## THE SYMMETRY GROUPS OF LIGHT

(Revised Feb., 2013) John A. Gowan

In the mathematical terms of Evariste Galois' "Group Theory", the <u>"Tetrahedron Model"</u> is a description of the *symmetry group of light*, including its destruction by <u>asymmetric weak force decays</u> (producing our matter-only Cosmos), and its on-going restoration in obedience to <u>Noether's Theorem of symmetry</u> <u>conservation</u> (as in the conversion of bound to free energy in stars). (See diagram: <u>"The Four Forces Produce Light"</u>.)

The usual symmetry group identified with light is that of local phase transformations, and it is designated as either SO(2) or U(1). However, I am suggesting here that light contains a very much larger (and more interesting) symmetry group associated with its transformation into particle-antiparticle pairs (and back again into light). I don't know what the formal designation of this group might be.

For an expert's explanation of the formal aspects of symmetry and group theory, See: Keith Devlin *The Language of Mathematics* Chap. 5 "The Mathematics of Beauty", 1998 W. H. Freeman & Co. (Holt Paperbacks); see also: Ian Stewart *Why Beauty is Truth* Chapt. 13 "The Five Dimensional Man", Basic Books 2007. See also: *Symmetry* by Roy McWeeny, 2002, Dover Pub. Inc. (highly mathematical).

A "symmetry group" consists (for one example) of a collection of figures that can be transformed into one another without changing the original. The symmetry group of an equilateral triangle (say) consists of all the triangles that can be created from an original by means of rotation, translation, reflection, etc. - provided the transformed articles are indistinguishable from the original. How do we apply this notion to the case of light? In what sense is there a symmetry group associated with (consisting of) transformations of light (free electromagnetic radiation)?

Beyond the simple phase transformations of the electromagnetic field, the examples of interest here are the particle-antiparticle pairs of the Dirac/Heisenberg "vacuum" of spacetime. These particle-antiparticle pairs are constantly produced from borrowed energy and instantaneously annihilate each other in an endless cycle of creation and destruction alternating between light and virtual particles, a cycle which has been ongoing throughout spacetime since its beginning in the "Big Bang" (when these particle-pairs were real rather than virtual). However, since today they are "virtual" rather than "real" particles, we do not notice them even though they are everywhere around us. Essentially, we do not notice them because their symmetry is so complete. We notice especially the asymmetric forces and particles which surround (and comprise) us: gravity, time, atomic matter, charge.

These virtual particle-antiparticle pairs consist of all known (and unknown) species of elementary particles, and their creation and annihilation cycles form (along with the aforementioned phase transformations) the primordial symmetry group of light or free electromagnetic energy. During the "Big Bang", the symmetry of light and its virtual particle-pairs was broken by <u>asymmetric high-energy weak force decays</u> which resulted in the creation of our matter-only universe. Our Cosmos consists of one-half of light's original symmetry group, the matter half. The antimatter half was annihilated along with most of the original matter. Hence the universe around us (including ourselves) is 1/2 of light's original symmetry group revealed in its low-energy, matter-only form: put it together with its vanished antimatter counterpart and you get back the original light. What we are seeing in the physical objects around us (plants, animals, planets, stars) is the long-term evolutionary consequence of the energy and information contained in the original light and its particle-antiparticle symmetry group, exposed to our view only because its original symmetry was broken (in half), unleashing powerful and inexorable forces which forever seek to restore (conserve) the original whole symmetry. Evolution is ultimately driven by matter's eternal search for antimatter - seen most directly and commonly in the electron-proton pairing - the primordial, powerful, and eternal attraction between positive

and negative electric charges. (See: "Light and Matter: A Synopsis of the Unified Field Theory".)

