 120$

$$|H| = |Q| + ||Q|_b|.$$

Let  $|Q| = 0$ ; then  $|H| = ||Q|_b|$ .

Thus if it is fulfilled the condition  $|H| = ||Q|_b|$  then  $|Q| = 0$

In the range of  $2 < 2n < 120$ .  $\square$

## 10 The solution of the Goldbach's binary problem

**Lemma 10.1**  $\forall 2 < 2n < 120$  exists at least one representation of type "H".

**Proof 10.1** Taking into account equation (42) we get:  $|H| = |Q| - |Q|_b$ . Since in the range of  $2 < 2n < 120$  by proposition(9.1)  $|Q|_b < 0$  then  $|H| = |Q| + ||Q|_b|$

Whence  $|H| > 0$  excepting  $2n = 94; 96; 100; 106; 118$ ; in which  $|Q|_b = 0$  (see subsection 12.1).

For this  $2n$  the truth of lemma follows from that for  $2n$  in which  $|Q|_b = 0$  then  $|Q| > 0$  in this points since the points of exclusion for  $|Q|$   $2n = 26, 28, 32, 38$  (see subsection 12.2) don't coincidence with points of exclusion  $|Q|_b$   $2n = 94, 96, 100, 106, 118$  (see subsection 12.1).

Thus  $|H| > 0 \forall 2 < 2n < 120$   $\square$

**Theorem 10.1**  $\forall 2n > 2$  exists at least one representation of even positive integer  $2n$  in the view of a sum of two odd prime positive integers or "1" and odd prime positive integer.

**Proof 10.2** Earlier by Lemma we proved that  $|H| > 0 \forall 2 < 2n < 120$ . For the full proof of the theorem we need to prove that

$$|H| > 0 \forall 2n > 120$$

Let  $|H| = 0$  for some value  $n > 60$  then by definition (7.2) we get:

$$\Delta|H| = |H|_m \tag{56}$$

Also from definition (7.1) we get:

$$|Q| = |Q|_m - \Delta|Q| \tag{57}$$

Taking into account that at  $|H| = 0$  from equation (42) it follows that:

$$|Q| = |Q|_b \tag{58}$$

we get:

$$|Q|_b = |Q|_m - \Delta|Q| \quad (59)$$

Taking into account proposition (7.1 ) and equation (56) we get:

$$|Q|_b = |Q|_m - (|Q|_m - |H|_m - |Q|_b + |H|_m) \quad (60)$$

Whence  $|Q|_b = |Q|_b$

Taking into account equation (38) we get identity for  $n$  is even,  $n > 1$  :

$$(n - 2p)/2 = (n - 2p)/2 \quad (61)$$

Taking into account equation (39) we get identity for  $n$  is odd or odd prime,  $n > 1$  :

$$(n - 2p - 1)/2 = (n - 2p - 1)/2 \quad (62)$$

Thus in the result of assumption that  $|H| = 0$  for some value  $n > 60$  we come to identities: (61 and 62) from them we transite to other identities which are correct too:

$$2n(n - 2p) = 2n(n - 2p); 2n^2 - 4pn = 2n^2 - 4pn; \text{ whence : } 4pn = 4pn; \text{ or } 2np = 2np .$$

$$2n(n - 2p - 1) = 2n(n - 2p - 1); 2n^2 - 4pn - 2n = 2n^2 - 4pn - 2n; \text{ whence : } 4pn = 4pn; \text{ or :}$$

$$2np = 2np \quad (63)$$

We fix value of  $p$  and will be substitute values of infinite series  $2n$  to(63) and identity (63) is correct for all values of  $2n$  then it is determine following relationship between  $n$  and  $p$  : each  $p$  correspond to infinite set of  $2n$  .

That contradict by proposition (8.5) which assert that each  $p$  correspond to finite set of  $2n$  included between neighboring values of  $p$ .

Hence our assumption that  $|H| = 0$  for some value  $n > 60$  is false and it is true that  $|H| > 0 \forall 2n > 120$ . Thus we proved that  $|H| > 0 \forall 2n > 2$  . Hence  $\forall 2n > 2$  exists at least one representation of even positive integer  $2n$  in the view of a sum of two odd prime positive integers or "1" and odd prime positive integer.  $\square$

