Confined Helical Wave Structure of Electron

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We show that the energy and the momentum of the standing waves formed by trapping electromagnetic waves between two reflecting surfaces transform exactly like those of a point particle. When such a standing wave system is given translational motion, it is observed that its time-dependent part transforms into a plane wave while its space dependent part becomes the amplitude wave which gets compacted into the internal coordinates. Using the idea that the electromagnetic waves can have spatial amplitude, we show that if such a wave is confined after imparting spinning motion, a system of two confined helical waves is formed with the one travelling forward in time forming electron and the one travelling backward in time forming positron. The confined helical wave picture of the particle gives a simple explanation of Pauli's exclusion principle.

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1 Introduction

We know that in the standard model all elementary particles are treated as point particles in spite of the fact that such an assumption can at best be a convenient mathematical abstraction. A new insight into the structure of particles like electron at this stage will offer a new perspective to the problems of the physics of the ultra small. The new solutions of Maxwell's equation proposed by us attributing spatial amplitude to the electromagnetic waves offer a lot of promise in this regard [1]. In this paper we shall show that the standing waves formed by the electromagnetic waves acquire mass and takes the form of a modified plane wave when it is given translational velocity. Such a standing wave system is seen to be ideally suited to represent the half integer spin of a particle like electron.

Although we propose to confine the electromagnetic waves by the artificial construct of two perfectly reflecting surfaces kept facing each other at a distance of half the wave length of the wave, the actual process may involve vacuum fluctuations. However, we do not propose to

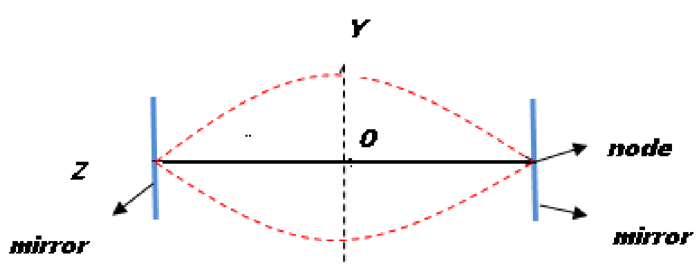


Fig.1- The figure represents a standing half wave formed by reflections on two mirrors kept facing each other. Note that the maximum amplitude is at the origin of coordinates

go into the interactions involved in the confinement of the electromagnetic waves in this paper. We shall initially study the standing wave formed by the confinement of plane progressive electromagnetic waves between two perfectly reflecting mirrors and then impart translational velocity to it. We may treat the standing wave having nodes at the ends as a linear combination of forward and reverse waves. Here we shall use the complex form of the wave representation for convenience which in the stationary frame of reference can be expressed as

$$\phi_o \quad = \quad \boldsymbol{\xi}_{\mathbf{o}} \left(e^{-i(E_o t_o - p_o z_o)/\hbar} + e^{-i(E_o t_o + p_o z_o)/\hbar} \right) \quad = \quad 2\boldsymbol{\xi}_{\mathbf{o}} \cos\left(p_o\, z_o/\hbar\right)\, e^{-i\hbar^{-1} E_o t_o} \qquad (1)$$

On imparting translational velocity v = βc to the standing wave, the forward and the reverse components will undergo Doppler shift and ϕ_o will transform to

$$\phi \quad = \quad \xi \left(e^{-i(E_1 t - p_1 z)/\hbar} + e^{-i(E_2 t + p_2 z)/\hbar} \right), \qquad (2)$$

where $$E_1 = \gamma E_o(1+\beta) \;\; ; \;\; E_2 \;=\; \gamma E_o(1-\beta) \; ; \; \gamma \;=\; (1-\beta^2)^{-\frac{1}{2}} \qquad (3)$$

Here we have assumed that ξ_o transforms to ξ on account of the relativistic transformation in the electric field. If we denote the average of the energy and momentum of the forward and the reverse waves taken together as E and *þ* respectively, where $p_1 = E_1$/c and $p_2 = E_2$/c, then we have

$$E \quad = \quad \tfrac{1}{2}\left(E_1 + E_2\right) \quad = \quad \gamma E_o \quad ; \quad þ \quad = \quad \tfrac{1}{2}(p_1 - p_2) \quad = \quad \gamma E_o\, \mathbf{v}/c^2 \qquad (4)$$

Note that this is exactly how the energy and the momentum of a particle behave under relativistic transformation. Therefore, the standing wave appears to be a quite promising representation of a particle. We may now express (2) as

$$\phi \quad = \quad \xi\, e^{-i\hbar^{-1}(Et - þz)} \left(e^{-i[(E_1 - E)t - (p_1 - þ)z)]/\hbar} + e^{-i[(E_2 - E) - (p_2 - þ)z]/\hbar} \right) \qquad (5)$$

On simplifying the equation keeping in mind that that $(p_1$-*þ*)/$(E_1$-$E)$ = $(p_2$+*þ*)/(E-$E_2)$ = 1/v and $(E_1$-$E)$ = (E-$E_2)$ = *þc* = Ev/c, we obtain

$$\phi \quad = \quad 2\xi \cos\left[E\left(z - \mathbf{v}t\right)/\hbar c\right] e^{-i\hbar^{-1}(Et - þz)} \qquad (6)$$

This moving standing wave with Doppler shifted components is mathematically equivalent to a modified plane wave whose amplitude is a wave propagating with the translational velocity. Here we may attach the electric vector ξ either to the amplitude wave or to the plane wave. Later in we shall show that the electric vector belongs to the amplitude wave and it will determine the inner structure of the particle. If we now take z = vt + Nλ, where N is an integer, the cosine function in (6) will become unity and ϕ will acquire the exact form of the plane wave. However as the amplitude of ϕ has a series of maxima it is ill-suited to represent a localized particle. But then we should note that the well accepted plane wave representation of a particle also suffers from the same debility. A comparison of (6) with (1) reveals that the amplitude wave and the plane wave are respectively the space and the time dependent components of the standing wave as viewed from a moving frame of reference.

It is interesting to note that the De Broglie wave has the same form as the exponential function in (6). De Broglie derived such a wave [2] based on the assumption that a particle is represented by a vibration of the form, ϕ_o = B exp[-iħ$^{-1}E_o t_o$]. Such a vibration on gaining translational velocity would acquire the form of the plane wave or the de Broglie wave. This can be shown by applying relativistic transformations on E_o and t_o to obtain

$$\phi \quad = \quad B\, e^{-i\hbar^{-1}(Et - þz)}$$

Note that while de Broglie had to introduce the artificial concept of a vibration which has no spatial spread, the same form of the wave is obtained in (6) by the confinement of a physical wave like the electromagnetic wave. We shall show that the amplitude wave which comes in the way of identifying ϕ in (6) with the plane wave actually represents the spin aspect of the particle.

