TWIN PARADOX 1938-2012

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The phenomenon of the transverse Doppler effect provides an opportunity to validate the twin paradox (or clock paradox). Using this approach, a contradiction can be shown between the theory and the experimental results. The nature of the paradox can be best described through experiments with atomic clocks, as these devices are practically used for experimental demonstration of the phenomenon.

1. The clock paradox according to relativity theory

The simplest version of the clock paradox is the following: A and B are two adjacent atomic clocks at rest ($v = 0$) in a closed inertial system K. After an acceleration period, the atomic clock A travels with constant velocity v_a at L distance, then stops and moves contrariwise with constant velocity at L distance before slows it down and stops next to clock B.

According to the standard theoretical statement, the clock in motion would run slower, therefore, after returning next to clock B, clock A shows less time elapsed as clock than clock B that remained at rest. This can be described with the frequency changes of the clocks. The frequency of the clock in motion is

$$f_a = f_0\sqrt{-v_a^2/c^2} \tag{1}$$

as a function of velocity v_a, where f_0 is the frequency of the clock at rest and c is light speed. The diagram of the process is shown in Figure 1, where the distance travelled (L) is showed on axis s.

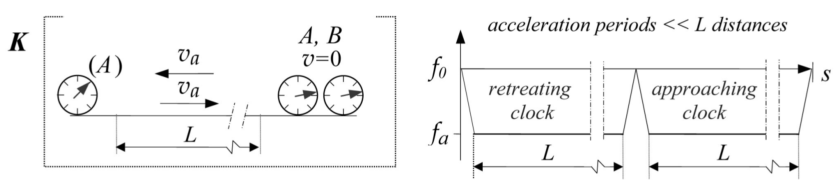

Figure 1. Diagram of the clock paradox in inertial system K

2. The true frequency changes of atomic clocks in the Ives–Stilwell experiment

In this experiment there is a true change in the frequency of the clock in motion, as the function of velocity, and this is not identical to the change in frequencies from the viewpoint of an observer in relative motion.

Ives and Stilwell [1] (**1938**) were the first to show that true changes can be observed in the frequency of atomic clocks moving with different velocities in an inertial system. [1]

[1] In the experiment the turning of the of Earth's surface is negligible compared to the high velocity of the atomic clocks.

(According to the phenomenon referred to as the transverse Doppler-effect – assuming ideal measurements – the frequency of an atomic clock measured at right angles to the direction of its motion is free of the Doppler–shift deriving from the relative differences in motion. [2])

The relation between velocity and frequency is described as $f = f_0\sqrt{1 - v^2/c^2}$, as already mentioned in the previous section.[2] Taking the elapsed time on the stationary clock as t_0, the time showed by a clock in motion with velocity v is

$$t = t_0/\sqrt{1 - v^2/c^2}. \tag{2}$$

In the next section we will test the clock paradox with the help of the Ives–Stilwell experiment, by modifying the former experimental setting in a way that now the inertial system K is moving inertially inside the system K' described by Ives–Stilwell. (The inertial system K' of Ives–Stilwell is an arbitrary chosen accessory system of reference here, like – as an analogy – the 0 Celsius degree of the temperature scale.)

3. Correlations between the Ives–Stilwell experiment and the frequencies of atomic clocks in system K

Using the results of Ives and Stilwell on true frequency change, we are going to show through an example that there is no time difference between clocks (twins) as predicted by the theory of relativity. The main point of this experiment is that the true, own frequencies of the atomic clocks in the closed system K can be determined by atomic clocks of the same type and state outside the system K, and are thereby suitable for validating theoretical conclusions.

Let's repeat the experiment described above with the system K moving with a velocity v_1 inside the system K' of the Ives–Stilwell experiment. K and K' Galilean (non–accelerated) systems!

A parallel experiment is conducted now by with placing the same type of atomic clocks, A' and B' in the system K'. We determine the frequencies of the clocks in system K (inside the system K') step by step by measuring the transverse Doppler-shifts of the clocks in system K', A' and B', which are moving with the same velocity and along the same axis (x) as the clocks A and B. (Inertial atomic clocks moving with different velocities have constant frequencies as long as external factors remain constant.) The frequencies of the clocks A, depending on its movements, are as follows: f_1 at rest, f_2 when retreating, f_3 when approaching (Figure 2).

