Derivation of a chiral SO(3)⊗SU(2)⊗U(1) quantum field theory from Kaluza-Klein theory

PACS
12.10.-g Unified field theories and models
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11.10.Kk Field theories in dimensions other than four

Abstract. A geometric theory in 10+1 dimensions is developed starting from a transition $S^{10} \rightarrow S^3 \times S^7$ followed by dynamical compactification in which S^7 becomes the compactified particle dimensions in a Kaluza-Klein theory, and the spatial S^3 inflates. The closed space acquires a vacuum winding $\pi_7(S^3)=\pi_4(S^3)=Z_2$ with SO(3), SU(2), U(1) eigenvalues (0,–½ 1) and chirality $Z_2 = \{L, R\}$. This vacuum breaks the symmetry of the particle space $SU(4)/SU(3) \cong S^7$ to $(Spin(3)\otimes SU(2)\otimes U(1))/Z_3$ giving 12 topological monopoles by $\pi_7(S^4) = Z\times Z_3\times Z_4$ with spin ½ giving 3 families of 4 fermionic monopoles that split into SO(3) coloured and colourless SU(2) doublets with the same charges as the fundamental particles. Topological conditions in the classical theory give a definition of Planck's constant $\hbar=c^3\chi^2/G$ as the physical scale of the topological spin charge, and define the Weinberg angle as $\tan^2\theta_W=5/16$. Closed formulae for e, g, g', m_Z, m_W, m_H are derived in the classical theory. The topological monopoles take the form of rotating compactified black holes in the dimensionally reduced theory, where their ergo-region can trap virtual-radiation sufficient to cancel the rest mass of the black hole. This leads to the derivation of a quantum field theory for the topological monopoles where the Kaluza-Klein dimensional reduction gives a Lagrangian containing the terms of the Standard Model, including a quartic scalar field term which gives the coupling constant value $\lambda=1/8$ for the Higgs term.

1. Introduction

The development of a pure geometrical Kaluza-Klein theory [1-5] for the known particle interactions in the original spirit of Kaluza [1] was apparently shown not to be possible by Witten [6] because the chirality of electroweak interactions couldn't be generated. However, this result no longer holds for a closed universe, where a chiral vacuum becomes possible with a topological winding at the level of the whole closed universe which breaks the symmetry of the particle dimensions in a Kaluza-Klein theory, and allows for topological monopoles as defects in the local structure of the space. The closure of the space also gives rise to a dynamical compactification [7] mechanism driven by the transfer of radiation from the non-spatial dimensions to the spatial dimensions [8]. This transfer of radiation gives a see-saw process where the compactification drives a form of inflatonless inflation [9] in the spatial dimensions.

This paper imposes the closure condition on the the assumption of 10+1 dimensions to give a unified universe of S^{10}, and so the geometrical theory will be referred to as S^{10} unified field theory (STUFT). This metric-field theory introduced in [10] contains no matter fields or any other fields, and so is a a pure metric field theory in which the higher dimensional universe is empty of matter, as was originally assumed by Kaluza [1] and envisaged by Einstein [11,12]. However, the field equations of Einstein gravity extended to any number of spatial dimensions support metric-wave solutions as the dimensional extension of gravitational waves in General Relativity, and so the universe is assumed to initially contain metric-wave radiation. This gives radiation that realises the dynamic compatification of dimensions through a compactification-inflation see-saw mechanism of radiation transfer.

The metric-field equations are assumed to describe a real "fabric of space" as a direct extension to the "fabric" conceptualisation of space-time in General Relativity. This specifically means that the 10 initially equal spatial dimensions are assumed to be physical dimensions, and are not just the effective dimensions of a projection theory [13-16]. This physicality assumption is further extended with the following three conditions:

1) if there is contact of the “fabric of space” at two distinct points in the space, then a bridge can form between those two points
2) the “fabric of space” can form holes when the topology allows
3) the space is physically closed

The first condition is that the Einstein-Rosen bridge [17] solution of General Relativity can be extended to 10+1 dimensions and can form upon self-contact of the “fabric of space” given a suitable stress-energy term. The second physical assumption is that when the topology allows, metric-field radiation of sufficient energy can create holes in the space. The critical assumption is that the universe is closed, as in combination with the previous two conditions it gives the topology for the formation of a non-trivial winding in the space and a spectrum of 12 topological defects [10].

The topological conditions of STUFT are outlined in section 2 and the compactification-inflation see-saw given in section 3 [10]. The standard dimensional reduction techniques of Kaluza-Klein theories are applied in section 4 to obtain the dimensionally reduced classical theory. The topological conditions applying in the dimensionally reduced theory gives non-singular topological monopoles, as is the case for the “solitons” of other Kaluza-Klein theories [18,19]. This no singularity condition is applied to the Kerr metric in section 5, where the properties of the space gives a geometric definition of Planck's constant, together with closed formulae for the coupling constants, the energy density of the topological winding of the space, the vector-field masses and the scalar-field mass [10], which all show agreement with those of the Standard Model. The classical monopole theory is then considered in section 6, where difficulties in the classical theory require a change in descriptive framework. It is shown in section 7 that assuming the topological monopoles possess a wave property enables the derivation of a quantum field theory with local symmetry SO(3)⊗SU(2)⊗U(1), but where the topological monopoles with SO(3) colour charge possess 1/3 electric charges. The Lagrangian terms of the quantum field theory derived from the dimensionally reduced classical theory of section 4 include those of the Standard Model, up to this change in local colour symmetry group.

2. Electroweak Vacuum and Particles

The metric-field equations in 10+1 dimensions will be taken to be that of Einstein gravity with a stress-energy tensor for the energy density of metric-waves

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{\kappa}{c^4} T_{\mu\nu} \tag{1}$$

and the universe is assumed to initially contain an energy density ρ_0 of metric-waves,. The Ricci scalar of the Einstein action encapsulates isotropy and homogeneity conditions, which also give the metric for a closed space in any number D of spatial dimensions as being the Friedman-Robertson-Walker metric

$$ds^2 = -c^2 dt^2 + a^2(t)\left[\frac{dr^2}{1-r^2} + r^2 d\Omega_{D-1}^2\right] \tag{2}$$

for a sphere S^D with time dependent radius $a(t)$ and generalised angle element Ω_{D-1}. For stress-energy tensor components $T_{tt} = \rho c^2$, $T_{ii} = p$, the field equations for $a(t)$ are given by

$$\frac{D(D-1)}{2}\left[\left(\frac{\dot{a}}{a}\right)^2 + \frac{c^2}{a^2}\right] - \Lambda c^2 = \kappa\rho$$
$$-(D-1)\left(\frac{\ddot{a}}{a}\right) - \frac{(D-1)(D-2)}{2}\left[\left(\frac{\dot{a}}{a}\right)^2 + \frac{c^2}{a^2}\right] + \Lambda c^2 = \frac{\kappa}{c^2} p \tag{3}$$

which gives the S^D cosmology the generic character of the radius of the sphere increasing up to some maximum radius and then decreasing. The point of maximum radius for the S^{10} cosmology will be taken to mark the time t_0=0 at which point the radiation energy density is ρ_0.

As the S^{10} shrinks from its maximum radius at t_0=0, the energy density of the metric-wave radiation ρ will increase, which in the classical metric field theory will be associated with an increase in the wave amplitude of the metric-waves. If any of these metric-wave modes are such that they oscillate orthogonal to the surface of the sphere S^{10}, then these two factors will inevitably lead to a situation where wave crests collide internally to the sphere. The "fabric of space" is assumed to be such that this can result in the formation of an internal Einstein-Rosen bridge [17], in which case the topology of the sphere S^D is then changed to that of a torus $T^{n+m}=S^n\times S^m$. This is the critical transition from which all else follows, and so it is assumed that either the given mechanism leads to such an internal bridge, or extending the Einstein action with additional curvature terms leads to the existence of some other mechanism by which an internal bridge forms. In any case, the net result would be the insertion of a tunnel through S^D, where the fusing would be topologically equivalent to two internal hemispherical depressions in the sphere meeting and fusing to form a tube. The homotopy group $\pi_n(S^{n-1}) = Z_2\ \forall n>2$ shows that the meeting of such hemispherical depressions in S^n at their equatorial sphere S^{n-1} can result in a torus T^{n+m} with a non-trivial winding, where $\pi_m(S^n) = Z_2$ is required for consistency

$$
\begin{aligned}
S^{10} &\rightarrow T^{2+8} = S^2\times S^8 && \text{as } \pi_8(S^2) = Z_2 \\
S^{10} &\rightarrow T^{3+7} = S^3\times S^7 && \text{as } \pi_7(S^3) = Z_2 \\
S^{10} &\rightarrow T^{4+6} = S^4\times S^6 && \text{as } \pi_6(S^4) = Z_2
\end{aligned}
$$

The second possibility is selected as it involves the S^3 of a closed spherical spatial cosmology given by the Friedman-Robertson-Walker metric. This gives S^7 as being the non-spatial dimensions, where the given homotopy group is for a map from S^7 to the whole S^3 spatial cosmology, and so would give a non-trivial vacuum structure. The sphere S^7 is a Hopf fibre-bundle [20,21] of an S^3 fibre over an S^4 base-space, and the non-trivial map of the given homotopy group involves a map from the S^4 base-space to the S^3 cosmology, as $\pi_7(S^3) = \pi_4(S^3) = Z_2$.

The map from S^4 to S^3 is an example of the homotopy group relation $\pi_n(S^{n-1}) = Z_2\ \forall n>2$, where the equatorial sphere S^3 of S^4 is mapped to the spatial cosmology S^3. This gives the topological transition as leading to a sphere decomposition sequence that is locally of the form

$$S^{10} \rightarrow S^3\times S^7 \rightarrow S^3\times(S^3\times S^4) \rightarrow S^3\times(S^3\times(S^3\times S^1))$$

The first step is the transition due to the formation of an Einstein-Rosen bridge, the second step is the separation of S^7 into S^3 fibre and S^4 base-space, and the last step is the sphere S^3 being split off by the map to the spatial S^3. As it is the sphere S^7 which is mapped to the spatial sphere S^3, the twisted torus $T^{3+7}=S^3\times S^7$ can be identified as having an inner sphere S^7 that is "rotated" in going around the outer sphere of S^3. This identification for the sphere composition of the torus T^{3+7} allows for the S^3 of the S^4 base-space to be identified with the group space of the symmetry group Spin(3) ≅ SU(2) and S^1 with that of U(1), such that group eigenvalues can be found for this non-trivial vacuum structure.

The dimensional reduction of the maps S^n to $S^{n-1}\ \forall n>2$ can generically be given by the coordinate parametrisation $x_{n-1} = x_\xi \cos\xi$, $x_n = x_\xi \sin\xi$ such that the radius of the sphere $r^2 = x_0^2 + \ldots + x_{n-2}^2 + x_\xi^2$ is reduced to that of S^{n-1}. For S^4, this parametrisation reduces the radius $r^2 = x_0^2 + x_1^2 + x_2^2 + x_\xi^2$ to that of S^3, where the x_3, x_4 coordinates define the S^1 group space of U(1) with group eigenvalue 1, and the coordinates x_0, x_1, x_2, x_ξ define the S^3 group space of SU(2) with group eigenvalue ½. As the S^3 fibre of the space S^7 doesn't participate in the map to the spatial S^3, the corresponding group eigenvalue is 0, which selects the group Spin(3) as the double cover of SO(3) with group eigenvalues including 0.

The map $\pi_7(S^3) = \pi_4(S^3) = Z_2$ specifies a non-trivial winding in the orientation of the S^3 sub-space of the S^4 base-space of S^7 in going around the outer sphere of S^3 in the torus $T^{3+7}=S^3\times S^7$, which implies that the rotation sense of the winding can be related to the sense of "going around" the spatial S^3. Such relative rotation sense is simpler for the case of non-trivial twists in the torus $T^2=S^1\times s^1$, where S^1 denotes the outer circle of the torus and s^1 the cross-section. Non-trivial twists in T^2 are given by the homotopy group $\pi_1(S^1) = Z$, but attention will be restricted to the cases $\{-1,+1\}$. In going clockwise around the outer S^1 of the torus T^2, the inner s^1 circle is either rotated clockwise or anti-clockwise by 2π, where using the right-hand rule as the definition of rotation sense gives the labels of left and right for $\{-1,+1\}$. It should be noted that the choice of clockwise and the right-hand rule to define rotation sense is arbitrary, but once that choice is made the two cases of the twisted T^2 torus can be given the chiral labels {L, R}. For the non-trivial vacuum of $\pi_4(S^3) = Z_2$ the rotation sense in the spatial S^3 is defined by way of the rotation group SU(2), where the right-hand rule gives the same chiral labels. A similar arbitrary choice of rotation sense has to be made for the internal particle base-space S^4, where following the convention of the Standard Model gives the S^4 space orientation as being (–½,1). This gives the twisted vacuum as having (SO(3),SU(2),U(1)) eigenvalues (0,–½,1) with the two possibilities of the non-trivial map $\pi_7(S^3) = \pi_4(S^3) = Z_2$ having spatial chiralities {L, R}. For the local colour symmetry group being SO(3) this would give the colour, isospin and hypercharge eigenvalues of the electroweak vacuum in the Standard Model, where the electroweak vacuum has chirality L. It was claimed in [6] that the chiral vacuum of the Standard Model cannot be generated for a Kaluza-Klein theory, but that result was for flat space-time whereas STUFT considers a closed cosmology.

The map from the S^4 base-space of S^7 to the spatial S^3 breaks the equivalence of the 7 non-spatial dimensions, leaving the symmetry of the S^3 (colour) fibre intact but breaks the symmetry of the S^4 (electroweak) base-space. This symmetry breaking leaves intact the symmetry of a closed S^1 embedded in the S^4 base-space, where the (SU(2), U(1)) eigenvalues (–½,1) show a compact embedding for the unbroken U(1) symmetry with group embedding angle $\tan\phi_W = ½$. For any transition where manifold G is reduced to H, the homotopy group relation $\pi_2(G/H) = \pi_1(H)$ shows that topological monopoles arise when $\pi_1(H)\neq 0$ [22], which will be true for the compact embedding of the unbroken U(1) symmetry.

