Reinterpreting Solutions of Maxwell's Equations in Vacuum

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Abstract: The authors show that the solutions of Maxwell's equations in vacuum admit electromagnetic waves having oscillations not only in the electromagnetic field but also in spatial displacement which leads to the helical structure of the wave. In the conventional approach the path of the electromagnetic wave is represented by a straight line, and the sinusoidal (non-mechanical) oscillations of the electric and magnetic fields at every point on the straight line constitute the wave. But with the introduction of the spatial amplitude, the straight line path will have to be modified into a helical one. The reason why the existence of such a spatial component of the electromagnetic wave has remained unknown may be due to the fact that it falls outside the limits of accuracy of observation. It is obvious that the existence of the spatial amplitude will warrant the introduction of a new basic field. It has to be examined if this basic field could be identified with the Higgs field.

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I INTRODUCTION

We know that electromagnetic field and its interactions have been thoroughly investigated during the last 150 years leaving hardly any scope for further improvement in our understanding. While classical electrodynamics has been able to explain a vast range of interactions, the accuracy obtained in predicting these phenomena using quantum electrodynamics has been truly astounding. Nevertheless, we should remember that there are certain problems in the theory of electromagnetic fields which have remained intractable to this day. For example, we still do not have a clear picture as to how the electric charge gets compacted to a point without causing the problems of infinity [1]. We still do not know what the fine structure constant actually is except that it some way or other represents the relative strength of the electromagnetic interactions [2]. We also do not have a clear idea how the amplitude of the oscillations of the electric and magnetic fields of the electromagnetic wave are related to its frequency. Possibly, the mystery of the fine structure constant may get unraveled if we could strike a connection between these properties of the electromagnetic wave.

As a first step we propose to re-examine the basic concepts which go to the conventional representation of the electromagnetic wave. It is quite possible that a slight tweaking of the conventional picture while not altering the situation where the current theories reign supreme may provide us with some new insights. The one idea we propose to examine in this paper is the implication of introducing spatial amplitude to the electromagnetic wave. Before we proceed further in this direction we shall have a brief review of the solutions of Maxwell's equation in vacuum and see how the conventional picture emerged from it.

We know that the beauty of the Maxwell's equations is that while they are very simple linear equations, all classical aspects of the electromagnetic phenomena can be explained by them. Since we propose to confine ourselves to the study of the transmission of electromagnetic waves in vacuum, we shall confine ourselves to Maxwell's equations in vacuum given by

$$\nabla \cdot \boldsymbol{\xi} = 0, \tag{1a}$$

$$\nabla \times \xi = -\frac{\partial \boldsymbol{B}}{\partial \boldsymbol{t}}. \tag{1b}$$

$$\nabla \cdot \boldsymbol{B} = 0\,, \tag{1c}$$

$$\nabla \times \boldsymbol{B} = \frac{1}{c^2}\frac{\partial \xi}{\partial t}. \tag{1d}$$

We know that for the electric and magnetic fields these equations have solutions in the form [3][4] (Appendix A)

$$\boldsymbol{\xi} = \boldsymbol{\xi_o}\, sin(\omega t - \boldsymbol{k}.\boldsymbol{r}) \ . \tag{2}$$

$$\boldsymbol{B} = \boldsymbol{B_o}\, sin(\omega t - \boldsymbol{k}.\boldsymbol{r}) \ . \tag{3}$$

Note that the magnetic field will always be perpendicular to the electric field and both will in turn be perpendicular to the direction of propagation. Another important point to be kept in mind is that the solutions represent not a single wave, but a wave front that has the same values for the electric (and magnetic) field at any instant in the transverse direction.

II INTRODUCING SPATIAL AMPLITUDE TO THE ELECTROMAGNETIC WAVE

The proposal that the electromagnetic wave possesses spatial amplitude was one of the most actively debated topics towards the end of the 19th century. Going by the analogy with the mechanical waves, it was generally believed by physicists during that period that the electromagnetic wave which is a transverse wave needs a physical medium to propagate in space[5]. They named the medium ether. It was assumed that the oscillations in the electric and magnetic fields were set off by similar spatial oscillations in ether. But the problem with such a medium was that it had to possess all sorts of properties which were quite often incompatible with each other. Besides, in a way ether functioned like an absolute frame of reference against which any motion could be reckoned. Finally when Einstein came up with his theory of relativity in 1905 to explain the constancy of the velocity of light in all inertial frames of references, the concept of ether was given a decent burial. It became generally accepted that the electromagnetic waves were generated by oscillations in electric and magnetic fields only. Such an approach has further justification in that it is based entirely on Maxwell's equations which deal only with variations in the electric and the magnetic fields. These equations do not involve variations in the spatial displacement or variations in any other field. Here it is worthwhile to note that after formulating the General theory of relativity in 1915, Einstein appears to have changed his views and was favorably disposed towards the existence of ether [6]. In recent years the idea of ether has undergone a revival and is proposed to be connected to the concept of dark matter [7].

