THE EXISTENCE OF THE TWIN PRIMES

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ABSTRACT

The author had published a paper on the solutions for the twin primes conjecture in an international mathematics journal in 2003. This paper approaches the twin primes problem through the analysis of the intrinsic nature of the prime numbers.

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Theorem:- The twin primes are infinite.

Proof:-

We note a very important intrinsic characteristic of the primes. Like all the houses in a neighbourhood or location which are separated from each other by the number of houses between them, the primes are also separated from each other by the number of integers separating them. The closest will of course be the prime neighbours separated by 2 integers (i.e., twin primes), followed next in proximity by the prime neighbours separated by 4 integers, then by the prime neighbours separated by 6 integers, the prime neighbours separated by 8 integers, the prime neighbours separated by 10 integers, the prime neighbours separated by 12 integers, and so on, by larger and larger intervals, to infinity, as is shown in the appendix. The twin primes are actually comparable to 2 closest neighbours living just next door to one another. There will always be 2 closest next-door neighbours, neighbours living 2 doors away, neighbours living 3 doors away, neighbours living 4 doors away, neighbours living 5 doors away, neighbours living 6 doors away, and so on, by greater and greater intervals, in any neighbourhood, any residential area; there will always be different intervals separating all the houses in a neighbourhood or location. Similarly, in the infinite list of the primes, there will always be different intervals separating all the primes, ranging from the smallest interval of 2 integers (in the case of the twin primes), 4 integers, 6 integers, 8 integers, 10 integers, 12 integers, and more and more integers, to an infinite number of integers, which is an intrinsic characteristic of the primes. In other words, there will always be intervals of various magnitudes or sizes (i.e., intervals of various numbers of integers) between, separating, all the primes in the infinite list of the primes, and, each of these intervals of various magnitudes or sizes should be intrinsically infinite in order that the list of the primes is infinite. The twin primes, which we are examining here, are not likely to be finite (as is evident from the appendix), and should of course be intrinsically infinite; in fact, to say that the twin primes are finite is like saying that next-door neighbours who are closest, in a neighbourhood or residential area, are rare and limited, which is absurd.

Hence, our conclusion that the twin primes are infinite.

APPENDIX

Anecdotal Evidence Of The Infinity Of The Twin Primes

TOP TWIN PRIMES IN 2000, 2001, 2007 & 2009

In the year 2000, $4648619711505 \times 2^{60000} \pm 1$ (18,075 digits) had been the top twin primes pair which had been discovered. In the year 2001, it only ranked eighth in the list of top 20 twin primes pairs, with $318032361 \cdot 2^{107001} \pm 1$ (32,220 digits) topping the list. In the year 2007, in the list of top 20 twin primes pairs, $318032361 \cdot 2^{107001} \pm 1$ (32,220 digits) ranked eighth, while $4648619711505 \times 2^{60000} \pm 1$ (18,075 digits) was nowhere to be seen; 2003663613*2^195000-1 and 2003663613*2^195000+1 (58,711 digits), which was discovered on January 15, 2007, by Eric Vautier (from France) of the Twin Prime Search (TPS) project in collaboration with PrimeGrid (BOINC platform), was at the top of the list. As at August 2009, $65516468355 \cdot 2^{333333}-1$ and $65516468355 \cdot 2^{333333}+1$ (100,355 digits) is at the top of the list of top 20 twin primes pairs, while $318032361 \cdot 2^{107001} \pm 1$ (32,220 digits) ranks 11th., and, 2003663613*2^195000-1 and 2003663613*2^195000+1 (58,711 digits) ranks second in this list.

We can expect larger twin primes than these extremely large twin primes, much larger ones, infinitely larger ones, to be discovered in due course.

LIST OF PRIMES PAIRS FOR THE FIRST 2,500 CONSECUTIVE PRIMES, 2 TO 22,307, RANKED ACCORDING TO THEIR FREQUENCIES OF APPEARANCE

S. No.	Ranking	Prime Pairs	No. Of Pairs	Percentage
(1)	1	primes pair separated by 6 integers	482	19.29 %
(2)	2	primes pair separated by 4 integers	378	15.13 %
(3)	3	primes pair separated by 2 integers (t. p.)	376	15.05 %
(4)	4	primes pair separated by 12 integers	267	10.68 %
(5)	5	primes pair separated by 10 integers	255	10.20 %
(6)	6	primes pair separated by 8 integers	229	9.16 %
(7)	7	primes pair separated by 14 integers	138	5.52 %
(8)	8	primes pair separated by 18 integers	111	4.44 %
(9)	9	primes pair separated by 16 integers	80	3.20 %
(10)	10	primes pair separated by 20 integers	47	1.88 %
(11)	11	primes pair separated by 22 integers	46	1.84 %
(12)	12	primes pair separated by 30 integers	24	0.96 %
(13)	13	primes pair separated by 28 integers	19	0.76 %
(14)	14	primes pair separated by 24 integers	16	0.64 %
(15)	15	primes pair separated by 26 integers	10	0.40 %
(16)	16	primes pair separated by 34 integers	9	0.36 %
(17)	17	primes pair separated by 36 integers	5	0.20 %
(18)	18	primes pair separated by 32 integers	2	0.08 %
(19)	18	primes pair separated by 40 integers	2	0.08 %
(20)	19	primes pair separated by 42 integers	1	0.04 %
(21)	19	primes pair separated by 52 integers	1	0.04 %

Total No. Of Primes Pairs In List: 2,498

It is evident in the above list that the primes pairs separated by 6 integers, 4 integers and 2 integers (twin primes), among the 21 classifications of primes pairs separated by from 2 integers to 52 integers (primes pairs separated by 38 integers, 44 integers, 46 integers, 48 integers & 50 integers are not among them, but, they are expected to appear further down in the infinite list of the primes), are the most dominant, important. There is a long list, an infinite list, of other primes pairs, besides those shown in the above list, which also play a part as the building-blocks of the infinite list of the integers.

The list of the integers is infinite. The list of the primes is also infinite. The infinite primes are the building-blocks of the infinite integers - the infinite odd integers are all either primes or composites of primes, and, the infinite even integers, except for 2 which is a prime, are all also composites of primes. Therefore, all the primes pairs separated by the integers of various magnitudes, as described above, can never all be finite. If there is any possibility at all for any of these primes pairs to be finite, there is only the possibility that a number of these primes pairs are finite (but never all of them). However, will it have to be the primes pairs separated by 2 integers or twin primes (which are the subject of our investigation here), which are the only primes pair, or, one among a number of primes pairs, which are finite? Why question only the infinity of the primes pairs separated by 2 integers, the twin primes? Are not the infinities of the primes pairs separated by 8 integers and more, whose frequencies of appearance are lower, as compared to those of the primes pairs which are separated by 6, 4 and 2 integers respectively, in the above list of primes pairs, more questionable? Why single out only the twin primes? (There are at least 18 other primes pairs, separated by from 8 integers to 52 integers, whose respective infinities should be more suspect, as is evident from the above list of primes pairs, if any infinities should be doubted. Evidently, the primes pairs separated by 2 integers (twin primes) are not that likely to be finite.)

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