The Rindler coordinate theory's expansion

Sangwha-Yi

Department of Math , Taejon University 300-716

ABSTRACT

In the general relativity theory, the Rindler coordinate theory's mathematics modernizes and the Rindler coordinate theory expands to be the Rindler coordinate theory of the accelerated observer that have the initial velocity. First, find the Rindler coordinate theory with initial velocity that used the tetrad on the new method and discover the new inverse-coordinate transformation of the Rindler coordinate theory with the initial velocity.

PACS Number:**04,04.90.+e**

Key words:The general relativity theory,

The Rindler theory,

The coordinate transformation,

The tetrad,

The initial velocity

e-mail address:sangwha1@**nate.com**

Tel:051-624-3953

I.Introduction

This theory is that the Rindler coordinate theory's mathematics modernizes and that the Rindler coordinate theory expands to be the Rindler coordinate theory of the accelerated observer that has the initial velocity.

Finding the Rindler's coordinate theory, use following the formula about the constant accelerated matter that moves in the line.

$$x+\frac{c^2}{a_0}=\frac{c^2}{a_0}\cosh(\frac{a_0\tau}{c}),t=\frac{c}{a_0}\sinh(\frac{a_0\tau}{c}) \quad (1)$$

x and t is the coordinate and the time in the inertial system about the constant accelerated matter. a_0 is the constant acceleration, τ is invariable time about the constant accelerated matter, c is light speed in the inertial system in the free space-time.

In the special relativity, if the matter that moves in the line is accelerated, the formula about inertial coordinate system $S(t,x,y,z)$ and $S'(t',x',y',z')$ is

$$V=\frac{u+v_0}{1+\frac{u}{c^2}v_0} \ , \ V=V_x=\frac{dx}{dt},u=u_x=\frac{dx'}{dt'} \ , \ dx=\frac{dx'+v_0dt'}{\sqrt{1-\frac{v_0^2}{c^2}}}, \ dt=\frac{dt'+\frac{v_0}{c^2}dx'}{\sqrt{1-\frac{v_0^2}{c^2}}},$$

$$y=y', \ z=z', \ \frac{dy}{dt}=\frac{dy'}{dt'}=0, \frac{dz}{dt}=\frac{dz'}{dt'}=0$$

$$a=\frac{d}{dt}(\frac{V}{\sqrt{1-\frac{V^2}{c^2}}}), \ a'=\frac{d}{dt'}(\frac{u}{\sqrt{1-\frac{u^2}{c^2}}}) \quad (2)$$

The velocity V has the initial velocity v_0 and the velocity u is the velocity by the acceleration a'.

$$a=\frac{d}{dt}(\frac{V}{\sqrt{1-\frac{V^2}{c^2}}})=\frac{\sqrt{1-\frac{v_0^2}{c^2}}}{1+\frac{v_0}{c^2}u}\frac{d}{dt'}(\frac{u+v_0}{\sqrt{1-\frac{v_0^2}{c^2}}\sqrt{1-\frac{u^2}{c^2}}})=\frac{1}{1+\frac{v_0}{c^2}u}\frac{d}{dt'}(\frac{u+v_0}{\sqrt{1-\frac{u^2}{c^2}}})$$

$$a(1+\frac{v_0}{c^2}u)=\frac{d}{dt'}(\frac{u}{\sqrt{1-\frac{u^2}{c^2}}})+\frac{d}{dt'}(\frac{v_0}{\sqrt{1-\frac{u^2}{c^2}}}) \quad (3)$$

In this time , the acceleration a' of the velocity u is

$$a'=\frac{d}{dt'}(\frac{u}{\sqrt{1-\frac{u^2}{c^2}}}) \ , \ u=\frac{\int a'dt'}{\sqrt{1+\frac{1}{c^2}[\int a'dt']^2}} \quad (4)$$