2 More on the Amplitude wave and the Plane Wave

Let us now modify (6) and express it as

$$\phi \quad = \quad \xi\left(e^{iE(z-vt)/\hbar c} + e^{-iE(z-vt)/\hbar c}\right)e^{-i\hbar^{-1}(Et-\text{þ}z)} \quad . \tag{7}$$

Of the two terms in the bracket representing the amplitude, we shall confine ourselves with only the first term and express it as

$$\phi \quad = \quad \xi\, e^{iE(z-vt)/\hbar c}\; e^{-i\hbar^{-1}(Et-\text{þ}z)} \quad = \quad \phi_A\phi_P \quad , \tag{8}$$

where ϕ_A represents the amplitude wave while ϕ_P stands for the plane wave. It can be easily shown that the wavelength λ_A of the amplitude wave and λ_P of the plane wave are given by

$$\lambda_A \quad = \quad \lambda_o\sqrt{\left(1-\mathbf{v}^2/c^2\right)} \quad , \qquad \lambda_P \quad = \quad \frac{c^2/\mathbf{v}}{\omega/2\pi} \quad = \quad (c/\mathbf{v})\lambda_A \tag{9}$$

where λ_o is the wavelength of the standing wave in the rest frame of reference. Note that the amplitude wave has got the same wave length as the trapped wave as the factor, $\sqrt{(1-v^2/c^2)}$ represents the relativistic contraction in length. Since $v < c$, we have $\lambda_P > \lambda_A$. The plane wave has the phase velocity c^2/v which is superluminal and therefore it cannot represent the movement of any physical entity. Besides, we saw that the amplitude of the modified plane wave given in (6) has a series of maxima at

$$z \quad = \quad \mathbf{v}t + Nhc/E \quad = \quad \mathbf{v}t + N\lambda_A \quad . \tag{10}$$

Therefore, it is not possible to localize the particle at any particular point. Surprisingly, in spite of the fact that the velocity and the wave length of the confined electromagnetic wave and the plane wave are different, it is observed from (9) that their periods are equal.

ie; $$T_P \quad = \quad \lambda_P/(c^2/\mathbf{v}) \quad = \quad 1/(\omega/2\pi) \quad = \quad 2\pi/(\gamma\omega_o) \quad = \quad \gamma^{-1}\,T_o$$

where T_o is the period of the confined electromagnetic wave in its proper reference frame. The factor γ^{-1} on the right hand side of the above equation will account for the relativistic time dilation.

Although a particle may be represented by the confined wave as discussed above, this is true only at one level. The plane wave represents particle at a different level. This is so because if we trap the moving particle within a larger box with totally reflecting walls, then the standing wave will be formed based on the wave length of the plane wave, and not that of the original electromagnetic wave. In other words, where the internal structure of a particle does not come into play (this is equivalent to treating it as a point particle), it could be well defined by the plane wave.

3 Compacting and the Creation of the Inner Spaces

We shall now show how the concept of the internal coordinates emerges when the electromagnetic wave is confined between two mirrors. To elucidate the concept, let us consider a point on the surface of a sphere with centre at P located at a distance $\hat{z}_o$ from the origin of axes. Let P′ be a point on the surface of the sphere which is at a distance z'_o from its centre. If we now treat the spherical body as a point particle, it is obvious that P′ will coincide with P and therefore z'_o will have to be taken as zero. But such an assumption will cause problems if P' were revolving around P. If P' is made to coincide with P, then it will become impossible to account for the spin angular momentum of the sphere unless it is treated as an intrinsic property of the particle. This is another way of stating that the point P' and the spin of

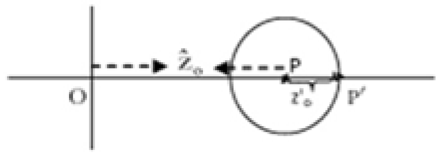


Fig.2 - P' is a point on the surface of a sphere centered at P at a distance $\hat{z}_o$ from O. If the sphere is taken as a point particle, P' will coincide with P and its angular momentum can be accounted for only by assuming existence of the internal space to the point particle.

the sphere are defined in the internal space of the sphere. Note that the internal space introduced here is just a mathematical construct which enables us to treat the sphere as a point particle while simultaneously accounting for its spin.

We shall now apply this concept of the internal space in the case of the standing wave. We may take $\hat{z}_o$ and $\hat{t}_o$ as the spatial and time coordinates of the standing wave assuming it to be compacted to a point (say P). In that case, z'_o and t'_o may be taken as the coordinates of a point (say P′) defined in the internal space of the standing wave. If we denote by z_o and t_o the space and time coordinates of the point P′ from the origin of the coordinates, then we have

$$z_o \quad = \quad \hat{z}_o + z'_o \quad \text{and} \quad t_o \quad = \quad \hat{t}_o \quad + t'_o \tag{11}$$

Let us now express the standing wave given in (1) in a more general way using the internal coordinates, z'_o and t'_o. Note that here z'_o occupies values in the range $0 < z'_o < \frac{1}{2}\lambda_0$ as the wave is confined between two mirrors separated by a distance $\frac{1}{2}\lambda_0$. Similarly, t' occupies values in the range $0 < t'_o < T_o$, where T_o is the period of the oscillations. Taking "NT_o" as the rounded off external time coordinate denoted by $\hat{t}_o$, and t_o' as the corresponding internal time coordinate, we have

$$z_o \quad = \quad z'_o \quad \text{and} \quad t_o \quad = \quad NT_o \quad + t'_o \tag{12}$$

and

$$\hbar^{-1}E_o t_o \quad = \quad \omega_o t_o \quad = \quad (2\pi N + \omega_o t_o^{'}) \quad = \quad \hbar^{-1}E_o\hat{t}_o + \hbar^{-1}E_o t_o^{'}$$

We may now express (1) as

$$\phi_o \quad = \quad 2\xi_o \cos\left[E_o z'_o/\hbar c\right] e^{-i\hbar^{-1}E_o t_o} \quad . \tag{13}$$

When the system is given a translational velocity, (1) gets transformed to (6) and (z-vt) in that equation can be written using (10) and the internal coordinates discussed above as

$$(z - \mathbf{v}t) \quad = \quad (\hat{z} - v\hat{t}) + (z' - \mathbf{v}t') \quad = \quad N\lambda_A + z' - \mathbf{v}t' \tag{14}$$

Therefore, (6) can be written as

$$\phi \quad = \quad 2\xi \cos\left[E\left(z'-vt'\right)/\hbar c\right] e^{-i\hbar^{-1}(E\,t - \text{þ}\,z)} \quad . \tag{15}$$

Note that by virtue of (14) we may switch from external spatial coordinates to internal spatial coordinates whenever the need arises. The interesting property of ϕ here is that the spatial part of the standing wave gets defined entirely in the internal coordinates while the time dependent part gets defined in the laboratory coordinates. Had we defined the plane wave entirely by $\hat{t}_o$, then it would not have been possible to bring out the periodic structure of the wave. Therefore, the plane wave has to be defined in t_o and not $\hat{t}_o$ so that the wave nature gets fully expressed by it.