The group embedding angle $\tan\phi_W=½$ for the unbroken U(1) symmetry differs from the Weinberg angle $\tan\theta_W\approx 0.55$ [23-25] in the Standard Model. However, there is a distinction between the physical spaces which possess a physical scale, and the group spaces of the unitary groups SU(2) and U(1), which are unit spheres. The group angle ϕ_W gives the embedding of the unbroken U(1) group in the broken SU(2) and U(1) symmetries, whereas the expression of the Weinberg angle $\tan\theta_W = g'/g$ is in terms of the SU(2) and U(1) coupling constants g and g'. In the dimensionally reduced Kaluza-Klein theory these coupling constants give the physical scale of the compactified dimensions. So in STUFT, the Weinberg angle will be expressed in terms of the physical scales of the isospin S^3 (radius r_I) and hypercharge S^1 (radius r_Y) spaces as

$$\tan\theta_W = \frac{r_Y}{r_I}\tan\phi_W$$

where $\tan\theta_W > \tan\phi_W$ indicates that $r_Y > r_I$. Although the vacuum map $S^4 \rightarrow S^3$ could be expected to distort the physical shape of the S^4 base-space, the dimensional reduction of S^4 to S^3 gives an anomaly in the definition of the physical scales of r_Y and r_I.

The surface of a sphere S^n is geometrically defined to be the set X of coordinate tuples $(x_0, \ldots, x_n)$ in $(n+1)$-dimensions for a given radius The coordinate parametrisation that picks out an S^1 from S^n to give the equatorial sphere S^{n-1} results in the subset $Y\subset X$ of coordinate tuples $(x_0, \ldots, x_{n-2}, x_\xi)$ in n-dimensions. In the set X, each coordinate x_i has the same range $[-r_n,+r_n]$ and the same mean square over the set X of $\langle x_i^2\rangle = r_n^2/(n+1)\ \forall i$. The subset Y of the equatorial sphere S^{n-1} consists of the same coordinates x_i for i=0, …, n–1 with the same root-mean square values, and so the calculated radius r_{n-1} of the equatorial sphere S^{n-1} in terms of the coordinates of S^n is

$$r_{n-1}^2 = \frac{n}{n+1} r_n^2$$

For the map $S^4 \rightarrow S^3$ where the hypercharge radius r_Y is the radius of the full S^4 and the isospin radius r_I is the radius of the equatorial sphere S^3, the Weinberg angle θ_W will be given by

$$\tan\theta_W = \frac{r_Y}{r_I}\tan\phi_W = \frac{1}{2}\sqrt{\frac{5}{4}} \approx 0.5590 \tag{4}$$

This closed formula for $\tan\theta_W$ gives $\sin^2\theta_W=5/21\approx0.2381$ and $\cos^2\theta_W=16/21$, which compares with the experimental range of $\sin^2\theta_W= 0.2312 - 0.2397$ [23-25].

With the value of the group embedding angle $\tan\phi_W=½$ being modified by a dimensional reduction anomaly in physical scale, the irrational value of the Weinberg angle $\tan\theta_W$ is compatible with a compact embedding for the unbroken U(1) symmetry, for which the theory will contain topological monopoles. However, there will not just be a single type of topological monopole because the initial apportioning of the dimensions of the S^7 space into S^3 fibre and S^4 base-space will not be unique. For the trivial vacuum, these different ways to apportion these non-spatial dimensions will be equivalent, but for the non-trivial vacuum they will not. The different ways for the space S^7 to be apportioned to the S^4 base-space is given by the homotopy group for the mapping of S^7 to S^4

$$\pi_7(S^4) = Z\times Z_{12} = Z\times Z_3\times Z_4$$

The space S^7 corresponds to the space of a quotient G/H of Lie Groups G and H, where of the possible group quotients giving S^7, $SU(4)/SU(3) \cong S^7$ is selected by the symmetry breaking of the non-trivial winding of the space. The SU(4) symmetry acting over S^7 is broken by the non-trivial vacuum, where the standard pattern of SU(n) symmetry breaking to $SU(n-p)\otimes SU(p)\otimes U(1)$ gives

$$SU(4) \rightarrow (SU(2) \cong Spin(3)) \otimes SU(2) \otimes U(1)$$

which has the same local form of $SO(3)\otimes SU(2)\otimes U(1)$ as found for the non-trivial vacuum. However, the Z_3 centre of the group quotient SU(4)/SU(3) gives the full symmetry group as being

$$(Spin(3) \otimes SU(2) \otimes U(1))/Z_3 \tag{5}$$

where this Z_3 of the colour fibre gives a 1/3 factor to the U(1) eigenvalues of the monopoles with SO(3) eigenvalue $|\lambda_C| = 1$. So the U(1) charge eigenvalues are given by

$$\lambda_Q = \lambda_I + \tfrac{1}{2}\lambda_Y \qquad \text{where } \lambda_Y = \begin{matrix} -1 \text{ for colourless monopoles} \\ \tfrac{1}{3} \text{ for coloured monopoles} \end{matrix} \tag{6}$$

which gives the particle identification for the 3 by 4 table of topological monopole eigenvalues shown in Table 1 [10].

Table 1: SO(3), SU(2), U(1) eigenvalues for the SU(3) co-sets with particle identification

SU(3) co-set			1/3	2/3	1
SO(3)	1	SU(2) +½	U(1) 2/3 ($u^{+2/3}$)	U(1) 2/3 ($c^{+2/3}$)	U(1) 2/3 ($t^{+2/3}$)
		SU(2) –½	U(1) –1/3 ($d^{-1/3}$)	U(1) –1/3 ($s^{-1/3}$)	U(1) –1/3 ($b^{-1/3}$)
SO(3)	0	SU(2) +½	U(1) 0 (ν_e)	U(1) 0 (ν_μ)	U(1) 0 (ν_τ)
		SU(2) –½	U(1) –1 (e^{-1})	U(1) –1 (μ^{-1})	U(1) –1 (τ^{-1})

These topological monopoles are not point defects with a singularity, as their topological basis gives them the form of a spatial sphere S^2 being wrapped around S^7 such that the configuration cannot be unwound. In any S^D space given by (2), space only exists within the surface and does not exist in either the interior or the exterior of the sphere S^D. So when the spatial S^2 of a topological monopole is wrapped around S^7 devoid of space, the S^2 surface of the topological monopole will mark the boundary of a real hole in space with the same finite radius as the S^7 dimensions. This surface boundary would be in the locally flat space-time of the full S^3 cosmology, and so this S^2 surface would have to be in one of the representations of the Poincaré group, which for massive objects in their rest frame are just in terms of the rotation group. There are two distinct topological maps from the S^3 group space of the SU(2) rotation group to the S^2 surface around a spatial hole: the first is the same dimensional reduction of S^n to S^{n-1} with homotopy group $\pi_n(S^{n-1})=Z_2\ \forall n>2$, for which the SU(2) group eigenvalues are ±½; and the second is the decomposition of the S^3 fibre-bundle into S^1 fibre and S^2 base-space, where h: $S^3 \rightarrow S^2$ is given by the parametrisation

$$h(x_0, x_1, x_2, x_3) = (x_0^2 + x_1^2 - x_2^2 - x_3^2,\ 2(x_0 x_3 + x_1 x_2),\ 2(x_1 x_3 - x_0 x_2))$$

The mapping from the S^2 base-space defined by this parametrisation to the S^2 spatial sphere gives the monopole "hedgehog" [26,27] with homotopy group $\pi_3(S^2)=Z$ and SU(2) group eigenvalues ±1.

The coordinate parametrisation $x_3 = x_\xi \cos\xi$, $x_2 = x_\xi \sin\xi$ for S^3 gives S^2, $r^2 = x_0^2 + x_1^2 + x_\xi^2$, with the U(1) rotation of x_2, x_3 in the group space being around the coordinate axis of x_ξ. When this is mapped to the spatial S^2 it specifies a rotation about one of the axes in 3 spatial dimensions, which gives the topological construction of an object with SU(2) rotation group eigenvalues of ±½, and so the topological monopoles of Table 1 will also have the topological spin charge of ½ for fermions.

These two decompositions of S^3 to S^2 also give two distinct classes of topological monopole because the embedding of the unbroken U(1) in SU(2) means that a circle S^1 must be selected from the isospin group space S^3. The fibre-bundle decomposition of S^3 where the S^2 is mapped to the spatial S^2 gives a configuration where the U(1) generator I_Q is mapped to the spines of the "hedgehog" to give an electric monopole. The coordinate parametrisation of S^3 that leads to the homotopy group $\pi_3(S^2)=Z_2$ gives the topological basis for the Dirac magnetic monopole [28], where I_Q is aligned with the z-axis. The Dirac string is removed by expressing I_Q as being aligned in opposite directions in the northern and southern hemispheres of the S^2 spatial sphere, with a 2π gauge rotation of U(1) at the equator joining the two hemispheres of the configuration together

$$A_\phi^N = \frac{q_m}{r\sin\theta}(1-\cos\theta)(I_Q\hat{z}) \qquad A_\phi^S = \frac{q_m}{r\sin\theta}(1+\cos\theta)(-I_Q\hat{z})$$

$$A_\phi^S = A_\phi^N - \frac{2q_m}{r\sin\theta} = A_\phi^N - \frac{i}{q_e}\lambda\nabla_\phi\lambda^{-1} \qquad\qquad \lambda = \exp(2iq_e q_m\phi)$$

This topological basis for the electric and magnetic monopoles gives the Dirac quantisation condition for the gauge rotation $q_e q_m = ½n$ in dimensionless units. It also means that STUFT will display electromagnetic duality and contain a spectrum of both electric and magnetic monopoles, which will be added to the energy-momentum tensor in the dimensionally reduced theory as a perfect fluid.

The monopoles of Table 1 also possess non-Abelian charges, where the issue of charge confinement is the same for both colour and isospin charges as their corresponding spaces are both S^3 sub-spaces of the full S^7 particle space. An electric monopole for the S^3 fibre-bundle is given by the S^2 base-space being mapped to the spatial S^2 such that S^1 fibre orientation is that of the monopole "hedgehog". Consider a topological monopole/anti-monopole pair separated along some line, which gives the following 3 distinct topological regions:

1) S^2 base-space maps to the spatial S^2 around the centre point of each monopole

2) S^1 fibre maps to the spatial S^1 around the line connecting the pair
3) Outer region not enclosing either the monopoles or their connecting line

The topology of regions 1 and 2 prevent any unwinding of the monopole configuration, but in region 3 the configuration gives both the S^2 base-space and the S^1 fibre with no constraints. So the symmetry of the S^3 particle space is free to act locally to unwind the configuration such that it is trivial in region 3. This is not the case for the Abelian charges where the particle space S^1 lacks the degrees of freedom required to locally unwind the configuration. The topological constraints of regions 1 and 2 limit this local unwinding of the configuration to the formation of tube such that the charge flux only flows along the connecting line between the pair. As this charge flux must be the same at all points between the pair, the field energy will increase linearly with separation, and so there-exists a constant force of attraction for all separation distances. This is the classical confinement of non-Abelian charges, where the topology giving the Z_3 factor in (5) for colour charges means that the same topological argument for classical confinement will also apply to configurations of 3 coloured monopoles.

3. Compactification-Inflation See-Saw

The previous section considered the consequences of the transition from S^{10} to $S^3 \times S^7$ due to the formation of an internal bridge, which included the appearance of matter in the form of topological monopoles. The effect of the transition on the metric-wave radiation assumed to be present in the initial S^{10} cosmology will be considered in this section, where the focus on metric-wave radiation within S^D reveals an issue with regards to the interpretation of the cosmological constant Λ. The cosmological term Λ is not constant in an absolute sense, but is specifically constant with respect to variation in the metric $g_{\mu\nu}$, which is clearly seen in the derivation of the metric field equations from the Einstein action as the variation is with respect to the metric $g_{\mu\nu}$. The Friedman-Robertson-Walker metric for a general S^D cosmology is parametrised by a radial scale factor $a(t)$, where the radius a of the sphere S^D is in the D+1 spatial dimension. So the S^D metric $g_{\mu\nu}(a)$ is parametrised by an extra-dimensional variable a outside of the D-dimensional surface, and the cosmological constant $\Lambda(a)$ can be similarly parametrised whist still being constant with respect to the metric $g_{\mu\nu}$ within the surface of the sphere S^D.

Taking the covariant derivative of the field equations (1) for constant Λ gives the conservation law $T^{\mu\nu}{}_{;\nu}=0$, but when the cosmological term is parametrised by the extra-dimensional variable a, the conservation law with only radiation in the space becomes

$$\Lambda(a)_{;\nu} g^{\mu\nu} = \frac{d\Lambda}{da} a_{;\nu} g^{\mu\nu} = \kappa T^{\mu\nu}_{;\nu} = \kappa\left(\dot{\rho} + D(\rho + p)\frac{\dot{V}}{V} \right)$$

For $a(t)$ just varying with time, only the energy conservation equation will be modified

$$\dot{a}\frac{d\Lambda}{da} = \kappa\left(\dot{\rho} + D(\rho + p)\frac{\dot{V}}{V} \right)$$

This can be rewritten in the infinitesimal form of the thermodynamic equation $dE+PdV+VdP=0$

$$Vd\rho + D(\rho + p)dV - \frac{1}{\kappa}Vd\Lambda = 0 \tag{7}$$

where setting $\Lambda(a) = \Lambda_p\, p(a)$ gives the term $(\Lambda_p/\kappa)Vdp$. The need for a cosmological pressure term is not particularly apparent in an open infinite space, but is more clearly necessary in a closed S^D space.