Eq(3) is

$$a(1+\frac{v_0}{c^2}u)=\frac{d}{dt'}(\frac{u}{\sqrt{1-\frac{u^2}{c^2}}})+\frac{d}{dt'}(\frac{v_0}{\sqrt{1-\frac{u^2}{c^2}}})=a'+v_0\frac{d}{dt'}(\sqrt{1+\frac{1}{c^2}[\int a'dt']^2})$$

$$=a'+v_0\frac{\int a'dt'}{\sqrt{1+\frac{1}{c^2}[\int a'dt']^2}}\frac{a'}{c^2}=a'(1+\frac{v_0}{c^2}\frac{\int a'dt'}{\sqrt{1+\frac{1}{c^2}[\int a'dt']^2}})$$

$$=a'(1+\frac{v_0}{c^2}u) \qquad (5)$$

Therefore, if the matter that moves in the line is accelerated, it is $\frac{dy}{dt}=\frac{dy'}{dt'}=0$, $\frac{dz}{dt}=\frac{dz'}{dt'}=0$, the acceleration a about the accelerated matter that has the initial velocity v_0 in the inertial coordinate system $S(t,x,y,z)$ and the other acceleration a' about the accelerated matter that has not the initial velocity v_0 in the inertial coordinate system $S'(t',x',y',z')$ are same.

In this time, if the acceleration a' is the constant acceleration a_0, the acceleration in the inertial coordinate system $S(t,x,y,z)$ and in the inertial coordinate system $S'(t',x',y',z')$ is the constant acceleration a_0.

$$a_0=a'=\frac{d}{dt'}(\frac{u}{\sqrt{1-\frac{u^2}{c^2}}})=a=\frac{d}{dt}(\frac{V}{\sqrt{1-\frac{V^2}{c^2}}}) \quad (6)$$

Therefore,

$$V=\frac{dx}{dt}=\frac{a_0t+C}{\sqrt{1+\frac{1}{c^2}(a_0t+C)^2}},\quad u=\frac{dx'}{dt'}=\frac{a_0t'}{\sqrt{1+\frac{1}{c^2}(a_0t')^2}},x'=\frac{c^2}{a_0}(\sqrt{1+\frac{1}{c^2}(a_0t')^2}-1)$$

$$=\frac{\gamma a_0(t'+\frac{v_0}{c^2}x')+C}{\sqrt{1+\frac{1}{c^2}(a_0\gamma(t'+\frac{v_0}{c^2}x')+C)^2}}$$, C is the constant number

$$=\frac{\gamma a_0(t'+\frac{v_0}{c^2}\cdot\frac{c^2}{a_0}(\sqrt{1+\frac{1}{c^2}(a_0t')^2}-1))+C}{\sqrt{1+\frac{1}{c^2}(a_0\gamma(t'+\frac{v_0}{c^2}\cdot\frac{c^2}{a_0}(\sqrt{1+\frac{1}{c^2}(a_0t')^2}-1))+C)^2}}$$

$$= \frac{\gamma a_0 t' + \gamma v_0 \sqrt{1+\frac{1}{c^2}(a_0 t')^2} - \gamma v_0 + C}{\sqrt{1+\frac{1}{c^2}(\gamma a_0 t' + \gamma v_0 \sqrt{1+\frac{1}{c^2}(a_0 t')^2} - \gamma v_0 + C)^2}}$$

$$= \frac{u+v_0}{1+\frac{u}{c^2}v_0} = \frac{\frac{a_0 t'}{\sqrt{1+\frac{1}{c^2}(a_0 t')^2}} + v_0}{1+\frac{v_0}{c^2}\frac{a_0 t'}{\sqrt{1+\frac{1}{c^2}(a_0 t')^2}}} = \frac{a_0 t' + v_0 \sqrt{1+\frac{1}{c^2}(a_0 t')^2}}{\sqrt{1+\frac{1}{c^2}(a_0 t')^2} + \frac{v_0}{c^2} a_0 t'} \quad (7)$$

In this time,

$$\sqrt{1+\frac{1}{c^2}(\gamma a_0 t' + \gamma v_0 \sqrt{1+\frac{1}{c^2}(a_0 t')^2})^2} = \gamma(\sqrt{1+\frac{1}{c^2}(a_0 t')^2} + \frac{v_0}{c^2} a_0 t') \quad (8)$$