Observe that although we started with the assumption that a particle is represented by a localized electromagnetic wave, ultimately the particle gets represented by an altogether different wave which is no more localized in space. This new wave has the form of the plane wave. The amplitude wave "2ξ cos[E(z`-vt`)ħc]" is defined in the internal coordinates and later we shall show that it determines the spin and the internal structure of the particle. In the external coordinates, ϕ represents a plane wave whose amplitude is a dimensionless number (unity). This means that the plane wave does not represent the oscillations of any particular field or any spatial displacement. Since we are dealing with a given state of the particle, we may take the amplitude to be unity. It is now very easy to make the generalization to case where the state of the particle is not given. This would lead us to the quantum mechanical representation of the wave function where the amplitude denotes the probability amplitude.

It is obvious that ϕ in (15) satisfies the Klein Gordon equation (note that the energy and momentum operators do not operate on the amplitude wave as it is now defined in the internal coordinates) and not the wave equation for the electromagnetic waves although ϕ is taken as a linear combination of these waves. This confirms that the confinement of the electromagnetic wave has resulted in the generation of mass [3]. We now observe that the standing wave picture is compatible with the plane wave representation of a particle in quantum mechanics.

4 Helical Structure of the Confined wave

In the above discussion we saw that a standing wave formed by the confinement of the electromagnetic wave acquires mass and takes on the form of a modified plane wave. But on detailed scrutiny of the standing wave, we observe that it cannot possess half integer spin. This is because the projection of the spin of the forward and the reverse waves along z-axis can take values of only ± ħ. Therefore, the system of the confined wave cannot possess half integer spin. Another problem is that the confinement of such a wave creates only particle and no antiparticle. This goes against the basic conservation principles. We shall resolve these issues now.

Till now we have treated the electromagnetic wave in the conventional manner without attributing any spatial amplitude to the wave. We shall now examine the issue based on the concept of the spatial amplitude [1]. Let us start with a plane polarized spatial wave travelling along z-axis. The first plane polarized wave may constructed by taking a linear combination of two circularly polarized helical waves, one travelling forward in time and the other travelling reverse in time and space with the opposite helicity. Note that for the wave travelling reverse in time, the direction of the electric (and magnetic) vector will be reversed. We should keep in mind that both these waves satisfy Maxwell's equations in vacuum and there is no way to distinguish which is moving forward in time and which is moving reverse in time. The property that photon is its own antiparticle arises from this behavior. Since these two waves have opposite helicity, we end up having a plane polarized wave both in spatial and electromagnetic oscillations. We may consider another plane polarized wave constructed similarly, but with its electric vector pointing in the opposite direction. A plane polarized spatial wave may be constructed by a linear combination of these two plane polarized waves and for such a system the electric and magnetic fields of the component waves get cancelled. Assuming the plane of spatial oscillations to be along the y-z plane, let us represent such a plane polarized spatial wave by

$$\begin{aligned} \phi'_o &= 2\mathbf{j}\,\eta_o \cos[\tfrac{1}{2}(E_o t_o - p_o z_o)/\hbar] \\ &= \phi_o + \phi_o^* \end{aligned} \tag{16}$$

where $\phi_o = \mathbf{j}\,\eta_o\, e^{-i\frac{1}{2}(E_o t_o - p_o z_o)/\hbar}$ *and* $\phi_o^* = \mathbf{j}\,\eta_o\, e^{i\frac{1}{2}(E_o t_o - p_o z_o)/\hbar}$.

Here we have intentionally used the cosine function, instead of the complex function to represent the electromagnetic wave because the particle and the antiparticle state of photon are the same. Therefore, it is impossible to say whether the electromagnetic wave is to be represented by ϕ_o or ϕ_o^* . The best option is to take the linear combination of both states. We shall now assume that in the presence of an interacting field, ϕ_o and ϕ_o^* undergo spinning motion in the opposite directions with angular velocity $\frac{1}{2}\omega_o = \frac{1}{2}E_o/\hbar$. In the next section we shall show that such a spin angular velocity will result in the system acquiring spin of $\frac{1}{2}\hbar$. Note

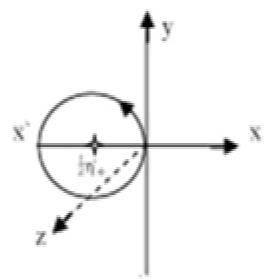
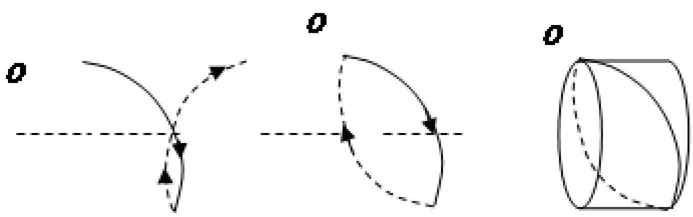


Fig.3 - The figure is the projection on the transverse plane of helical wave formed by gaining spin angular velocity $\frac{1}{2}\omega_o$ (in the anticlockwise direction) to a plane polarized spatial wave propagating along the z-axis. The wave travelling in the reverse time is not shown here.

that the net spin angular momentum gained will be zero as the angular velocities of ϕ_o and ϕ_o^* are in the opposite directions. While the spin angular velocity of ϕ_o is imparted in the anticlockwise direction, that of ϕ_o^* will be imparted in the clockwise direction as it is travelling backward in time. In other words, the spin angular momentum imparted to the two waves will have opposite signs.

We should keep in mind that such a helical wave formed by acquiring angular velocity as shown in figure 3 will not possess electromagnetic field. This cancellation of the electric and the magnetic fields may occur only for a very short duration allowed by the uncertainty principle and the same field will reappear shortly in the final configuration. We may now express the helical wave as a linear combination of the two waves, one moving forward in time and the other moving reverse in time. We may now imagine that the wave in the forward time acquires electric field (and also magnetic field) in one direction while the wave in the reverse time acquires it in the opposite direction with the result that the resultant field at any point remains zero. We shall shortly show that the wave in the forward time and the reverse time occupy the same spatial region. The helical waves so formed may not be resolvable in to two plane-polarized spatial waves orthogonal to each other because in that case it would be no different from the electromagnetic wave as far as its spin is concerned which we know is unity. In other words, if the property of the half spin is to be retained, the confined helical wave has to be taken as a fundamental state which cannot be resolved in terms of plane polarized states. In a separate paper we shall show that unlike the electromagnetic wave such a wave cannot progress freely in space as vacuum fluctuations may confine it forming a standing wave system. This means that the full frequency wave constituting the particle state can be treated as a half frequency wave that possesses half integer spin.