### Corollary 10.1

$$|Q| > |Q|_b; \forall n > 1 \quad (64)$$

**Proof 10.3** From equation (42) we get:

$$|Q| = |Q|_b + |H|$$

Then for  $|H| > 0$  we get:

$$|Q| > |Q|_b \quad \forall n > 1. \quad \square$$

**Corollary 10.2**

$$|L| < p; \quad \forall n > 1 \tag{65}$$

**Proof 10.4** From equation (2.7) we get:

$$|L| = p - 2|H|$$

Then for  $|H| > 0$  we get:

$$|L| < p; \quad \forall n > 1. \quad \square$$

## 11 The computation of the real values of $|Q|, |H|$

### 11.1 The relative accuracy of computation of $|Q|, |H|$

**Definition 11.1** The relative accuracy of computation of  $|Q|$  as follows below :

$$\delta_Q = (\Delta|Q|/|Q|_m)100\% \tag{66}$$

**Definition 11.2** The relative accuracy of computation of  $|H|$  as follows below:

$$\delta_H = (\Delta|H|/|H|_m)100\% \tag{67}$$

#### 11.1.1 The estimation of $\delta_Q$

**Proposition 11.1**

$$100(p^2)/(n-p)^2 > \delta_Q \tag{68}$$

for  $n$  is even ,  $n > 2$ ;

**Proof 11.1** By definition (7.1) we have  $|Q| = |Q|_m - \Delta|Q|$ . By definition. (11.1) we have:

$\Delta|Q| = (\delta_Q|Q|_m)/100$  then we get :  $|Q| = |Q|_m - (\delta_Q|Q|_m)/100$  .

By corollary (10.9)  $|Q|_m - (\delta_Q|Q|_m)/100 > |Q|_b$  .

Whence it follows that  $100((|Q|_m - |Q|_b)/|Q|_m) > \delta_Q$ .

Taking into account equations (46 and 38 ). Then we get:  $100((n - p)^2)/2n - (n - 2p)/2)/((n - p)^2)/2n > \delta_Q$ .

Hence  $100(p^2)/(n - p)^2 > \delta_Q$  for  $n$  is even ,  $n > 2$ ;  $\square$

### Proposition 11.2

$$100(p^2 + 2p - n + 1)/(n - p - 1)^2 > \delta_Q \quad (69)$$

for  $n$  is odd or oddprime,  $n > 4$

**Proof 11.2** By definition (7.1) we have  $|Q| = |Q|_m - \Delta|Q|$ . By definition. (11.1) we have:

$\Delta|Q| = (\delta_Q|Q|_m)/100$  then we get :  $|Q| = |Q|_m - (\delta_Q|Q|_m)/100$  .

By corollary (10.9)  $|Q|_m - (\delta_Q|Q|_m)/100 > |Q|_b$  .

Whence it follows that  $100((|Q|_m - |Q|_b)/|Q|_m) > \delta_Q$ .

Taking into account equations ( 47 and 39 ) then we get:

Then we get:  $100((n - p - 1)^2)/2n - (n - 2p - 1)/2)/((n - p - 1)^2)/2n > \delta_Q$  .

Hence  $100(p^2 + 2p - n + 1)/(n - p - 1)^2 > \delta_Q$  . for  $n$  is odd or odd prime,  $n > 4$  .  $\square$

#### 11.1.2 The dependence of $\delta_Q(2n)$

**Theorem 11.1** For  $n$  is even  $> 2$  If  $n \rightarrow \infty$  then  $\delta_Q \rightarrow 0$ .

**Proof 11.3** We equate  $\delta_Q$  with its estimation by equation(68)

then for  $n$  is even we have:  $\delta_Q = 100(p^2)/(n - p)^2$  .

We replace  $p$  with its first order approximation by equation (19) we get:

$$\delta_Q = (100 \cdot 4n^2/\ln^2 2n)/(n^2 \ln^2 2n - 4n^2 \ln 2n + 4n^2) \ln^2 2n = 400/\ln^2 2n(1 - 4/\ln 2n + 4/\ln^2 2n)$$

Whence follows that if  $n \rightarrow \infty$  then  $\delta_Q \rightarrow 0$ .

for  $n$  is even,  $n > 2$ ;