The periodic table of the elements is a basic (and astounding) example of the potential for information contained in (one half of) light's symmetry group. In some sense the elements of the periodic table are a symmetry group of light - since they do transform into one another (via the "W" IVBs), and when mixed with their anti-particles they return to light. But atoms and elements are themselves already compounded objects. At a deeper level we discover this information content is fractal - beyond the U, D (up, down) quarks of the proton and neutron lies another (heavier) level of C, S (charm, strange) quarks, and beyond them, a still heavier family of T, B (top, bottom) quarks. The three quark families are likewise accompanied by three successively heavier levels of leptonic families: the electron, muon, tau and their respective neutrinos. (Leptons, neutrinos, and mesons serve as alternative charge carriers for the quarks and for each other replacing the original antimatter charge carriers.) The symmetry group of light turns out to be complex indeed, a complexity revealed only when its symmetric form is cut in half - like exposing the seeds, cells, nuclei, and DNA of a cantaloupe when we cut in half its smooth and simple sphere. (See: "The Particle Table".) (See also: "A List of the Possible Quark Combinations of the Baryons and Their Charges"; See also: "Flavor Combinations of Baryons Containing U.D or C.S Quarks"; See also: "Flavor Combinations of Baryons Containing C,S or T,B Quarks"; See also: "Flavor Combinations of Baryons Containing U,D or T,B Quarks".)

The quark and lepton series (including the neutrinos) are obvious examples of light's symmetry groups. They can be transformed into other members of their kind (via the "W" IVBs), and when combined with their antiparticles, restore the light from which they were originally created. But this is not all. Single particle transformations within these symmetry groups are (must be) precisely controlled by the weak force IVB (Intermediate Vector Boson) mechanism, such that every electron or other elementary particle created today is exactly the same as those created in the "Big Bang". Going back to the equilateral triangle analogy, the triangle can rotate only through 60 degree "quantum" steps; these fixed points correspond to the fixed mass, spin, and charge of the various elementary particles and particle-antiparticle pairs. The mass hierarchy of the leptons and quarks perhaps corresponds to rotations or scale changes in our triangle analogy. The creation of single particles (rather than particle-antiparticle pairs) is especially difficult, and is the sole purview of the weak force and the reason for its elaborate and massive mechanism. (See: <u>The "W" IVB and the Weak Force Mechanism</u>"; see also: <u>"The Strong and Weak 'Particle' Forces": Part 2</u>.)

And there is more. At the high energy levels of the early "Big Bang", the particle species lose their individuality and combine in ever more inclusive categories - analogously to the biological classification hierarchy of species, genus, family, and order. These particle classes come together as the forces unite - first the electric and weak forces combine, the "electroweak" unification bringing together all the lepton species and (separately) all the quark species into a lepton "genus" and a hadron "genus". At the electroweak unification energy level all leptons can freely transform one into another, and likewise all quarks can freely transform among themselves - having given up their individual identities for a more inclusive "generic" identity. Hence this is a higher energy and symmetry state of force unification, another category of light's symmetry groups (the "electroweak" symmetry group - a simpler group than the ground state, having fewer distinct members). Indeed, these force-unity states are also fractal, like the three quark and lepton energy levels before them. There are three of these force-unity states as well, the second being the GUT (Grand Unified Theory) unification level of the combined electroweak and strong force, unifying all the leptons with all the quarks (the "family" group of fermions including leptoquarks - a further simplification of light's symmetry group). Third and finally, the TOE (Theory of Everything) unification comprising all four forces, including gravity, unites the bosons (field vectors) and fermions (particles) in a single grand electromagnetic energy "order". (See: "The Higgs Boson and the Weak Force IVBs".) This is the ultimate simplification of light's symmetry group, perhaps corresponding (in our geometric analogy) to a circle. In this final state of ultra-high energy and symmetry (seen only at the beginning of the Cosmos in the "Creation Event"), free electromagnetic energy is transformed into bound electromagnetic energy, and vice versa, setting the stage

for weak force symmetry-breaking and the emergence of the matter-only universe of light's energy and information content. This is the Cosmos we now occupy, light's symmetry group revealed in its low-energy, conserved, asymmetric, bound and temporal form as massive atomic matter, charge, gravity, time, and information. (See: <u>"Symmetry Principles of the Unified Field Theory"</u>.)