Right now we shall not worry about the details of the vacuum interactions which confine it as it will be taken up in a separate paper. When confined, such a helical wave will have its plane of incidence always perpendicular to the spatial displacement at any point on it in normal incidence. This means that the reflected wave will not undergo any phase change. However, to form a standing wave one more condition has to be satisfied. The reflections should take place at such a point that when it comes back to original starting point the wave should be in phase again. The simplest case for such a condition to be satisfied is when the helical wave gets reflected at the half wavelength mark. The situation will be clear from figure 4. The closed helical wave formed by the reflection could be seen as forming a path on the surface of an imaginary cylinder as shown in figure 4(c). Note that the successive helical loops undergo precession on the surface of this imaginary cylinder. Here we should keep in mind that as per



(a) (b) (c)

Fig.4 - (a) denotes one full helical wave moving towards the right with the first half of the wave shown in full line while the second half is shown in dotted lines. (b) shows the helical wave that is reflected at the half-wavelength mark to get back to the starting point to be in phase for the next cycle. (c) shows that the closed helical wave is located on the surface of an imaginary cylinder.

convention while the forward wave is left handed, the reverse wave will be right handed. This will mean that for the system of the confined helical wave, the net spin will not be zero as the spin of the forward and the reverse waves will be pointing in the same direction. We may express the forward and the reverse waves as

$$\begin{aligned} \phi_1 &= \eta_o \left(\mathbf{i}\, e^{-i\hbar^{-1}(E_o t_o - p_o z_o)} + \mathbf{j}\, e^{-i\hbar^{-1}(E_o t_o + p_o z_o)} \right) \\ \phi_2 &= \eta_o \left(\mathbf{i}\, e^{-i\hbar^{-1}(E_o t_o + p_o z_o)} + \mathbf{j}\, e^{-i\hbar^{-1}(E_o t_o + p_o z_o)} \right) \end{aligned} \tag{17}$$

Here we should keep in mind that there is no interference between the forward and the reverse waves and for the same reason it is meaningless to take the linear combination of ϕ_1 and ϕ_2. We should also assume that the formation of the particle antiparticle pair takes place when the plane polarized wave is near an interacting field so that the requirement of the momentum conservation is taken care of by it.

Note that the helical wave obtained by imparting spin to ϕ_o^* is a wave travelling backward in time and the spin angular momentum of such a system would be in the opposite direction. Such a confined helical wave travelling backward in time can be identified with the antiparticle state. It becomes obvious that these two confined waves (one spinning counterclockwise and the other spinning clockwise or vice versa) represent the particle and the antiparticle states.

In section 1 we showed that the standing (electromagnetic) wave can represent a particle as it acquires mass on confinement and also its average energy and momentum behave just like the energy and momentum of a particle under relativistic transformations. It can be easily seen that these properties will equally well apply to the case of the confined helical waves introduced in this section. Let us now compare the properties of the standing electromagnetic wave and the confined helical wave discussed above. Let us take the case of the circularly polarized electromagnetic wave forming the standing wave. The spatial displacement of the forward and the reverse waves can be expressed as the real part of the complex functions given by

$$\begin{aligned} \phi_1 &= \eta_o \left(\mathbf{i}\, e^{-i\hbar^{-1}(E_o t_o - p_o z_o)} + i\,\mathbf{j}\, e^{-i\hbar^{-1}(E_o t_o + p_o z_o)}\right) \\ \phi_2 &= \eta_o \left(\mathbf{i}\, e^{-i\hbar^{-1}(E_o t_o - p_o z_o)} - i\,\mathbf{j}\, e^{-i\hbar^{-1}(E_o t_o + p_o z_o)}\right) \end{aligned} \tag{18}$$

Since the forward and the reverse waves undergo interference, we may re-express (18) as

$$\begin{aligned} \phi_x &= \mathbf{i}\eta_o \left(e^{-i\hbar^{-1}(E_o t_o - p_o z_o)} + e^{-i\hbar^{-1}(E_o t_o + p_o z_o)}\right) \\ \phi_y &= i\,\mathbf{j}\eta_o \left(e^{-i\hbar^{-1}(E_o t_o - p_o z_o)} - e^{-i\hbar^{-1}(E_o t_o + p_o z_o)}\right) \end{aligned}$$

Note that the phase of the wave does not undergo any change on reflection in the z-x plane while in the y-z plane there is a phase shift of π on reflection. Because of this phase shift, the reflected wave also has the same helicity as the forward wave. The standing wave formed as a result will be one in which each point on it will be oscillating in the transverse direction. Since the helicity of the forward wave and the reverse wave are the same, the net spin of the standing wave system as a whole will be zero. Here we should keep in mind that since we are dealing with luminal waves, the helicity can be taken as an intrinsic property which is observer independent.

5 More on the Confined Helical Wave

Let us take the function ϕ_o in the range $-\lambda_o \le z_o \le \lambda_o$. Note that since ϕ_o represents a half frequency wave, the range specified above covers only one wave length. Let us now denote by $\phi_o^{L_1'}$ the spatial displacement of the forward wave belonging to ϕ_o and introduce a spin angular velocity of $\frac{1}{2}\omega_o = \frac{1}{2}E_o/\hbar$ in the counterclockwise direction on its spatial amplitude as discussed in the previous section to obtain

$$\phi_o^{L_1'} = \{\mathbf{j}\,\eta_o' \cos[\tfrac{1}{2}(E_o t_o - p_o z_o)/\hbar] - \mathbf{i}\,\eta_o' \sin[\tfrac{1}{2}(E_o t_o - p_o z_o)/\hbar]\}\, e^{-i\frac{1}{2}(E_o t_o - p_o z_o)/\hbar} \tag{19}$$

Note that only the amplitude gets transformed and is given by the two terms within the curly brackets. The right hand side of the above equation could be simplified to give

$$\phi_o^{L_1'} = \tfrac{1}{2}(\mathbf{j} - i\mathbf{i})\,\eta_o'\, e^{-i(E_o t_o - p_o z_o)/\hbar} + \tfrac{1}{2}(\mathbf{j} + i\mathbf{i})\,\eta_o' \tag{20}$$

Note that (19) and (20) do not represent the electromagnetic waves. They can be termed as a new type of helical waves created out of the plane polarized spatial electromagnetic waves by imparting spin motion to them. Now shifting the origin of the coordinate axes to $\frac{1}{2}(\mathbf{j}+i\mathbf{i})\grave{\eta}_o$ and taking $\eta_o = \frac{1}{2}\grave{\eta}_o$, we obtain

$$\phi_o^{L_1} = (\mathbf{j}-i\mathbf{i})\,\eta_o\; e^{-i(E_o t_o - p_o z_o)/\hbar}$$

$$= \eta_o(\mathbf{j}-i\mathbf{i})\, e^{i\hbar^{-1}p_o z'_o} e^{-i\hbar^{-1}E_o t_o} \quad . \qquad (21)$$

Here we have introduced the internal coordinate z′ based on the discussion in section 3.

If we now keep in mind that in (21) the exponential function "exp[$-i\hbar^{-1}E_o t_o$]" represents the wave aspect of the particle, the remaining amplitude part defined in the internal coordinates can be shown to represent the spin. Note that the vector properties are confined to the amplitude part of the function which is defined in the internal coordinates. Therefore, to understand the internal structure of the particle we have to study the amplitude part in isolation. Now in (21) taking only the real part of the function representing the amplitude, we obtain

$$\phi_{Ao}^{L_1} = \eta_o[\mathbf{j}\cos(p_o z'_o/\hbar) + \mathbf{i}\ \sin(p_o z'_o/\hbar)] \qquad (22)$$

Needless to say, ϕ_{Ao}^{L1} stands for a helical wave which is independent of time.

