New theory of plasma magnetic moment may explain features of solar jets and spicules

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Abstract

Current theory states that the magnetic moment of a charged particle is constant, or invariant in a slowly changing magnetic field. It also states that the magnetic flux through a Larmor orbit is constant.

The current theory is closely examined, and found to have inconsistencies. A new theory is developed with new results for both the energy and magnetic moment of a charged particle. The new theory predicts particle behaviour which is opposite to convention: for example, that a plasma will decelerate as it moves through a magnetic field which is weakening. The magnetic field of the sun would then act as a restraining influence on solar plasma ejections.

The theoretical results are compared with experimental results for the velocity, deceleration and height of solar jets and spicules.

Introduction

When a charged particle moves at right angles to a uniform magnetic field, it rotates in Larmor circles. If this magnetic field is steadily increased the current conventional theory claims that the magnetic flux enclosed by the orbit remains constant, and also that the kinetic energy of the particle increases in proportion to the field.

It is suggested here that these statements are incompatible, and that this is due to inconsistencies in the conventional theory, both in its derivation and conclusions.

It is well known that when a charged particle, moving at velocity V at right angles to a magnetic field B, then the radius, r, of the Larmor circle is given by

$$r = \frac{mV}{Be} \qquad (1)$$

where m=particle mass and e=particle charge

If the magnetic field slowly increases then the radius r will slowly decrease as shown in figure 1.

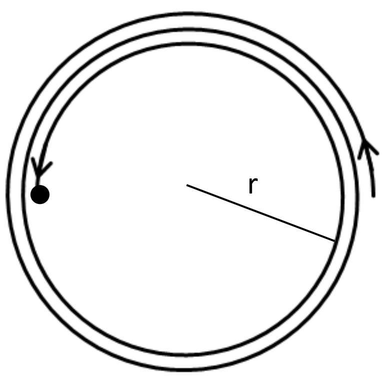


Figure 1

As the magnetic field increases
the radius, r, decreases.

A changing magnetic field will also cause an induced emf around each orbit, due to electromagnetic induction, so that the energy and velocity, V, of the particle will also change

Conventional theory

This was first suggested by Alfven (1950), and the proof is reproduced on the following page.
The main condition is that rate of change of B must be slow, that is the relative change in B during one single orbit must be much less than one, so that the orbits are almost circular.
The main results of conventional theory state that:

- **The magnetic flux, Φ, through the Larmor orbit is constant. ie**

$$\Phi = \text{constant} \tag{2}$$

- **The kinetic energy of the particle, $W_\perp$, is proportional to the applied field, B, ie**