There is one more important reason for taking the spatial amplitude of the electromagnetic wave as zero. If the electromagnetic wave possessed spatial amplitude, then two identical waves in phase will have twice the spatial amplitude as compared to a single one. This would mean that the spatial amplitude of the electromagnetic waves in phase will be directly proportional to the intensity of the waves. Therefore, the light transmitted through a small aperture should behave differently when the intensity of light is changed. For example, a low intensity coherent light beam may travel through a small aperture without any loss of energy while a high intensity beam would get obstructed substantially by the aperture. But we know light does not behave in that manner. On the other hand, it is observed that the oscillations in the electric (also magnetic) field add up when two such waves occupy the same space. The only way this difference can be resolved is by treating the amplitude of the spatial oscillations as zero.

We know that at any point on the path of the electromagnetic wave the divergence of the electric field (also magnetic field) in the transverse direction at any instant is zero. In fact the electromagnetic wave has to be seen as progressing as a wave front, rather than as a single wave progressing along a straight line. The progression of the wave may be represented by successive wave fronts which are normal to the direction of motion. Note that the wave front is assumed to extend to infinity in the transverse plane. In other words the introduction of the spatial amplitude does not appear to be incompatible with Maxwell's equations. In the next section we shall go a step further and show that the electromagnetic wave could be attributed a helical structure which will provide us with a simple picture of photon that accounts for its spin angular momentum.

It is reasonable to assume that the spatial oscillations propagating at luminal velocities constitute the basic wave that transports the electromagnetic oscillations and these two oscillations could be having a phase difference of zero orπ. If we denote the spatial amplitude by $\boldsymbol{\eta_o}$, then we may represent the plane polarized spatial and electromagnetic waves by

$$\boldsymbol{\eta} = \pm\, \boldsymbol{\eta_o} \sin(\omega t - \boldsymbol{k}.\boldsymbol{r}) \quad ; \tag{4a}$$

$$\xi = \xi_{\boldsymbol{o}} \sin(\omega t - \boldsymbol{k}.\boldsymbol{r}) \tag{4b}$$

The reason why the oscillations in the electric and the spatial displacement could combine only in two ways may be due to the fact that they represent two different fields. If the spatial oscillations are allowed with the oscillations of the electric (and magnetic) field in all possible phases, then in the case where the wave is trapped between two reflecting mirrors we may have a situation with the standing wave formed is entirely by the oscillations in the electromagnetic field while the spatial oscillations get destroyed completely. It could be the other way round also. Such joining of the oscillations may be disallowed in order to avoid such situations.

Note that the introduction of the spatial oscillations in (4a) does not alter the situation represented by the solutions given in (2) and (3). The electromagnetic wave represented by (2) and (3) continues to represent the electric and magnetic fields in the transverse planes and their variations with time. In other words, the spatial wave defined by equations (4a) and (4b) is consistent with Maxwell’s equations.

III PHOTON AS A HELICAL CIRCULARLY POLARIZED WAVE

We saw that the solution of Maxwell’s equations in vacuum is expressed as a sine function in (2). We could have taken the solution as a cosine function also. In fact, we could have taken the linear combination of the sine and cosine waves which would represent a circularly polarized wave. In that case, treating the wave as progressing along the Z-direction, we may represent the circularly polarized (left handed when viewed head on) spatial and electromagnetic waves by

$$\boldsymbol{\eta_x} = \boldsymbol{\eta_o} \cos(\omega t - kz) \tag{5a}$$

$$\boldsymbol{\eta_y} = \boldsymbol{\eta_o} \sin(\omega t - kz) \tag{5b}$$

$$\xi_{\boldsymbol{x}} = \xi_{\boldsymbol{o}} \cos(\omega t - kz) \tag{5c}$$

$$\xi_{\boldsymbol{y}} = \xi_{\boldsymbol{o}} \sin(\omega t - kz) \tag{5d}$$

Here we have taken only the case where the electric and spatial oscillations are in phase. Initially when the concept of photon was introduced by Einstein, it was treated as the particle aspect of the electromagnetic waves. The wave nature becomes relevant when we take a large group of the photons which can be studied classically. But the structure of photon itself has remained an enigma. The classical picture of the wave and the quantum picture of the particle have remained irreconcilable. There is one more reason for this incompatibility. The classical picture deals with only the propagation of the wave front. The idea of a wave train is not properly defined in the classical approach. With the introduction of the spatial amplitude as explained in the previous section, photon could be attributed the internal structure of a helical wave. To be exact, photon has to be treated as a helical wave train with the electric (and magnetic) field riding on it making it circularly polarized.

We shall now show that the helical structure of the electromagnetic wave will force spatial amplitude $\boldsymbol{\eta_o}$ to possess only a certain value. We know that the projection of the helical wave given in equations (5a) to (5d) on to the transverse X-Y plane will be a circle (*fig.*1). Let us now estimate the velocity of the circular motion. The simplest case that comes to our mind is one where the circular motion occurs at the velocity of light. Any other velocity will involve introduction of anew attribute to the electromagnetic wave. Since the wave is also progressing with velocity c along Z-axis, it is obvious that by the time the wave travels one wave length along Z-axis, it would have executed one full circle in the transverse directions.