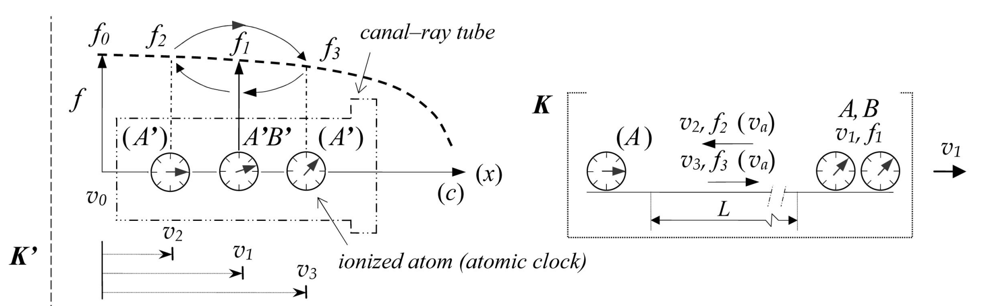

Figure 2. Movements of the clocks in system K inside the system K' of Ives and Stilwell

[2] As we will see, this relation is not symmetrical for all stationary clocks in an inertial system, as in the case of this experiment as well. However, it does not influence the deduction and therefore can be disregarded here.

The velocities of clocks A and B at rest are v_1, and the respective frequencies measured on clocks A' and B' are f_1. After acceleration (that is a negative acceleration since clock A drops behind clock B) clock A moves with a constant velocity v_2. At velocity v_2 the frequency of the clock A, determined with the transverse Doppler shift on clock A', is f_2, a frequency higher than frequency f_1 of the clock at rest. After travelling to distance L, clock A reduces its speed in system K, and reaches a stationary state. Now the frequency of clock A (f_1) will be again identical to the frequency of clock B, as both are moving with the same velocities.

In order to return next to clock B, clock A have to be accelerated to a velocity higher than v_1. Let this velocity be v_3 and let $v_1 - v_2 = v_3 - v_1$. The corresponding frequency f_3 can be determined again with the transverse Doppler shift of clock A' of the Ives–Stilwell system. Assuming velocities much lower than the speed of light, the relevant segment of the velocity–frequency diagram can be regarded as linear, therefore, frequency differences and can be considered with a good approximation as equal. The scheme of the frequency-distance diagram of the experiment is shown on Figure 3.

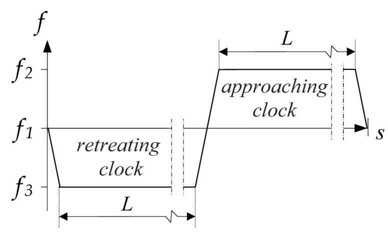

Figure 3. Frequency changes of clock A

3.1. **The method of clock frequency measurements in system K and its suitability in the experiment.** *In the theory of relativity the same phenomenon changes in two separate relations* [3]. *In one case, the phenomenon depends on the frame of reference chosen, while in the other it depends on the physical state. To separate the effects of the two relations, one has to choose an adequate frequency measuring method.*

The frequency measurements in system K apply to the periods when the atomic clocks move at constant velocity along an axis. For these measurements a counter attached to the clock, and therefore moving with the clock, is suitable for many reasons.

This was the method used in the Hafele–Keating [4] *experiments as well. One advantage of this approach is that because of the integration of the clock and the counter (detector+counter), no relative movements occur between the two (where the counter serves as the observer).*

Other advantage is the exclusion of aberration, since there are no relative movements between the clock and the counter even during acceleration periods. The setting is suitable for the detachment of acceleration periods if its operation is restricted to L–L sections.

The velocity of the clocks in system K are known on sections L; therefore, the frequencies of the clocks from different directions can be counted based on the frequencies measured from distances L.

The frequency shift measured at right angles changes with the acceleration of the clock. When making measurements in these inertial systems (e.g. atomic clocks) one has to consider the consequences of acceleration effects (known or unknown) causing *changes in the physical state of the system* besides the *changes caused by the frame of reference chosen* in the own values of kinetic energy, mass and *frequency*.[3] The effects of the *changes in physical state* are usually overseen ("merged" with changes derived from the chosen frame of reference) when working with relativity theory; however, as the present study shows, these effects can have a crucial role in determining the physical properties of inertial systems.