$\square$

**Theorem 11.2** For  $n$  is odd or odd!  $> 4$  If  $n \rightarrow \infty$  then  $\delta_Q \rightarrow 0$ .



**Proof 11.4** We equate  $\delta_Q$  with its estimation by equation (69) then for  $n$  is odd or odd prime we have:  $\delta_Q = 100(p^2 + 2p - n + 1)/(n - p - 1)^2$ . We replace  $p$  with its first order approximation by equation (19) we get:  
 $\delta_Q = (4n^2 + 4n \ln 2n - n \ln^2 2n + \ln^2 2n)/(n^2 \ln^2 2n - 4n^2 \ln 2n + 4n^2 - 2n \ln^2 2n + 4n \ln 2n + \ln^2 2n)$  ;  
 $\delta_Q = 4n^2 \ln^2 2n (1/\ln^2 2n + 1/n \ln 2n - 1/4n + 1/4n^2)/4n^2 \ln^2 2n (1/4 - 1/\ln 2n + 1/\ln^2 2n - 1/2n + 1/n \ln 2n + 1/4n^2)$   
 $(1/\ln^2 2n + 1/n \ln 2n - 1/4n + 1/4n^2)/(1/4 - 1/\ln 2n + 1/\ln^2 2n - 1/2n + 1/n \ln 2n + 1/4n^2)$   
 Whence follows that if  $n \rightarrow \infty$  then  $\delta_Q \rightarrow 0$ .  
 for  $n$  is odd or odd prime,  $n > 4$ ;  
 Thus we proved that if  $n \rightarrow \infty$  then  $\delta_Q \rightarrow 0 \forall n > 4$

□

## 11.2 The character of dependence of $|Q|(2n)$

**Theorem 11.3** If  $n \rightarrow \infty$  then  $|Q| \rightarrow |Q|_m$ .

**Proof 11.5** By Theorems (11.1 and 11.2)  $\delta_Q \rightarrow 0$  if  $n \rightarrow \infty$  then by Definition 11.1  $\Delta|Q| \rightarrow 0$  since  $\delta_Q \rightarrow 0$ . Whence by Definition 7.1 we have:  
 If  $n \rightarrow \infty$  then  $|Q| \rightarrow |Q|_m$ . □

## 11.3 The formulas for computation of the real values of $|Q|, |H|$

$$|Q| = \text{round}((n - p)^2)/2n \quad (70)$$

for  $n$  is even,  $n > 16$

$$|Q| = \text{round}((n - p - 1)^2)/2n \quad (71)$$

for  $n$  is odd or odd prime,  $n > 17$

Where:  $p = \text{round}(2n/\ln 2n + 2n \ln^2 2n)$ . By (20)

The computed value of  $|Q|$  it can be used for computation of  $|H|$  by equations(30 and 31):

$$|H| = (2|Q| + 2p - n)/2 \text{ for } n \text{ is even, } n > 16$$

$$|H| = (2|Q| + 2p - n - 1)/2 \text{ for } n \text{ is odd or odd prime, } n > 17$$

## 12 Appendices

### 12.1 The numerical solution of $|Q|_b = 0$ in the range $2 < 2n < 138$

**Remark 12.1** If  $n$  is prime then value of  $p$  is decremented by one .  
 $p^* = p - 1$  by corollary 2.3 .

- $2n = 4; n = 2; p = 2; |Q|_b = -1$  by (38).
- $2n = 6; n = 3; p^* = 2; |Q|_b = -1$  by (39).
- $2n = 8; n = 4; p = 4; |Q|_b = -2$  by (38).
- $2n = 10; n = 5; p^* = 3; |Q|_b = -1$  by (39).
- $2n = 12; n = 6; p = 5; |Q|_b = -2$  by (38).
- $2n = 14; n = 7; p^* = 5; |Q|_b = -2$  by (39).
- $2n = 16; n = 8; p = 6; |Q|_b = -2$  by (38).
- $2n = 18; n = 9; p = 7; |Q|_b = -3$  by (39).
- $2n = 20; n = 10; p = 8; |Q|_b = -3$  by (38).
- $2n = 22; n = 11; p^* = 7; |Q|_b = -2$  by (39).
- $2n = 24; n = 12; p = 9; |Q|_b = -3$  by (38).
- $2n = 26; n = 13; p^* = 8; |Q|_b = -2$  by (39).
- $2n = 28; n = 14; p = 9; |Q|_b = -2$  by (38).
- $2n = 30; n = 15; p = 10; |Q|_b = -3$  by (39).
- $2n = 32; n = 16; p = 11; |Q|_b = -3$  by (38).
- $2n = 34; n = 17; p^* = 10; |Q|_b = -2$  by (39)
- $2n = 36; n = 18; p = 11; |Q|_b = -2$  by (38).
- $2n = 38; n = 19; p^* = 11; |Q|_b = -2$  by (39).
- $2n = 40; n = 20; p = 12; |Q|_b = -2$  by (38).
- $2n = 42; n = 21; p = 13; |Q|_b = -3$  by (39).
- $2n = 44; n = 22; p = 14; |Q|_b = -3$  by (38).
- $2n = 46; n = 23; p^* = 13; |Q|_b = -2$  by (39).
- $2n = 48; n = 24; p = 15; |Q|_b = -3$  by (38).
- $2n = 50; n = 25; p = 15; |Q|_b = -3$  by (39).
- $2n = 52; n = 26; p = 15; |Q|_b = -2$  by (38)
- $2n = 54; n = 27; p = 16; |Q|_b = -3$  by (39).
- $2n = 56; n = 28; p = 16; |Q|_b = -2$  by (38).
- $2n = 58; n = 29; p^* = 15; |Q|_b = -1$  by (39).
- $2n = 60; n = 30; p = 17; |Q|_b = -2$  by (38).
- $2n = 62; n = 31; p^* = 17; |Q|_b = -2$  by (39).
- $2n = 64; n = 32; p = 18; |Q|_b = -2$  by (38).
- $2n = 66; n = 33; p = 18; |Q|_b = -2$  by (39).
- $2n = 68; n = 34; p = 19; |Q|_b = -2$  by (38).