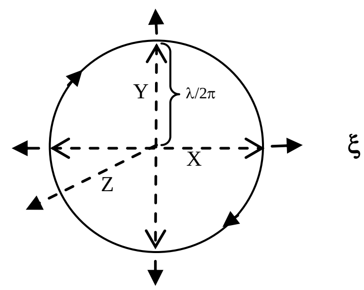


Fig.1. This is the projection of a circularly polarized wave on a transverse plane. The vertical and the horizontal lines stand for two spatial waves having a phase difference of $\frac{1}{2}\pi$. Here we have also shown arrows pointing inward representing the electric field at these points having a phase difference of π with the spatial oscillations.

Therefore, we may conclude that the radius, R of the circle will be given by

$$R \;=\; \lambda/2\pi \;=\; 1/k \tag{6}$$

Here we have a clearer idea of the term helicity used in the case of the electromagnetic wave because now we have the case of the spatial amplitude of the wave spinning around an axis. This allows us to understand the concept of spin of the electromagnetic wave classically on the basis of the helical structure of the wave. Since a point on the electromagnetic wave executes rotational motion in the transverse direction, we may take the magnitude of its angular momentum for this motion to be

$$S \;=\; |\boldsymbol{r} \times \boldsymbol{p}| \;=\; (\lambda/2\pi)(h/\lambda) \;=\; \hbar \tag{7}$$

Note that the momentum $\boldsymbol{p}$ used in the above expression represents the momentum of the circular motion in the transverse direction. Of course, the magnitude of the momentum in the direction of progression also has the same value.

IV COMPACTING OF THE SPATIALAMPLITUDEINTO THE INTERNAL SPACE

We shall now examine how the superposition of two waves affects the spatial amplitude. Let us consider two identical waves both in phase represented by equations (5a), (5b), (5c) and (5d). If we go by the classical theory of the mechanical waves, resultant wave will be given by

$$\boldsymbol{\eta_x}' \;=\; 2\boldsymbol{\eta_o}\cos(\omega t - kz) \tag{8a}$$

$$\boldsymbol{\eta_y}' \;=\; 2\boldsymbol{\eta_o}\sin(\omega t - kz) \tag{8b}$$

$$\boldsymbol{\xi_x}' \;=\; 2\boldsymbol{\xi_o}\cos(\omega t - kz) \tag{8c}$$

$$\boldsymbol{\xi_y}' \;=\; 2\boldsymbol{\xi_o}\sin(\omega t - kz) \tag{8d}$$

We observe that the resultant wave has amplitude which is twice that of the individual component. Although prima-facie (8a) and (8b) appear to be in order, on detailed scrutiny we encounter a serious problem. We observe that while the spatial amplitude has doubled, the wave length of the wave has remained unchanged. This means that when we project the helical wave on to the transverse plane, the circle so obtained will have a radius of λ/π. Therefore, if the integrity of the wave is to be retained in propagation, the velocity of the circular motion will have to be twice that of light which is not acceptable. Note that this problem arises only in the case of the spatial oscillations. It does not arise in the case of the oscillations in the electric and magnetic fields.

The solution to this problem can be found if we treat electromagnetic waves as constituted by a large number of photons. Note that we represent a photon as a wave train of circularly polarized helical waves. Here the spatial oscillations of the waves do not interfere with others by superposition. Only the electric and magnetic fields need to be affected by superposition. The spatial amplitude remains unaffected by superposition for reason that the velocity of circulation in the transverse plane can only be c. In other words two photons travelling in the same direction cannot be represented by waves in the classical sense by adding up their spatial amplitudes. Note that this does not disallow spatial displacement of waves interfering with each other destructively as the resultant velocity in that case will be less than c. As regards the oscillations in the electric and magnetic fields, they will behave in the classical manner interfering with each other constructively or destructively according to the phase difference between the waves.

The picture described above will be identical to the conventional picture provided we treat the diameter of the helical wave as negligibly small and equate it to zero for all practical purposes. But once the radius is treated as zero, it becomes impossible to account for the spin angular momentum of the wave. Therefore, spin will have to be accounted for by introducing an internal space to the wave and defining it there. Note that this procedure is nothing but compacting the radius of the helical wave into the internal coordinates. In such a mathematical construct the helical path of the electromagnetic wave becomes a straight line and the internal space will have the form of a cylindrical tube.

We should understand that compacting the spatial amplitude into the internal space is a mathematical program which resolves the problem of the superluminal velocity of the wave in the transverse plane. Since compacting involves treatment of the spatial amplitude as zero it enables us to treat the wave propagation based on the laws of classical mechanics and electromagnetism. We should keep in mind that the so called internal space is actually an innate part of the external space which is spanned by the laboratory coordinate system. The process of compacting of the amplitude into the internal coordinates should be understood as just a mathematical procedure to account for the peculiar nature of the spatial oscillations in the electromagnetic wave propagation.