All the conservation laws and forces of the Cosmos work continuously to maintain, conserve, and/or restore its original symmetric energy state, even as the information content of matter evolves (in the biological realm) toward a fractal iteration of its Creator. (See: <u>"Teilhard de Chardin: Prophet of the Information Age"</u>.)

Returning the material system to symmetry in the absence of antimatter is the central problem of the Universe, requiring the creation (by gravity) of a new (alternative) entropy-carrying dimension: time. The historical maintenance of charge invariance in a world of relative rather than absolute motion, composed of both light and matter, space and time, is a challenge met by the field vectors of the four forces, which are themselves compounded of matter and anti-matter (or are their own anti-particles) expressly for this purpose. (See: "Global vs Local Gauge Symmetry in the Tetrahedron Model".)

The Symmetry Groups of Light

John A. Gowan (Revised Feb., 2013)

Cosmic Transformation: Multiverse --> Universe (creation of our Universe as a conserved electromagnetic subset of the Multiverse). Gravitational negative temporal energy balances electromagnetic positive spatial energy; charge balances anti-charge. The universe is created from a condition of zero net energy and zero net charge: therefore and thereafter the total system must be conserved (must continue to sum to zero net energy and charge).

Before Symmetry-Breaking: "Theory of Everything" ("TOE" - all forces unified).

Planck Era "Order" level: fermions combined with bosons - all electromagnetic (EM) energy forms unified.

Group 1) Primordial electromagnetic group: *charged* elementary leptons x anti-leptons. Electron, muon, tau, leptoquark, plus a neutrino for each (and antiparticles). Only elementary leptonic particles are distinguished by neutrino "identity" charges. Electric charge, "identity" or "flavor" charge, spin, and left-right handedness (parity) are also present as conserved parameters. Another possible "metric" group includes spacetime, light, and gravity (the latter as the temporal component of spacetime).

Quarks are produced from leptoquarks where they originate as triplets (the leptoquark splits into three subunits). In the primordial "quark soup" they recombine in quark-antiquark pairs (mesons) or again as electrically charged triplets (baryons). Electrically neutral baryon combinations (produced by the "Y" IVBs) persist into the next lower (GUT) energy level.

Group 2a) Primordial quarks - 6 quarks x 6 anti-quarks (from split leptoquarks);
Group 2b) mesons - many quark-antiquark combinations, both neutral and charged ("quark soup").
Group 2c) Electrically charged baryons and anti-baryons (many combinations of quark triplets).
Group 2d) gluons - 3 colors x 3 anti-colors (9 combinations, of which one is not functional as it is doubly

neutral);

Energy Transformations: Free (massless) EM energy (waveform) --> particle-antiparticle form of bound (massive) EM energy. Symmetry parameters are transformed to and conserved as charges. *The charges of matter are the symmetry debts of light*. All four physics forces (including gravity) are involved in the conversion of massless light to primordial massive leptonic particles (leptoquarks).

Elementary particle transformations: Primordial leptoquarks are converted from electrically charged to electrically neutral by "Y" IVBs. This transformation is considerably facilitated by the merger of all fermion identities at the TOE energy level. These electrically neutral leptoquarks persist into the symmetry-breaking realm of the "X" IVBs (see GUT Era below). Leptoquarks are the heavy end of the leptonic spectrum of elementary particles, split in thirds (the nascent quarks) due to the self-repulsion of its own electric charge in the overly-massive corpus of the particle - producing a natural upper limit to the leptonic mass spectrum.

Higgs(y), "Y" family of IVBs. The Higgs(y) gauges the energy level of the TOE and the "Y" IVBs. The Higgs is to the mass relations of the "particle zoo" what "c" is to the metric relations of spacetime. (See: <u>"Table of the Higgs Cascade"</u>.)

**During Symmetry-Breaking: "Grand Unified Theory" (GUT - strong and EW forces unified).** Leptoquark Era "Family" level: all fermions combined with one another (= leptons combined with hadrons); bosons separate.