Therefore,

$$C = \gamma v_0 \quad (9)$$

Hence,

$$x = \frac{c^2}{a_0}(\sqrt{1+\frac{1}{c^2}(a_0 t + \gamma v_0)^2} - \sqrt{1+\frac{1}{c^2}(\gamma v_0)^2})$$

$$= \frac{c^2}{a_0}(\sqrt{1+\frac{1}{c^2}(a_0 t + \gamma v_0)^2} - \gamma) = \frac{c^2}{a_0}(\frac{1}{\sqrt{1-\frac{V^2}{c^2}}} - \frac{1}{\sqrt{1-\frac{{v_0}^2}{c^2}}}), V = \frac{a_0 t + \gamma v_0}{\sqrt{1+\frac{1}{c^2}(a_0 t + \gamma v_0)^2}}$$

$$x' = \frac{c^2}{a_0}(\sqrt{1+\frac{1}{c^2}(a_0 t')^2} - 1) = \frac{c^2}{a_0}(\frac{1}{\sqrt{1-\frac{u^2}{c^2}}} - 1) \ , \ u = \frac{a_0 t'}{\sqrt{1+\frac{1}{c^2}(a_0 t')^2}},$$

$$\gamma = \frac{1}{\sqrt{1-\frac{{v_0}^2}{c^2}}} \quad (10)$$

And

$$d\tau = \sqrt{1-V^2/c^2}\,dt = \frac{dt}{\sqrt{1+\frac{1}{c^2}(a_0 t + \gamma v_0)^2}}, \ d\tau = \sqrt{1-u^2/c^2}\,dt' = \frac{dt'}{\sqrt{1+\frac{1}{c^2}(a_0 t')^2}}$$

$$\tau = \frac{c}{a_0}\sinh^{-1}(\frac{a_0}{c}t + \gamma\frac{v_0}{c}) - \frac{c}{a_0}\sinh^{-1}(\gamma\frac{v_0}{c}) = \frac{c}{a_0}\sinh^{-1}(\frac{a_0}{c}t + \gamma\frac{v_0}{c}) - \tau_0$$

$$\tau + \tau_0 = \frac{c}{a_0}\sinh^{-1}(\frac{a_0}{c}t + \gamma\frac{v_0}{c}), \quad \tau = \frac{c}{a_0}\sinh^{-1}(\frac{a_0 t'}{c})$$

$$\tau_0 = \frac{c}{a_0}\sinh^{-1}(\gamma\frac{v_0}{c}), \ \gamma = \frac{1}{\sqrt{1-\frac{v_0^{\ 2}}{c^2}}} \qquad (11)$$

Therefore,

$$t + \gamma\frac{v_0}{a_0} = \frac{c}{a_0}\sinh(\frac{a_0}{c}\tau + \frac{a_0}{c}\tau_0)$$

$$= \frac{c}{a_0}[\sinh(\frac{a_0\tau}{c})\cosh(\frac{a_0\tau_0}{c}) + \cosh(\frac{a_0\tau}{c})\sinh(\frac{a_0\tau_0}{c})]$$

$$\gamma = \frac{1}{\sqrt{1-\frac{v_0^{\ 2}}{c^2}}} \qquad (12)$$

In this time,

$$\tau = \frac{c}{a_0}\sinh^{-1}(\frac{a_0 t'}{c}) \ \rightarrow \ \sinh(\frac{a_0\tau}{c}) = \frac{a_0 t'}{c},$$

$$x' = \frac{c^2}{a_0}(\sqrt{1+\frac{1}{c^2}(a_0 t')^2} - 1) \ \rightarrow \ \cosh(\frac{a_0\tau}{c}) = \sqrt{1+\frac{a_0^{\ 2}t'^2}{c^2}} = 1 + \frac{a_0}{c^2}x'$$

$$\tau_0 = \frac{c}{a_0}\sinh^{-1}(\gamma\frac{v_0}{c}) \rightarrow \sinh(\frac{a_0\tau_0}{c}) = \frac{\gamma v_0}{c}, \ \cosh(\frac{a_0\tau_0}{c}) = \sqrt{1+\frac{\gamma^2 v_0^{\ 2}}{c^2}} = \gamma,$$

$$\gamma = \frac{1}{\sqrt{1-\frac{v_0^{\ 2}}{c^2}}} \qquad (13)$$

Therefore, Eq(12) is

$$t + \gamma\frac{v_0}{a_0} = \frac{c}{a_0}\sinh(\frac{a_0}{c}\tau + \frac{a_0}{c}\tau_0)$$

$$= \frac{c}{a_0}[\sinh(\frac{a_0\tau}{c})\cosh(\frac{a_0\tau_0}{c}) + \cosh(\frac{a_0\tau}{c})\sinh(\frac{a_0\tau_0}{c})]$$

$$= \frac{c}{a_0}[\gamma\sinh(\frac{a_0\tau}{c}) + \cosh(\frac{a_0\tau}{c})\frac{\gamma v_0}{c}]$$

$$=\frac{c}{a_0}[\frac{a_0 t'}{c}\cdot\gamma+(1+\frac{a_0}{c^2}x')\cdot\frac{\gamma v_0}{c}]=\gamma(t'+\frac{v_0}{c^2}x')+\gamma\frac{v_0}{a_0}, \gamma=\frac{1}{\sqrt{1-\frac{{v_0}^2}{c^2}}} \quad (14)$$