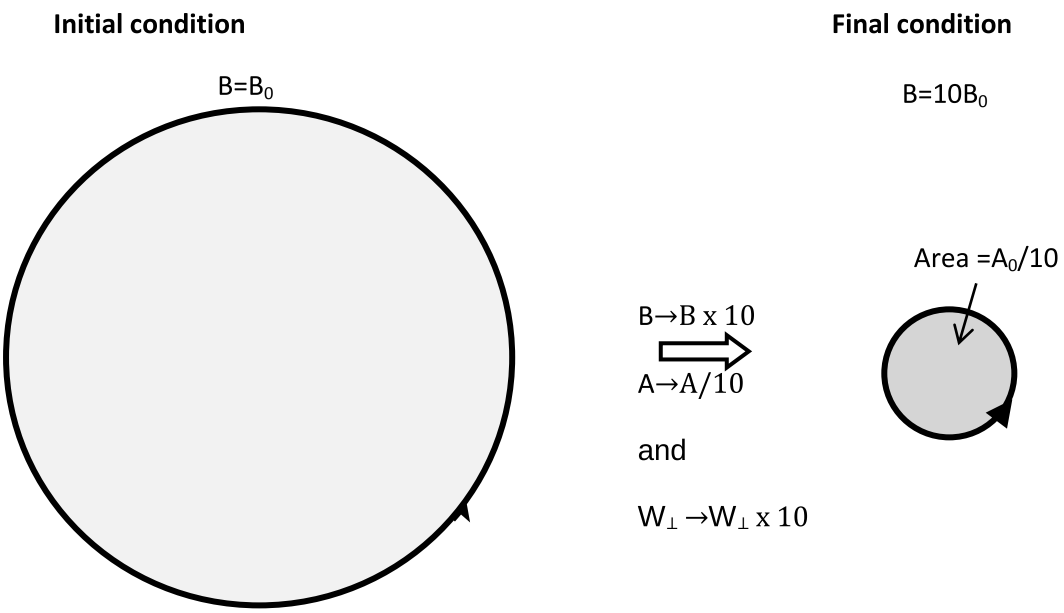
$$W_\perp = k\,B \tag{3}$$

(Where k is a constant, depending on the initial conditions and $W_\perp$ is the kinetic energy at right angles to the field, in the plane of the Larmor orbit.)

The change in energy is caused by the change in magnetic flux through the orbit of the particle.

Current theory – an example

This theory can be outlined by considering the effect of a large change in magnetic flux density, B , from B_0 to 10 B_0, for example. If B is changing slowly, then the particle will then execute a large number of revolutions during this time, of gradually decreasing radius.



For a uniform magnetic field, B, at right angles to the area, A,
the magnetic flux, Φ, is given by: $\Phi = B\,A$

$$So \quad \delta\emptyset = B\delta A\delta + A\delta B$$

Now if equation (2) is correct then $\boldsymbol{\delta\emptyset = 0}$ and so current theory requires that

$$\frac{\boldsymbol{\delta A}}{\boldsymbol{A}} = \frac{\boldsymbol{-\delta B}}{\boldsymbol{B}} \tag{5}$$

So Alfven's result means that, in order for the magnetic flux to remain constant, the relative change in area of a Larmor orbit must be equal (and opposite to) the relative change in magnetic flux density.

Derivation of the conventional Alfven theory

Alfven's proof starts with the emf induced around the particle's orbit which, to quote:

"changes the energy of the particle. We have

$$\oint E.dl = -\frac{\partial \emptyset}{\partial t} \qquad (6)$$

$\emptyset$ $(= \boldsymbol{\pi r^2 B})$ ***is the flux through the circular path of the particle and the integral is to be taken*** *along the periphery of the same circle."*

"The gain in energy in one turn is

$$\delta W_{\perp} = -e\oint E.dl = e\pi r^2 \frac{dB}{dt} \qquad (7)$$

"(The negative sign derives from the fact that a positive particle goes in a direction opposite to that in which the integral is to be taken.) Thus the rate of increase in energy is given by":

$$\frac{dW_{\perp}}{dt} = \frac{\delta W_{\perp}}{T} = \frac{W_{\perp}}{B}\frac{dB}{dt}$$

Alfven then uses equation (1), and the formula for the periodic time in a Larmor orbit,

$$T = \frac{2\pi}{\omega} = \frac{2\pi m}{Be} \qquad (8)$$

and by eliminating dt and integrating, gets the conventional result:

$$W_{\perp} = \mu B \qquad (9)$$

where μ, the magnetic moment, is a constant.
(Note that the magnetic moment, μ is defined as the ratio $W_{\perp}/B$)
It then follows that the magnetic flux through a gyro-orbit, Φ must also be constant.

This can easily be seen as follows:

$$\Phi = BA = B\pi r^2 = B\pi \frac{m^2 {V_{\perp}}^2}{B^2 e^2} = \pi \frac{m^2 {V_{\perp}}^2}{Be^2} \qquad (10)$$

which is clearly constant if $\frac{m{V_{\perp}}^2}{B} =$ constant, i.e. if the magnetic moment is constant.

Contradictions and errors in the current theory

It can be seen in equation (7) that Alfven gives $\frac{\partial \emptyset}{\partial t}$ as $\pi r^2 \frac{dB}{dt}$ ie as $A\frac{dB}{dt}$, rather than $A\frac{dB}{dt} + B\frac{dA}{dt}$, where A is the area of the Larmor orbit.

Two problems with current theory

1. The change in area between two orbits has clearly been neglected. Alfven himself does not mention this assumption, although other authors who reproduce this proof do mention this point, for example Gartenhaus (1964). However, as shown here the magnetic flux can only be constant if the relative change in area ($=2\pi r\delta r/A$) is equal to (but opposite in sign) to the relative change in flux density,B, as shown in equation (5).