Since this compacting is a mere mathematical construct one may presume that the spatial amplitude of the electromagnetic wave could be experimentally measured to the required accuracy. In the light of the above discussion we are tempted to conjecture that a coherent beam of light passing through a circular aperture of diameter less than λ/π (being the spatial amplitude of the electromagnetic wave) should undergo cut off transmission thereby firmly establishing the existence of the spatial oscillations in the electromagnetic wave. But unfortunately the situation is not that simple as there is a cut off in the intensity of the transmitted wave when the aperture has a diameter of only $\lambda/2$ due to certain other properties of the electromagnetic wave propagation [4]. Therefore, it will not be possible to observe the attenuation of the wave transmission at the aperture radius of λ/π. In fact, this could be one of the reasons why the existence of the spatial oscillations in the electromagnetic wave has remained unnoticed.

V DEFINING VECTOR POTENTIAL IN THEREAL SPACE-TIME

In the conventional approach the vector potential is introduced as a mathematical construct. However, the Aharanov-Bohm effect that has been experimentally observed confirms the physical reality of the vector potential. We shall show that with the introduction of the spatial amplitude to the electromagnetic wave the physical reality of the vector potential can be visualized. We know that the electric and magnetic fields can be expressed in terms of the vector potential as

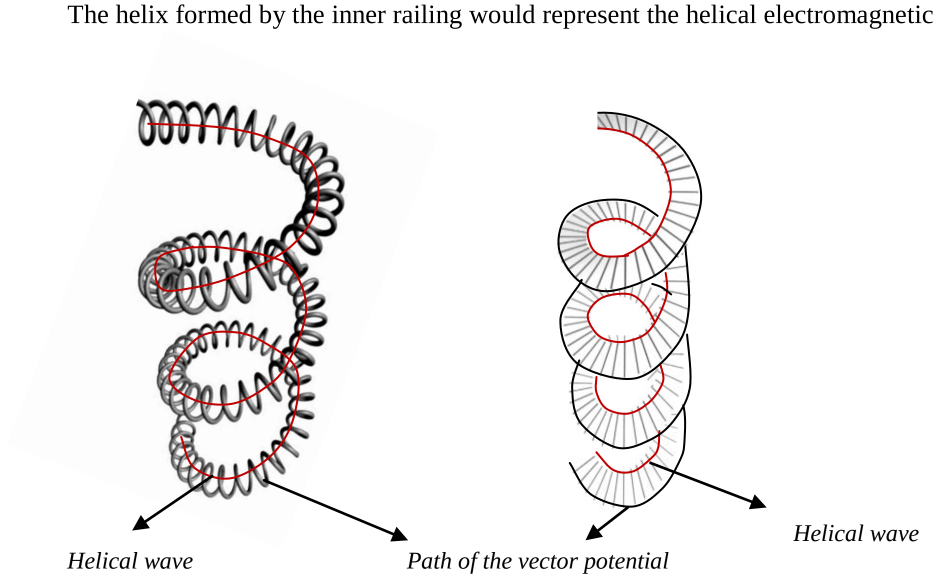
$$\boldsymbol{\xi} = -\nabla\phi - \frac{\partial \mathbf{A}}{\partial t}, \tag{9a}$$

$$\mathbf{B} = \nabla \times \mathbf{A} \tag{9b}$$

Note that we have to ignore the gradient of the scalar potential in the expression for the electric field as we are dealing with vacuum which is a charge free state. Let us now examine how the vector potential will fit into the helical structure of the electromagnetic wave.

The best way to introduce vector potential is to attribute it a path along a solenoid which wraps tightly around the helical path. In other words, the path of the vector potential will be along a solenoid which in turn forms a helix (fig. 2a). But the problem with the figure given in 2(a) is that the direction of $-\partial \mathbf{A}/\partial t$ will be always directed to the axis of the solenoid and therefore in terms of equations (9a) and (9b) the electric field (we may take ϕ to be zero as we are dealing with charge free case) will keep undergoing rotation at a higher frequency than the frequency of the helical wave itself with the result that the electric field cannot remain pointed in the radial direction of the helical wave as demanded by Maxwell's equations. However, this problem can be resolved if we assume that the vector potential completes only one solenoid in one wavelength of the helical wave. In that case the path of the helical wave and the vector potential could be represented by a helical stair case (fig. 2b).

The helix formed by the inner railing would represent the helical electromagnetic wave



(a) (b)

Fig.2. (a) shows a solenoid that completes a number of turns around a helical path as it completes one full rotation. In (b) the solenoid completes only one turn by the time the helical path completes one turn. The helical path and the path of the solenoid run parallel to each other like the two sides of a helical stair case.

while the helix formed by the outer railing could represent the path of the vector potential. Note that fig.(2b) is obtained from fig.(2a) by taking the number of solenoid contained in one helix to be unity. The direction of the electric field will be parallel to the steps which are represented by lightly shaded thin lines in the radial direction. It can be easily shown (Appendix B) that the curl of **A** which represents the magnetic field(not shown in fig.2b) will be directed perpendicular to both the electric field and the direction of propagation and will be therefore in line with the conventional solutions of Maxwell's equations.