Therefore, Eq(10) is

$$x=\frac{c^2}{a_0}(\sqrt{1+\frac{1}{c^2}(a_0 t+\gamma v_0)^2}-\gamma), \quad x'=\frac{c^2}{a_0}(\sqrt{1+\frac{1}{c^2}(a_0 t')^2}-1)$$

$$=\frac{c^2}{a_0}(\sqrt{1+\frac{1}{c^2}(a_0\gamma(t'+\frac{v_0}{c^2}x')+\gamma v_0)^2}-\gamma)$$

$$=\frac{c^2}{a_0}(\sqrt{1+\frac{1}{c^2}(a_0\gamma(t'+\frac{v_0}{c^2}\cdot\frac{c^2}{a_0}(\sqrt{1+\frac{1}{c^2}(a_0 t')^2}-1))+\gamma v_0)^2}-\gamma)$$

$$=\frac{c^2}{a_0}(\sqrt{1+\frac{1}{c^2}(\gamma a_0 t'+\gamma v_0\sqrt{1+\frac{1}{c^2}(a_0 t')^2})^2}-\gamma)$$

$$=\frac{c^2}{a_0}(\sqrt{(\gamma\sqrt{1+\frac{1}{c^2}(a_0 t')^2}+\gamma a_0\frac{v_0}{c^2}t')^2}-\gamma)$$

$$=\gamma\frac{c^2}{a_0}(\sqrt{1+\frac{1}{c^2}(a_0 t')^2}-1)+\gamma v_0 t'=\gamma(x'+v_0 t') \quad , \quad \gamma=\frac{1}{\sqrt{1-\frac{{v_0}^2}{c^2}}} \quad (15)$$

or by Eq(13),Eq(14)

$$x=\frac{c^2}{a_0}(\sqrt{1+\frac{1}{c^2}(a_0 t+\gamma v_0)^2}-\gamma)$$

$$=\frac{c^2}{a_0}(\sqrt{1+\sinh^2(\frac{a_0}{c}\tau+\frac{a_0}{c}\tau_0)}-\gamma)=\frac{c^2}{a_0}(\cosh(\frac{a_0}{c}\tau+\frac{a_0}{c}\tau_0)-\gamma)$$

$$=\frac{c^2}{a_0}(\cosh(\frac{a_0}{c}\tau)\cosh(\frac{a_0}{c}\tau_0)+\sinh(\frac{a_0}{c}\tau)\sinh(\frac{a_0}{c}\tau_0)-\gamma)$$

$$=\frac{c^2}{a_0}(\cosh(\frac{a_0}{c}\tau)\gamma+\sinh(\frac{a_0}{c}\tau)\frac{\gamma v_0}{c}-\gamma)$$

$$= \frac{c^2}{a_0}\gamma\sqrt{1+\frac{1}{c^2}(a_0 t')^2} + \gamma v_0 t' - \frac{c^2}{a_0}\gamma\,, \quad \gamma = \frac{1}{\sqrt{1-\frac{v_0{}^2}{c^2}}} \qquad (16)$$

Hence, Eq(1) is in the inertial coordinate system $S'(t',x',y',z')$

$$x' = \frac{c^2}{a_0}(\cosh(\frac{a_0\tau}{c})-1)$$

$$t' = \frac{c}{a_0}\sinh(\frac{a_0\tau}{c}) \qquad (17)$$

Therefore, in the inertial coordinate system $S(t,x,y,z)$

$$t = \gamma(t'+\frac{v_0}{c^2}x') = \gamma(\frac{c}{a_0}\sinh(\frac{a_0}{c}\tau)+\frac{v_0}{a_0}(\cosh(\frac{a_0}{c}\tau)-1))$$

$$x = \gamma(x'+v_0 t') = \gamma(\frac{c^2}{a_0}(\cosh(\frac{a_0\tau}{c})-1)+\frac{v_0 c}{a_0}\sinh(\frac{a_0\tau}{c}))\,, \gamma = \frac{1}{\sqrt{1-\frac{v_0{}^2}{c^2}}}$$