2. In Alfven's equations (6) and (7), the energy gain δW over one Larmor orbit is calculated (using Faraday's induction law), by finding the rate of change in magnetic flux, $\emptyset/\partial t$. But according to the **result** of conventional theory, (equation 2), the magnetic flux does not change – it is constant even for large changes in B. So on the one hand, the theory requires a small flux change between orbits, while on the other hand, the result of the theory is that there is no flux change! In fact equation (3) permits **very large** changes in the kinetic energy of the particle, but equation (2) states that this happens with **no change** in magnetic flux: so where does the induced electric field come from?

These contradictions in Alfven's theory arise because of the assumption that the change in area, from the end of one orbit to the end of the next, can be ignored. This area is shown in figure 2b:

Figure 2a

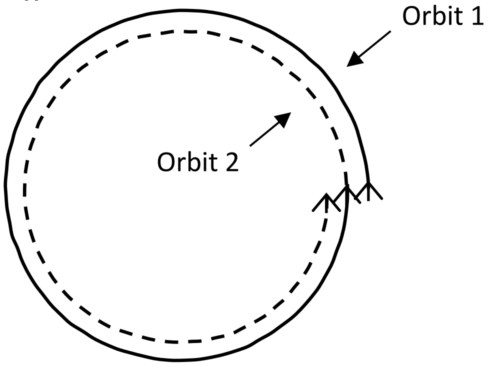
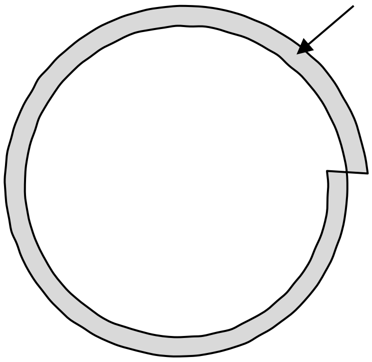


Figure 2b



An alternative derivation and theory

The following derivation tries to take into account the effect not only of a change in magnetic flux, but also of the associated change in area of the Larmor orbit.

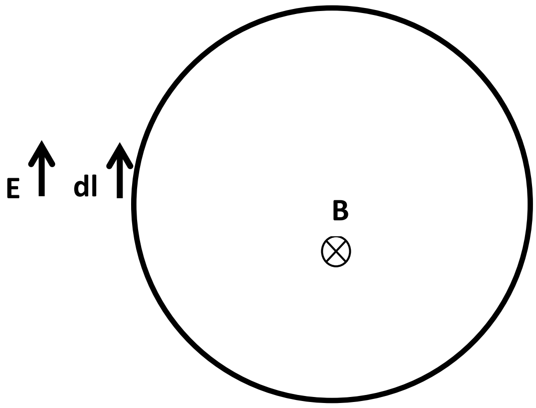
The main equation is the Fararday/Maxwell Induction law:

$$\oint \boldsymbol{E}.\boldsymbol{dl} = -\frac{\boldsymbol{d}}{\boldsymbol{dt}}\int \boldsymbol{B}.\boldsymbol{ds} \qquad (11)$$

B and d**s** are the magnetic flux density and elemental area vectors, both pointing away from the viewer. The right hand rule then means that the positive direction for **E** and d**l** are clockwise, as indicated in figure 9.

The left hand side of equation (11) is the induced emf , ε, around the Larmor orbit. The right hand side is often expressed simply as $\partial\emptyset/\partial t$. The effect of the minus sign is that if $\partial\emptyset/\partial t$ is positive, then the induced electric field and the emf act in an anti-clockwise direction in the case shown in figure 9, where the direction of B is away from the viewer, and at right angles to the plane of the orbit.

Figure 9



If the magnetic field, B is uniform over the area A of the Larmor orbit we simply get:

$$\int \boldsymbol{B}.\boldsymbol{ds} = \boldsymbol{B}.\boldsymbol{A} = BA \qquad (12)$$

The flux through a Larmor orbit is given by $\Phi = BA$
Then the rate of change of flux is given by

$$\frac{d\Phi}{dt} = B\frac{dA}{dt} + A\frac{dB}{dt}$$

In conventional theory the rate of change of area is taken as zero, but it has been shown here that this factor may be as important as the change in field, B: it cannot be neglected.