The stress term $T_{ii} = p$ in the field equations of general relativity apparently says that pressure acts as a gravitational source, which is a bit misleading. For a gas inside some volume, the pressure of the gas is given by the force exerted against the bounding surface, which for the gas in the centre of the volume is not a local description. As the force on the enclosing boundary is due to the momentum change of the gas particles, the local description of pressure is as a momentum density and this is what the stress term $T_{ii} = p$ is denoting. So it is momentum density which acts a gravitational source in general relativity, not pressure as such. For a space with a boundary, the metric-wave equations will possess a boundary term which would necessarily have to account for the pressure of radiation exerted against the boundary, whereas for a closed S^D space there is no such boundary term, and the global pressure of radiation being applied to the space is not being accounted for by the stress-energy tensor. This would leave the global pressure effect of radiation within the space unaccounted for, but this omission is remedied by introducing a cosmological term $\Lambda(a)=\Lambda_p p(a)$. Viewing radiation within a closed S^D space from the perspective of the D+1 dimensions within which the sphere resides, gives the path of radiation as being curved as it follows the surface of the sphere. The gravitational effect of the stress-energy density within the surface of the sphere does not account for this deflection, because the gravitational effect acts within the confines of the surface, which is orthogonal to the deflection of the radiation in D+1 dimensions. A cosmological pressure term $\Lambda(a)$ that is "constant" within the surface is the additional term required to account for the global effects of radiation pressure on the surface of S^D.

This outward radiation pressure of metric-wave radiation within an S^D surface will be demonstrated for the topological monopoles of the previous section, which have an S^2 surface of the compactified S^7 particle dimensions and encompass a real hole in 3+1 dimensional space-time. Consider metric-waves in the surface of the compactified sphere, where the radius r of the sphere S^2 gives a long-wavelength cut-off and the diameter 2χ of the compactified S^7 gives a short-wavelength cut-off. This combination means that if the radius decreases, some of the metric-waves in the S^2 surface will be excluded from the surface into the exterior volume of space. From the perspective of 3 spatial dimensions this radiation would appear to come from the volume of the sphere, where the change in energy would be given by

$$dE = TdS - PdV \tag{8}$$

Continuing with the 3-dimensional perspective of the radiation, if the energy density $\rho=E/V$ is used as the value of the rate of energy change in sphere, then

$$\frac{E}{V} = \left(\frac{\partial E}{\partial V}\right)_T = T\left(\frac{\partial S}{\partial V}\right)_T - P$$

Using the 3-dimensional equation of state $\rho=E/V=3P$ for radiation and the Maxwell relation

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V \qquad \text{gives}$$

$$T\left(\frac{\partial P}{\partial T}\right)_V = 4P \qquad \text{or} \qquad P = \tfrac{1}{3}\rho = CT^4 \tag{9}$$

where C is some constant. As this is the characteristic relation for the energy density of radiation emitted from a black body at temperature T, it implies that the shrinking S^2 would appear to be a black body from the perspective of the exterior 3D space. The 2-dimensional perspective of the metric-wave radiation within the S^2 surface is that of radiation in 2 spatial dimensions, for which the equation of state $\rho=2p$ gives energy conservation in the sphere S^2 as being $pV=const$. So the outward radiation pressure gradient for the shrinking S^2 will be $\mathrm{d}p/\mathrm{d}r \propto 1/r^4$, which equates to the radiation pressure P in the 3-dimensional perspective, $P=\mathrm{d}p/\mathrm{d}r$. This then gives the apparent temperature T of the sphere S^2 in 3-dimensions as being inversely proportional to radius, as for a black hole.

This derivation of $T\propto 1/r$ involves equating the perspective in 2 spatial dimensions within the S^2 surface with the exterior perspective in 3 spatial dimensions, where the 3-D radiation pressure $p_3=P$ is distinct from the 2-D radiation pressure $p_2=p$. The thermodynamic equation (8) implies that the entropy S_D and temperature T_D should similarly be labelled with the number of spatial dimensions D in which they are defined. The entropy of equations (8) and (9) is S_3, and the black body temperature is $T_3 \propto 1/r$, whereas the S^2 energy conservation relation $p_2V=const.$ is for the condition of 2-D adiabatic changes $dS_2=0$ in the sphere. Equating the 3-D radiation pressure with the 2-D radiation pressure gradient is consequently based upon the adiabatic condition $\partial S_2/\partial r=0$ for the sphere. It has just been shown that the shrinking S^2 emits radiation, and so the total amount of radiation energy E in the sphere is proportional to the radius r, but the adiabatic condition $\partial S_2/\partial r=0$ implies that the surface entropy S_2 doesn't change with energy E, $\partial S_2/\partial E=0$. As the thermodynamic temperature T_D in D spatial dimensions is defined by

$$\frac{\partial S_D}{\partial E} = \frac{1}{T_D}$$

this implies that the 2-D temperature T_2 of radiation within S^2 is infinite for all values of the radius r. This is in turn implies that the surface entropy density of S^2 has reached its maximum value, and so it is simply proportional to the area A of the sphere, $S_2=const.A$, as for a black hole. For a short-wavelength cut-off of 2χ, the maximum number of wave modes per dimension will be proportional to $1/2\chi$, where for $r>>\chi$ the number of configurations given by the multinomial coefficient will increase exponentially, and so the maximum density of states per dimension g_1 will be given by $g_1 \propto \exp(1/2\chi)$. As the configurational entropy is given by $\ln(g_1)$, this maximum density of configurations per dimension gives the entropy S_2 for the radiation in the sphere as being

$$S_2 = \frac{k_B}{(2\chi)^2} A \tag{10}$$

This maximum entropy bound would be expected to be displayed by the sphere S^2 for $r>>\chi$ because it is at the upper limiting temperature of $T_2=\infty$. It can be noted that simply setting the compactification scale χ to the Planck length $l_p=\sqrt{(\hbar G/c^3)}$ in (10) gives the entropy expression for a black hole derived by considering Hawking radiation [29]

$$S_2 = \frac{k_B}{4\chi^2} A = \frac{k_B c^3}{4\hbar G} A$$

The significant difference is that (10) was derived without reference to quantum theory, but on the basis of metric-wave radiation within a sphere S^D exerting an outward pressure. From the perspective within S^2, the thermodynamic equation (8) gives the outward heat flow from the surface of the sphere

$$dE_2 + P_2 dV_2 = T_2 dS_2 = dq$$

For a closed S^D cosmology in which there is no space exterior into which radiation can be excluded, the corresponding thermodynamic equation will be

$$dE_D + P_D dV_D = (\Lambda_p/\kappa) V_D dP_D$$

The significance of this for STUFT is that after the transition to the torus $T^{3+7}=S^3 \times S^7$ there will be an outward radiation pressure $P=dp_7/dr$ from S^7 and an outward pressure $p=dp_3/dr$ from S^3. In the torus T^{3+7} the two spaces are "exterior" spaces to each other, and so if $P>p$ radiation will be driven out of the particle dimensions S^7 into the spatial S^3. This situation will be realised for the scenario of the initial transition occurring during the contraction of S^{10}, which would be expected to initially continue into the $S^3 \times S^7$ phase. The radiation pressure p_D in an S^D cosmology without a cosmological term $\Lambda(a)$ scales as $p_D(a) \propto 1/a^{D+1}$ for radial scale factor $a(t)$, whereas with a $\Lambda(a)$ term the scaling can be denoted as $p_D(a) \propto 1/a^{D+\varepsilon}$, where $0<\varepsilon<1$. The radiation pressure gradients will then be (for some $0<\delta<1$)

$$P = \frac{\partial p_7}{\partial \chi} \propto \frac{1}{\chi^{8+\delta}} \qquad\qquad p = \frac{\partial p_3}{\partial a} \propto \frac{1}{a^{4+\varepsilon}}$$

and so the radiation pressure from S^7 to S^3 will increase faster than in the reverse direction. This pressure difference is free to drive metric-wave radiation from the S^7 particle dimensions of the torus and into the outer S^3 spatial cosmology. The earlier conclusions then imply that the radiative resistance to further contraction of S^7 would be reduced, and the radiation pushed into the spatial S^3 would increase the outward radiative pressure term against the S^3 surface and resist further contraction. As the scale factor $\chi(t)$ continued to decrease, more radiation would be pushed from S^7 into S^3, where the field-equations (3) for a cosmological term of the form $\Lambda(a)=\Lambda_p p(a)$ imply that this will not only halt further contraction of S^3, but reverse it into an expansion. As the scale factor $a(t)$ increases and that of $\chi(t)$ decreases, the quantity of radiation transferring from S^7 to S^3 will decrease until it is insignificant in comparison to the radiation already within S^3, and the S^3 cosmology exits this inflationary epoch.

The thermodynamic analysis given for S^2 shows that this conclusion essentially follows from the physicality assumption for the "fabric of space", where the surface of space acts like any other closed physical surface with respect to outward pressures. This leads to the conclusion that the cosmological "constant" is a cosmological radiation pressure term $\Lambda(a)=\Lambda_p p(a)$, where the currently low radiation pressure of the cosmic microwave background at T_3=2.725K [30] would give a low current value for $\Lambda(a)$. If the spatial S^3 is assumed to be approximately described by the Friedman-Robertson-Walker metric, where the dynamics are dominated by the relativistic content of the universe for the majority of its history, then the small $a(t)$ result of $a^2 \propto t$ could be assumed to be a rough approximation up to the current age of the universe t_U. Assuming that the radiation scaling is approximately given by $p(a) \propto a^{-4}$ leads to the approximation $\Lambda(a) \propto a^{-4} \propto t^{-2}$, where $\Lambda \propto t_U^{-2}$ in Planck units approximately gives the low current value for the cosmological term of $\Lambda=1.7\times10^{-121}$ [31-33]. This would seem to provide additional confirmation that the cosmological term is a cosmological pressure term, where it should be noted that this line of reasoning is for a closed S^D cosmology.

As this compactification-inflation see-saw is driven by the relative radiation pressures, the extent of the increase in the spatial scale factor $a(t)$ due to the see-saw alone will be given by a constant radiation pressure condition for metric-wave radiation within the torus T^{3+7}. For the generic scaling form of the radiation pressure within S^D with a cosmological term $\Lambda(a)$, the constant pressure condition will be of the form $a^{\varepsilon+1}\chi^{\delta+1}=const.$ where the values $0<\varepsilon<1$ and $0<\delta<1$ depend upon the exact form of the full metric and the constants κ, Λ_p. The absolute physical scale of the radius a in any S^D cosmology is undefined, but the scale of a can be meaningfully defined relative to χ such that a/χ is a physically defined quantity. Given that the two scales would initially be given by $a=2\chi$ for a simple transition of a sphere to a torus, the estimated increase in this measurable quantity will be given by:

$$\frac{a}{\chi} = 2\chi^{-(10+\varepsilon+\delta)/(3+\varepsilon)} \tag{11}$$

Assuming that the scale of χ is now given by the Planck length l_p the constant area condition of $\varepsilon=0$, $\delta=0$ gives an inflationary factor of $a/\chi = 2(l_p)^{-10/3} = 1.87\times10^{116}$, whereas the constant volume condition of $\varepsilon=0$, $\delta=1$ gives an inflationary factor of $a/\chi = 2(l_p)^{-11/3} = 7.41\times10^{127}$. It should be noted that in all cases l_p^{-1} of the increase will be due to the unit χ being used to measure physical distances decreasing.

4. Kaluza-Klein Dimensional Reduction

The compactification of the previous section means that the scale χ of the particle dimensions will not be fixed in an absolute sense. If the torus $T^{3+7}=S^3\times S^7$ could be viewed in the 11 spatial dimensions in which it notionally resides, then the scale factor $\chi(t)$ of the compactified dimensions would decrease as the spatial scale factor $a(t)$ increased. The scale $\chi(t)$ would only be stationary at the moment when the S^3 spatial cosmology reached its maximum radius, and $\chi(t)$ reached its minimum. After this, the scale factor $a(t)$ of S^3 would decrease due to the gravitational attraction of the content of the universe, and this decrease in S^3 would start to increase the radiation pressure. Radiation in the torus $S^3\times S^7$ is free to transfer between the S^3 and S^7 depending upon the relative radiation pressures, and so the increasing radiation pressure of a contracting S^3 is free to transfer to S^7, such that the pressure forces the S^7 particle dimensions to reflate and the see-saw runs in reverse. In this way, the scale χ will be forced to remain non-zero at all times, which is the physical condition required for a Kaluza-Klein theory.

All physical scales in the metric-field theory are relative to χ because it is also the radius of the spatial S^2 wrapped around the compactified S^7 in the monopoles. In normal electrodynamics, changing the radius r of a sphere bearing charge q on its surface would not change the electric far-field. So if two such electrically charges objects formed a stable configuration where they were separated by some distance d, such as in some physical material, the distance d could be used to measure the change in the radius r. This will not be possible for charged topological monopoles of radius χ, as a change in χ will also change the electric far-fields in the Kaluza-Klein theory, because the gauge fields are in terms of the compactified particle dimensions with radius χ. So when the scale χ changes, all physical means of measuring length scales will change with it, such that χ remains immeasurable. This only leaves the arbitrary definition of a length scale, and this is the form of the meter in the S.I. unit system, which will be given by $1\text{m}=M\chi$ in this theory, where the value chosen for M was fundamentally arbitrary.