VI GAUGE INVARIANCE OF THE SPATIAL AMPLITUDE AND THE REAL NATURE OF THE VECTOR POTENTIAL

We know that Maxwell's equations possess the gauge freedom in terms of vector potential and scalar potential which satisfy the relations

$$A` \;=\; A + \nabla\chi \qquad (10a)$$

$$\phi` \;=\; \phi - \partial\chi/\partial t \qquad (10b)$$

where χ satisfies the wave equation

$$\nabla^2\chi \; - \; \left(\frac{1}{c^2}\right)\frac{\partial^2\chi}{\partial^2 t} \; = \; 0 \tag{11}$$

Needless to say equation (11) is the direct result of the introduction of the Lorenz gauge condition [2]

$$\nabla . \boldsymbol{A} + \left(\frac{1}{c^2}\right)\frac{\partial \phi}{\partial t} \; = \; 0 \qquad . \tag{12}$$

It can be easily shown that the values of the electric field and the magnetic field given by equations (9a) and (9b) are independent of χ. In fact, till recently it was presumed that only $\boldsymbol{\xi}$ and **B** represent whatever is observable in an electromagnetic field. The vector potential **A** and the scalar potential ϕ were not considered to be observable entities. However, we know that this assumption had to be changed after the discovery of Aharonov-Bohm effect [6].

Let us now examine the gauge transformation given in (10a) and (10b) to find out what χ stands for in the light of the helical wave structure of the electromagnetic wave. In the previous section we had taken **A** to be a vector with constant magnitude defined at every point on the solenoid which wraps around the helical path of the electromagnetic wave directed along its tangent. In the more general case we may attribute a radial component at every point on the solenoid. Therefore, we may resolve the vector potential A' as

$$\boldsymbol{A'} \quad = \quad \boldsymbol{A_T} \; + \; \boldsymbol{A_r} \tag{13}$$

where $\mathbf{A}_T$ is the component tangential to the solenoid while $\mathbf{A_r}$ is normal to it. We ignore the component of **A** along the direction of propagation as it will have a constant value and therefore will be of no interest for the purpose on hand. Since $\mathbf{A}_r$ is in the direction of the radial vector $\boldsymbol{\eta}$, we may express it as a gradient in the form

$$\boldsymbol{A}_r \quad = \; \nabla\chi \tag{14}$$

provided the amplitude of the χ wave is symmetric around the axis of the helical wave in the transverse direction. We shall express the exact form of the χ wave shortly. On the basis of the above equation we have

$$\nabla \times \mathbf{A}' \quad = \quad \nabla \mathrm{x} \mathbf{A}_T \tag{15}$$

since

$$\nabla \times \mathbf{A}_r \; = \; 0 \tag{16}$$

As far as the electric field is concerned, the component $\mathbf{A}_r$ gets canceled from the right hand side of (9a) and therefore it will not contribute to the observable electric field. This means that whatever be its value, $\mathbf{A}_r$ does not contribute to the electric and magnetic fields. We may now re-express (13) as

$$\boldsymbol{A}' \quad = \quad \boldsymbol{A}_T \; + \; \nabla\chi \tag{17}$$

In the light of the above analysis we also observe that the second term of the Lorenz gauge condition given in (12) will always be zero because ϕ is zero everywhere in the charge free situation. This means that $\nabla.\mathbf{A} = 0$. Such a result is possible only if the vector potential **A** has no contribution from $\mathbf{A}_r$ and is constituted entirely by A_T. This is the essence of the Coulomb gauge condition in the charge free case. Here we see that Coulomb gauge condition and Lorenz gauge condition coincide.

We know that the electric and the magnetic fields are defined only on the helical path of the electromagnetic wave. But this does not mean that **A** and χ are also defined only on the helical path of the wave. We shall show that even points removed from the helical path may be identified with specific values of **A** and χ. Here we should keep in mind that A and χ possess directional symmetry in a plane transverse to the direction of propagation of the helical wave. Therefore, the value of **A** at a point could be taken as a function of r, where r is the radial distance of the point from the axis of the helical wave. In other words, if we take a transverse cross section of the helical electromagnetic wave, each point on the plane could possess vector potential **A**(r,θ) where r and θare the polar coordinates of the point. For the sake of convenience we shall take the transverse cross section to be in the X-Y plane assuming that the electromagnetic wave is propagating in the Z-direction. We may now re-express (13) as

$$\boldsymbol{A}' \quad = \quad \boldsymbol{A}_T \; + \quad \nabla_r \chi(r) \tag{18}$$

This requirement $\nabla_r \chi(r) \;=\; \boldsymbol{A}_r$ will be met if we express χ(r) as

$$\chi(r) \quad = \quad \pm\frac{1}{r} g(t,z) \;\; = \;\; \pm\, g(t,z)(x^2 +\, y^2)^{-½} \tag{19}$$

where g(t,z) is a function of t and z. Remember that χ(r) satisfies the wave equation (11) and therefore g(t,z) represents a wave propagating in the Z-direction. It is interesting to note that eχ has the dimension of action and therefore (e/ħ)g(t,z) has the dimension of length. In other words, (e/ħ)g(t,z) would represent the length of the arc of the circle with radius $r = (x^2+y^2)^{½}$. It is obvious that in such a situation eχ/ħ which is dimensionless number could represent a rotation through a certain angle in the X-Y plane. The gauge freedom ultimately boils down to this invariance in an arbitrary rotation. Recall that in section iv we have shown that the spatial amplitude could be compressed into the internal space. Therefore, the rotation described here pertains to the internal space. In the forthcoming paper we shall show that this freedom to choose any value for χ (gauge freedom) actually translates into the invariance of the wave function in an arbitrary phase change.