For the moment, for simplicity we will consider a (hypothetical) positive particle that is rotating clockwise in figure 9, that is, one with a positive velocity vector in the same positive direction as both **E** and **dl**.
If the velocity of the particle is taken as V then the area A is found by substituting from equation (1):

$$A = \pi r^2 = \pi \frac{m^2 V^2}{B^2 e^2}$$

$$\text{so} \qquad \frac{dA}{dt} = 2\pi r\frac{dr}{dt} \qquad (11)$$

Taking B and V, as usual, to be time dependent

$$\frac{dA}{dt} = 2\pi\frac{m^2 V}{Be^2}\left(-\frac{V}{B^2}\frac{dB}{dt} + \frac{1}{B}\frac{dV}{dt}\right)$$

so

$$B\frac{dA}{dt} = \frac{\pi m^2 V}{Be^2}\left(-2\frac{V}{B}\frac{dB}{dt} + 2\frac{dV}{dt}\right)$$

And the emf is given by

$$\varepsilon = -(B\frac{dA}{dt} + A\frac{dB}{dt}) = \frac{\pi m^2 V}{Be^2}\left(\frac{V}{B}\frac{dB}{dt} - 2\frac{dV}{dt}\right) \qquad (12)$$

The energy gain over one rotation is eε, and using the period as given by equation (8), the rate of change of energy is

$$\frac{d}{dt}\left(\frac{mV^2}{2}\right) = \frac{e\varepsilon}{T} = \frac{\varepsilon Be^2}{2\pi m}$$

Substituting into (12) gives:

$$\frac{d}{dt}\left(\frac{mV^2}{2}\right) = \frac{mV}{2}\left(\frac{V}{B}\frac{dB}{dt} - 2\frac{dV}{dt}\right)$$

Or

$$\frac{dW_\perp}{dt} = \frac{W_\perp}{B}\frac{dB}{dt} - \frac{dW_\perp}{dt}$$

And so

$$\frac{dW_\perp}{dt} = \frac{W_\perp}{2B}\frac{dB}{dt} \qquad (13)$$

However, this is for a hypothetical positive particle, rotating clockwise in figure 9. A real positive particle will rotate in the **opposite** direction, so will experience an electric field in the opposite direction. If the hypothetical particle is accelerating, then a real particle will be decelerating by the same amount, and vice-versa. So for a real particle we must have:

$$\frac{dW_\perp}{dt} = -\frac{W_\perp}{2B}\frac{dB}{dt} \qquad (14)$$

Solving this gives:

$$W_\perp = \frac{k}{B^{\frac{1}{2}}} \qquad (15)$$

where k is a constant, given by the initial conditions for W and B.

This means that an increase in the magnetic flux density, B, will lower the energy of the particle. This is completely opposite to the conventional theory, where an increase in B produces an increase in W.

This means that the magnetic moment, defined as $\mu \equiv \frac{W_\perp}{B}$, is not constant, as conventional theory requires. From (15) it can be seen that

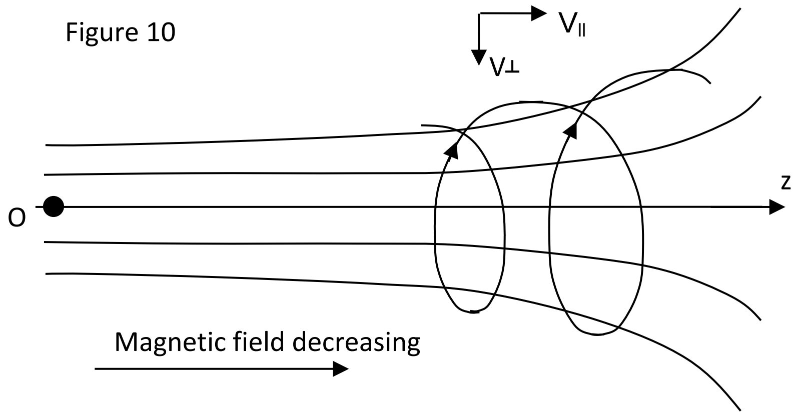
$$\mu = \frac{k}{B^{3/2}} \qquad (16)$$

Implications of the new theory

Deceleration of plasmas moving into lower field regions

Figure 10 shows a charged particle moving in the positive direction of the z-axis, into a magnetic field that is getting weaker. This is indicated by the magnetic field lines, which are diverging. Conventional theory requires that particles moving into lower field regions accelerate along the field lines, gaining in energy $W_\parallel$, but losing equally in energy perpendicular to the field, $W_\perp$. In conventional theory the velocity $V_\parallel$ along the z-axis should increase.