$$y = y'\,,\ z = z' \qquad (18)$$

$$dt = \gamma(\cosh(\frac{a_0}{c}\tau)+\frac{v_0}{c}\sinh(\frac{a_0}{c}\tau))d\tau\,,$$

$$dx = \gamma(c\sinh(\frac{a_0}{c}\tau)+v_0\cosh(\frac{a_0}{c}\tau))d\tau\,,$$

$$dy = dy' = 0\,,\ dz = dz' = 0$$

$$V = \frac{dx}{dt} = (c\tanh(\frac{a_0}{c}\tau)+v_0)/(1+\frac{v_0}{c}\tanh(\frac{a_0}{c}\tau))\,, \gamma = \frac{1}{\sqrt{1-\frac{v_0{}^2}{c^2}}} \qquad (19)$$

II. Additional chapter-I

The tetrad $e_a{}^{\mu}$ is the unit vector that is each other orthographic and it use the following formula.

$$e_a{}^{\mu} e_b{}^{\upsilon} g_{\mu\upsilon} = \eta_{ab} \qquad (20)$$

$e^a{}_{\mu}$ is

$$e^a{}_{\mu} = \eta^{ab} g_{\mu\upsilon} e_b{}^{\upsilon} \; (21)$$

and it is $e_a{}^{\mu}$'s inverse-matrix. And it is

$$e^a{}_\mu e_b{}^\mu = \delta^a{}_b \ , \ e^a{}_\mu e_a{}^\upsilon = \delta_\mu{}^\upsilon$$

$$e^a{}_\mu e^b{}_\upsilon \eta_{ab} = g_{\mu\upsilon} \qquad (22)$$

The $e^\alpha{}_\mu(\tau)$ is the tetrad that if $\xi^1 = \xi^2 = \xi^3 = 0, d\xi^1 = d\xi^2 = d\xi^3 = 0$ .It is not the accelerated system and it is that the point's the accelerated motion is in the line in the inertial coordinate system. In this time, in Eq(22) it does $g_{\mu\upsilon} = \eta_{\mu\upsilon}$.

Therefore, Eq(22) is

$$\eta_{\alpha\beta} e^\alpha{}_0(\tau) e^\beta{}_0(\tau) = \eta_{00} = -1$$

$$d\tau^2 = -\frac{1}{c^2}\eta_{\alpha\beta} dx^\alpha dx^\beta$$

$$\rightarrow \quad -1 = \eta_{\alpha\beta}(\frac{1}{c}\frac{dx^\alpha}{d\tau})(\frac{1}{c}\frac{dx^\beta}{d\tau}) = \eta_{\alpha\beta} e^\alpha{}_0(\tau) e^\beta{}_0(\tau) \, (23)$$

According to Eq(19),Eq(23)

$$e^\alpha{}_0(\tau) = \frac{1}{c}\frac{dx^\alpha}{d\tau}$$

$$= (\gamma \cosh(\frac{a_0}{c}\tau) + \frac{v_0}{c}\gamma \sinh(\frac{a_0}{c}\tau),0,0) \ , \gamma \sinh(\frac{a_0}{c}\tau) + \frac{v_0}{c}\gamma \cosh(\frac{a_0}{c}\tau),0,0) \, , \gamma = \frac{1}{\sqrt{1 - \frac{v_0{}^2}{c^2}}}$$

(24)

About y -axis's and z -axis's orientation

$e^\alpha{}_2(\tau) = (0,0,1,0)$ (25), $e^\alpha{}_3(\tau) = (0,0,0,1)$ (26)

And the other unit vector $e^\alpha{}_1(\tau)$ has to satisfy the tetrad condition, Eq (22)

$$e^\alpha{}_1(\tau) = (\gamma \sinh(\frac{a_0}{c}\tau) + \frac{v_0}{c}\gamma \cosh(\frac{a_0}{c}\tau),$$

$$\gamma \cosh(\frac{a_0}{c}\tau) + \frac{v_0}{c}\gamma \sinh(\frac{a_0}{c}\tau),0,0) \, , \gamma = \frac{1}{\sqrt{1 - \frac{v_0{}^2}{c^2}}} \qquad (27)$$