- $2n = 70; n = 35; p = 19; |Q|_b = -2$  by (39).  
 $2n = 72; n = 36; p = 20; |Q|_b = -2$  by (38).  
 $2n = 74; n = 37; p^* = 20; |Q|_b = -2$  by (39).  
 $2n = 76; n = 38; p = 21; |Q|_b = -2$  by (38).  
 $2n = 78; n = 39; p = 21; |Q|_b = -2$  by (39).  
 $2n = 80; n = 40; p = 22; |Q|_b = -2$  by (38).  
 $2n = 82; n = 41; p^* = 21; |Q|_b = -1$  by (39).  
 $2n = 84; n = 42; p = 23; |Q|_b = -2$  by (38).  
 $2n = 86; n = 43; p^* = 22; |Q|_b = -1$  by (39).  
 $2n = 88; n = 44; p = 23; |Q|_b = -1$  by (38).  
 $2n = 90; n = 45; p = 24; |Q|_b = -2$  by (39).  
 $2n = 92; n = 46; p = 24; |Q|_b = -1$  by (38).  
 $2n = 94; n = 47; p^* = 23; |Q|_b = 0$  by (39).  
 $2n = 96; n = 48; p = 24; |Q|_b = 0$  by (38).  
 $2n = 98; n = 49; p = 25; |Q|_b = -1$  by (39).  
 $2n = 100; n = 50; p = 25; |Q|_b = 0$  by (38).  
 $2n = 102; n = 51; p = 26; |Q|_b = -1$  by (39).  
 $2n = 104; n = 52; p = 27; |Q|_b = -1$  by (38).  
 $2n = 106; n = 53; p^* = 26; |Q|_b = 0$  by (39).  
 $2n = 108; n = 54; p = 28; |Q|_b = -1$  by (38).  
 $2n = 110; n = 55; p = 29; |Q|_b = -2$  by (39).  
 $2n = 112; n = 56; p = 29; |Q|_b = -1$  by (38).  
 $2n = 114; n = 57; p = 30; |Q|_b = -2$  by (39).  
 $2n = 116; n = 58; p = 30; |Q|_b = -1$  by (38).  
 $2n = 118; n = 59; p = 29; |Q|_b = 0$  by (39).  
 $2n = 120; n = 60; p = 30; |Q|_b = 0$  by (38).  
 $2n = 122; n = 61; p = 29; |Q|_b = 1$  by (39).  
 $2n = 124; n = 62; p = 30; |Q|_b = 1$  by (38).  
 $2n = 126; n = 63; p = 30; |Q|_b = 1$  by (39).  
 $2n = 128; n = 64; p = 31; |Q|_b = 1$  by (38).  
 $2n = 130; n = 65; p = 31; |Q|_b = 1$  by (39).  
 $2n = 132; n = 66; p = 32; |Q|_b = 1$  by (38).  
 $2n = 134; n = 67; p = 31; |Q|_b = 2$  by (39).  
 $2n = 136; n = 68; p = 32; |Q|_b = 2$  by (38).  
 $2n = 138; n = 69; p = 33; |Q|_b = 1$  by (39).