Figure 10



However the new theory indicates that in this figure, the horizontal velocity $V_{\parallel}$ is **decreasing**, while the perpendicular velocity $V_{\perp}$ is increasing. This is exactly the opposite to conventional theory. This particle will decelerate, and in the right circumstances, will reverse its direction and accelerate back into the higher field regions. Alternatively, if the field begins to increase again, as occurs half way around a magnetic loop, the particle can start to accelerate again along the loop.

This deceleration is similar to that seen in solar spicules.

Acceleration/Deceleration equation

The acceleration/deceleration of a charged particle can be found as follows:

The total energy of the particle is constant: $W_{\perp} + W_{\parallel} = W_{\text{total}}$

So $\frac{dW_{\perp}}{dz} = -\frac{dW_{\parallel}}{dz}$ **,** where z is the distance along the horizontal.

Since
$$\frac{dW_{\parallel}}{dz} = m\frac{dV_{\parallel}}{dt}$$

we get
$$m\frac{dV_{\parallel}}{dt} = \frac{{}^{1}\!/_{2}k}{B^{3}\!/_{2}}\frac{dB}{dZ} \tag{18}$$

using equation (15).

The acceleration $dV_{\parallel}/dt$ will clearly be constant if $\frac{1}{B^{3}\!/_{2}}\frac{dB}{dZ}$ is constant.

(It is shown later that this term is approximately constant in particular circumstances)

k is found from equation (15) using the initial conditions, $k = W_{\perp 0}{B_0}^{1/2}$

The initial perpendicular energy, $W_{\perp 0}$ can be expressed in terms of $W_{\parallel 0}$ if we assume for simplicity that the energy of particles is initially isotropic. Then since $W\perp$ is the energy in the two dimensional plane perpendicular to the field, compared to the single dimensional energy $W_{\parallel 0}$, we have $W_{\perp 0} = 2 W_{\parallel 0}$.

Using $= \frac{1}{2}mV^2$, equation (18) can then be rewritten as

$$\frac{dV_{\parallel}}{dt} = \frac{\frac{1}{2}{V_{\parallel 0}}^2{B_0}^{1/2}}{B^{3}\!/_{2}}\frac{dB}{dZ} \tag{19}$$

The mass m of the particle cancels, indicating this formula does not depend on the species of particle. Note that if dB/dz is negative, so is $dV_{\parallel}/dt$, indicating a deceleration. If we let $dV_{\parallel}/dt$ be denoted by 'a', this equation can be written:

$$a = \frac{{B_0}^{1/2}}{2B^{3/2}}\frac{dB}{dZ}{V_{\parallel 0}}^2 \qquad (20)$$

Constancy of the term $\boldsymbol{\frac{B_0^{1/2}}{2B^{3/2}}\frac{dB}{dZ}}$

If we assume that the magnetic field can, over a limited region, be represented by an inverse square law, then the field is given by

$$B = \frac{m}{z^2}$$ where m is a constant.

Then it follows, perhaps surprisingly, that $\frac{B_0^{1/2}}{2B^{3/2}}\frac{dB}{dZ} = -\left(\frac{B_0}{m}\right)^{1/2}$ = constant (21)

This can be obtained simply by substitution and differentiation of the inverse square law. Equation (20) can then be written

$$a = -\left(\frac{B_0}{m}\right)^{1/2} \; {V_{\parallel 0}}^2 \qquad (22)$$

It is interesting to note that this simple relationship applies only in the case of an inverse square law. In reality the field only follows this law at larger solar distances, but it can be a reasonable approximation over a restricted distance.

The minus sign indicates that if the field is decreasing as z increases, the acceleration is in the negative direction of z, in other words it corresponds to a deceleration.

Application of the acceleration/deceleration equation to solar jets and spicules.

Equation (20) can be applied to the phenomenon of solar spicules, fibrils and jets.