III. Additional chapter-II

According to the tetrad $e^a{}_\mu$,in the flat Minkowski space, the inertial coordinate system $S(t, x, y, z)$ transform the accelerated system $\xi(\xi^0, \xi^1, \xi^2, \xi^3)$ .In this time, the accelerated observer of the accelerated system $\xi(\xi^0, \xi^1, \xi^2, \xi^3)$ and the accelerated matter that has the initial velocity v_0 in the inertial coordinate system $S(t, x, y, z)$ are same. Therefore, by Eq(22)

$$d\tau^2 = dt^2 - \frac{1}{c^2}[dx^2 + dy^2 + dz^2]$$

$$= -\frac{1}{c^2}\eta_{ab}\frac{\partial x^a}{\partial \xi^\mu}\frac{\partial x^b}{\partial \xi^\upsilon}d\xi^\mu d\xi^\upsilon$$

$$= -\frac{1}{c^2}\eta_{ab}e^a{}_\mu e^b{}_\upsilon d\xi^\mu d\xi^\upsilon = -\frac{1}{c^2}g_{\mu\upsilon}d\xi^\mu d\xi^\upsilon \quad (28)$$

$$e^a{}_\mu = \frac{\partial x^a}{\partial \xi^\mu} \quad (29)$$

For saving the Rindler coordinate theory in the new mathematical way, the $e^\alpha{}_\mu(\xi^0)$ is used by Eq (25),Eq(26),Eq(27) that used ξ^0 instead of τ .In this time, $dy = d\xi^2 \neq 0$, $dz = d\xi^3 \neq 0$, because it is the matter that the accelerated observer of the accelerated system $\xi(\xi^0, \xi^1, \xi^2, \xi^3)$ observes .

The unit vector $e^\alpha{}_1(\xi^0)$ is

$$e^\alpha{}_1(\xi^0) = \frac{\partial x^\alpha}{\partial \xi^1} = (\gamma\sinh(\frac{a_0}{c}\xi^0) + \frac{v_0}{c}\gamma\cosh(\frac{a_0}{c}\xi^0),$$
$$\gamma\cosh(\frac{a_0}{c}\xi^0) + \frac{v_0}{c}\gamma\sinh(\frac{a_0}{c}\xi^0),0,0)\ ,\gamma = \frac{1}{\sqrt{1-\frac{v_0^2}{c^2}}} \quad (30)$$

$$\frac{\partial e^\alpha{}_1(\xi^0)}{c\partial \xi^0} = \frac{\partial^2 x^\alpha}{\partial \xi^1 c\partial \xi^0} = \frac{\partial e^\alpha{}_0(\xi^0)}{\partial \xi^1} \quad (31)$$

Therefore, the vector $e^\alpha{}_0(\xi^0)$ is

$$e^\alpha{}_0(\xi^0) = \frac{\partial x^\alpha}{c\partial \xi^0}$$
$$= ((1 + \frac{a_0}{c^2}\xi^1)(\gamma\cosh(\frac{a_0}{c}\xi^0) + \frac{v_0}{c}\gamma\sinh(\frac{a_0}{c}\xi^0)),$$
$$(1 + \frac{a_0}{c^2}\xi^1)(\gamma\sinh(\frac{a_0}{c}\xi^0) + \frac{v_0}{c}\gamma\cosh(\frac{a_0}{c}\xi^0)),0,0)\ ,\gamma = \frac{1}{\sqrt{1-\frac{v_0^2}{c^2}}} \quad (32)$$

About y -axis's and z -axis's orientation, the unit vector $e^\alpha{}_2(\xi^0)$ and $e^\alpha{}_3(\xi^0)$ is

$$e^\alpha{}_2(\xi^0) = \frac{\partial x^\alpha}{\partial \xi^2} = (0,0,1,0) \quad (33), \quad e^\alpha{}_3(\xi^0) = \frac{\partial x^\alpha}{\partial \xi^3} = (0,0,0,1) \quad (34)$$

