# What Causes Tides?

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## Abstract
It has been known for thousands of years that the waters of coastal seas ebb and flow on a daily basis. This movement is long-term explained to be from the gravitational attraction of the Moon and Sun. However, a strict investigation shows this understanding unreachable. Here we propose, the Earth's curved motions about the centre of mass of the Earth-Moon system and around the Sun may generate some centrifugal effects to stretch the Earth's body. The resulting elongated Earth, together with its daily rotation, drives each part of ocean basin to regularly rise and fall, generating water transferring back and forth, and also the fall and rise of water level regionally.

## 1 Introduction
### 1.1 A brief introduction of tidal ideas
From antiquity it has been familiar that coastal seas always perform daily regular movements of water rise and fall. Since these movements are closely related to the frequently coastal activities, explaining them has undoubtedly tested human wisdom for millennia. Aristotle (384-322 BC) was highly perplexed and vaguely attributed it to the rocky nature of the coast. Early Chinese considered tides as the beating of the Earth's pulse and alternatively, it was believed to be caused by the Earth's breathing. Some people thought tides were caused by the different depths of ocean water. Galileo considered that the rotations of the Earth around the Sun and about its axis induced motions of the sea to generate the tides. However, the majority certainly linked tidal action to the influence of the Moon and of the Sun. Seleucus (lived in the 2nd century BC) was the first to consider this connection. He concluded the height of tide was correlated with the Moon's position relative to the Sun. However, the exact determination of how the Moon and Sun cause tides is unknown. A few Arabic explanations proposed that the Moon uses its rays to heat the water and then expand it. Descartes argued that space was full of ethereal substance and the resulting stresses between the ether and the Earth's surface gave birth to tides when the Moon travelled round the Earth. In contrast, Kepler and Newton represent those who expressly define this influence as the attraction of the Moon and Sun on the water. Newton formulated an attractive mechanism to account for the tide, within it the gravity gradient of the Moon produces a pair of bulges of water on Earth (that is in the line of the Earth-Moon system). As Newton and the well known 1740's Essayists (such as Daniel Bernoulli, Leonhard Euler, Colin Maclaurin, and Antoine Cavalleri, for instance) had assumed the oceans' response to the tidal driving force to be quasi-static, considering the complexity of actual oceans and currents, Laplace developed a set of hydrodynamic equations of continuity and momentum for a fluid on a rotating earth. Together with the following endeavors (including William Thomson, Baron Kelvin, Henri Poincaré, Arthur Thomas Doodson, etc.,), the idea of gravitation as the cause of tide (the attractive mechanism) was increasingly strengthened and became the cornerstone of modern tidal theory. A fuller review of the tidal history is shown in these works [1,2,3]. From then on, it seems that the physics of tide had been satisfactorily resolved. However, a strict investigation shows there still is a big step towards the final fact.

### 1.2 Problems of the attractive mechanism
The attractive mechanism may be outlined with such a paradigm: the Earth orbits about the centre of mass of the Earth-Moon system, this makes all particles of the Earth travel around in circles which have the same radius. The force necessary to give each particle of the Earth the acceleration to perform this revolution is the same as for the particle at the centre of the Earth, for such a particle the Moon's gravitational force provides this necessary acceleration. For those particles nearer the Moon than the centre particle, the Moon's gravitational

attraction is greater than that necessary to maintain the orbit. For those further away the forces are weaker. The differences between the forces necessary for orbit and the forces actually experienced generate the tides on the surface of the Earth [1]. Practically, the tide-generating force is treated as the gradient of the gravity field of the Moon (Sun) and can further be decomposed into two components respectively perpendicular and parallel to sea surface, the horizontal component ultimately produces tides. Speculate an elastic Earth covering with a self-gravitating ocean, no land and no the influences of inertia and currents, the tidal potential gives the Earth two symmetric bulges of water that is always in the Earth-Moon line [2,4]. For a site of the Earth's surface it will pass through the two bulges as the Earth revolve about its axis, this results in two cycles of high and low water per day.