Group 3) Leptoquarks - electrically neutral only: particle-antiparticle pairs - 9 possible pairs (leptoquark neutrinos and anti-neutrinos are part of this group).

Elementary particle transformations: matter-only hyperons created from the asymmetric weak force decay of electrically neutral leptoquarks (mediated by "X" IVBs, with the <u>emission of leptoquark anti-neutrinos</u>). Leptoquarks may be transformed at the GUT level but are first created at the TOE level.

Alternative charge carriers: leptons, neutrinos (carrying electric and "identity" charges); mesons (carrying partial quark charges). Alternative charge carriers act in place of antiparticles, permitting decays rather than annihilations. Alternative charge carriers play a crucial role in breaking the primordial symmetry of light and its particle-antiparticle pairs, and afterward, in balancing charges between matter-only dissimilar charge partners (as in the familiar electron-proton pair).

Higgs(x), "X" family of IVBs. "X" IVBs mediate the asymmetric decay of leptoquarks to hyperons; "X" IVBs also mediate "proton decay". Higgs(x) gauges the energy level of the GUT and the "X" IVBs. (See: "The Origin of Matter and Information".)

# After Symmetry-Breaking, "Electroweak" (EW - electric and weak forces unified):

Matter-only Hyperon Era "Genus" level: all leptons combined with themselves, all hadrons combined with themselves, but leptons and hadrons remain separate from each other.

Elementary particle transformations: heavy hyperons decay to less massive and ground state baryons, leptons, and leptonic neutrinos (via the "W" IVB); likewise, the "W" IVB mediates the decay of heavy leptons. Creation, destruction, and transformation of *single* elementary particles (quarks and leptons); transformations of baryons. Baryons may be created or destroyed only at higher energy ("GUT" level).

Various asymmetric or partial "matter-only" quark and lepton groups (missing antiparticles, which now exist only virtually, excepting neutrinos). Hence these groups have only half their original members, and cannot spontaneously transform to light. Alternative charge carriers: leptons, neutrinos, mesons (the latter carrying quark partial charges). Color charge, gluons, "asymptotic freedom" (quark confinement); gluons function to maintain whole quantum unit charges <u>in baryons and mesons</u>.

Partial groups 4a, 4b). Baryons (composed of 3 quarks in all flavors and colors) = <u>partial group 4a</u>. Leptons and neutrinos, including antineutrinos (all flavors except leptoquarks) = partial group 4b.

Higgs(w), "W" family of IVBs. The Higgs "W" gauges the EW energy level and the "W" family of IVBs.

(See: <u>"The 'W' IVB and the Weak Force Mechanism"</u>; see also: <u>"The Higgs Boson and the Weak Force IVBs"</u>.)

# "Rebound" to Symmetry: EM Ground State of "Ordinary" Matter "Electromagnetic" (EM - all forces separate).

Atomic, Chemical, Information, and Biological Era; "Species" level: electrons, protons, neutrons. Chemical (electron shell) transformations only; all nuclear transformations belong to EW level (above). Rebound phase begins (restoring the original symmetric energy state of the universe and light); gravitational creation of planets, stars, galaxies and cosmic megastructure.

Group 5) Various mixed partial groups of atomic matter (molecules, chemicals, minerals, crystals, etc.). All charges and forces present and separate. Nuclear and chemical (electron shell) partial groups; the Periodic Table of the Elements presents a fundamental ordering of these partial groups. Radioactive elements are in transitional stage between EW and EM levels. Only stable nuclei belong in EM groups. Classification of partial groups beyond the periodic table is largely arbitrary. Life is a biological partial group derived from the information content of carbon, water, and organic chemistry.

*The charges of matter are the symmetry debts of light* (Noether's Theorem). Charge conservation and charge invariance = symmetry conservation; <u>local gauge symmetry</u>.

Chemical/molecular transformations; electron shell transformations; information transformations; creation of life and the biological realm. Higher-order "emergent" information processing and creation, fractal iterations of fundamental forces, including evolution and creative drives. (See: <u>"The Information Pathway"</u>.)