Although the scale χ will be continually changing in an absolute sense, as all physical scales are defined relative to χ, the physical scale χ will be constant in terms of physical measurement. So the pre-condition of a Kaluza-Klein theory that there-exist compactified dimensions of some fixed size is met in physically measurable terms. The dimensional reduction procedures for Kaluza-Klein theories can now be applied to the torus $T^{3+7}=S^3\times S^7$, where those of this section come from the Kaluza-Klein review [4]. The space-time coordinates of the space S^3 will be denoted x^μ and those of the compactified S^7 denoted y^m. The non-Abelian gauge fields of the theory are given by adopting the anastz for the metric:

$$g_{XY}=\begin{pmatrix} g_{\mu\nu}(x)-\Phi_{mn}(y)B_\mu^m B_\nu^n & B_\mu^n \\ B_\nu^m & -\Phi_{mn}(y) \end{pmatrix} \tag{12}$$

where

$$B_\mu^n=\xi_a^n(y)A_\mu^a(x)$$

and $g_{\mu\nu}$ is the metric of space-time and Φ_{mn} is the metric of the S^7 particle dimensions. Transformations of the y^m coordinates that have an x-dependence of the form

$$y^m \rightarrow y^m + \xi_a{}^m(y)\,\varepsilon^a(x)$$

induce the usual non-Abelian transformation for a gauge field

$$A_\mu^a \rightarrow A_\mu^a+\partial_\mu\varepsilon^a(x)+C_{abc}\varepsilon^b(x)A_\mu^c$$

The structure constants C_{abc} can be related to the structure constants f_{abc} of a non-Abelian group through the introduction of a gauge coupling constant g

$$C_{abc}=g f_{abc} \qquad \text{and} \qquad t_a=gT_a \qquad \text{so that} \qquad [T_a, T_b]=if_{abc}T_c$$

The action for Einstein gravity in 10+1 dimensions in given by

$$\overline{S}=-\frac{1}{2\kappa}\int d^{11}\overline{x}\left|\det\overline{g}\right|^{1/2}\overline{R} \tag{13}$$

where the cosmological term and stress-energy tensor have been left out for simplicity. Substituting the ansatz (12) into this action and integrating over the y variables gives the effective 3+1 dimensional action for general relativity and non-Abelian gauge fields of the standard form

$$S = -\frac{1}{16\pi G}\int d^4x\left|\det g_{\mu\nu}\right|^{1/2} R - \frac{1}{4}\int d^4x\left|\det g_{\mu\nu}\right|^{1/2} F^a_{\mu\nu}F^{\mu\nu}_a \tag{14}$$

where the standard normalisation of the gauge fields in Kaluza-Klein theories has been used [4].

The group quotient SU(4)/SU(3) gives the differential manifold S^7 of the compactified dimensions, and so the symmetry group in this action is SU(4), with SU(3) co-sets. This symmetry is broken by the non-trivial vacuum winding of homotopy group $\pi_7(S^3) = \pi_4(S^3) = Z_2$, where the parametrisation of the sphere S^4, $x_4 = x_\xi \cos\xi$, $x_3 = x_\xi \sin\xi$, gives the S^3 group space of SU(2) which is then mapped to the spatial sphere S^3. In terms of the gauge fields of (14), this mapping corresponds to an instanton-like configuration [34] for which the gauge field is pure gauge, $A_\mu=U^{-1}\partial_\mu U$ for $U\in$SU(2)

$$U = i\hat{\boldsymbol{x}}\cdot\boldsymbol{\sigma} \tag{15}$$

where the spatial variables $\boldsymbol{x}$ in the surface S^3 have been normalised to unit vectors and there is no explicit dependence upon a 4$^{\text{th}}$ Euclideanised time variable. However, this configuration has an implicit time dependence where it is pure gauge at all times in the $S^3\times S^7$ phase, but doesn't exist in the S^{10} phase before and after the $S^3\times S^7$ phase of the current universe. This pure gauge configuration has no field strength $\boldsymbol{F}_{\mu\nu}$, but breaks the symmetry of the gauge fields in (14) to (Spin(3)⊗SU(2)⊗U(1))/Z_3 as in section 2. This gives the Lagrangian for the gauge fields with this gauge field background as being

$$L_G = -\frac{1}{4}G^a_{\mu\nu}G^{\mu\nu}_a - \frac{1}{4}W^a_{\mu\nu}W^{\mu\nu}_a - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \tag{16}$$

where $G_{\mu\nu}$ is the SO(3) colour field strength, $W_{\mu\nu}$ is the SU(2) isospin gauge fields and $B_{\mu\nu}$ is the U(1) hypercharge gauge field.

The action for the scalar sector of the dimensionally reduced theory is derived from the ansatz

$$g = \begin{pmatrix} g_{\mu\nu}(x) & 0 \\ 0 & -\Phi_{mn}(x,y) \end{pmatrix} \tag{17}$$

which differs from (12) by including x dependence in the metric Φ_{mn}. Substituting this into (13) gives the action [4,35]

$$\begin{aligned} S = -\frac{1}{16\pi G}\int d^4x\int d^7y\left|\det g_{\mu\nu}\right|^{1/2}\left|\det\Phi_{mn}\right|^{1/2}\{R+\widetilde{R} \\ -\Phi^{mn}D_\mu D^\mu\Phi_{mn} - \tfrac{1}{2}D^\mu\Phi_{mn}D_\mu\Phi^{mn} \\ -\tfrac{1}{4}\Phi^{mn}D^\mu\Phi_{mn}\Phi^{pq}D_\mu\Phi_{pq} + \tfrac{1}{4}\Phi^{mn}\Phi^{pq}D_\mu\Phi_{pm}D^\mu\Phi_{qn}\} \end{aligned} \tag{18}$$

It is only the electroweak S^4 base-space of the compactified S^7 that possesses the spatial x dependence through the non-trivial vacuum winding, and so the indices of the metric $\Phi_{mn}(x, y)$ in (18) will only run over 1-4. Furthermore, the parametrisation $x_4 = x_\xi \cos\xi$, $x_3 = x_\xi \sin\xi$ of S^4 gives S^3, and so the non-zero elements of the metric for the vacuum map S^4 to S^3 can be denoted as the scalar terms

$$\phi_1 = \Phi_{11},\quad \phi_2 = \Phi_{22},\quad \phi_3 = \Phi_{33},\quad \phi_4 = \sqrt{2}\Phi_{34} = \sqrt{2}\Phi_{43} \tag{19}$$

Substituting this form for the metric into (18) gives a quadratic term for m=1-4

$$L_2 = \phi_m D_\mu D^\mu \phi_m + \tfrac{1}{2} D^\mu \phi_m D_\mu \phi_m$$

and a quartic term

$$\begin{aligned} L_4 = &\tfrac{1}{2}\left(\phi_1 D^\mu \phi_1\right)\left(\phi_2 D_\mu \phi_2\right) + \tfrac{1}{2}\left(\phi_1 D^\mu \phi_1\right)\left(\phi_3 D_\mu \phi_3\right) + \tfrac{1}{2}\left(\phi_2 D^\mu \phi_2\right)\left(\phi_3 D_\mu \phi_3\right) \\ &- \tfrac{1}{4}\left(\phi_4 D^\mu \phi_4\right)\left(\phi_3 D^\mu \phi_3\right) + \tfrac{1}{8}\left(\phi_4 D^\mu \phi_4\right)\left(\phi_4 D^\mu \phi_4\right) \end{aligned} \tag{20}$$

Combining the scalar terms into the representation of SU(2)⊗U(1)

$$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} \tag{21}$$

allows the quadratic term in the action to be rewritten as the local Lagrangian term

$$L_S = (D_\mu \Phi)^\dagger (D^\mu \Phi) \tag{22}$$

for the closed S^3 space containing no net topological charge. For the background vacuum configuration with (SU(2), U(1)) eigenvalues (–½,1) the form of the scalar field derivative will be

$$D_\mu \Phi = \left(\partial_\mu + \tfrac{i}{2} g \sigma^a W^a_\mu + \tfrac{i}{2} g' B_\mu\right)\Phi$$

The definition of the eigenvalues for the unbroken U(1) charge symmetry given in (6) and that of the Weinberg angle in (4) gives the basis for the field definitions of the Standard Model

$$A_\mu = \sin\theta_W W^3_\mu + \cos\theta_W B_\mu \qquad Z_\mu = \cos\theta_W W^3_\mu - \sin\theta_W B_\mu \tag{23}$$

and the isospin g and hypercharge g' coupling constants in terms of the electric coupling constant e

$$g\sin\theta_W = g'\cos\theta_W = e \tag{24}$$

In terms of the scalar field Φ denoting the 4 non-zero terms of the metric Φ_{mn}, the non-trivial winding of the electroweak vacuum corresponds to a spatial variation in Φ such that the bottom term of the SU(2) doublet always points in the same direction in going around the spatial S^3. This gives the local form ϕ_1=ϕ_2=0 and ϕ_3=ϕ_4=ϕ at all spatial locations $\boldsymbol{x}$ in S^3 at all times in the $S^3 \times S^7$ phase. Substituting ϕ=η into the local Lagrangian term (22) gives mass terms for the W and Z gauge fields

$$m_W^2 = \left(\frac{g}{2}\right)^2 \eta^2 \qquad m_Z^2 = \left(\frac{g}{2\cos\theta_W}\right)^2 \eta^2 \tag{25}$$

The spatial variation in Φ going around S^3 is accompanied by the pure gauge variation arising from (15), which in the local Lagrangian term (22) for ϕ_3=ϕ_4=ϕ will give $D_\mu\phi$ = ½ϕ and a mass term in the perturbative expansion ϕ=η+$\delta\phi$

$$D_\mu \Phi D^\mu \Phi \rightarrow \frac{1}{2}\left(\frac{\eta^2}{4}\right)\delta\phi^2 = \frac{1}{2} m_H^2 \delta\phi^2 \tag{26}$$

for which the scalar field mass is half the electroweak scale η. This gives an explicit demonstration that the dimensionally reduced theory gives the mass terms for the vector and scalar fields as claimed in [10]. In addition to the quadratic Lagrangian term (22), the quartic term (20) also gives a term in the local Lagrangian with $\phi_1=\phi_2=0$ and $\phi_3=\phi_4=\phi$ and the pure gauge configuration from (15)

$$L_4 = -\frac{1}{8}\left(\phi D^\mu \phi\right)\left(\phi D^\mu \phi\right) \to -\frac{1}{8}\left(\frac{\phi^2}{2}\right)^2 \tag{27}$$

This is the form of the quartic term of the Higgs potential in the Standard Model, with scalar coupling constant $\lambda=1/8$, as claimed would be the case in [10].

5. Compactified Black Holes

In the dimensionally reduced theory, the topological monopoles of section 2 will be rotating objects bearing electric or magnetic charges. The surface topology of the monopoles is given by $S^2\times S^7$, where the spatial configuration is unwound within the compactified surface, such that the interior of the monopole is devoid of space. Such monopole "solitons" with no spatial singularity are feature of some Kaluza-Klein theories [18,19], but they do not always reduce to the black holes of General Relativity. In STUFT, the condition that the S^2 surface encloses a hole in space demands the existence of a spatial surface for the space-time metric of the topological monopoles in the dimensionally reduced theory. For the neutrino-like monopoles of Table 1 with no electric charge and a topological spin charge, this spatial surface condition in the dimensionally reduced theory can only be met by a black hole with a physical surface at the event horizon. In the General Relativity portion of the dimensionally reduced theory, this gives the condition that the Kerr metric for this neutral compactified black hole monopole possesses a real value for the radius of the event horizon. So consider the Kerr metric for a rotating black hole with mass m and angular momentum j in natural units G=1, c=1

$$ds^2 = -\frac{\Delta - a^2\sin^2\theta}{R^2}dt^2 - 2a\frac{2mr\sin^2\theta}{R^2}dtd\phi + \frac{\left(r^2+a^2\right)^2 - a^2\Delta\sin^2\theta}{R^2}\sin^2\theta d\phi^2 + \frac{R^2}{\Delta}dr^2 + R^2 d\theta^2$$

where

$$a \equiv \frac{j}{m} \qquad \Delta \equiv r^2 - 2mr + a^2 \qquad R^2 \equiv r^2 + a^2\cos^2\theta$$

The horizon for the rotating black hole occurs at $g_{rr}=\infty$, i.e. $\Delta=0$, which in physical units is given by:

$$\Delta = r^2 - \frac{2Gm}{c^2}r + \frac{j^2}{m^2c^2} = 0 \qquad r = \frac{Gm}{c^2} \pm \sqrt{\frac{G^2m^2}{c^4} - \frac{j^2}{m^2c^2}}$$

The condition that the monopole has no singularity and an S^2 surface requires the event horizon of the black hole to have a real-valued radius, which will only be true for angular momentum given by

$$j \le \frac{Gm^2}{c}$$

A topological monopole with radius given by the compactification scale χ could be expected to satisfy this bound, for which the angular momentum is

$$j_\chi = \frac{c^3}{G}\chi^2 \tag{28}$$

and the event horizon radius is

$$\chi = \frac{Gm_\chi}{c^2} \tag{29}$$

The compactification scale χ is the radius of the S^7 particle dimensions and so is the smallest physical scale in the space, which implies that this j_χ is the smallest value for angular momentum in the space.

To demonstrate the effect of this limit, consider a test particle of mass m travelling along the surface of a compactified tube of radius χ with linear momentum $p_z = mv_z$ parallel to the compactified tube, and angular momentum $j_\chi = m\chi v_\phi$ around the tube. As the mass m is unspecified, it will be replaced by the angular momentum j_χ, and the angular velocity $v_\phi = \chi\omega$ then used to eliminate χ

$$p_z = \frac{j_\chi v_z}{\chi\, v_\phi} = \frac{v_z}{v_\phi^2} j_\chi \omega$$

Considering the limit of $v_z = v_\phi = c$ for a circular wave mode around the compactified tube gives

$$E = p_z c = j_\chi \omega \tag{30}$$

From this we have the obvious identification $\hbar = j_\chi$, with χ being the Planck length l_p and m_χ being the Planck mass m_p

$$\hbar = j_\chi = m_p l_p c = \frac{c^3\chi^2}{G} \qquad \chi = l_p = \frac{Gm_p}{c^2} = \sqrt{\frac{\hbar G}{c^3}} \qquad m_\chi = m_p = \sqrt{\frac{\hbar c}{G}} \tag{31}$$

This identification $\hbar = j_\chi$ would appear to give the rotating black hole a rotation group eigenvalue of 1, but this would not be the measured angular momentum because a rotating black hole causes an angular rotation of a reference frame in the vicinity of the black hole given by (in dimensionless units)

$$\omega(r,\theta) = \frac{d\phi}{dt} = \frac{g^{t\phi}}{g^{tt}} = \frac{2Jr}{\left(r^2+a^2\right)^2 - a^2\Delta\sin^2\theta}$$

At the surface of the event horizon, this frame-dragging is given by

$$\omega(\chi) = c/(2\chi) \qquad \text{or} \qquad v = \omega\chi = \tfrac{1}{2}c$$

Whereas the angular momentum bound (28) corresponds to $j_\chi = m_\chi \chi c$, this frame-dragging will give the measured angular momentum of the horizon as $j = \tfrac{1}{2} m_\chi \chi c$. The moment of inertia for a mass shell with mass m and radius r is $I_m = \tfrac{1}{2}mr$, giving the angular momentum for rotational velocity v of $j = \tfrac{1}{2}mrv$. So the measured angular momentum $j = \tfrac{1}{2}m_\chi\chi c$ of the event horizon would appear to be due to a mass shell m of radius χ rotating at the maximum velocity of c, giving a measured rotation group eigenvalue of ½.