Let us now examine how the reverse wave will transform when spin is introduced. Let us denote it by $\phi_o^{L_2}$ given by

$$\phi_o^{L_2{}'} = \{\mathbf{j}\,\eta'_o\cos[\tfrac{1}{2}(E_o t_o + p_o z_o)/\hbar] - \mathbf{i}\,\eta_o\sin[\tfrac{1}{2}(E_o t_o + p_o z_o)/\hbar]\}e^{-i\frac{1}{2}(E_o t_o + p_o z_o)/\hbar} \qquad (23)$$

$$= \tfrac{1}{2}(\mathbf{j}-i\mathbf{i})\,\eta'_o\; e^{-i(E_o t_o + p_o z_o)/\hbar} + \tfrac{1}{2}(\mathbf{j}+i\mathbf{i})\,\eta'_o$$

Shifting the origin to $\frac{1}{2}(\mathbf{j}+i\mathbf{i})\,\grave{\eta}_o$ and taking $\eta = \frac{1}{2}\grave{\eta}_o$, we obtain

$$\phi_o^{L_2} = \eta_o(\mathbf{j}-i\mathbf{i})\, e^{-i\hbar^{-1}p_o z'_o} e^{-i\hbar^{-1}E_o t_o} \qquad (24)$$

Taking the real part of the amplitude of the above wave in isolation and replacing $\frac{1}{2}\eta_o$ by η_o, we obtain

$$\phi_{Ao}^{L_2} = \eta_o[\mathbf{j}\cos(p_o z'_o/\hbar) - \mathbf{i}\sin(p_o z'_o/\hbar)] \qquad (25)$$

Note that the forward wave given in (21) and the reverse wave given in (25) together will form a closed helical half wave or a closed loop.

Let us now examine the confined wave formed by ϕ_o^* given in (16). Since ϕ_o^* is moving in the reverse time, the spatial displacement of the corresponding confined wave which has clockwise rotation could be attributed to the antiparticle state denoted by $\phi_o^{R^*}$ and following the steps similar to the ones taken for ϕ_o^{L1} we obtain

$$\phi_o^{R_1{}'*} = \{\mathbf{j}\,\eta'_o\cos[\tfrac{1}{2}(E_o t_o - p_o z_o)/\hbar] + \mathbf{i}\,\eta'_o\sin[\tfrac{1}{2}(E_o t_o - p_o z_o)/\hbar]\}e^{i\frac{1}{2}(E_o t_o - p_o z_o)/\hbar}$$

$$= \tfrac{1}{2}(\mathbf{j}-i\mathbf{i})\,\eta'_o\; e^{i(E_o t_o - p_o z_o)/\hbar} + \tfrac{1}{2}(\mathbf{j}+i\mathbf{i})\,\eta'_o \qquad (26)$$

Shifting the origin to $\frac{1}{2}(\mathbf{j}+i\mathbf{i})\grave{\eta}_o$ and replacing $\frac{1}{2}\eta_o$ by η_o, we obtain

$$\phi_o^{R_1^*} \quad = \quad \eta_o(\mathbf{j} - i\mathbf{i})\, e^{-i\hbar^{-1} p_o z'_o} e^{i\hbar^{-1} E_o t_o} \tag{27}$$

The real part of the amplitude of the above wave taken in isolation gives us

$$\phi_{Ao}^{R_1^*} \quad = \quad \eta_o[\mathbf{j}\cos(p_o z'_o/\hbar) + \mathbf{i}\sin(p_o z'_o/\hbar)] \tag{28}$$

Similarly we may express the reverse wave in the reverse time as

$$\phi_{Ao}^{R_2^*} \quad = \quad \eta_o[\mathbf{j}\cos(p_o z'_o/\hbar) - \mathbf{i}\sin(p_o z'_o/\hbar)] \tag{29}$$

Note that (22) and (25) pertains to the particle state while (28) and (29) pertains to the anti-particle state.

An interesting aspect one may observe in the above analysis is that the confinrd helical wave representing both particle and antiparticle states the origin is being shifted to the same point. This means at the instant of creation of the particle-antiparticle pair the net electric (also magnetic) field is zero at all points on the confined helical waves. However electric field of ξ_o can reappear on the confined helical wave representing the particle state in such a way that it gets compensated by the electric field $-\xi_o$ that reappears on the corresponding wave representing the antiparticle state. Remember that the electric field on the wave travelling forward in time was having electric field ξ_o while that on the wave travelling reverse in time was having electric field $-\xi_o$.

Till now we have studied the forward and reverse waves constituting the confined helical waves in their proper reference frame. If a translational velocity is given along z-direction, then, based on the discussions in section 1, (19) and (22) will transform to

$$\phi^{L_1} \;=\; (\mathbf{j} - i\mathbf{i})\,\eta_1\; e^{iE(z-vt')/\hbar c}\,\phi \quad ; \quad \phi^{L_2} \;=\; (\mathbf{j} - i\mathbf{i})\,\eta_2\, e^{-iE(z-vt')/\hbar c}\,\phi \tag{30}$$

where $\phi \;=\; e^{-i\hbar^{-1}(Et - \text{þ} z)}$. Since η_o may undergo relativistic transformation on acquiring translational motion we denote the amplitude of the forward wave by η_1 and that of the reverse wave by η_2. Using (4), (30) can be re-expressed (see appendix) as

$$\phi^{L_1} \;=\; (\mathbf{j} - i\mathbf{i})\,\eta_o\, e^{i\theta}\; e^{-i\hbar^{-1}(E_1 t - \mathrm{p}_1 z)} \quad ; \quad \phi^{L_2} \;=\; (\mathbf{j} - i\mathbf{i})\,\eta_o\, e^{-i\theta}\; e^{-i\hbar^{-1}(E_2 t + \mathrm{p}_2 z)} \tag{31}$$

where θ is given by $\sin\theta = -i\gamma\beta$, $\gamma = (1-\beta^2)^{-\frac{1}{2}}$, β being equal to v/c.

6 Confined Helical Wave and its Spin

In quantum mechanics, spin is taken as an intrinsic property of electron. The idea of electron spinning around its own axis does not make sense as it is taken as a point particle without any internal structure. This is so different from the macroscopic situation where the direction of the spin of a body is exactly determined and is observer independent. Therefore, it was believed that the spin of a particle like electron cannot be understood in terms of the classical analogues of rotational motion in the three dimensional space. It is treated as a property defined in the internal space (of the particle) that cannot be analyzed further. But in our approach, the plane wave representing an electron is attributed an internal structure of the confined helical wave. This means that spin can also be traced to some property of the confined helical wave. We shall see how this can be done.