It is useful, initially, to use an inverse square law for the magnetic field. The actual field is more complex than this, but over part of the photosphere and corona, it may provide a reasonable approximation to the real magnetic field. The benefit of using an inverse square law is that it simplifies the acceleration relationship to the relatively simple equation (22). The deceleration is then a constant, given only by magnetic field parameters and the initial velocity.

Estimation of $\boldsymbol{\frac{B_0^{1/2}}{2B^{3/2}}\frac{dB}{dZ}}$

By taking this as a constant, it can be evaluated at any height, z. Choose $z = z_0$, where is taken to be the bottom of a spicule.

Then the constant becomes $\frac{1}{2B_0}\left(\frac{dB}{dZ}\right)_{z=z_0}$

This quantity, which has the dimensions of a length^{-1}, is assumed to be related simply to the magnetic scale height.

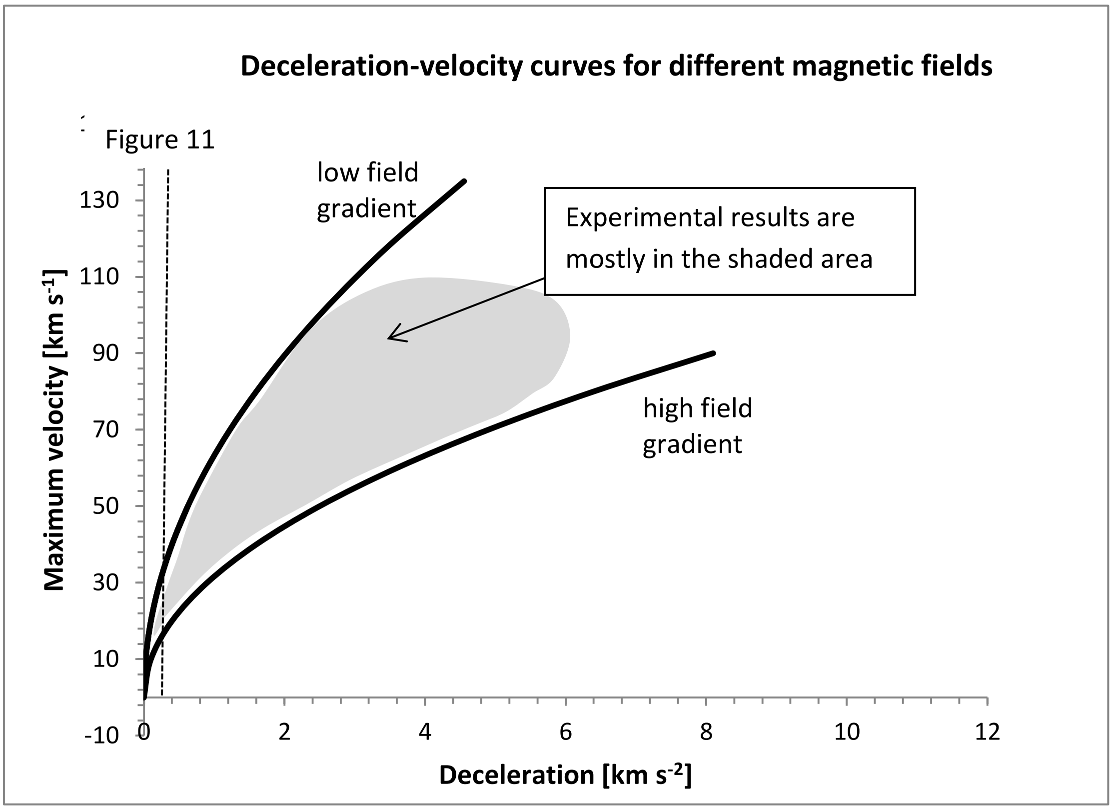
Brosius et al (2002) give one estimate of this as 0.5 x 10^9 cm (0.5 x 10^4 km). However it is well known that the solar magnetic field is quite variable, and stronger fields are often quite localised. For this reason, we will examine the result of two scale heights, 0.5 x 10^4 km and a more localised one, 0.1 x 10^4 km.