At first glance, this paradigm is considerably self-contained, but its expectation is against observation. Firstly, the two bulges produce an equality of two low waters if a site of the Earth's surface rotates within the resulting two bulges. As shown in Figure 1 (a), in the middle of the two bulges there will be mechanically a belt (marked with white line) of lowest water, a site $P$ will pass this belt at $P_1$ and $P_2$ as the Earth rotates about its axis, two low waters of same size are generated per day. From a view of the globe, other sites such as $N$ and $Q$, which are at different latitudes, will also pass this belt respectively at $N_1(N_2)$ and $Q_1(Q_2)$ to form two low waters of same size. In general, the two bulges request the high waters of all sites around the globe to be different in size whereas the low waters to be same in size within a fixed day. Contrary to this, observation shows that not only the high waters of all sites are different in size but also the low waters are different in size. Secondly, the two bulges also request the size of the two successive high waters of a site (which is at higher latitude) to be developed reversely, namely, the size of one high water is increased (decreased) whereas the size of another is decreased (increased). Refer to Figure 1 (a), the size of one high water at site $N(Q')$ increases whereas the size of another at site $N'(Q)$ decreases as the two bulges transfer from the north to the south.

Some could keep suspicion on this argumentation because it is only deduced from the Moon's influence. Let's further consider a combined effect of the Moon and Sun. Refer to Pugh's work [1], the attractive mechanism employs a formula (the equilibrium model) below to describe the elevation of the sea surface.

$$H_m=a(M_m/M_e)[C_0(t)(3\sin^2\varphi_p/2-1/2)+C_1(t)\sin 2\varphi_p+C_2(t)\cos^2\varphi_p]$$

$$C_0(t)=(a/R_m)^3(3\sin^2 d/2-1/2)$$

$$C_1(t)=(a/R_m)^3(3\sin 2d\cos C_p/4)$$

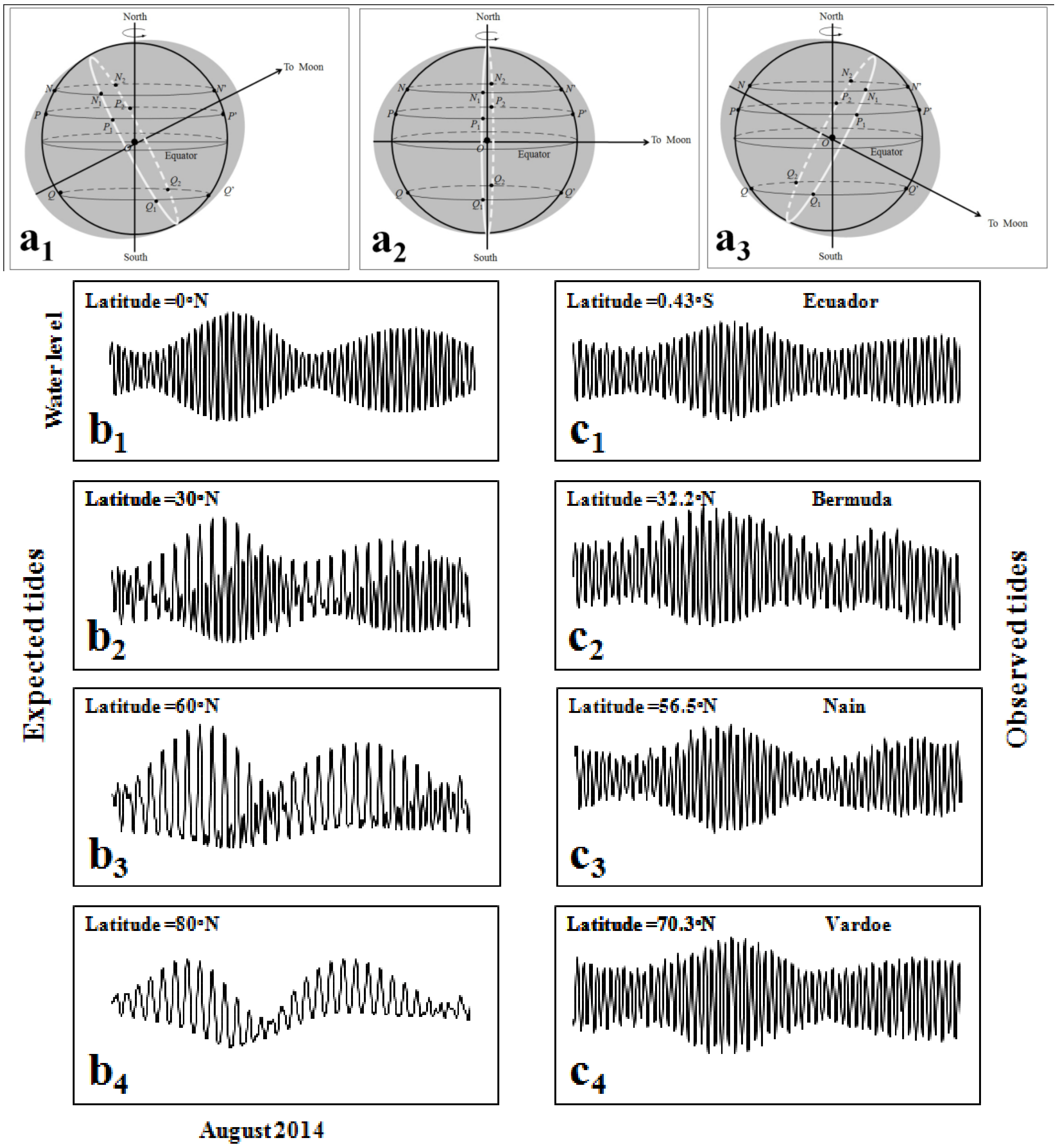
$$C_2(t)=(a/R_m)^3(3\cos^2 d\cos 2C_p/4)$$