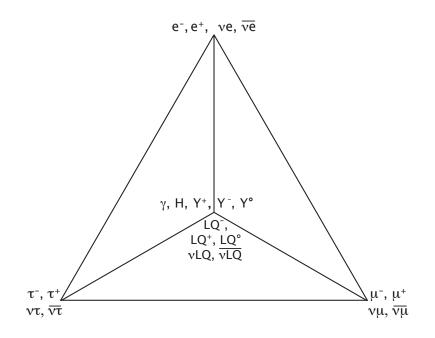
All forces act to return matter to light: fission, fusion, radioactive particle and proton decay, matter x antimatter annihilations, the nucleosynthetic pathway of stars, quasar conversion of gravitational potential energy, "Hawking radiation" of black holes. Note the similarity between the black hole and the initial TOE state, in that gravity is equivalent in strength to, or united with, the other forces. Gravity simplifies and completes the mixed partial symmetry groups of matter either through proton decay within a black hole's event horizon, or through the extraction of antimatter from spacetime outside the event horizon, producing Hawking's "quantum radiance", the ultimate fulfillment of Noether's symmetry conservation theorem. (See: "Symmetry Principles of the Unified Field Theory".)

The appended diagrams suggest solutions to certain of light's symmetry groups in the format of the "Tetrahedron Model". These diagrams are also intended to illustrate the deep connections between the "Standard Model" of physics and the "Tetrahedrom Model" as presented in these web pages. (See also: <u>"The Higgs Boson and the Weak Force IVBs"</u>.)

(See: Fig. 1 <u>"The Symmetry Groups of Light: The Leptonic Spectrum"</u>. (Theory of Everything (TOE) Energy Level - all forces unified.)

Fig. 1: The Symmetry Groups of Light: The Leptonic Spectrum

THE SYMMETRY GROUPS OF LIGHT: THE LEPTONIC SPECTRUM Creation of mass during the T.O.E. or Plank Era of the "Big Bang"



John A. Gowan and August T. Jaccaci February, 2013

The leptoquark is the heaviest member of the leptonic spectrum—so heavy that it splits into three quarks bearing partial charges (as a more stable solution to the self-repulsion of its own electric charge). The leptoquark is therefore the physical upper mass limit to the leptonic series of elementary particles. As an elementary particle, the leptoquark has its own neutrino (vLQ), which is a natural candidate for the "dark matter" WIMP. Leptoquarks originally form as matter–antimatter charged pairs, but the "Y" IVBs can produce a decay to an electically neutral quark configuration (see below). These electrically neutral leptoquarks go on to decay asymmetrically via the "X" IVB of the next lower (G.U.T.) energy level, producing an excess of matter baryons, which decay in turn (via the "W" IVBs) to produce the particles of the electroweak domain. Every lepton can transform into every other given sufficient energy and the mediation of the appropriate IVB.

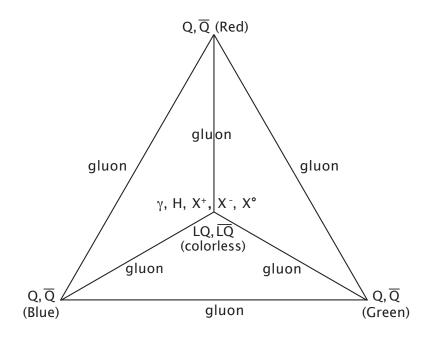
 $LQ^{-} \xrightarrow{(Y^{-})} LQ^{\circ} + e^{-} + \overline{ve}$ 

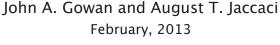
See: "The Origin of Matter and Information": http://www.johnagowan.org/origin.html

(See: Fig. 2 <u>"The Symmetry Groups of Light: leptoquarks"</u>. (Grand Unified Theory (GUT) Energy Level - strong and electroweak forces unified.)

