**In an adjacency matrix which encodes for a directed Hamiltonian path, a non-zero determinant value certifies the existence of a directed Hamiltonian path when no zero rows (columns) and no similar rows (columns) exist in the adjacency matrix.**

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## ABSTRACT

The decision version of Directed Hamiltonian path problem is an NP-complete problem which asks, given a directed graph G, does G contain a directed Hamiltonian path? In two separate papers, the author expresses the graph problem as an adjacency matrix and a proof given to show that under two special conditions relating to theorems on the determinant of a square matrix, a non-zero determinant value certifies the existence of a directed Hamiltonian path. Here, a brief note is added to repair a flaw in the proof. The result, as expressed in the paper title is a more defensible proposition.

## INTRODUCTION

In theoretical computer science, the decision version of the directed Hamiltonian path problem is among the class of problems referred to as NP-complete. For this class of problems, there exists no known efficient algorithm for deciding them. NP-complete problems give rise to many scheduling and routing problems with industrial importance [1].

In [2] the author advanced the argument of a non-zero determinant value as a verifier of the existence of a directed Hamiltonian path and in [3], a proof by means of deductive logic is given. In both papers, the graph problem is expressed as an adjacency matrix, a common representation of a directed graph. The purpose of this paper is to repair a flaw in the proof and to present a more defensible proposition.

The proof from [3] is given by means of deductive logic where six preceding axioms were linked to reach a conclusion:

1. An adjacency matrix is a square matrix representing the combination of edges of a directed graph across *n* rows and *n* columns of adjacency matrix of size *n*. An adjacency matrix has a fixed zero diagonal from left to right.

2. The rows (columns) of an adjacency matrix are listed as row (column) 1, 2, 3, …, (n-3), (n-2), (n-1), n in an adjacency matrix of size *n*.

3. The presence of edges per row (column) of an adjacency matrix is denoted by 1’s while their absence by 0’s, so a combination of edges per row (column) of adjacency matrix size *n* is actually a combination of 0’s and 1’s.

4. Since the combination of edges per row (column) of an adjacency matrix is represented by a combination of 0’s and 1’s, in order to satisfy the dual conditions of the absence of a zero row (column) and the absence of similar rows (columns) across the *n* rows (columns) of adjacency matrix, each row (column) is assigned at least one edge (1) and a different combination of 0’s and 1’s is applied across row (column) 1, 2, 3, …, (n-3), (n-2), (n-1), n of adjacency matrix size *n*.

5. Different combinations of 0’s and 1’s applied across row (column) 1, 2, 3, …,(n-3), (n-2), (n-1), n of an adjacency matrix size *n* implies different permutations or sequences of 0’s and 1’s (permutation of edges) across the respective rows (columns).

6. In an arbitrary adjacency matrix, a directed Hamiltonian path is a sequence of one way edges across rows (columns) 1, 2, 3, …, (n-3), (n-2), (n-1), n. In effect, a direct Hamiltonian path is a unique permutation of edges across row (column) 1, 2, 3, …, (n-3), (n-2), (n-1), n.

7. Therefore for an adjacency matrix of size *n* satisfying the dual conditions of the absence of a zero row (column) and the absence of similar rows (columns) – implying a non-zero determinant value for the adjacency matrix, there exists a directed Hamiltonian path.

**A CORRECTION**

A flaw in the proof above from [3] is perceivable in the $5^{th}$ and $6^{th}$ axioms. A directed Hamiltonian path is a sequence of one-way, compatible edges; therefore a mere permutation of edges in an adjacency matrix does not necessarily imply a Hamiltonian path as there might be a repetition of edges in a mere permutation of edges. Here therefore is a more defensible proposition (set against the proposition in [3]): In an adjacency matrix which encodes for a directed Hamiltonian path, a non-zero determinant value certifies the existence of a directed Hamiltonian path when there are no zero rows (columns) and when no similar row (column) exists in the adjacency matrix.

The correctness of the proposition seems self-evident; nevertheless, a short proof is demonstrated by the following statements:

1. An adjacency matrix is a square matrix representing the combination of edges of a directed graph.

2. The determinant of a square matrix gives a non-zero value when there no zero rows (columns) and no similar rows (columns).

3. A directed Hamiltonian path is a sequence of one-way compatible edges in an adjacency matrix, a representation of a directed graph.

4. A sequence of one-way compatible edges is applied in an arbitrary adjacency matrix ensuring that there are no zero rows (columns) and no rows (columns) are similar.

5. In this adjacency matrix, a non-zero determinant value verifies the existence of a directed Hamiltonian path.

**DISCUSSION AND CONCLUSION**

An important premise of the proof in [3] is that different combinations of 0'^s^ and 1'^s^ in rows (columns) of an adjacency matrix automatically lead to different combinations of edges in the adjacency matrix. Apparently this is incorrect. Consider as an example a simple 4 × 4 adjacency matrix (Figure 1), where a different combination of 0'^s^ and 1'^s^ in column 1 and 4 leads to the same combination (1→4, 4→1) of edges in the adjacency matrix.

| | | | |
|---|---|---|---|
| 0 | 1 | 0 | 1 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 0 |

Figure 1

In addition, it is trivial to show that the proof demonstrated for a Hamiltonian path can be modified to that for a Hamiltonian cycle. A Hamiltonian path becomes a Hamiltonian cycle when the sequence of one-way, compatible edges in a Hamiltonian path terminates with the edge which starts the sequence.

A proof for the proposition “In an adjacency matrix which encodes for a Hamiltonian cycle, a non-zero determinant value certifies the existence of the Hamiltonian cycle when there are no zero rows (columns) and no similar rows (columns) is similar to the first:

1. An adjacency matrix is a square matrix and a representation of a directed graph.

2. The determinant of a square matrix is non-zero if there are no zero rows (columns) and no similar rows (columns) in the adjacency matrix.

3. A Hamiltonian cycle is applied in the adjacency matrix in such a way that there are no zero rows (columns) and no similar rows (columns).

4. In this adjacency matrix, a non-zero determinant value certifies the existence of a Hamiltonian cycle.

Representing a directed graph as an adjacency matrix presents a route to an efficient method for verifying the existence of a Hamiltonian path in an adjacency matrix with no zero row (column) and no similar row (column) because of two key theorems regarding the determinant of a matrix with proofs given in [2] which state that the determinant of a matrix is non-zero when there are no zero rows (columns) and when no rows (columns) are similar. Therefore in an adjacency matrix which encodes a directed Hamiltonian path, a non-zero determinant value verifies the existence of a Hamiltonian path when the adjacency matrix exhibits these two properties.

**REFERENCES**

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