## 12.2 The numerical solution of $|Q| = 0$ in the range $8 < 2n < 134$

$2n = 10;  Q  = 0; (10 - 9 = 1) .$
$2n = 12;  Q  = 0; (12 - 9 = 3) .$
$2n = 14;  Q  = 0; (14 - 9 = 5) .$
$2n = 16;  Q  = 0; (16 - 9 = 7) .$
$2n = 18;  Q  = 0; (18 - 9 = 9) .$
$2n = 20;  Q  = 0; (20 - 9 = 11) .$
$2n = 22;  Q  = 0; (22 - 9 = 13) .$
$2n = 24;  Q  > 0; (24 - 9 = 15) .$
$2n = 26;  Q  = 0; (26 - 9 = 17) .$
$2n = 28;  Q  = 0; (28 - 9 = 19) .$
$2n = 30;  Q  > 0; (30 - 9 = 21) .$
$2n = 32;  Q  = 0; (32 - 9 = 23) .$
$2n = 34;  Q  > 0; (34 - 9 = 25) .$
$2n = 36;  Q  > 0; (30 - 9 = 21) .$
$2n = 38;  Q  = 0; (38 - 9 = 29) .$
$2n = 40;  Q  > 0; (40 - 15 = 25) .$
$2n = 42;  Q  > 0; (42 - 9 = 33) .$
$2n = 44;  Q  > 0; (44 - 9 = 35) .$
$2n = 46;  Q  > 0; (46 - 21 = 25) .$
$2n = 48;  Q  > 0; (48 - 9 = 39) .$
$2n = 50;  Q  > 0; (50 - 15 = 35) .$
$2n = 52;  Q  > 0; (52 - 25 = 27) .$
$2n = 54;  Q  > 0; (54 - 9 = 45) .$
$2n = 56;  Q  > 0; (56 - 21 = 35) .$
$2n = 58;  Q  > 0; (58 - 9 = 49) .$
$2n = 60;  Q  > 0; (60 - 9 = 51) .$
$2n = 62;  Q  > 0; (62 - 9 = 21) .$
$2n = 64;  Q  > 0; (64 - 9 = 55) .$
$2n = 66;  Q  > 0; (66 - 9 = 57) .$
$2n = 68;  Q  > 0; (68 - 33 = 35) .$
$2n = 70;  Q  > 0; (70 - 15 = 55) .$
$2n = 72;  Q  > 0; (72 - 9 = 63) .$
$2n = 74;  Q  > 0; (74 - 9 = 21) .$
$2n = 76;  Q  > 0; (76 - 21 = 55) .$
$2n = 78;  Q  > 0; (78 - 9 = 69) .$
$2n = 80;  Q  > 0; (80 - 15 = 65) .$
$2n = 82;  Q  > 0; (82 - 25 = 57) .$
$2n = 84;  Q  > 0; (84 - 9 = 75) .$

$$\begin{aligned}
2n = 86; |Q| > 0; (86 - 9 = 77) . \\
2n = 88; |Q| > 0; (88 - 25 = 63) . \\
2n = 90; |Q| > 0; (90 - 9 = 81) . \\
2n = 92; |Q| > 0; (92 - 15 = 77) . \\
2n = 94; |Q| > 0; (94 - 9 = 85) . \\
2n = 96; |Q| > 0; (96 - 9 = 87) . \\
2n = 98; |Q| > 0; (98 - 21 = 77) . \\
2n = 100; |Q| > 0; (100 - 15 = 85) . \\
2n = 102; |Q| > 0; (102 - 9 = 93) . \\
2n = 104; |Q| > 0; (104 - 9 = 95) . \\
2n = 106; |Q| > 0; (106 - 15 = 91) . \\
2n = 108; |Q| > 0; (108 - 9 = 99) . \\
2n = 110; |Q| > 0; (110 - 15 = 95) . \\
2n = 112; |Q| > 0; (112 - 21 = 91) . \\
2n = 114; |Q| > 0; (114 - 9 = 105) . \\
2n = 116; |Q| > 0; (116 - 21 = 95) . \\
2n = 118; |Q| > 0; (118 - 25 = 93) . \\
2n = 120; |Q| > 0; (120 - 9 = 111) . \\
2n = 122; |Q| > 0; (122 - 35 = 87) . \\
2n = 124; |Q| > 0; (124 - 9 = 115) . \\
2n = 126; |Q| > 0; (126 - 9 = 117) . \\
2n = 128; |Q| > 0; (128 - 9 = 119) . \\
2n = 130; |Q| > 0; (130 - 9 = 121) . \\
2n = 132; |Q| > 0; (132 - 9 = 123) .
\end{aligned}$$

### 13 RESUME

*With help of the "Arithmetic of binary representations of even integer  $2n$ " (ABR $2n$ ) it is got one of possible solutions of the Goldbach's binary problem.*

*With help of the ABR $2n$  it can also be solved other arithmetical problems. In ABR $2n$  are given formulas for computation of the number of binary representations even integer  $2n$  for basic types of BR $2n$ . Particularly for values inaccessible for computer programs.*

## 14 References

### References

- [1] *J.B.Zeldovich , A.D.Myshkis: The elements of applied mathematics, pgs: 562-567, publishers "Nauka",Moscow (1965)*