where $H_m$, $a$, $M_m$, $M_e$, $\varphi_p$, $R_m$, $d$, and $C_p$ are respectively the elevation of the sea surface, the Earth's radius, the Moon's mass, the Earth's mass, the latitude of a particle at the sea surface, the distance of the Earth and Moon, the declination of the Moon, and the hour angle of the particle relative to the Moon.

Replace $M_m$ and $R_m$ respectively with $M_s$ (the Sun's mass) and $R_s$ (the distance of the Earth and Sun), the elevation of the sea surface $H_s$ due to the effect of the Sun may be got. $H_m+H_s$ therefore represents the final elevation of sea surface that is due to a combined effect of the Moon and Sun. Figure1(b) shows several tides of different latitudes expected from this model. Related parameters for this expectation is listed in Table 1. It can be seen that, at latitude 30°N the size of one high water of the tide increases at the time when the size of another decreases; towards higher latitudes (60°N and 80°N, for instance) one high water and one low water are gradually disappearing, moreover, the shapes of the high and low waters are no longer symmetric. Contrary to these, refer to Figure 1(c), the two successive high waters of each of the observed tides are alternately increased or decreased, this behavior is also applicable for the two successive low waters. On the whole, there is a major discrepancy in morphology between the expected tides and the observed tides.

Lastly, refer to Figure 2(b), the attractive mechanism requests the semidiurnal tides (two high waters and two low waters per day) to occur primarily at lower latitudes while the diurnal tides (one high water and one low water) to occur primarily at higher latitudes. Contrary to this, refer to 1(e), the observation shows that the semidiurnal tides are almost dominant around the globe and the diurnal tides are relatively spare. There are semidiurnal tides at Greenland and Norway, while there are also diurnal tides at the Malaysia's seas and the Karumba Gulf.

Some could argue that, as the response of the ocean to the tidal force is complicated and depends on the shape of ocean basins, water depth, the Earth's rotation, orientation of coastline, and so on, there is no value in trying to understand the tidal force using observed tides. This is actually not the matter. Throughout the tides around the globe, we see, the high and low waters of all tides are uniformly increased or decreased at the same time, even if the geographic surroundings around these tidal sites are greatly different. This means the evolution of the high and low waters of all tides is independent of these factors. Contrary to this, refer to Figure 1(c and d), this evolution of the high and low waters performs a strong coupling with the positions of the Moon and Sun. So far, this relationship is defined as a result of the gravitational attraction between these bodies, therefore, employing the results from the equilibrium mode, which only considers the gravitational attraction, to examine the attractive mechanism itself is completely appropriate.