The differential coordinate transformation is

$$dx^\alpha = \frac{\partial x^\alpha}{\partial \xi^\mu}d\xi^\mu = \frac{\partial x^\alpha}{c\partial \xi^0}cd\xi^0 + \frac{\partial x^\alpha}{\partial \xi^1}d\xi^1 + \frac{\partial x^\alpha}{\partial \xi^2}d\xi^2 + \frac{\partial x^\alpha}{\partial \xi^3}d\xi^3$$
$$= e^\alpha{}_0(\xi^0)cd\xi^0 + e^\alpha{}_1(\xi^0)d\xi^1 + e^\alpha{}_2(\xi^0)d\xi^2 + e^\alpha{}_3(\xi^0)d\xi^3$$

$$cdt = \gamma[(1+\frac{a_0}{c^2}\xi^1)\{\cosh(\frac{a_0\xi^0}{c})+\frac{v_0}{c}\sinh(\frac{a_0\xi^0}{c})\}cd\xi^0$$

$$+\{\sinh(\frac{a_0\xi^0}{c})+\frac{v_0}{c}\cosh(\frac{a_0\xi^0}{c})\}d\xi^1] \quad (35)$$

$$dx = \gamma[(1+\frac{a_0}{c^2}\xi^1)\{\sinh(\frac{a_0\xi^0}{c})+\frac{v_0}{c}\cosh(\frac{a_0\xi^0}{c})\}cd\xi^0$$

$$+\{\cosh(\frac{a_0\xi^0}{c})+\frac{v_0}{c}\sinh(\frac{a_0\xi^0}{c})\}d\xi^1] (36), \gamma = \frac{1}{\sqrt{1-\frac{v_0^2}{c^2}}},$$

$$dy = d\xi^2, dz = d\xi^3 \quad (37)$$

Therefore if Eq(35), Eq(36) and Eq(37) integrate, finally the Rindler coordinate theory's coordinate transformation of the accelerated observer with the initial velocity is found.

$$ct = \gamma(\frac{c^2}{a_0}+\xi^1)\{\sinh(\frac{a_0\xi^0}{c})+\frac{v_0}{c}\cosh(\frac{a_0\xi^0}{c})\} - \gamma\frac{v_0 c}{a_0} \quad (38)$$

$$x = \gamma(\frac{c^2}{a_0}+\xi^1)\{\cosh(\frac{a_0\xi^0}{c})+\frac{v_0}{c}\sinh(\frac{a_0\xi^0}{c})\} - \gamma\frac{c^2}{a_0} \quad (39),$$

$$y = \xi^2, z = \xi^3 \quad (40), \gamma = \frac{1}{\sqrt{1-\frac{v_0^2}{c^2}}}$$

Therefore, the new inverse-coordinate transformation of the Rindler coordinate theory of the accelerated observer with the initial velocity is

$$\frac{(ct+\gamma\frac{v_0 c}{a_0})}{(x+\gamma\frac{c^2}{a_0})} = \frac{\tanh(\frac{a_0\xi^0}{c})+\frac{v_0}{c}}{1+\frac{v_0}{c}\cdot\tanh(\frac{a_0\xi^0}{c})}$$

$$\xi^0 = \frac{c}{a_0}\tanh^{-1}[\frac{\frac{(ct+\gamma\frac{v_0 c}{a_0})}{(x+\gamma\frac{c^2}{a_0})}-\frac{v_0}{c}}{1-\frac{v_0}{c}\cdot\frac{(ct+\gamma\frac{v_0 c}{a_0})}{(x+\gamma\frac{c^2}{a_0})}}] (41), \quad \gamma = \frac{1}{\sqrt{1-\frac{v_0^2}{c^2}}}$$

$$(x+\gamma\frac{c^2}{a_0})^2 - (ct+\gamma\frac{v_0 c}{a_0})^2 = (\frac{c^2}{a_0}+\xi^1)^2\gamma^2(1-\frac{v_0^2}{c^2}) = (\frac{c^2}{a_0}+\xi^1)^2$$

$$\xi^1 = \sqrt{(x+\gamma\frac{c^2}{a_0})^2 - (ct+\gamma\frac{v_0 c}{a_0})^2} - \frac{c^2}{a_0}\,(42), \xi^2 = y, \xi^3 = z\,(43), \gamma = \frac{1}{\sqrt{1-\frac{v_0{}^2}{c^2}}}$$

IV. Additional chapter-III

Therefore, the invariable time $d\tau$ of the Rindler coordinate theory of the accelerated observer with the initial velocity is by Eq(35),Eq(36)Eq(37)

$$d\tau^2 = dt^2 - \frac{1}{c^2}[dx^2 + dy^2 + dz^2]$$

$$= (1+\frac{a_0}{c^2}\xi^1)^2(d\xi^0)^2 - \frac{1}{c^2}[(d\xi^1)^2 + (d\xi^2)^2 + (d\xi^3)^2]\,(44)$$

Hence, the invariable time $d\tau$ of the new accelerated system theory of the accelerated observer that has the initial velocity v_0 is not related to the initial velocity v_0.