This gives Planck's constant as being geometrical in origin, due to the physical compactification scale χ inside which the Poincaré group of relativity does not apply. The enclosing surfaces of such spatial regions must be describable in terms of representations of the rotation group with eigenvalues of 1 or ½, and where the physical scale of the angular momentum is given by $\hbar$. So for any representation

Ψ of the rotation group the action of the angular momentum operator is given by

$$J_z\Psi = n\hbar\Psi \qquad \text{or} \qquad J_z\Psi = \tfrac{1}{2}\, n\hbar\Psi$$

and this adds the scale $\hbar$ to the Lie Algebra of the rotation group

$$[J_i, J_j] = i\hbar\ \varepsilon_{ijk}J_k \tag{32}$$

This scale propagates throughout the Lie Algebra of the Poincaré group, and adds $\hbar$ to the commutator relations of the Hamiltonian formulation of classical physics through the infinitesimal generators

$$[x_\mu, P_\nu] = [x_\mu, i\hbar\partial_\nu] = i\hbar\delta_{\mu\nu} \tag{33}$$

As the Poincaré invariants are mass and spin, the physical scale factor is given by $\hbar$, and the physical length scale of χ only appears in its invariant form χ^2, as in the definition of $\hbar$ given in (31).

The scale $\hbar$ gives the physical scale factor between the compactified particle dimensions of radius χ and the unit sphere S^3 of the rotation group applying to the S^2 surface of the topological monopoles with unbroken U(1) charge symmetry. The physical scale of the circle S^1 giving the underlying group space of the unbroken U(1) symmetry group is also given by the compactification scale χ, where the scale factor q for U(1) is related to $\hbar$ of the SU(2) rotation group by the area ratio between S^1 and S^3

$$q = \frac{A_1}{A_3}\hbar = \frac{2\pi(r=1)}{2\pi^2(r=1)^3}\hbar = \frac{\hbar}{\pi}$$

Adding in the factor of c for an electric charge e with a non-zero gauge field A_0 term, the physical scale of the Dirac quantisation condition is given by

$$eq_m = \tfrac{1}{2}n\hbar c$$

and the unit circle S^1 of the U(1) group is given by group elements of the form

$$\lambda = \exp\left(i\frac{e}{\hbar c}\phi\right)$$

where the physical scale of the electric charge e is given by

$$\frac{e}{\hbar c} = \frac{1}{\pi} \approx 0.3183 \tag{34}$$

With the value of the Weinberg angle given by (4) the values of the isopin and hypercharge coupling constants g and g' given by (24) will be

$$\frac{g}{\hbar c} = \frac{e}{\hbar c\sin\theta_W} = \frac{1}{\pi}\sqrt{\frac{21}{5}} \approx 0.6523 \tag{35}$$

$$\frac{g'}{\hbar c} = \frac{e}{\hbar c\cos\theta_W} = \frac{1}{\pi}\sqrt{\frac{21}{16}} \approx 0.3647 \tag{36}$$

which compare with the Standard Model values of 0.652 and 0.357 respectively [36]. The SO(3) colour coupling constant g_c has an S^3 group space, and so has an area ratio factor of 1

$$\frac{g_c}{\hbar c} = 1 \tag{37}$$

The scalar field terms of the dimensionally reduced theory in section 4 give expressions for the masses of the W and Z fields (25), which can now be evaluated using the closed formulae for the Weinberg angle and the value η for the electroweak vacuum [10]

$$m_W^2 = \left(\frac{g}{2\hbar c}\right)^2 \eta^2 = \frac{21}{20}\left(\frac{e}{\hbar c}\right)^2 \eta^2 = \frac{21}{20}\frac{\eta^2}{\pi^2} \qquad m_Z^2 = \left(\frac{g}{2\hbar c}\right)^2 \frac{\eta^2}{\cos^2\theta_W} = \frac{21}{20}\frac{21}{16}\left(\frac{e}{\hbar c}\right)^2 \eta^2 = \frac{441}{320}\frac{\eta^2}{\pi^2}$$

These are not particle masses, but are instead just mass terms for classical physics waves, i.e. the W and Z waves of the dimensionally reduced theory take the form of evanescent waves. These mass terms for the W and Z fields will limit their range in the classical field theory, such that they will not give the classical confinement of charges discussed in section 2. This is in contrast to the massless colour fields with Spin(3) symmetry, which by the topological arguments of section 2 will display classical field theory configurations of coloured monopoles with a colour flux tube between them.

6. Classical Monopole Theory

The derivations of the previous section give a geometric origin for Planck's constant $\hbar$ in terms of being the physical scale factor for all the topological charges: spin, electric charge, isospin and colour. As the topological spin charge, Planck's constant enters into the Lie Algebra of the Poincaré group and then the Hamiltonian formulation of mechanics (33). Despite this being a feature of quantum theory, this is still strictly a classical physics theory.

The spin invariant $\hbar$ also applies to the gauge fields of the dimensionally reduced metric, where it is included in the wave uncertainty relations $\Delta f \Delta t \geq ½$ and $\Delta\lambda\Delta x \geq ½$ of classical physics to give uncertainty relations $\Delta E \Delta t \geq \hbar/2$ and $\Delta p \Delta x \geq \hbar/2$ for wave radiation in the dimensionally reduced theory. The radius χ of the particle dimensions defines the physical scale of the natural unit system (m_p, l_p, t_p) in (31), where mass, spin $\hbar$ and the speed of light c are the invariants of the dimensionally reduced space. Since the intrinsic error of measurement for a measuring scale is ±½ the units used, the error of distance and time measurement will be $\pm ½ l_p$ and $\pm ½ t_p$, which limits the accuracy of measurement for the canonical variables of Hamiltonian mechanics

$$(\Delta x \geq \tfrac{1}{2} l_p)\left(\Delta p \geq m_p \frac{\frac{1}{2} l_p}{\frac{1}{2} t_p}\right) = \tfrac{1}{2} m_p \frac{l_p^2}{t_p} \geq \frac{\hbar}{2} \qquad (\Delta t \geq \tfrac{1}{2} t_p)\left(\Delta E \geq m_p \left(\frac{\frac{1}{2} l_p}{\frac{1}{2} t_p}\right)^2\right) = \tfrac{1}{2} m_p \frac{l_p^2}{t_p} \geq \frac{\hbar}{2} \tag{38}$$

by (31). As the topological spin charge $\hbar$ is an invariant of the space, changing the unit system will not change this intrinsic measurement limit of $\hbar/2$ for the product of the measurement errors. So the Heisenberg uncertainty relations will apply to measurements of both wave radiation and the topological defects, because of the compactification of the particle dimensions [10].

Many of the major features of the Standard Model have now been derived at the classical physics level, but notably not the following:

1) The topological monopoles are not deduced to display wave motion
2) There are no continuous matter fields, as the topological monopoles are discrete objects
3) There is no quantisation of the metric field, or gauge fields, to give particles

It is these features which mark the transition to quantum field theory, where [10] showed that the form of quantum theory can be derived in terms of a change in descriptive framework due to difficulties in the classical physics theory. The physical cause for these difficulties in the classical monopole theory is that the black hole monopole of the Planck mass in section 5 has an ergo-region which traps virtual-radiation capable of totally cancelling the Planck mass. The ergo-region of a black hole is defined by the boundary where $g_{tt} = 0$, which for a rotating black hole is outside of the event horizon with a radius given in dimensionless units by

$$r_+ = m^2 + \sqrt{(m^2 - a^2 \cos^2 \theta)}$$

which for the limit $r_h = \chi$ of the compactified black hole monopole becomes

$$r_+ = \chi(1 + \sin\theta) \tag{39}$$

Inside the ergo-region $\chi < r < r_+(\theta)$ the sign of g_{tt} is reversed, and so consequently any radiation within the ergo-region will have the mass invariant $m^2 < 0$ of virtual-radiation. Propagating radiation is trapped in orbit by the sign reversal of the metric term g_{tt}, which can conceptually be viewed in terms of total internal reflection at the ergo-region boundary. Such a view gives a physical basis for the classical physics wave equations possessing a non-propagating radiation tail beyond the ergo-region boundary, as for evanescent waves with total internal reflection in optics.

In the dimensionally reduced theory, the energy of radiation is $E=\hbar\omega$, which for ω being the inverse of the Planck time $t_p=l_p/c$ gives $E = m_p c^2$ by (31), implying that radiation trapped within the ergo-region could cancel the Planck mass of the bare topological monopole by the Penrose process [37], leaving the rest mass of the monopole largely determined by the charge fields [10]. In which case, the coloured monopoles would be expected to be more massive than the non-coloured monopoles, $m_{c=1}>m_{c=0}$, and the masses greater for monopoles with greater electric charge, $m_{q=2/3}>m_{q=1/3}$ and $m_{q=1}>m_{q=0}$. It should be noted that although the neutrino-like monopoles have no electric far-field because the W and Z fields cancel, as $m_Z > m_W$ there is a small region where the fields haven't cancelled yet, and so by this simple argument the neutrinos would be expected to have a non-zero mass $m_\nu > 0$. A similar argument can be made for increasing mass to be expected for increasing SU(3) co-set, and the 12 fundamental fermions fit this mass hierarchy except for the down quark mass being greater than the up quark mass.

Whereas the macroscopic black holes of section 3 decrease in mass through the emission of radiation, at the limit of the compactification radius χ the topological monopole has a spin charge of $\hbar/2$ which cannot be radiated away. The mass of this frustrated black hole is instead decreased through a Penrose process [37] of virtual radiation being trapped in the ergo-region, where the task in hand is to calculate what the Planck mass of the bare black hole is reduced to when the surface bears the charges of Table 1. The issue with this calculation is revealed by considering a monopole and anti-monopole collision, where the topological charges of the pair cancel and the resulting black hole configuration is free to radiate all its mass away. Applying time reversal to this process gives radiation of sufficient energy creating a monopole and anti-monopole pair, which was the second physicality condition given at the beginning. For the black hole monopole, the energy of virtual radiation in the ergo-region is more than sufficient for this process, being of the Planck energy scale $E=m_p c^2$.

Now consider calculating the energy of the field configuration around this black hole monopole using the Hamiltonian given by the dimensionally reduced Lagrangian of section 4. As the gauge fields and scalar fields propagate as waves in the classical theory, a series expansion approach can be adopted for the fields of the propagating radiation trapped in the ergo-region, the non-propagating radiation tail, and the charge field for the charges residing on the surface of the monopole black hole. However, the energy of the wave terms in this series expansion can be large enough to create a monopole/anti-monopole pair for any of the 12 monopoles of Table 1. This process must also be included in the series expansion because a monopole/anti-monopole pair has a dipole moment which will effect the charge field of the bare monopole. So the series expansion must also include this vacuum polarisation effect of virtual-radiation creating monopole/anti-monopole pairs, where these monopoles also possess the $m^2 < 0$ mass invariant of the ergo-region, and so constitute virtual-matter.

The series expansion for the dimensionally reduced Hamiltonian will be denoted as being of the form $H(\boldsymbol{n}, \boldsymbol{A}, \boldsymbol{\Phi})$, where $\boldsymbol{n}$ denotes the monopoles of each of the 12 types, $\boldsymbol{A}$ denotes the gauge fields and $\boldsymbol{\Phi}$ denotes the scalar fields. The field energy of the monopole in the dimensionally reduced theory will be given by integrating the Hamiltonian over the spatial volume surrounding the monopole, and so the theory $H(\boldsymbol{n}, \boldsymbol{A}, \boldsymbol{\Phi})$ will have no explicit dependence on space-time coordinates. Each monopole term $n_i(\boldsymbol{x}, t)$ denotes the space-time coordinates of a monopole of type i, where the motion of the monopole is given by the geodesic transport of the covariant derivative D_μ in the dimensionally reduced theory in section 4. In generic terms, the form of the Hamiltonian of the dimensionally reduced theory will be

$$\begin{aligned} H = \int d^3x \left|\det g_{uv}\right|^{1/2} \{ & a(\boldsymbol{A},\boldsymbol{A}) + b(\boldsymbol{A},\boldsymbol{A},\boldsymbol{A}) + c(\boldsymbol{A},\boldsymbol{A},\boldsymbol{A},\boldsymbol{A}) \\ & + d(\boldsymbol{\Phi},\boldsymbol{\Phi}) + e(\boldsymbol{A},\boldsymbol{A},\boldsymbol{\Phi}) + f(\boldsymbol{A},\boldsymbol{A},\boldsymbol{\Phi},\boldsymbol{\Phi}) \\ & + s(\boldsymbol{\Phi},\boldsymbol{\Phi},\boldsymbol{\Phi}) + t(\boldsymbol{\Phi},\boldsymbol{\Phi},\boldsymbol{\Phi},\boldsymbol{\Phi}) + u(\boldsymbol{A},\boldsymbol{A},\boldsymbol{\Phi},\boldsymbol{\Phi},\boldsymbol{\Phi}) + v(\boldsymbol{A},\boldsymbol{A},\boldsymbol{\Phi},\boldsymbol{\Phi},\boldsymbol{\Phi},\boldsymbol{\Phi}) \\ & + \alpha(\boldsymbol{n},\boldsymbol{n}) + \beta(\boldsymbol{n},\boldsymbol{n},\boldsymbol{A}) + \gamma(\boldsymbol{n},\boldsymbol{n},\boldsymbol{\Phi}) + \sigma(\boldsymbol{n},\boldsymbol{n},\boldsymbol{A},\boldsymbol{A}) + \tau(\boldsymbol{n},\boldsymbol{n},\boldsymbol{\Phi},\boldsymbol{\Phi}) \} \end{aligned} \tag{40}$$

The terms a, b, c come from the gauge field strengths of the gauge field Lagrangian (16), the terms d, e, f come from the quadratic scalar field Lagrangian term (22), and the terms s, t, u, v come from the quartic scalar field Lagrangian term (20). The terms α, β, γ would come from linear covariant derivative terms for the monopoles, and the terms σ and τ from quadratic covariant derivative terms. All that will be required is that at least one of the terms β, γ, σ and τ be present in the Hamiltonian, because these terms change the numbers of discrete monopoles through the creation of monopole/anti-monopole pairs from wave radiation, and their annihilation to radiation.