The acceleration equation for each of these regions then reduces, respectively to

$$a = -2x10^{-4}\ {V_{\parallel 0}}^2 \qquad (23a)$$

$$a = -10x10^{-4}\ {V_{\parallel 0}}^2 \qquad (23b)$$

Figure 11 shows these two curves. The top curve is for the lower scale height. The vertical dotted line is the value of solar acceleration at the surface.



Spicule deceleration – experimental results

There is now substantial evidence that many plasma jets, or spicules behave ballistically after ejection from the solar surface. They have a wide range of initial velocities and usually experience a constant deceleration. The deceleration is often many times greater than that due to gravity and so cannot be explained simply by solar gravity. (Solar deceleration is shown by the vertical dotted line in figure 11). At other times the deceleration is small, much less than solar gravity.
The deceleration has been found to increase with the maximum (that is the initial) velocity with which the plasma jet is launched from the solar surface.

This phenomenon has been described by a number of researchers: Hansteen et al (2006), De Pontieu (2007), Langangen et al (2008) Anan et al, (2009) and Zhang et al (2012). They made

measurements of hundreds of specular features, recording their deceleration, initial velocity and length.
Each produced scattergrams of deceleration and velocity, and showed a clear correlation between the deceleration of a spicule and its initial velocity.
Anan et al recorded accelerations up to 2.3 ms^{-2} for specular jets over a plage area, close to the limb. Zhang et al had a larger range of velocities and decelerations, up to 5.5 ms^{-2}.
The great majority of the points in the scattergrams fitted into the shaded area shown in figure11, between the two curves. There were, however a number points that did not fit, showing high velocity and low acceleration, but these were in a minority.

Zhang et al (2012), examined over 100 spicules in both quiet sun and coronal hole regions. Table 1 gives their results for velocity and deceleration and shows the quantity a/V^2.

Table 1

	Quiet sun	Coronal hole
Number of spicules	105	102
Mean acceleration, a, (km s^{-2})	-0.14	-1.04
Mean vertical velocity, V, (km s^{-1})	15.5	40.5
a/V^2 (km^{-1})	**5.8x 10^{-4}**	**6.3x10^{-4}**

This quantity a/V^2 has been calculated from Zhang's results. Although there is almost a factor of 10 difference between the accelerations of quiet sun and coronal hole situations, this "acceleration constant" is essentially the same in both cases. The mean value is approximately 6.0 x 10^{-4} km-1 This will produce a curve between the two curves in figure 11 and is a reasonable fit to most of their data.

Characteristic length

The constant in equation (22) has the unit L^{-1}. It is also related to the maximum length that a plasma can travel into a decreasing field before its velocity reaches zero, that is it stops and starts to travel in the opposite direction.

By combining equation (22) with the standard equation $V^2 = 2as$ where s is the distance travelled

We get $s = \left(\frac{V_{\parallel 0}{}^2}{2a}\right)$

Using the values from equation (23a) this gives s = 2000 km, which is a typical value for the spicule length found by Anan (2010), so the new theory does provide some explanation for the length of spicules.

Variation of magnetic moment

Conventional theory states that the magnetic moment of a charged particle is constant, provided the magnetic field changes slowly. But there is some evidence that the magnetic moment of a plasma such as the solar wind increases as the magnetic field decreases, which may support the new theory. In www.livingreviews.org/lrsp-2006-1, for example Marsch reports that the magnetic moment of protons increases by a factor of about 3 in moving out from 0.4 R_0 to 0.98 R_0, approximately. This may be partly due to the increase in magnetic moment caused by decreasing fields, as seen in equation (16).

Discussion and summary

The new theory of magnetic moment predicts behaviour which is almost opposite to the old theory. Despite this, it does seem to produce a reasonable explanation for the velocity- deceleration relationship and length of solar spicules, despite a number of simplifying assumptions. One of these assumptions is that the plasma is largely collisionless: collisions will tend to reduce $W_{\perp}$ and increase $W_{\parallel}$ making the particle's energy more isotropic. This could be one of the reasons for the minority of measurements that lie well above the curves in figure 11. The other effect ignored here is that of solar gravity.

This may be partly justified as solar gravitational acceleration is much less than the values shown in figure 11, and the fact that many of the spicules are not vertical. The effect of solar acceleration will be to add an extra term to the overall deceleration, depending on the inclination of the spicule.

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