We know that the accepted practice in quantum mechanics is to represent the angular momentum component, M_z along the z-axis by the relation

$$M_z \quad = \quad i\hbar\, \partial\psi \big/ \partial\theta \qquad where \quad \psi \quad = \quad B e^{i\hbar^{-1}A} \ , \tag{32}$$

Here $\partial\theta$ represents an infinitesimal rotation around z-axis in the x-y plane and while A represents the action function. Accordingly we have

$$M_z \quad = \quad -\partial A \big/ \partial\theta \ . \tag{33}$$

We should keep in mind that if a particle were to have a non-zero value for angular momentum (with regard to a point in the path) it should progress along a curved path which would be evident from the variation in the phase of the wave given by $-\partial A/\partial\theta$. But we know that the spin angular momentum is independent of the path of the particle. This leaves us with only the rotation of the amplitude of the wave to account for the spin. We shall now take the forward wave given in (32) and examine if its amplitude can exhibit the spin properties of the particle. Expressing $\phi^{L_1}(\eta)$ as ϕ^{L_1} and taking into account the relativistic transformations in η_1 and η_2 (see appendix), we obtain

$$\phi^{L_1} \quad = \quad \phi_A^{L_1} \phi \tag{34}$$

$$where \quad \phi_A^{L_1} = (\mathbf{j} - i\mathbf{i})\, \eta_o\, e^{i\theta}\, e^{iE(z' - vt')/\hbar c} \quad and \quad \phi \, = \, e^{-i\hbar^{-1}(Et - \text{þ}z)} \ .$$

The amplitude wave introduced here has got vector properties and is defined in the internal coordinates. Therefore it appears to be ideally suited to represent spin. The phase of the amplitude wave can be expressed as

$$\begin{aligned} E(z' - vt')/\hbar c \quad &= \quad \hbar^{-1}\left[\tfrac{1}{2}(E_1 + E_2) z'/c - \tfrac{1}{2}(E_1 - E_2) t'\right] \\ &= \; -\tfrac{1}{2}\hbar^{-1} E_1 (t' - z'/c) + \tfrac{1}{2}\hbar^{-1} E_2 (t' + z'/c). \end{aligned} \tag{35}$$

Therefore, the amplitude wave in (34) can be expressed as

$$\phi_A^{L_1} \quad = \quad (\mathbf{j} - i\mathbf{i}) \eta_o^{\frac{1}{2}} \; e^{\frac{1}{2}i\theta} \; e^{\frac{1}{2}i\hbar^{-1}A'_1} \; \eta_o^{\frac{1}{2}} \; e^{\frac{1}{2}i\theta} e^{-\frac{1}{2}i\hbar^{-1}A'_2} \ . \tag{36}$$

Here A_1' and A_2' are respectively the action of the forward and the reverse waves in terms of the internal coordinates. These waves represented in (36) should not be confused with the electromagnetic wave confined between the two mirrors. We shall show later that these represent spin rotations which created the confined wave that represents a particle out of the original wave with frequency $\frac{1}{2}E_o/\hbar$. Note that these waves undergo a phase change of only π when the plane wave undergoes a phase change of 2π. In (36) we should remember that the vector (**j**-*i***i**) represents the rotation in the x-y plane in the anti-clockwise direction and for the same reason it is logical to denote it by the unit vector **k** in the z-direction. We may now express $\phi_A^{L_1}$ as

$$\phi_A^{L_1} \quad = \quad w^{2*} u^1 , \tag{37}$$

where $\qquad u^1 \; = \; \eta_o^{\frac{1}{2}}\, e^{i\frac{1}{2}\theta} e^{i\frac{1}{2}\hbar^{-1}A'_1}, \quad w^2 \; = \; \eta_o^{\frac{1}{2}}\, e^{\frac{1}{2}i\theta} e^{i\frac{1}{2}\hbar^{-1}A'_2}$

Here we have suppressed the unit vector **k** because it is taken for granted that the direction of the spin could be along the direction of the momentum. Since θ is imaginary, the function "exp($i\frac{1}{2}\theta$)" is a real number and is invariant to the conjugation process. Here the product of u^1 and w^{2*} forms the amplitude of the plane wave. It can be easily seen that u^1 rotates the original half frequency wave into a full fledged forward wave.

We may now express the spin angular momentum, S of the forward wave by

$$S\phi_A^{L_1} \;=\; i\hbar\,\frac{\partial \phi_A^{L_1}}{\partial(\omega_1 t')} \quad=\quad i\hbar\; w^{2*}\,\frac{\partial u^1}{\partial(\omega_1 t')} \quad=\quad \tfrac{1}{2}\,\hbar\,\phi_A^{L_1}. \tag{38}$$

Note that by the time the phase of the forward component of the confined helical wave rotates by an angle $\omega_1 t'$, u^1 would have rotated through only half of that. As the operator $\partial/\partial(\omega_1 t)$ does not operate on w^{2*}, it is obvious that for the purpose of the spin of the forward wave, we may ignore its existence completely. This would mean that we may represent the forward wave given in (34) as

$$\phi^{L_1} \quad=\quad u^1\,\phi \tag{39}$$

Here we have dropped the factor "(**j**-**i**)" from the right hand side as it represents the direction of the spin which any way can be accounted by suitably defining u^1 to represent the correct spin rotation. We should keep in mind that the rotation has been reckoned with regard to luminal velocity of the wave which for the forward wave is in the z-direction and in the opposite direction for the reverse wave. This gives the value of the spin as $\frac{1}{2}\hbar$. Note that the right hand side of the above equation does not bring out the true picture of the inner structure of the particle. But since we are interested only in the spin of the system that emerges from the inner structure, u^1 is adequate for the purpose. In the above discussion we have taken the helical wave rotating in the counterclockwise direction.

Let us now take the reverse wave ϕ^{L_2} from (24) and express it as

$$\begin{aligned}\phi^{L_2} \quad &= \quad (\mathbf{j}-i\mathbf{i})\eta_o^{\frac{1}{2}}\; e^{-\frac{1}{2}i\theta}\; e^{\frac{1}{2}i\hbar^{-1}A'_2}\eta_o^{\frac{1}{2}}\; e^{-\frac{1}{2}i\theta}\; e^{-\frac{1}{2}i\hbar^{-1}A'_1}\;\phi \\ &= \quad (\mathbf{j}-i\mathbf{i})w^{1*}u^2\;\phi \end{aligned} \tag{40}$$

where $\quad u^2 \;=\; \eta_o^{\frac{1}{2}}\; e^{-\frac{1}{2}i\theta}\; e^{i\frac{1}{2}\hbar^{-1}A'_2} \quad$ and $\quad w^1 \;=\; \eta_o^{\frac{1}{2}}\; e^{-\frac{1}{2}i\theta}\; e^{i\frac{1}{2}\hbar^{-1}A'_1}$

Following the same steps as taken from (34) to (38), we will obtain

$$S\phi_A^{L_2} \;=\; -i\hbar\,\frac{\partial \phi_A^{L_2}}{\partial(\omega_2 t')} \quad=\quad -i\hbar w^{1*}\,\frac{\partial u^2}{\partial(\omega_2 t')} \quad=\quad -\tfrac{1}{2}\,\hbar\,\phi_A^{L_2}. \tag{41}$$