Fig. 2: The Symmetry Groups of Light: Leptoquarks

# THE SYMMETRY GROUPS OF LIGHT: LEPTOQUARKS. G.U.T. ENERGY DOMAIN (GRAND UNIFIED THEORY: UNION OF STRONG AND ELECTROWEAK FORCES)





See: http://www.johnagowan.org/galois.html

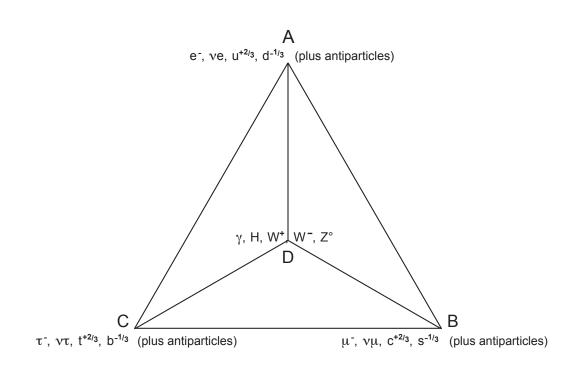
A primordial heavy lepton (or anti-lepton) splits into 3 quarks, forming a leptoquark (or anti-leptoquark). Quarks are transformed to red, blue or green "colors" by gluon exchange (strong force confinement via color charges, perimeter "vertices"). "X" IVBs compress quark and gluon fields, creating a colorless leptoquark ("central" vertex). Subsequent asymmetric weak force decays (via the "X°" IVB) of electrically neutral leptoquarks result in an excess of matter baryons (and leptoquark anti-neutrinos), creating the electroweak domain (anti-particles with overbar):

$$LQ^{\circ} \times \overline{LQ}^{\circ} \xrightarrow{(X^{\circ})} \overline{\nu LQ} + e^{+} + e^{-} + N^{\circ}$$

See: "The Origin of Matter and Information": http://www.johnagowan.org/origin.html

(See: Fig.3 <u>"The Symmetry Groups of Light: Particles"</u>. (Electroweak (EW) Energy Level - electromagnetic and weak forces unified.)

# THE SYMMETRY GROUPS OF LIGHT: PARTICLES



John A. Gowan and August T. Jaccaci Feb. 12, 2013 http://www.johnagowan.org/partable.html

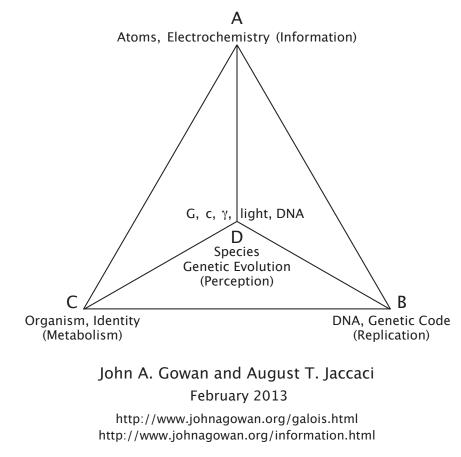
The particles of the "Standard Model" represented as a symmetry group in the format of the "Tetrahedron Model." The four vertices are: A) electron, electron neutrino, up and down quarks (plus antiparticles); B) muon, muon neutrino, charm and strange quarks (plus antiparticles); C) tau, tau neutrino, top and bottom quarks (plus antiparticles); D) photon, Higgs, IVBs (W<sup>+</sup>, W<sup>-</sup>, Z°). Each vertex can "rotate" or transform into every other. IVBs facilitate and control all transformations. The diagram illustrates the convergence of the "Tetrahedron" and "Standard" models in terms of particles within the electroweak energy domain.

(See: Fig. 4: "The Symmetry Groups of Light: The Electromagnetic Ground State".)

Fig. 4: The Symmetry Groups of Light: The Electromagnetic Ground State

# THE SYMMETRY GROUPS OF LIGHT: Electromagnetic Ground State (chemical interactions only)

Life, biology, animate Information, consciousness. The Universe awakens to and explores itself. The Body and Brain Electric. Life is the sacred mystery of the Universe and the Electromagnetic Domain.