VII DISCUSSION

From the above analysis it becomes quite clear that the introduction of the spatial amplitude to the electromagnetic wave not only simplifies many aspects of the theory but also provides us with a consistent physical picture. We further observe that the vector and scalar potentials are no more just mathematical constructs, but real fields which are defined in the three dimensional space.

We observe that the approach based on the classical mechanics has to treat vector potential **A** as a mathematical construct for two reasons. The first one is that the spatial amplitude cannot be introduced as there is no way to limit its value in the classical approach. We saw the limit to the spatial amplitude is imposed due to the fact that relativistically the velocity of light is the limiting velocity for any physical entity. The second requirement for a solution of the problem is the introduction of the concept of photons as the basic state of the electromagnetic waves. Without the concept of photons it will be impossible to account for the oscillations of two waves at a given spatial point as their displacements do not add up.

The introduction of the spatial amplitude to the electromagnetic wave implies the existence of a new field. It will be shown in a separate paper that electron-positron pair can be created from the confinement of the electromagnetic waves and they could be attributed the structure of confined helical half wave. The generation of the rest (mass) energy of the particle is seen to be the direct result of the localization of the energy of the electromagnetic wave contained in the spatial oscillations. It is observed that the fine structure constant is the ratio of the energy of the electromagnetic field of the electron to its rest mass. Since the fine structure constant represents the relative strength of the electromagnetic interactions, we are tempted to conclude that the rest mass of electron obtains its contribution from some other field. This leads to the suggestion that the energy of the spatial oscillations belongs to a basic field. It is worthwhile to note that the existence of the Higgs field has been mooted for quite some time as the field that creates mass [8]. We do not propose to identify the basic field with the Higgs field at this stage. If the proposed approach is to hold good, the electromagnetic wave will have to be treated as a composite wave having oscillations both in the basic field and in the electromagnetic field.

The introduction of spatial amplitude forces us to review the concept of vacuum in greater detail. In fact, it appears that we may have to attribute elastic properties to vacuum to explain the transverse spatial oscillations generated by the electromagnetic wave which may take us back to the earlier concept of ether. In this connection it is worthwhile to note that in quantum field theory the electromagnetic wave is replaced by its particulate form, photons which are represented by oscillators. In quantum field theory, these oscillators are taken as the most basic constituents of any field. Therefore when we introduce spatial amplitude, we are in way imputing internal structure to these oscillators. Such an internal structure has already been put in place in quantum field theory by way of internal space of photon where its spin is defined. Therefore compacting spatial amplitude into the internal space of the oscillators may be the most convenient and logical option. In other words, the concept of the spatial amplitude could be seamlessly joined to quantum field theory obviating the need for ether as a material medium to account for the elastic forces.

VIII CONCLUSION

Although prima-facie the idea that the electromagnetic waves possess spatial oscillations goes against all long established concepts, its existence does not warrant any modification in the conventional approach. This is because the concept of the spatial amplitude has already been accounted for in the conventional approach by the introduction of the internal space. That apart, the existence of the spatial amplitude of $\lambda/2\pi$ cannot be directly observed because even by the conventional approach where the electromagnetic wave is supposed to propagate along a straight line, a circular aperture of $\lambda/2$ will cut off transmission of waves through it. In a series of papers we shall show that we could treat electron as a confined helical wave formed from the electromagnetic wave with its spatial amplitude playing a crucial role in explaining the spin and the electric charge of a particle. Therefore, it is a comforting thought that the introduction of the spatial amplitude will in no way affect the results validated by the conventional approach. The idea that a major part of the energy of the electromagnetic wave is constituted by the oscillations in the basic field, if found acceptable, may result in a completely new way of looking at the basic structure of particles.

IX. APPENDIX - A

Maxwell's equations in vacuum is given by

$$\nabla \cdot \boldsymbol{\xi} = 0, \tag{20a}$$

$$\nabla \times \boldsymbol{\xi} = -\frac{\partial \boldsymbol{B}}{\partial \boldsymbol{t}}. \tag{20b}$$

$$\nabla \cdot \boldsymbol{B} = 0\,, \tag{20c}$$

$$c^2 \nabla \times \boldsymbol{B} = \frac{\partial \xi}{\partial t}. \tag{20d}$$

We shall now solve these equations using Feynman's insightful approach to understand how the concept of the electromagnetic wave emerges from them [2]. We know that the equation (20a) could be expanded to obtain

$$\nabla \cdot \boldsymbol{\xi} = \frac{\partial \xi_x}{\partial x} + \frac{\partial \xi_y}{\partial y} + \frac{\partial \xi_z}{\partial z} = 0 \tag{21}$$