4. Consequences of merging the clock paradox and the Ives-Stilwell experiment.

As revealed by counting and as shown in Figure 3, the time differences between clocks – derived from frequency changes of clock A – level off (disregarding the negligible differences). *The time difference between clocks or twins predicted by the theory does not exist.* (The acceleration periods are negligible compared to the length of L distances.)

The retreating clock runs faster than the clock at rest, on the contrary, it runs slower on the way back. Therefore, the time differences between clocks seen on sections L level off, and finally clock A will show the same time as the stationary clock B. The same can be observed when the clock moves in the opposite direction, only with opposite signs.

This conclusion is supported by the short section of the Hafele–Keating experiment, where the movement of the clocks can be considered as along a line, where the frequencies of the clocks were shown to be dependent on the direction of movement. According to Einstein [7], the frequency change of the clock is proven for moving in a polygonal line, and he assumes that these results are also valid for a continuously curved line. Based on these assumptions, the short periods of movement of the clocks with constant velocities in the Hafele–Keating experiment can be regarded as along a line.[4]

According to Einstein's argumentation, the short periods of the Hafele–Keating (and similar) experiments correspond to the clock paradox experiment showed above, and therefore the consequences described above can be regarded as proven without any further experiments needed. Based on the theory, the twin who travels ages slower during the whole trip, and therefore will be younger than the other upon returns.

On the contrary, measurements show that when retreating, the process of aging accelerates, while on the way back the travelling twin ages slower than the one at rest.

If the clocks move with symmetrical velocities, the differences in direction–dependent frequency changes $f_2 - f_1$ and $f_1 - f_3$ and are asymmetrical (this is the difference that was considered as negligible previously). However, different kinetic energies pertain to these symmetric velocities, resulting in asymmetric changes in the physical states of the clocks. Here we have to note the results of Kaufmann [8] published in 1901 on the changes of electron charge for different velocities.

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[3]a/ "Nobody doubts the 'reality' of kinetic energy, otherwise the very reality of energy would have to be denied." [5]

b/ "2. On the Gravitation of Energy: One result yielded by the theory of relativity is that the inertia mass of a body increases with the energy it contains; if the increase of energy amounts to E, the increase in inertial mass is equal to E/c, when c denotes the velocity of light.

Now is there an increase of gravitating mass corresponding to this increase of inertia mass? If not, then a body would fall in the same gravitational field with varying acceleration according to the energy it contained. That highly satisfactory result of the theory of relativity by which the law of the conservation of mass is merged in the law of conservation of energy could not be maintained, because it would compel us to abandon the law of the conservation of mass in its old form for inertia mass, and maintain it for gravitating mass." [6]

[4] "If we assume that the result proved for a polygonal line is also valid for a continuously curved line, we arrive at this result: If one of two synchronous clocks at A is moved in a closed curve with constant velocity until it returns to A, the journey lasting t seconds, then by the clock which has remained at rest the travelled clock on its arrival at A will be $\frac{1}{2}t\,v^2/c^2$ second slow." [7]

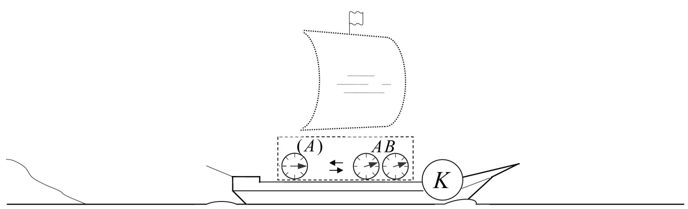
Later on, measurements proved that this phenomenon is not exclusively applies for electrons, and verified that the changes in mass can be described with the equation

$$m = m_0/\sqrt{1 - v^2/c^2}. \tag{3}$$

This is an inverse ratio change compared to the relation for frequency change. If the mass of an atom increases, its radiation frequency decreases and vice versa.

Considering the asymmetry in the frequency changes (based on considerations not described in details here) leads to the conclusion that the twin paradox as it was described in special relativity theory is valid only for a system at absolute rest. However, this leads beyond the scope of this study [9, 10].

It has to be added to the conclusions, that an experiment can be conducted in a closed inertial system that is capable of demonstrating the movement of the system. If Galileo had possessed our present knowledge concerning atomic clocks, would we speak principle of relativity today?



K non–acceleration systems

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