The wave expansion for the gauge fields $\boldsymbol{A}$ and scalar fields $\boldsymbol{\Phi}$ will be of the form

$$\boldsymbol{A}_\mu = \int_\omega d\omega \int_{\boldsymbol{k}} \boldsymbol{A}_\mu^{\boldsymbol{k}} e^{i(\omega t - \boldsymbol{k}\cdot\boldsymbol{x})} dk^3 \qquad\qquad \boldsymbol{\Phi} = \int_\omega d\omega \int_{\boldsymbol{k}} \boldsymbol{\Phi}_{\boldsymbol{k}} e^{i(\omega t - \boldsymbol{k}\cdot\boldsymbol{x})} dk^3 \tag{41}$$

where the integral over ω is required because $\omega \neq |\boldsymbol{k}|$ for virtual-radiation in the ergo-region of the Kerr metric. The ranges of the ω and $\boldsymbol{k}$ integrals are linked by the dispersion relation for virtual-radiation in the ergo-region, where $E = \hbar\omega \leq m_p c^2$ by (30), (31) and the bare monopole mass. When the Hamiltonian contains at least one of the terms β, γ, σ and τ, waves in the expansion (41) will create a monopole/anti-monopole pair for $E \geq 2m_i$ where m_i is the reduced mass of the type i monopole with bare mass m_p. This gives recursion in (40), where the calculation of the reduced mass m_i of the type i monopole from the bare mass m_p includes the term being calculated. So the wave expansions of (41) will give a recursive series expansion in terms of the monopoles/anti-monopoles created in the Hamiltonian.

The form of the terms in the classical physics of (40) defines 3 and 4 point reaction vertices between the gauge fields, the scalar fields and the discrete monopoles. However in the wave expansion of the terms β, γ, σ and τ, the created monopole/anti-monopole pairs must annihilate back to radiation because they carry conserved topological charges which cannot change the topological charge of the original monopole that the Hamiltonian expansion is about. This means that the terms β, γ, σ and τ will give closed loops over monopole/anti-monopole creation and annihilation, where the ω integral of (41) for the closed loop will be over the range of energies for the created pair. The same Hamiltonian expansion of (40) must then be recursively repeated for every monopole and anti-monopole created in the initial term H_0 of the expansion. This gives an infinite series expansion for the recursion

$$H = \sum_{n=0}^{\infty} \int^{\omega_E} d\omega_0 \int^{\omega_0} d\omega_1 \ldots \int^{\omega_{n-1}} d\omega_n H_0(\omega_0) H_1(\omega_1) \ldots H_n(\omega_n) \tag{42}$$

where ω_E is the upper limit given by the dispersion relation for virtual-radiation in the ergo-region. The sign reversal of g_{tt} in the ergo-region also impacts space-time separations such that causal events involving the ergo-region can have space-like separations, giving a distance equivalent to the mass invariant $m^2 < 0$ in the ergo-region. So when the integrals of (42) are switched from phase-space to real-space, the time integrals will be subject to a corresponding ergo-region constraint, and so be of the form

$$H = \sum_{n=0}^{\infty} \int^{t_E} dt_0 \int^{t_0} dt_1 \ldots \int^{t_{n-1}} dt_n H_0(t_0) H_1(t_1) \ldots H_n(t_n) \tag{43}$$

This recursive series expansion can be expressed in terms of the Hamiltonian density by including the spatial integrals of (40) to give

$$H = \sum_{n=0}^{\infty} \int^{t_E} d^4x_0 \left|\det g_{uv}\right|^{1/2} \int^{t_0} d^4x_1 \left|\det g_{uv}\right|^{1/2} \ldots \int^{t_{n-1}} d^4x_n \left|\det g_{uv}\right|^{1/2} \overline{\mathrm{H}}_0(\boldsymbol{x}_0)\, \overline{\mathrm{H}}_1(\boldsymbol{x}_1) \ldots \overline{\mathrm{H}}_n(\boldsymbol{x}_n) \tag{44}$$

Although this is of the same form as the path integral expansion in quantum field theory [38], here it is a recursive expansion for the mass reduction of the bare monopole mass by virtual-radiation trapped in the ergo-region of the compactified black hole that is a monopole. This recursive expansion for the terms of the theory $H(\boldsymbol{n}, \boldsymbol{A}, \boldsymbol{\Phi})$ will be expressed as

$$H(\boldsymbol{n}, \boldsymbol{A}, \boldsymbol{\Phi}) = \sum_{n=0}^{\infty} \sum_a T_n^a(\boldsymbol{n}, \boldsymbol{A}, \boldsymbol{\Phi}) \tag{45}$$

where the variable n gives the recursion depth, and the variable a runs over the combinations of reaction sequences possible at that recursive depth. Each term $T_n^a(\boldsymbol{n}, \boldsymbol{A}, \boldsymbol{\Phi})$ is of the form of a reaction sequence that starts with a single monopole m_i that emits virtual-radiation w_j, and then both m_i and w_j take part in a reaction sequence that ends with the initial monopole m_i again.

Since (44) has the same form as the path integral expansion of quantum field theory, the same visual aid of Feynman diagrams [38] can be used to graphically denote the reaction sequences, where the terms of a Feynmann diagram are related to the terms of the Hamiltonian (44) to give an expression for $T_n^a(\boldsymbol{n}, \boldsymbol{A}, \boldsymbol{\Phi})$. The vertex terms of the Hamiltonian are given by the terms $T_0^a(\boldsymbol{n}, \boldsymbol{A}, \boldsymbol{\Phi})$, where the following 4 generic monopole reactions are of primary relevance to changes in monopole numbers:

1) $m_i + V_n \rightarrow m'_i + w + V_n'$
2) $m_i + w_j + V_n \rightarrow m'_i + V_n'$
3) $w_j + V_n \rightarrow m_- + m_+ + V_{n+1}'$
4) $m_- + m_+ + V_{n+1} \rightarrow w_j + V_n'$

Every monopole reaction sequence will start with the first vertex term as it denotes the emission of virtual-radiation, and ends with the second vertex term denoting the absorption of virtual-radiation. The creation and annihilation of monopole/anti-monopoles pairs is given by vertex terms 3 and 4. The term V_n is a running accounting term for the construction of the terms $T_n^a(\boldsymbol{n}, \boldsymbol{A}, \boldsymbol{\Phi})$, imposing the condition that the monopole reaction sequence ends with the initial monopole only, and also imposes the energy conditions of the integrals in the wave expansion (42).

The terms of $T_1^a(\boldsymbol{n}, \boldsymbol{A}, \boldsymbol{\Phi})$ are generated from combinations of $T_0^a(\boldsymbol{n}, \boldsymbol{A}, \boldsymbol{\Phi})$, and this pattern is repeated at each recursive level so that the terms $T_n^a(\boldsymbol{n}, \boldsymbol{A}, \boldsymbol{\Phi})$ are generated from combinations of the terms $T_0^b(\boldsymbol{n}, \boldsymbol{A}, \boldsymbol{\Phi})$, …, $T_{n-1}^c(\boldsymbol{n}, \boldsymbol{A}, \boldsymbol{\Phi})$. In this process, the accounting term V_n starts at zero and records the numbers of monopoles produced by the reaction vertices, such that balancing reaction vertices are added to return the term V_n to zero when the reaction sequence ends with the initial monopole. In this way, each successive term $T_n^a(\boldsymbol{n}, \boldsymbol{A}, \boldsymbol{\Phi})$ in the series expansion is an ever larger reaction network involving more reaction vertices, where (45) is an infinite series.

Despite the complexity of the space-time integrals over the gauge fields $\boldsymbol{A}$ and scalar fields $\boldsymbol{\Phi}$, the difficulties of $H(\boldsymbol{n}, \boldsymbol{A}, \boldsymbol{\Phi})$ are with the countable numbers of monopoles $\boldsymbol{n}$, because $H(\boldsymbol{n}, \boldsymbol{A}, \boldsymbol{\Phi})$ is such that Gödel's proof of incompleteness [39] can be constructed solely within the scope of $H(\boldsymbol{m}, \boldsymbol{A}, \boldsymbol{\Phi})$, using only terms that denote physical monopoles, fields and reactions within the theory itself. The consistency required of $H(\boldsymbol{n}, \boldsymbol{A}, \boldsymbol{\Phi})$ by the proof is also just in terms of the monopoles, where their topological basis ensures that a monopole either exists or it doesn't, and so the dimensionally reduced theory $H(\boldsymbol{n}, \boldsymbol{A}, \boldsymbol{\Phi})$ will have the required consistency over $\boldsymbol{n}$. The incompleteness discussed in [10] applies to the virtual-monopole reaction network as follows:

- Logical truth values are physically realised by existence in $H(\boldsymbol{n}, \mathbf{A}, \mathbf{\Phi})$, where a term A denoting a physical property is true if the property exists, and false if it doesn't.
- Logical operations can be physically realised in $H(\boldsymbol{n}, \mathbf{A}, \mathbf{\Phi})$ in terms of physical terms A and B.
- Logical implication in $H(\boldsymbol{n}, \mathbf{A}, \mathbf{\Phi})$ is physically realised in terms of causation, where physical state A causing physical state B gives a realisation of A implies B.
- Logical induction is present in $H(\boldsymbol{n}, \mathbf{A}, \mathbf{\Phi})$ in terms of induction from a true statement about a property of one monopole n_i=1 to a true statement about any number of monopoles.

This gives a physical realisation of the logical operations required for $H(\boldsymbol{n}, \mathbf{A}, \mathbf{\Phi})$ to constitute a formal deductive system in its own terms, i.e. without the external application of mathematical operations.

- Successor function is physically realised by monopole creation, $s(n_i)$: $n_i \rightarrow n_i + 1$.
- Predecessor function is physically realised by monopole annihilation, $p(n_i)$: $n_i \rightarrow n_i - 1$.
- Zero function is physically realised by annihilation of all anti-monopoles created about a monopole and vice-versa, $z(n_i)$: $n_i \rightarrow 0$.
- Projection functions are given by the identification of monopole types, and this identification can be extended to any combination of monopole reactions, $P_i(n_1, \ldots, n_m) \rightarrow n_i \; \forall i$.

This gives a physical realisation within $H(\boldsymbol{n}, \mathbf{A}, \mathbf{\Phi})$ of the initial functions required for the operations of arithmetic and partial recursive functions over the natural-numbers to be realised in physical terms.

- Addition over the number of monopoles n_i of some type i is realised in the construction of the term $T_n^{a}(\boldsymbol{n}, \boldsymbol{A}, \boldsymbol{\Phi})$ by the accounting term V_n controlling how many times the successor function $s(n_i)$ is applied.
- Multiplication over the number of monopoles n_i of some type i is realised in the construction of the term $T_n^{a}(\boldsymbol{n}, \boldsymbol{A}, \boldsymbol{\Phi})$ by the accounting term V_n controlling how many times a term $T_m^{b}(\boldsymbol{n}, \boldsymbol{A}, \boldsymbol{\Phi})$, with $m<n$, containing addition over n_i is added to the term $T_n^{a}(\boldsymbol{n}, \boldsymbol{A}, \boldsymbol{\Phi})$.
- Substitution is physically realised in terms of creating a reaction network term $T_n^{a}(\boldsymbol{n}, \boldsymbol{A}, \boldsymbol{\Phi})$ from pre-existing reaction network terms $T_0^{b}(\boldsymbol{n}, \boldsymbol{A}, \boldsymbol{\Phi}), \ldots, T_{n-1}^{c}(\boldsymbol{n}, \boldsymbol{A}, \boldsymbol{\Phi})$.
- Recursion is physically realised in terms of creating a term $T_n^{a}(\boldsymbol{n}, \boldsymbol{A}, \boldsymbol{\Phi})$ denoting a new reaction network from pre-existing reaction networks, which can subsequently be included some number m of times in further reaction networks. In this way a new variable m is added to the theory.
- A recursive number theoretic function $f(n_0, \ldots, n_m)$ is physically realised in terms of the numbers of monopoles and monopole reaction sub-networks (denoted $n_0, \ldots, n_m$) giving some number of monopoles or monopole sub-networks, i.e. $f(n_0, \ldots, n_m) \rightarrow n$
- The infinite recursion that gives the infinite series expansion of (45) means that the function creation process can be repeated indefinitely, and so every recursive number-theoretic function is realised within the scope of $H(\boldsymbol{n}, \boldsymbol{A}, \boldsymbol{\Phi})$ in physical terms.

This gives a physical realisation of arithmetic over the natural-numbers of $\boldsymbol{n}$ and all number theoretical functions within $H(\boldsymbol{n}, \boldsymbol{A}, \boldsymbol{\Phi})$, such that Gödel's incompleteness proof can be constructed solely within the scope of $H(\boldsymbol{n}, \boldsymbol{A}, \boldsymbol{\Phi})$ in physically-real terms denoting monopole numbers and reactions.

- Gödel number g can be calculated for any term within $H(\boldsymbol{n}, \boldsymbol{A}, \boldsymbol{\Phi})$ because all number-theoretic functions are defined within $H(\boldsymbol{n}, \boldsymbol{A}, \boldsymbol{\Phi})$.
- Diagonal function D can be defined for the same reason.
- Gödel and Rosser sentences can be expressed within the scope of $H(\boldsymbol{n}, \boldsymbol{A}, \boldsymbol{\Phi})$ using only terms contained within $H(\boldsymbol{n}, \boldsymbol{A}, \boldsymbol{\Phi})$.