Here we assume that there are no variations in the field variables with respect x and y, so that the first two terms could be taken as zero. Hence, we have

$$\frac{\partial \xi_z}{\partial z} = 0 \qquad (22)$$

This means that ξ_z is a constant in the Z-direction. If we study Maxwell's equation (20d), assuming that just as in the case of the electric field, the magnetic field also has no variation in X and Y directions, then it can be seen that ξ_z is also a constant in time. Such a field could be conveniently taken as zero as we are interested in only dynamic fields. Therefore we may take $\xi_z = 0$. In other words, the electric field exists only in the X and Y directions. Now as a first step, for the sake of simplicity, we may assume that the electric field has a component only in the X-direction and obtain a solution on that basis. Later we may take up the case where the electric field has a component only in the Y-direction and get the corresponding solution. Then, the general solution could always be expressed as the superposition of the two cases.

Let us take the Maxwell's equation (20b) and express the components along the three coordinate axes as

$$(\nabla \times \boldsymbol{\xi})_x = \frac{\partial \xi_z}{\partial y} - \frac{\partial \xi_y}{\partial z}, \qquad (23a)$$

$$(\nabla \times \boldsymbol{\xi})_y = \frac{\partial \xi_x}{\partial z} - \frac{\partial \xi_z}{\partial x}, \qquad (23b)$$

$$(\nabla \times \boldsymbol{\xi})_z = \frac{\partial \xi_y}{\partial x} - \frac{\partial \xi_x}{\partial y} \qquad (23c)$$

Here $(\nabla x \boldsymbol{\xi})_z$ will be zero because the derivatives with regard to x and y are zero. Note that from equation (21) we have already taken ξ_x as a constant while ξ_y is taken as zero. $(\nabla x \boldsymbol{\xi})_x$ is zero because the first term which is a derivative of ξ_z is zero while the second term is zero for reasons already stated. The only component which is not zero is $(\nabla x \boldsymbol{\xi})_y$ which is equal to $\partial\xi_x/\partial z$. Setting the three components of $(\nabla x \boldsymbol{\xi})$ equal to the corresponding components of $-\partial \mathbf{B}/\partial t$, we obtain

$$\frac{\partial B_z}{\partial t} = 0, \qquad (24a)$$

$$\frac{\partial B_x}{\partial t} = 0 \qquad (24b)$$

$$\frac{\partial B_y}{\partial t} = -\frac{\partial \xi_x}{\partial z}. \qquad (24c)$$

Since the z and x components of the magnetic field have zero time derivatives, they represent constant fields. Such a field could be conveniently taken as zero as we are interested in only dynamic fields. Therefore, we may take $B_z = B_x = 0$. The last equation in (24c) shows that the electric field has only the x-component while the magnetic field has only the y-component. This means $\boldsymbol{\xi}$ and $\mathbf{B}$ are perpendicular to each other.

Let us now take the last Maxwell's equation whose components along X, Y and Z directions could be written as

$$c^2 \left(\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}\right) = \frac{\partial \xi_x}{\partial t} \qquad (25a)$$

$$c^2 \left(\frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x}\right) = \frac{\partial \xi_z}{\partial t} \qquad (25b)$$

$$c^2 \left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}\right) = \frac{\partial \xi_y}{\partial t} \qquad (25c)$$

On the left hand side of these equations, except for $\partial B_y/\partial z$ all other terms are zero. Therefore we have

$$-c^2 \frac{\partial B_y}{\partial z} = \frac{\partial \xi_x}{\partial t}. \qquad (26)$$

Now taking partial differentiation with regard to t and using the equation (24c), we obtain the wave equations

$$\frac{\partial^2 \xi_x}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 \xi_x}{\partial t^2} = 0 \, . \tag{27}$$

and

$$\frac{\partial^2 B_y}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 B_y}{\partial t^2} = 0 \tag{28}$$

Note that the above equations represent waves having polarization in one plane. Similarly, we can obtain the equations for waves having polarization in a perpendicular plane involving only ξ_y and B_x as

$$\frac{\partial^2 \xi_y}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 \xi_y}{\partial t^2} = 0 \tag{29}$$

and

$$\frac{\partial^2 B_x}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 B_x}{\partial t^2} = 0 \tag{30}$$

The solutions for these wave equations can be written as

$$\xi_x = \xi_{xo} \, sin(\omega t - kz) \, , \tag{31a}$$

$$\xi_y = \xi_{yo} \, sin(\omega t - kz), \tag{31b}$$

$$B_y = B_{yo} \, sin(\omega t - kz) \, , \tag{31c}$$

$$B_x = -B_{xo} \, sin(\omega t - kz) \, . \tag{31d}$$

where ω is the angular frequency and k is the wave vector. Actually we could have as well taken cosine function or even a complex function of the type "$\xi_o$exp[-i(ωt-kz)]". It is a matter of convenience. However, the fact that the sine function could be expressed as a linear combination of two waves, one travelling forward in time and the other travelling reverse in time is an added advantage as the wave equation given by equation (29) and (30) possess functions representing both waves as its solutions. Combining both, the wave equation in a general direction will be given by

$$\boldsymbol{\xi} = \boldsymbol{\xi_o} \, sin(\omega t - \boldsymbol{k}.\boldsymbol{r}) \, . \tag{32}$$