Note that for the reverse wave the spinning will appear right handed which explains the introduction of the minus sign. This means that the spin of the forward and the reverse waves will be aligned in the same direction which will be the direction of the translational velocity. An interesting property that emerges from the above description is that by the time the phase of the forward wave given in (32) changes by $\omega_2 t'$, u^2 would have rotated through only half of that. Since the operator $\partial/\partial(\omega_2 t`)$ does not operate on the component w^{1*}, it is obvious that for the purpose of the spin of the reverse wave, we may ignore its existence completely. This would mean that we may represent the reverse wave given in (40) as

$$\phi^{L_2} \quad=\quad u^2\,\phi \tag{42}$$

Since the rotation of the reverse wave is aligned in the same direction as that of the forward wave, spin of the forward and the reverse waves will be in the same direction with regard to the translational velocity of the confined wave or the particle it represents. Therefore, taking the average of the spin of the forward wave, S_1 and that of the reverse wave, S_2, we obtain

$$S = \tfrac{1}{2}\left[S_1 + S_2\right] = \tfrac{1}{2}\left[\tfrac{1}{2}\hbar + \tfrac{1}{2}\hbar\right] = \tfrac{1}{2}\hbar \quad . \tag{43}$$

The above treatment attributing spin of the particle to the amplitude wave defined in the internal coordinates gives a consistent result.

7 From the Confined Helical Wave to the Wave Function

In the approach followed till now we have been representing a particle in terms of a confined helical wave having a specific value for its energy and momentum. We saw that in external coordinate system the confined helical wave has the form as that of a plane wave. In fact, it is reasonable to assume that the plane wave representation of particle in quantum mechanics has its origin in the confined helical structure of particle as described above. In other words, we are attributing a substructure to the plane wave state. The spin of the particle emerges from the rotation acquired by the amplitude of the half frequency wave. Using (30) and (31) let us represent the confined helical wave as

$$\phi^{L} = \phi^{L_1} + \phi^{L_2} = 2(\mathbf{j} - i\mathbf{i})\cos[E(z' - vt')/\hbar c + \theta]\,\phi \tag{44}$$

Here the amplitude wave represented by the function "$2(\mathbf{j}\text{-}i\mathbf{i})\cos[E(z`\text{-}vt`)/\hbar c+\theta]$" is defined in the internal coordinates of the particle. Therefore, the amplitude of the plane wave in the external coordinate has to be taken as unity. Since we were dealing with a given state of the particle, it is obvious that the probability of finding the particle in that state has to be unity. But if the plane wave states formed by the confined helical waves are occupied in a virtual manner by the principle of quantum superposition, then in the place of unity, it becomes logical to introduce the probability amplitude [4]. Let us now denote the plane wave state by ϕ_i where "i" denotes its energy state which may take values from 1 to n. If the i^{th} state is occupied "a_i" number of times in the process of superposition, then we may express it by

$$\psi = \Sigma\,\psi_i = \Sigma\,a_i\,\phi_i \tag{45}$$

Here the function $\psi_i^*\psi_i = a_i^* a_i$ denotes the probability that the i^{th} state is occupied. Note that here spin aspect of the particle is not expressed explicitly.

We saw that although the electromagnetic wave exists in real space, the moment it gets confined it acquires the form of the plane wave which is no more defined in real space. This is made possible by compacting the spatial properties of the electromagnetic wave into the inner coordinates. It is now quite clear that the wave function as defined in quantum mechanics is a logical extension of the function representing the confined helical wave. De Broglie believed that ψ_i is defined in the real space [5] and now we know that his intuition was not very far from the truth. An important property of Ψ as defined in quantum mechanics is that it treats the eigen state as the fundamental state which is not required to be analyzed any further. Note that this self imposed restriction emerges from the assumption that the plane wave state is the most basic state. But we saw that the plane wave state can be attributed a sub-structure in terms of the confinement of electromagnetic wave. In this paper we have not clarified what happens to the electric and magnetic fields associated with the confined electromagnetic wave as it gets compacted to the internal coordinates. It will be shown in a separate paper that in the process of confinement the electric field generates the electric charge of the particle while the magnetic field gets canceled.

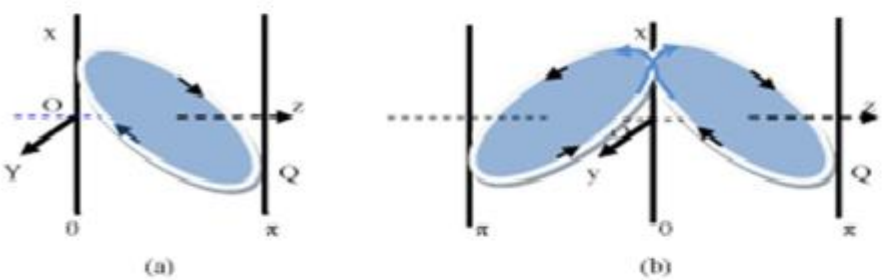
The particle state denoted by the confined helical wave has to be either an electron or a muon as only the electromagnetic field is involved here. It should be possible to treat other particles like quarks in terms of confinement of a more complex wave which has oscillations not only in space and electromagnetic field but also in other relevant fields. Note that in the process of confinement all the properties pertaining to the fields will get compacted into the internal coordinates while the time dependent part of the wave will get converted into the plane wave. In other words, in spite of the differences in the internal structures, it will be possible to represent all particles in terms of plane waves.

8 The Structure of Electron and Pauli's Exclusion Principle

In the light of the above discussion, it is quite clear that the electron can be represented by a helical half wave confined between two mirrors. Let us take the case of the electron in the spin up state which can be represented by ϕ^L. From (22) and (25) we know that in the rest frame of reference, the real part of the amplitudes of the forward helical half wave and the reverse helical half wave can be represented respectively by ϕ_{Ao}^{L1} and ϕ_{Ao}^{L2} given by

$$\begin{aligned} \phi_{Ao}^{L_1} &= \eta_o[\mathbf{j}\cos(p_o z_o'/\hbar)+\mathbf{i}\ \sin(p_o z_o'/\hbar)] \\ \phi_{Ao}^{L_2} &= \eta_o[\mathbf{j}\cos(p_o z_o'/\hbar)-\mathbf{i}\sin(p_o z_o'/\hbar)] \end{aligned} \tag{47}$$

The confined wave formed by these two waves represents a helical half wave defined in the internal coordinates as shown in figure 5. It should be noted that this closed helical half wave undergoes precession on the surface of an imaginary cylinder as already discussed in section 4.



Fig,5 - (a) represents the standing helical half wave formed between two reflecting mirrors. (b) shows that two such standing waves can be joined only when their spins are oriented in the opposite directions.

On the basis of this confined helical half wave structure of electron, it is possible to understand the Pauli's exclusion principle. It should be noted that two such standing waves can be joined together only at O as shown in figure 5(b). Joining these waves at Q with another wave is not possible as the wave gets completed only at O. But to be in phase at the point O, the other standing waves will have to have the opposite helicity (spin). Once one standing wave couples with another one with the opposite helicity (spin), then it is obvious that no further connections could be made with any other standing wave. It should be noted that for the confined helical wave structure or two particles of opposite spin to couple, their plane waves should remain in phase. This, we know, is possible only if the energy and momentum of the particle are equal. This explains the Pauli's exclusion principle.