So the theory $H(\boldsymbol{n}, \boldsymbol{A}, \boldsymbol{\Phi})$ is proven to be mathematically incomplete using only the terms denoting monopoles and wave radiation. As the undecidable propositions within $H(\boldsymbol{n}, \boldsymbol{A}, \boldsymbol{\Phi})$ are also expressed in such terms, they could correspond to observables. If this were the case, there would exist an observable property p of a monopole that could not be derived within the classical theory $H(\boldsymbol{n}, \boldsymbol{A}, \boldsymbol{\Phi})$. This would be the case for the topological monopoles of Table 1 being the particles, as particles are observed to display a wave property. This property is not derivable in $H(\boldsymbol{n}, \boldsymbol{A}, \boldsymbol{\Phi})$ as it crosses the classification divide between a hole and metric-waves in the "fabric of space".

7. Quantum Field Theory

In this section, it will be assumed that the topological monopoles of Table 1 are the particles, and so possess a wave property p that cannot be derived in the classical physics theory $H(\boldsymbol{n}, \boldsymbol{A}, \boldsymbol{\Phi})$. This gives a scientific description problem as there-exists a physical property p that can neither be predicted from $H(\boldsymbol{n}, \boldsymbol{A}, \boldsymbol{\Phi})$, nor included in $H(\boldsymbol{n}, \boldsymbol{A}, \boldsymbol{\Phi})$ as it will result in an inconsistent theory with wave-particle duality. There-exists no hidden variable that can be added to $H(\boldsymbol{n}, \boldsymbol{A}, \boldsymbol{\Phi})$ to account for p, as the problem lies with incompleteness over $\boldsymbol{n}$. So adding an extra field to $\boldsymbol{A}$ or $\boldsymbol{\Phi}$ won't work as it misses the source of the problem, and the infinite recursion of (45) means that all possible $\boldsymbol{n}$ variables have already been considered in the course of the incompleteness proof. Furthermore, the wave property p cannot simply be attached to the terms in $H(\boldsymbol{n}, \boldsymbol{A}, \boldsymbol{\Phi})$ as the character of the incompleteness proof for p means that it is a property of the infinite virtual-monopole reaction network given by the expansion in $H(\boldsymbol{n}, \boldsymbol{A}, \boldsymbol{\Phi})$. This would make the wave property p a physical property of an infinite set that cannot be reduced to the monopole content of the set.

The replacement procedure identified in [10] to convert the incomplete theory $H(\boldsymbol{n}, \boldsymbol{A}, \boldsymbol{\Phi})$ into a scientifically complete theory $H_C(\boldsymbol{\Psi}, \boldsymbol{A}, \boldsymbol{\Phi})$ is to replace the natural-number valued terms $\boldsymbol{n}(x_\mu)$ for the countable number of monopoles with real-number valued continuous field terms $\boldsymbol{\Psi}(x_\mu)$, to which the wave property p can be attached without causing an inconsistency or set-theoretic type conflict. This procedure works because Gödel's incompleteness theorems only apply to formal systems over the natural-numbers, not over the real-numbers. As the incompleteness of $H(\boldsymbol{n}, \boldsymbol{A}, \boldsymbol{\Phi})$ individually applies to each of the 12 monopoles, the replacement $\Psi^i(x_\mu)$ must be performed for all of them, where the spin charge $\hbar/2$ for the monopoles means that the field term Ψ^i must be a relativistic spinor. The wave property attached to Ψ^i then implies that the relativistic spinor must satisfy the Dirac equation

$$i\gamma^\mu D_\mu \Psi^i - m_i \Psi^i = 0 \tag{46}$$

where D_μ is the covariant derivative for the geodesic transport of the corresponding discrete monopole in the dimensionally reduced classical theory of section 4, and m_i is the monopole mass.

As the topological monopoles are discrete objects, the second part of the replacement procedure [10] is the addition of an ancillary function M that converts the Ψ^i into the observed values n_i for the countable number of particles of type i. The mathematical conditions of the incompleteness proof mean that M cannot be derived within $H(\boldsymbol{n}, \boldsymbol{A}, \boldsymbol{\Phi})$. If M were derivable in $H(\boldsymbol{n}, \boldsymbol{A}, \boldsymbol{\Phi})$ that would imply M was a partial recursive number-theoretic function that also holds over the real-numbers, but as all such functions can be expressed within $H(\boldsymbol{n}, \boldsymbol{A}, \boldsymbol{\Phi})$ the inverse M^{-1} would be derivable within $H(\boldsymbol{n}, \boldsymbol{A}, \boldsymbol{\Phi})$. This inverse could then be used to reverse the replacement procedure to give an apparently consistent and complete theory $H'(\boldsymbol{n}, \boldsymbol{A}, \boldsymbol{\Phi})$ over the same terms as the incomplete theory $H(\boldsymbol{n}, \boldsymbol{A}, \boldsymbol{\Phi})$. But such a modification to an incomplete theory cannot be both consistent and complete, so the assumption that M was derivable in $H(\boldsymbol{n}, \boldsymbol{A}, \boldsymbol{\Phi})$ was incorrect. The $\boldsymbol{x}$ variation of the continuous field Ψ^i description of some number of discrete objects must be integrated over to give n_i, and the wave property of Ψ^i implies that M must be of the form $n_i = M(\Psi_i^\dagger \Psi^i)$, so

$$n_i = \int d^4x \left|\det g_{\mu\nu}\right|^{1/2} \overline{\Psi}^i \Psi^i \tag{47}$$

The condition of local causality for the monopoles described by observables n_i, and the binary character of this operator provide the basis for deriving the spin statistics for the fermionic fields Ψ^i [38]. Consistency of the complete theory then requires that the same procedure be applied to the continuous vector gauge fields $\boldsymbol{A}$ and scalar fields $\boldsymbol{\Phi}$, where the equivalent expression of (47) for these fields similarly provides the basis for deriving the spin statistics for these bosonic fields [38]. It is this step required for consistency of $H_C(\boldsymbol{\Psi}, \boldsymbol{A}, \boldsymbol{\Phi})$ which gives the quantised gauge particles and quantised scalar particles in this quantum field theory.

The theory $H_C(\boldsymbol{\Psi}, \boldsymbol{A}, \boldsymbol{\Phi})$ obtained by the replacement $\boldsymbol{n} \rightarrow \boldsymbol{\Psi}$ retains all the topological and geometric features found for the classical physics $H(\boldsymbol{n}, \boldsymbol{A}, \boldsymbol{\Phi})$, and so all the values $\hbar$, θ_W, e, g, g', g_c, λ, m_Z, m_W, m_H carry over to $H_C(\boldsymbol{\Psi}, \boldsymbol{A}, \boldsymbol{\Phi})$. The Lagrangian terms for the classical physics $H(\boldsymbol{n}, \boldsymbol{A}, \boldsymbol{\Phi})$ found in section 4 carry over to give a local Lagrangian with local symmetry SO(3)⊗SU(2)⊗U(1) for the 12 fermions of Table 1

$$L = L_F + L_G + L_S + L_V \tag{48}$$

$$L_F = \overline{\Psi}^i i\gamma^\mu D_\mu \Psi^i - \overline{\Psi}^i m_i \Psi^i \qquad \text{(from 46)}$$

$$L_G = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a - \frac{1}{4} W^a_{\mu\nu} W^{\mu\nu}_a - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \qquad \text{(from 16)}$$

$$L_S = D_\mu \Phi D^\mu \Phi \qquad \text{(from 22)}$$

where the non-zero terms of the particle space metric Φ_{mn} for the electroweak vacuum are written as a representation of the SU(2)⊗U(1) symmetry

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} \qquad \text{(from 21)}$$

In these terms, the quartic term of the Lagrangian gives the apparent potential term

$$\begin{aligned} L_V = {} & \tfrac{1}{2}(\phi_1 D^\mu \phi_1)(\phi_2 D_\mu \phi_2) + \tfrac{1}{2}(\phi_1 D^\mu \phi_1)(\phi_3 D_\mu \phi_3) + \tfrac{1}{2}(\phi_2 D^\mu \phi_2)(\phi_3 D_\mu \phi_3) \\ & - \tfrac{1}{4}(\phi_4 D^\mu \phi_4)(\phi_3 D^\mu \phi_3) + \tfrac{1}{8}(\phi_4 D^\mu \phi_4)(\phi_4 D^\mu \phi_4) \end{aligned} \qquad \text{(from 20)}$$

which for the electroweak vacuum with ϕ_1=ϕ_2=0 and ϕ_3=ϕ_4=ϕ is

$$L_V = -\frac{1}{8}(\phi D^\mu \phi)(\phi D^\mu \phi) \qquad \text{(from 27)}$$

The masses for the W and Z bosons arise in L_S from the scalar field ϕ=η at all spatial locations in S^3, whereas the Higgs boson mass arises in L_S from the gauge field of the instanton-like configuration (15). The apparent local potential term L_V is not responsible for the electroweak vacuum in STUFT, where it instead arises from the global topology of the particle space S^7 being twisted in going around the full S^3 of the spatial universe. This global origin of the symmetry breaking makes the pursuit of unification in a local Lagrangian with symmetry group SU(4) and SU(3) co-sets somewhat pointless, as such a local theory would necessarily fail to represent the global topology of the space.

The chirality of the instanton-like configuration of (15) is only an issue for the spinor fields Ψ^i denoting the 12 fermions of Table 1, where it will give different covariant derivative terms in (46) for left and right chiralities of Ψ^i. This chirality of the electroweak vacuum also gives unequal contributions to the fermion mass terms of L_F, as the pure gauge field gives a factor of ½Ψ^i in the covariant derivatives of the left-handed spinor fields. This different behaviour of the left and right spinor fields Ψ^i creates a problem in L_F as the form of the mass terms m_i varies with chirality and SU(2)⊗U(1) eigenvalues. The pure gauge field of the instanton-like configuration (15) gives the gauge variation in S^3 such that the scalar field Φ of the electroweak vacuum is given by $\phi_3=\phi_4=\phi$ at all spatial locations $\boldsymbol{x}$ in S^3 at all times in the $S^3\times S^7$ phase. This means that the local Lagrangian term L_F can be given SU(2)⊗U(1) invariance by replacing the mass terms m_i with

$$g_i(\Psi^i{}_L{}^\dagger\,\Phi\,\psi^i{}_R + \psi^i{}_R{}^\dagger\,\Phi^\dagger\,\Psi^i{}_L) \tag{49}$$

This term is of the form of the γ term in (40) and gives the impression that the electroweak vacuum is directly the source of mass for the fermions, which is not the case in STUFT. The electroweak vacuum is the source of the topological conditions that give fermionic monopoles, but their masses are given by the Hamiltonian expansion of the previous section, and this is the source of the underlying problem with fermion masses in (48).

The assumption that the topological monopoles of the classical physics possess a wave property p that cannot be derived in the mathematically incomplete theory $H(\boldsymbol{n}, \boldsymbol{A}, \boldsymbol{\Phi})$, gives a description problem that is resolved by the replacement $\boldsymbol{n} \rightarrow \boldsymbol{\Psi}$ leading to the quantum field theory $H_C(\boldsymbol{\Psi}, \boldsymbol{A}, \boldsymbol{\Phi})$, but the mass calculation problem of $H(\boldsymbol{n}, \boldsymbol{A}, \boldsymbol{\Phi})$ remains. The theory $H_C(\boldsymbol{\Psi}, \boldsymbol{A}, \boldsymbol{\Phi})$ does not resolve this problem, but instead the form the Hamiltonian expansion (44) constrains the validity of the replacement $\boldsymbol{n} \rightarrow \boldsymbol{\Psi}$. This is because the integrals of (44) are over the space-time of the Kerr metric of a compactified black hole, which possesses an event horizon, rotational frame-dragging and an ergo-region, none of which can be denoted by the continuous field $\Psi^i(x_\mu)$ in space-time. However, far from the compactified black hole space is flat and a monopole can be approximated as a point. In this far-field limit $r{>>}\chi$ the point monopoles counted by the natural-number variable $\boldsymbol{n}$ can be replaced with $\boldsymbol{\Psi}$, and the Kerr metric dropped from the Hamiltonian expansion (44) to give

$$H = \sum_{n=0}^{\infty}\int^{t_E} d^4x_0 \int^{t_0} d^4x_1 \ldots \int^{t_{n-1}} d^4x_n\, \overline{\mathrm{H}}_0(\boldsymbol{x}_0)\,\overline{\mathrm{H}}_1(\boldsymbol{x}_1)\ldots\overline{\mathrm{H}}_n(\boldsymbol{x}_n) \tag{50}$$

This Hamiltonian expansion about a point particle is only valid in the far-field limit $r{>>}\chi$, and so this gives the range of validity for the replacement $\boldsymbol{n} \rightarrow \boldsymbol{\Psi}$ that yields the quantum field theory $H_C(\boldsymbol{\Psi}, \boldsymbol{A}, \boldsymbol{\Phi})$. This specifically means that $H_C(\boldsymbol{\Psi}, \boldsymbol{A}, \boldsymbol{\Phi})$ is only valid in the limit that the gravitational effects of particles are negligible, and so the quantum field theory cannot be re-combined with General Relativity. The limits on the integrals arise in (47) from the ergo-region limit $E \leq m_pc^2$ which will not be present in a theory in the far-field limit which omits the ergo-region. However, all the terms in $H_C(\boldsymbol{\Psi}, \boldsymbol{A}, \boldsymbol{\Phi})$ are now waves subject to the Heisenberg uncertainty relation $\Delta E\Delta t \geq \tfrac{1}{2}\hbar$, where setting the time variation to its minimum $\Delta t = \tfrac{1}{2}t_p = \tfrac{1}{2}l_p/c$ gives $\Delta E \geq c\hbar/l_p = m_pc^2$ by (31), which for the equality bound recovers the ergo-region bound of the classical physics theory. So using Heisenberg's uncertainty relation in place of the ergo-region constraint in the Hamiltonian expansion (50) gives a consistent quantum field theory for the far-field limit of $|\boldsymbol{x}|{>>}\chi$ and for energies $E{<<}m_pc^2$.