Similarly, we may obtain the wave equation for the magnetic component also which may be written as

$$\boldsymbol{B} = \boldsymbol{B_o} \, sin(\omega t - \boldsymbol{k}.\boldsymbol{r}) \, . \tag{33}$$

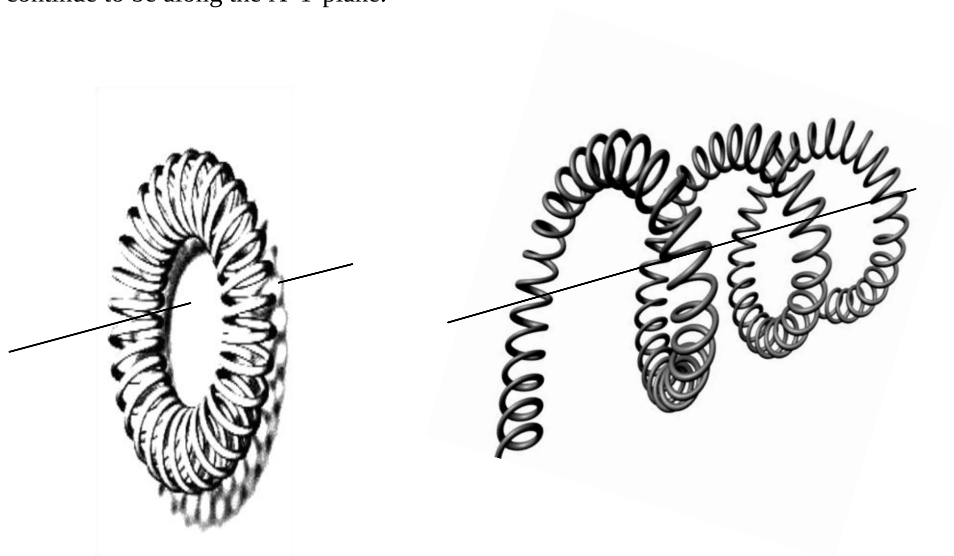
where $\boldsymbol{B_o}$ will always be perpendicular to $\boldsymbol{\xi_o}$

X. APPENDIX - B

To understand the implication of the vector potential in this picture we have to analyze its path in depth. To begin with let us consider a particle undergoing circular motion in the X-Y plane around the Z-axis. The angular momentum of such a particle in circular motion will be along the Z-axis. If such a particle is viewed by an observer who is having translational velocity along the negative direction of the Z-axis, then for him the path of the particle will be a helix. Nope that this translational motion will have no effect on the angular momentum of the circular motion which will continue to be directed along the Z-axis although the arc of the path of the particle will not be in the X-Y plane.

We shall now take the case of a toroid formed by a tightly wound solenoid bent into a circle in the X-Y plane as shown in figure 3a below with the Z-axis going through its centre.

Let us now imagine a particle moving along such a solenoid. We know that the curl of the path which is vector denoting the angular momentum of such a motion will trace a path along with the axis of the toroid which s circle. If such a particle is viewed by an observer moving along the negative side of the Z-axis with a constant translational velocity, then for him the path of the particle will be a helix formed by a solenoid as shown in fig.3b. We know that the translational motion will not have any effect on the direction of the angular momentum which will continue to be along the X-Y plane.



(a) (b)

Fig.3. (a) represents the path of point moving along a solenoid which is bent into a circle. (b)is the same path as viewed by an observer having a translational velocity towards the left.

We shall now take the solenoid helix as the path of the vector potential while the axis of the solenoid can be taken as the path of the electromagnetic wave which is a helix with its axis in the Z-direction. The magnetic field which is given by the curl of the vector potential will always be perpendicular to the Z-axis in the X-Y plane as required in terms of the solutions of Maxwell's equations. Note that the translational velocity has no effect on the direction of the curl of the vector potential which will continue to be in the X-Y plane. Now if the solenoid has a N number of windings in one wave length of the electromagnetic wave, then we will have a situation where $-\partial\mathbf{A}/\partial t$ which represents the electric field undergoing N complete rotations in one wave length of the electromagnetic wave which is not acceptable. Note that since **A** is a vector in the tangential direction $-\partial\mathbf{A}/\partial t$ will be pointing towards the axis of the solenoid which is along the helical wave. The situation can be understood if we take the analogous case of circular motion where $-\partial\mathbf{v}/\partial t$ represents the acceleration which is directed towards the centre.

We know that the electric field vector would complete only one rotation when the electromagnetic wave travels one wave length. This problem can be resolved if we assume that the solenoid in the solenoid helix is wound only once in one wave length of the helical electromagnetic wave. In that case the electric field represented by $-\partial\mathbf{A}/\partial t$ would complete only one rotation when the helical wave travels one wave length. In such a situation we can have the direction of $-\partial\mathbf{A}/\partial t$ always coinciding with the radial vector of the helical wave (see fig.2b given in section V). This picture is perfectly compatible with the solutions of Maxwell's equations.

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