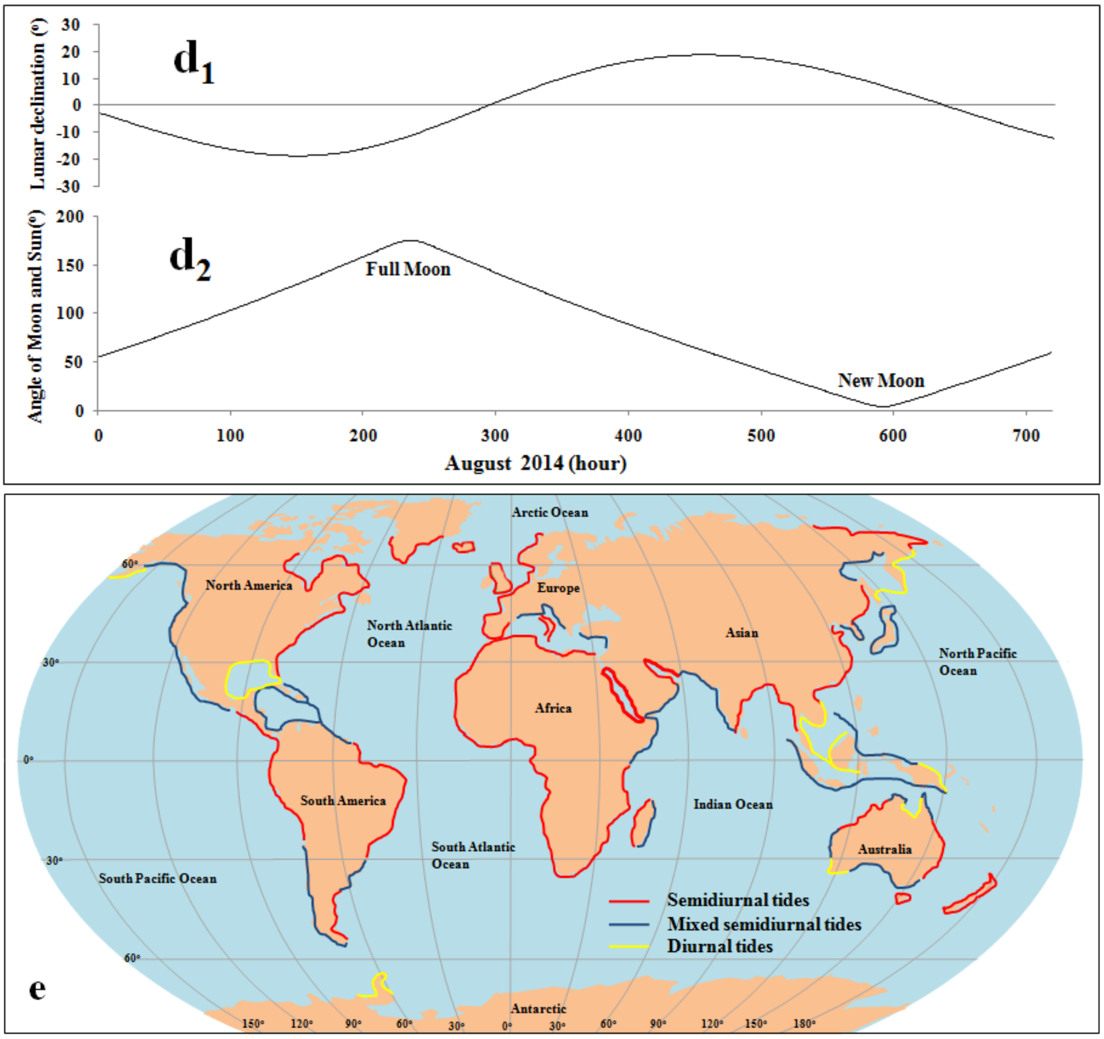



**Figure 1: Comparison of the expected tides from the attractive mechanism and the observed tides. a**, showing how the unequal high waters and the equal low waters are produced at various sites under the resulting two bulges. From $\mathbf{a_1}$, $\mathbf{a_2}$, to $\mathbf{a_3}$, it orderly represents the positions of the two bulges as the Moon transfers from the north to the south. *O* is the Earth's centre. Big circle represents undisturbed watery planet. Note the shapes of the two bugles are seriously exaggerated; **b**, the shapes of the expected tides at different latitudes in August 2014; **c**, the shapes of the observed tides at different latitudes during the month; **d**, the positions of the Moon and Sun relative to the Earth in the month; **e**, the distributions of the observed tidal patterns around the globe. Data supporting this map are from U.S. NOAA, GLOSS database (University of Hawaii Sea Level Center), and Bureau National Operations Centre (BNOC) of Australian government.

## 2 An analytic treatment of the deformed Earth and the resulting tide

The Earth may be mechanically thought to be a solid sphere that is enveloped by water and atmosphere [5,6,7]. The structure of solid Earth, from surface to interior, is sequentially divided into crust, mantle, outer core, and inner core[8]. A large number of works confirmed that these layers are filled with various materials [9-14] and denser materials are gravitationally concentrated towards the interior[15]. Notwithstanding, solid Earth is strictly not a rigid body, both experiments and measurements proved it to be elastic[16, 17] and to had been stretched into an oblate spheroid because of a centrifugal effect of the Earth's rotation about its axis [18,19]. This suggests, the Earth can be deformed by a centrifugal effect. It is well known that the Earth orbits about the centre of mass of the Earth-Moon system and the Earth-Moon system orbits about the Sun. A fuller details of the movements of the Earth, Moon, and Sun may be seen in these works [20-25]. These two curved movements inevitably generate two centrifugal effects $F_1$ and $F_2$ for the Earth (Figure 2(a)). $F_1$ and $F_2$ are respectively balanced by the gravitational attraction from the Moon and from the Sun. If we define the centrifugal effect of the Earth's rotation about its axis as $F_3$, then the ratio between $F_1$, $F_2$, and $F_3$ will be 1:178:505 according to established parameters such as orbital radius, orbital period, mass of each body, and so on. Practically, $F_2$ is far greater than $F_1$, but the working point of $F_2$ is not at the Earth's centre, compared to the working point of $F_1$ that is at the Earth's centre. In consideration of this matter, we suppose the effective part of $F_2$, which is able to stretch the Earth, is relatively small and lies at the Earth centre ($O_1$).