The Hamiltonian expansion (50) gives the form of the path integral expansions of quantum field theory [38], and the rest of the standard development of a quantum field theory proceeds in exactly the same way here. This includes the renormalization procedure [40] to handle the recursion that gives rise to (50), so as to normalise the calculation to the measured masses of particles. Cut-off regularization will have a physical basis in this quantum field theory, as the compactification scale χ gives a physical small scale cut-off, and the radial scale factor $a(t)$ of the spatial universe gives a large scale cut-off. It can be noted that the term (49) is required to ensure SU(2)⊗U(1) invariance of the Lagrangian (48), so that the theory can be renormalized.

A quantum field theory with local symmetry SO(3)⊗SU(2)⊗U(1), a non-trivial vacuum with the eigenvalues of the electroweak vacuum, and 12 fermionic monopoles with the eigenvalues of the fundamental particles has been derived from Einstein gravity (1) in 10+1 dimensions. Plank's constant $\hbar$ was derived, the values of the coupling constants θ_W, e, g, g', g_c and λ have all been derived with the values of the Standard Model, and a scalar (Higgs) boson mass predicted to be given by $m_H = \tfrac{1}{2}\eta$ [10]. The reason for the derivation of this quantum field theory is the mathematical incompleteness of the classical physics theory $H(\boldsymbol{n}, \boldsymbol{A}, \boldsymbol{\Phi})$ for the calculation of the mass reduction of a monopole with bare mass m_p, and this limitation still remains in that the fermion masses are unpredicted by this work. In addition, there are classical physics interactions of monopoles that are not included by the far-field limit of the quantum field theory. The first is SU(3) co-set transitions where a particle changes between co-set family, and so the values of the Cabibbo-Kobayashi-Maskawa (CKM) and Pontecorvo-Maki-Nakagawa-Sakata (PMNS) family transition matrices remain unpredicted, where it should be noted that these can only be calculated in STUFT from the classical physics of monopole interactions. The gravitational effects of the topological monopoles can be approximated by a Newtonian potential and a spin field for the rotational frame-dragging. The spin field will allow spin-flip interactions for near direct collisions of classical physics monopoles that are not possible under the Lagrangian (48)

$$m_L^a + m_R^b \to m_R^a + m_L^b$$

Such interactions will have a very small collision cross-section, but because the cross-section is non-zero, STUFT predicts that right-handed neutrinos exist [10].

8. Cosmology

As a pure metric field theory, STUFT will be strictly invariant under the joint action of time reversal $T(t) \rightarrow -t$ and parity $P(x) \rightarrow -x$ across all 10 dimensions, i.e. $P_{10}T$ invariant. After the topological transition $S^{10} \rightarrow S^3 \times S^7$ and compactification, the spatial parity operator is dimensionally reduced to P_3 and the remaining 7 parity operators become charge reversal operators $C(q) \rightarrow -q$, such that $P_{10}T$ invariance becomes global C_7P_3T invariance. However, the electroweak vacuum breaks the symmetry of the electroweak base-space, leaving the colour and electromagnetic symmetries intact to give local C_4P_3T invariance for these charges, but local C_3 and P_3 violation for isospin charges. This discrepancy between global and local invariance arises because the global operation of C_7P_3T reverses the chirality of the electroweak vacuum (L→R) and the time development of the whole cosmology, whereas the local operators C_3 and P_3 reverse the isospin charge and chirality of a particle, but not that of the electroweak vacuum. Furthermore, the local time reversal operator T will change a monopole into an anti-monopole, but not reverse the time development of the universe, and this implies local T violation. As the positive energy of matter is associated with temporal translation in the positive time direction, and the negative energy of anti-matter with temporal translation in the negative time direction, the positive time bias of an expanding cosmology implies a T violation bias in favour of matter over anti-matter [10]. This origin for local T violation implies that the extent of T violation will be correlated with the expansion rate $\mathrm{d}a(t)/\mathrm{d}t>0$ of the S^3 universe, and so will be low in the current epoch, but would have been higher in the inflationary epoch and its immediate aftermath when the monopoles formed.

This matter bias for an expanding cosmology predicts a net matter content for the STUFT universe, but this slight bias would be expected to be dominated by those monopoles and anti-monopoles which wouldn't have annihilated to radiation. The predicted non-zero mass of the neutrino-like electric monopoles gives a form of hot dark matter that would largely retain its primordial distribution of both matter and anti-matter, as neutrinos are so weakly interacting not to have annihilated. In addition, the magnetic duals of electric neutrinos would be expected to be of a higher mass and so possibly give a form of cold dark matter, where again the weakness of their interactions could be expected to have left a distribution of both monopoles and anti-monopoles.

In STUFT, monopole/anti-monopole annihilation is to a neutral black hole with a compactified surface that then emits its mass as radiation, which implies that the time taken for annihilation will be related to monopole mass. The relation between electric and magnetic charges in the Dirac quantisation condition, and the correlation between monopole charge and mass, implies that magnetic monopoles would be expected to have a much greater mass than their electromagnetic duals. This implies that magnetic monopoles would have frozen out before the electric monopoles in the early universe of the $S^3 \times S^7$ phase, and their annihilation time-scale is much longer than that of the electric monopoles. Under the conditions of high temperature and density in the early STUFT universe, there can exist a magnetic monopole mass above which the annihilation time-scale is such that the resulting neutral black hole acquires in-falling energy faster than it is emitted as radiation, and so grows. This would be the most significant form of mass perturbation in the early STUFT universe. It should be noted that the derivation of quantum field theory in section 7 was for the far-field limit of negligible spatial curvature, and so the application of quantum field theory in the vicinity of black holes and the high curvature of the early STUFT universe might not be entirely valid. Some of these primordial black holes of monopole/anti-monopole origin could have persisted and continued to grow. The super-massive black holes at the centres of galaxies would seem to indicate a potential role for such primordial black holes of magnetic monopole origin in the formation of galaxies. An estimation of the magnetic monopole masses in STUFT would be required to determine whether it does give such primordial black holes. If there is a distribution of macroscopic primordial black holes resulting from magnetic monopole/anti-monopole annihilation, they would also give a cold dark matter contribution.

The multi-stage pattern of sphere decomposition in STUFT would have occurred as the early STUFT universe expanded, and the sequence would explain the multi-stage pattern of intersection for the running coupling constants in the Standard Model:

$$S^{10} \rightarrow_{CI} S^3\times S^7 \rightarrow_{CF} S^3\times(S^3\times S^4) \rightarrow_{BS} S^3\times(S^3\times(S^3\times S^1)) \rightarrow_{EW} S^3\times(S^3\times S^1)$$

CI is the de-unification transition that starts the compactification-inflation see-saw driven by radiation transfer from S^7 to S^3 and gives the non-trivial vacuum winding of S^7 around S^3. The CF transition marks the colour-fibre separation from the particle space S^7 and corresponds to the intersection point of the isospin coupling g and the colour coupling g_c. The BS transition marks the separation of the S^4 electroweak base-space into S^3 and S^1, corresponding to the intersection point of the isospin coupling g and hypercharge coupling g'. The electroweak transition EW is then just a dynamic transition when the energy level drops below the electroweak energy scale η.

The closed universe S^3 will expand up to some maximum radius and then contract back down again as the see-saw process of section 3 runs in reverse. As the two scales $a(t)$ and $\chi(t)$ become comparable again the above sphere sequence would be reversed, ending in the initial transition being reversed with the restoration of the unified S^{10} phase. This gives the pattern of a cyclical universe, which then raises concerns about continuously increasing entropy with each cycle. However, these concerns are covered by the conclusion of section 3 that the definition of entropy is specific to the number of dimensions. So any closed cycle that crosses a dimensional reduction will encounter an entropy anomaly as the definition of entropy changes. For a black hole the entropy goes from S_3 for matter falling into a black hole, to S_2 for metric waves in the surface of the black hole, and back to S_3 when the energy is emitted as radiation. Attempting to directly compare the entropies S_3 and S_2 will give an apparent entropy anomaly because the comparison is not like with like. When matter falls into a black hole and is re-emitted as radiation, the radius of the black hole remains the same and so the S_2 entropy is unchanged. Outside of the black hole the conversion of matter to radiation increases the S_3 entropy as to be expected by the laws of thermodynamics. So there is no entropy problem with this closed cycle as long as the entropies in the different numbers of dimensions are not directly compared against each other. A similar situation arises for a cyclical universe between the S_{10} entropy in the S^{10} phase and the S_3 entropy of the $S^3\times S^7$ phase. In this case, the increase in S_3 entropy during the expansion and contraction of the universe requires the transition back to S^{10} to occur at a larger radius than that of the initial transition so as to give the same S_{10} entropy density in the unified S^{10} phase. Under these conditions there would be no entropy problem with a cyclical universe.

9. Discussion

The incompleteness proof of section 6 and the subsequent derivation of quantum field theory in section 7 on the basis of the wave property being undecidable in the classical physics is a significant result, as it questions the physical justification for matter fields. With the compactification of particle dimensions and the spatial inflation of the universe being powered by a radiation transfer mechanism in section 3, the physical justification for inflaton fields is also questionable. This would then make the addition of symmetry breaking fields in a field theory appear somewhat arbitrary. Without a convincing physical basis for matter fields being fundamental and arbitrary fields being of questionable physicality, this would have the effect of leaving geometric field theories in classical physics as the primary remaining route to the unification of physics. The assumptions of closure and physicality for the "fabric of space" of a metric field theory open up a different path to symmetry breaking and particles, where the symmetry breaking in the space is by a topological transition that allows for a non-trivial winding of the space, such that monopoles arise as stable topological defects.

Although in general there will exist a range of metric-field theories displaying these characteristics, STUFT appears to possess an underlying mathematical structure that identifies it as being unique. This is because the closure condition in 10+1 dimensions leads to STUFT being characterised by the closed spaces S^0, S^1, S^3 and S^7 in the 4 normed division algebras. The closure of space in Einstein gravity means that the spatial expansion will eventually be reversed, and the transition $S^{10} \rightarrow S^3{\times}S^7$ reversed $S^3{\times}S^7 \rightarrow S^{10}$ to restore the initial space, which necessarily gives a cosmology that is cyclical in time (S^1). The transition then leads to topological monopoles and anti-monopoles which are characterised by $S^0 = \{-1, 1\}$, and this gives a realisation of all 4 spheres. Furthermore, the wave property assumed for the topological monopoles in section 7 would then mean that they in representations of all 4 spheres:

- S^0: monopoles and anti-monopoles
- S^1: cyclical waves
- S^3: representations of the rotation group with group manifold S^3, i.e. spin
- S^7: representations of the group quotient $SU(4)/SU(3) \cong S^7$, i.e. Table 1

As the wave property of a particle cannot be derived in a classical physics theory because it crosses the classification divide between particle and wave, this simultaneous representation of the spheres of the 4 normed division algebras is unexpected in classical mechanics. As there are only 4 normed division algebras, the 4 spheres S^0, S^1, S^3 and S^7 are uniquely defined and only realised for unification of spatial and particle dimensions in a geometric theory with the topology of STUFT. These characteristics seem to uniquely identify STUFT as being the one and only possible geometric theory that yields a quantum field theory for 12 fermionic monopoles with the eigenvalues of the fundamental particles [10].

The obvious difficulty with the quantum field theory of section 7 is that the local SO(3) colour group differs from the SU(3) colour group of the Standard Model, but the coloured monopoles were shown in section 2 to nonetheless possess the same 1/3 electric charges as the quarks. It can also be noted that the vacuum eigenvalues (0,–½,1) gives the SO(3) colour angle θ_{QCD}=0. The SO(3) colour group is related to the cosmological difficulty of the closed spatial universe of STUFT, whereas the current evidence appears to point to an open universe. The problem for STUFT is that the closure of the universe is the direct reason for the chiral vacuum with the characteristics of the electroweak vacuum, which then gives the topological conditions for the 12 topological monopoles of Table 1. This means that in STUFT, the existence of fermionic matter is directly linked to the universe being closed S^3. The closure of the universe also underlies the compactification-inflation see-saw of section 3, which is further dependent upon the physicality assumption of the "fabric of space". This physicality assumption also gives the basis for identifying the group spaces of the symmetry groups Spin(3), SU(2) and U(1) with the physical particle spaces S^3, S^3 and S^1, where the closed formulae for the coupling constants and boson masses were derived on the basis of this identification. This directly links the S^3 fibre of the S^7 particle space with the group space of the colour group in STUFT, and so the colour group can only be Spin(3) unless the physicality assumption is dropped. However, if the physical assumption is dropped then all the other results of STUFT are lost.

So perhaps the real issue with STUFT is its unreasonable success in deriving closed formulae for θ_W, $\hbar$, e, g, g', λ, m_Z, m_W, m_H in classical physics, as the values given by these formulae for a compactification scale of the Planck length are to within 1-2% of experimental values. Even in the classical physics, it would be expected for these formulae to be modified because their derivation ignored any distortion to the S^7 particle space due to the non-trivial vacuum structure. All these classical values would be further expected to be subject to quantum corrections, which would be the explanation sought for the experimental up-down quark mass reversal relative to the simple heuristic classical physics argument in section 6. The calculation of the masses of both the electric and magnetic monopoles is the significant outstanding issue in the development of STUFT, and whether they are actually calculable in the classical physics. To the geometrical success of STUFT in deriving the parameters of the bosonic sector of electroweak theory, must be added the topological success in the fermionc sector of deriving the topological charges of the fermions, despite the colour group issue. This specifically includes the derivation of 3 particle families. It seems difficult to square these unreasonable successes with the local colour group not being SO(3) and the universe apparently not being closed.

A final point is that the principle of maximal symmetry would select the Ricci scalar for the action for 10+1 dimensional Einstein action, where the topology of STUFT is the same whatever the curvature terms. It should be noted that the derivations of section 5 require the dimensionally reduced gravitational action to be that of General Relativity based upon the Ricci scalar, and this may limit the possible form of the curvature terms in the 10+1 dimensional action to the corresponding Ricci scalar.

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