About x-axis's light speed,

$$dy = d\xi^2 = dz = d\xi^3 = 0, y = \xi^2 = z = \xi^3 = 0$$

$$cdt = dx \ , \ ct = x \ ,$$

$$cd\xi^0 = \frac{d\xi^1}{(1+\frac{a_0}{c^2}\xi^1)} ,$$

$$c\xi^0 = \frac{c^2}{a_0}\ln|1+\frac{a_0}{c^2}\xi^1| \rightarrow (1+\frac{a_0}{c^2}\xi^1) = e^{\frac{a_0}{c}\xi^0} \rightarrow (\frac{c^2}{a_0}+\xi^1) = \frac{c^2}{a_0}e^{\frac{a_0}{c}\xi^0} \quad (45)$$

In this time, if use the accelerated system's coordinate transformation, Eq(38),Eq(39)

$$ct = \gamma(\frac{c^2}{a_0}+\xi^1)\{\sinh(\frac{a_0\xi^0}{c}) + \frac{v_0}{c}\cosh(\frac{a_0\xi^0}{c})\} - \gamma\frac{v_0 c}{a_0}$$

$$= \gamma\frac{c^2}{a_0}e^{\frac{a_0}{c}\xi^0}\{\frac{e^{\frac{a_0}{c}\xi^0} - e^{-\frac{a_0}{c}\xi^0}}{2} + \frac{v_0}{c}\frac{e^{\frac{a_0}{c}\xi^0} + e^{-\frac{a_0}{c}\xi^0}}{2}\} - \gamma\frac{v_0 c}{a_0}$$

$$= \gamma\frac{c^2}{a_0}\{\frac{e^{2\frac{a_0}{c}\xi^0} - 1}{2} + \frac{v_0}{c}(\frac{e^{2\frac{a_0}{c}\xi^0} + 1}{2} - 1)\}$$

$$= \gamma\frac{c^2}{a_0}\{\frac{e^{2\frac{a_0}{c}\xi^0} - 1}{2} + \frac{v_0}{c}\frac{e^{2\frac{a_0}{c}\xi^0} - 1}{2}\}$$

$$= x = \gamma(\frac{c^2}{a_0}+\xi^1)\{\cosh(\frac{a_0\xi^0}{c}) + \frac{v_0}{c}\sinh(\frac{a_0\xi^0}{c})\} - \gamma\frac{c^2}{a_0}$$

$$= \gamma \frac{c^2}{a_0} e^{\frac{a_0}{c}\xi^0} \{ \frac{e^{\frac{a_0}{c}\xi^0} + e^{-\frac{a_0}{c}\xi^0}}{2} + \frac{v_0}{c} \frac{e^{\frac{a_0}{c}\xi^0} - e^{-\frac{a_0}{c}\xi^0}}{2} \} - \gamma \frac{c^2}{a_0}$$

$$= \gamma \frac{c^2}{a_0} \{ (\frac{e^{2\frac{a_0}{c}\xi^0} + 1}{2} - 1) + \frac{v_0}{c} \frac{e^{2\frac{a_0}{c}\xi^0} - 1}{2} \}$$

$$= \gamma \frac{c^2}{a_0} \{ \frac{e^{2\frac{a_0}{c}\xi^0} - 1}{2} + \frac{v_0}{c} \frac{e^{2\frac{a_0}{c}\xi^0} - 1}{2} \} \quad (46)$$

V. Conclusion

It found the Rindler coordinate theory with the initial velocity that used the tetrad on the new method. And the Rindler coordinate theory expanded to be the Rindler coordinate theory of the accelerated observer that have the initial velocity. And the Rindler coordinate theory's mathematics modernized.

Reference

[1]S.Weinberg,Gravitation and Cosmology(John wiley & Sons,Inc,1972)

[2]W.Rindler, Am.J.Phys.**34**.1174(1966)

[3]P.Bergman,Introduction to the Theory of Relativity(Dover Pub. Co.,Inc., New York,1976),Chapter V

[4]C.Misner, K,Thorne and J. Wheeler, Gravitation(W.H.Freedman & Co.,1973)

[5]S.Hawking and G. Ellis,The Large Scale Structure of Space-Time(Cam-bridge University Press,1973)

[6]R.Adler,M.Bazin and M.Schiffer,Introduction to General Relativity(McGraw-Hill,Inc.,1965)