If a fermion is accorded the confined helical wave structure, it is obvious that a boson having unit spin should be attributed the structure of confined helical full wave. On the basis of such a structure, it is quite clear that a boson could be attached to other bosons on either side, and a chain of bosons could be formed without limit.

9 Spin and the Symmetry in the Relativistic Transformation

Although spin emerges naturally from the solutions of the Dirac equations, its relation to the relativity principle is not quite evident from the approach. We shall now show that spin can be treated as the conserved quantity for the symmetry associated with the relativity principle. Let us take the equation

$$(\gamma_\nu þ_\nu)(\gamma_\mu x_\mu) = (\mathbf{þ.r} - \mathrm{Et}) - \mathrm{i}\boldsymbol{\Sigma}\cdot(\mathbf{r}\times\mathbf{þ}) - \boldsymbol{\alpha}\cdot(\mathbf{þ}ct - E\,\mathbf{r}/c)\ . \tag{48}$$

where γ_v stands for the Dirac gamma matrices while Σ and α are the familiar 4x4 matrices which could be expressed in terms of the Pauli's spin matrices [6]. The terms on the right hand side of the above equation can be treated as the sum of the scalar and vector products of the four-vector momentum and spatial coordinate. The first term on the right hand side of (48) gives us the action of the particle while the second term gives us the angular momentum. We saw from section 6 that the third term, which is the "cross product" involving energy and time (note that [$\mathbf{þ}$ct - E$\mathbf{r}$/c] = [$x_o þ_k$ - $x_k þ_o$]) is actually defined in the internal coordinates, and represents the spin angular momentum. If the particle is moving along z-axis, then we may express (48) as

$$(\gamma_\nu þ_\nu)(\gamma_\mu x_\mu) = (þ_z z - \mathrm{Et}) - \mathrm{i}\boldsymbol{\Sigma}_z\cdot(\mathbf{r}\times\mathbf{þ})_z + \boldsymbol{\alpha}_z\cdot E(\mathbf{z}' - \mathbf{v}t')/c\ . \tag{49}$$

This takes us to an interesting insight. We know that the invariance of the angular momentum can be attributed to the directional isotropy of the space. Or in other words, the physical property of a system will not change whatever be its direction in space. Now a rotation in the plane containing the time-axis and z-axis can be effected only by introducing a translational velocity to the system. Therefore, the invariance of the spin of a particle can be attributed to the invariance of the physical properties of the system on introducing a velocity transformation. In other words, spin is one of the invariant quantities that emerge out of the relativity principle which is a fundamental symmetry. This explains why spin is an intrinsic part of the Dirac equation. Note that the concept of the internal coordinates is essential to show that last term on the right hand side of (49) represents spin angular momentum of the system. This explains why it cannot be understood in classical terms.

10 Conclusion

From the above discussion, the representation of electron by a confined helical wave appears to be a viable proposition. It provides us with a new insight into the spin and the structure of the elementary particle. Needless to say one important advantage of this approach is that it gives a simple explanation for Pauli's exclusion principle and the non-classical behavior of the spin angular momentum.

In quantum mechanics the state of a particle is represented by a plane wave which is an eigen state of the four-momentum in the coordinate representation. Note that the eigen state is the ultimate level of reality in quantum mechanics, beyond which no measurement is assumed to be possible. In relativistic quantum mechanics spin is introduced as an internal degree of freedom and is not directly related to the plane wave representation of the particle. But in the approach followed in this paper, it is observed that the confinement of the electromagnetic wave results in its spatial oscillations getting compacted into the inner coordinates where the spin of the particle is defined while the time dependent component becomes the plane wave. Therefore, we are effectively assuming that the plane wave representation of a particle is not the dead end for the investigation into the structure of the particle. Here we are attributing an inner structure to the eigen state represented by the plane wave.

The confinement of the electromagnetic wave is shown to create mass as well as the spin of the particle. However, this approach cannot be considered complete unless it is shown that the electric charge can also created just like mass from the confinement of the helical wave. This will be attempted in the next paper. The confined helical wave representation of the electron promises to be an exciting new way of understanding the basic structure of matter without invoking the existence of strings.

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Appendix

Let us now consider the case where the confined helical wave is formed along z-axis with its spatial amplitudes in the x-y plane. Let us assume that it has a translational velocity v along the z-direction. If the spatial amplitude of the helical wave is spinning in the anticlockwise direction when viewed head on, we may treat it as left handed and represent it by ϕ^L. We know from section 4 that ϕ^L can be represented in terms of the forward wave and the reverse wave in the more general complex form as

$$\phi^{L_1} = (\mathbf{j} - i\mathbf{i})\,\eta_1\, e^{-i\frac{1}{2}\hbar^{-1}(E_1 t' - p_1 z')}\; e^{i\frac{1}{2}\hbar^{-1}(E_2 t' + p_2 z')}\; e^{-i\hbar^{-1}(Et - þ z)} \qquad \text{(A1)}$$

$$\phi^{L_2} \quad = \quad (\mathbf{j}-i\mathbf{i})\eta_2 \; e^{-i\frac{1}{2}\hbar^{-1}(E_2t'+p_2z')} \;\; e^{i\frac{1}{2}\hbar^{-1}(E_1t'-p_1z')} \; e^{-i\hbar^{-1}(Et-\text{þ}z)} \tag{A2}$$

But since the spatial amplitude of the helical wave $\eta_1 = \lambda_1/2\pi$ and $\eta_2 = \lambda_2/2\pi$, where λ_1 and λ_2 are the Doppler shifted wave lengths of the forward and the reverse waves respectively given by

$$\lambda_1 \quad = \quad \gamma\lambda_o(1+\beta) \;\; ; \quad \lambda_2 \quad = \quad \gamma\lambda_o(1-\beta)\,, \tag{A3}$$

we have
$$\eta_1 \quad = \quad \gamma\eta_o(1+\beta) \;\; ; \quad \eta_2 \quad = \quad \gamma\eta_o(1-\beta)\,, \tag{A4}$$

where $\eta_o = \lambda_o/2\pi$ and $\beta = v/c$. Taking $\gamma = \cos\theta$ and $-i\beta\gamma = \sin\theta$ and simplifying further, keeping in mind that the amplitude wave is defined in the internal coordinates, we may express the forward wave as

$$\phi^{L_1} \quad = \quad (\mathbf{j}-i\mathbf{i})\,\eta_o\, e^{i\theta}\; e^{i\hbar^{-1}E(z'-vt')} \;\; e^{-i\hbar^{-1}(Et-\text{þ}z)} \tag{A5}$$

where $\phi \;=\; e^{-i\hbar^{-1}(Et-\text{þ}z)}$. Similarly, the reverse wave can be expressed as

$$\phi^{L_2} \quad = \quad (\mathbf{j}-i\,\mathbf{i})\,\eta_o\, e^{-i\theta}\; e^{-i\hbar^{-1}E(z'-vt')}\,\phi \tag{A6}$$

$$\longleftrightarrow$$