- A. Electron shell orbitals, electron, photon, electromagnetic field, electric charge, electric and magnetic currents, "velocity c".
- B. Chemical bonding, molecular information.
- C. Integrated functional systems information (genome).
- D. Electrochemical brain; perceptual information; consciousness (Gravity modifies the EM metric, creating galaxies, stars, planets.)

#### Links:

**Unified Field Theory and Symmetry** 

Section I: Introduction to Unification

Symmetry Principles of the Unified Field Theory (a "Theory of Everything") - Part I; Part II; Part III (summary) Unified Field Table: Simple Form Principles of the Unified Field Theory: A Tetrahedral Model Synopsis of the Unification Theory: The System of Spacetime Synopsis of the Unification Theory: The System of Matter Global-Local Gauge Symmetries and the "Tetrahedron Model" - Part I (introduction); Part IIa (gravity); Part IIb (gravity); Part III (weak force); Part IV (general); Part V (survey) The "Tetrahedron Model" vs the "Standard Model" of Physics: A Comparison A Short Course in the Unified Field Theory Section X: Introduction to Conservation Section IX: Symmetry: Noether`s Theorem and Einstein's "Interval" Section XIV: Causality

# Gravitation

Section II: Introduction to Gravitation Why Gravity? A Rationale for Gravitation A Description of Gravitation The Double Conservation Role of Gravitation: Entropy vs Symmetry Extending Einstein's "Equivalence Principle" The Conversion of Space to Time "Dark Energy" and the "Accelerating Universe": Does Light Produce a Gravitational field?

## Entropy

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## Information

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The Destruction of Information
The Information Ladder (table)
"A List of the Possible Quark Combinations of the Baryons and Their Charges"
"Flavor Combinations of Baryons Containing U.D or C.S Quarks"
"Flavor Combinations of Baryons Containing C.S or T.B Quarks"
"Flavor Combinations of Baryons Containing U,D or T,B Quarks".)
"The Symmetry Groups of Light: The Electromagnetic Ground State" (EM ground state energy level)
"The Symmetry Groups of Light: The Leptonic Spectrum" (TOE energy level)
The Symmetry Groups of Light: Particles (electroweak energy level)
"The Symmetry Groups of Light: Leptoquarks" (GUT energy level)
"The Four Forces Produce Light"

# Weak Force, Intermediate Vector Bosons ("IVBs"); Strong Force

Section IV: Introduction to the Weak Force The "W" IVB and the Weak Force Mechanism (html file) The Weak Force: Identity or Number Charge The Weak Force "W" Particle as the Bridge Between Symmetric (2-D) and Asymmetric (4-D) Reality The Strong and Weak Short-Range Particle Forces Section XVIII: The Strong Force: Two Expressions Section XVI: Introduction to the Higgs Boson The "Higgs" Boson and the Spacetime Metric The "Higgs" Boson and the Weak Force IVBs - Part I; Parts II, II, IV The Higgs Boson and the Evolutionary Eras of the Cosmos The Particle Table

## References

Pierre Teilhard de Chardin *The Phenomenon of Man*. French: Editions du Seuil, Paris, 1955. English: Harper and Row, New York, 1959.

Books by my late father Prof. John Curtis Gowan "Trance, Art, Creativity" An Investigation of the Numinous Element and the Metaphysical Realm. A Book by Prof. John C. Gowan, Sr. "Operations of Increasing Order" Further Investigations of the Numinous Element and the Metaphysical Realm. A Book by Prof. John C. Gowan, Sr. "Development of the Psychedelic Individual". A Book by Prof. John C. Gowan, Sr. "Development of the Creative Individual". A Book by Prof. John C. Gowan, Sr.

Keith Devlin *The Language of Mathematics* Chapt. 5 "The Mathematics of Beauty", 1998 W. H. Freeman & Co. (Holt Paperbacks).

Ian Stewart *Why Beauty is Truth* Chapt. 13 "The Five Dimensional Man", Basic Books 2007. Roy McWeeny *Symmetry*, 2002, Dover Pub. Inc.