To better run the following deduction, we assume solid Earth to be a standard sphere and use the centrifugal effects $F_1$ and $F_2$ to independently stretch solid Earth along the Earth-Moon line ($O_1M$) and the Earth-Sun line ($O_1S$). Suppose a case, if an elastic ball is physically elongated along a fixed direction, and then, the ball will be accordingly shortened at another direction vertical to the fixed direction. For the Earth (which is elastic) it can bear such a behavior. Under the centrifugal effect $F_1$, section GIHJ (passing through a site M that is at the surface of solid Earth) and section KQLP respectively become section G'I'H'J' and section K'Q'L'P', site M turns to site M', section GIHJ is vertical to section KQLP, GH is the intersecting line between them (Figure 2(b)); Under the centrifugal effect $F_2$, section VSWR (also passing through site M) and section TYUX become sections V'S'W'R' and T'Y'U'X', site M turns to site M'', section VSWR is vertical to section TYUX, VW is the intersecting line between them (Figure 2(c)). It can be seen, solid Earth simultaneously undergoes two elongations (in the direction of the Earth-Moon line and in the direction of the Earth-Sun line) and two compressions (in the direction of section KQLP and in the direction of section TYUX). Therefore, the final position of site M is a result of the combination of these adjustments (elongation and compression). According to the geometry of ellipse, the distance between site M and the Earth's centre may be expressed as

$$H = O_1\mathrm{M}' + O_1\mathrm{M}'' - O_1\mathrm{M} \quad (1)$$

$$O_1\mathrm{M}' = [\mathrm{I}'O_1^2\cos^2\beta + \mathrm{G}'O_1^2\sin^2\beta]^{1/2} \quad (2)$$

$$O_1\mathrm{M}'' = [\mathrm{S}'O_1^2\cos^2\gamma + \mathrm{V}'O_1^2\sin^2\gamma]^{1/2} \quad (3)$$

Where

I'$O_1$ and G'$O_1$ are the semi-major and semi-minor axis of ellipse G'I'H'J', respectively, and G$O_1$= I$O_1$, G'$O_1$= G$O_1$-$k_m$', I'$O_1$= I$O_1$+$k_m$, $k_m$ is the elongation of solid Earth in the direction of the Earth-Moon line ($O_1M$) that is due to the centrifugal effect $F_1$, $k_m$' is the compression of solid Earth in the direction of section KQLP, G$O_1$ (I$O_1$) is the mean radius of solid Earth, equal to $O_1$M. β is the angle of site M and the Moon relative to the Earth's centre.

S'$O_1$ and V'$O_1$ are the semi-major and semi-minor axis of ellipse V'S'W'R', respectively, and V$O_1$= S$O_1$, V'$O_1$= V$O_1$-$k_s$', S'$O_1$= S$O_1$+$k_s$, $k_s$ is the elongation of solid Earth in the direction of the Earth-Moon line ($O_1M$) that is due to the centrifugal effect $F_2$, $k_s$' is the compression of solid Earth in the direction of section TYUX, V$O_1$ (S$O_1$) is the mean radius of solid Earth, equal to $O_1$M. γ is the angle of site M and the Sun relative to the Earth's centre.

β and γ are related to other astronomical constants. $\cos\beta = \sin\sigma\sin\delta_{\mathrm{m}} + \cos\sigma\cos\delta_{\mathrm{m}}\cos C_{\mathrm{mm}}$, $\cos\gamma = \sin\sigma\sin\delta_{\mathrm{s}} + \cos\sigma\cos\delta_{\mathrm{s}}\cos C_{\mathrm{ms}}$, where $\sigma$, $\delta_{\mathrm{m}}$, $\delta_{\mathrm{s}}$, $C_{\mathrm{mm}}$, and $C_{\mathrm{ms}}$ are respectively the geographic latitude of site M, the declination of the Moon, the declination of the Sun, the hour angle of site M relative to the Moon, and the hour angle of site M relative to the Sun.

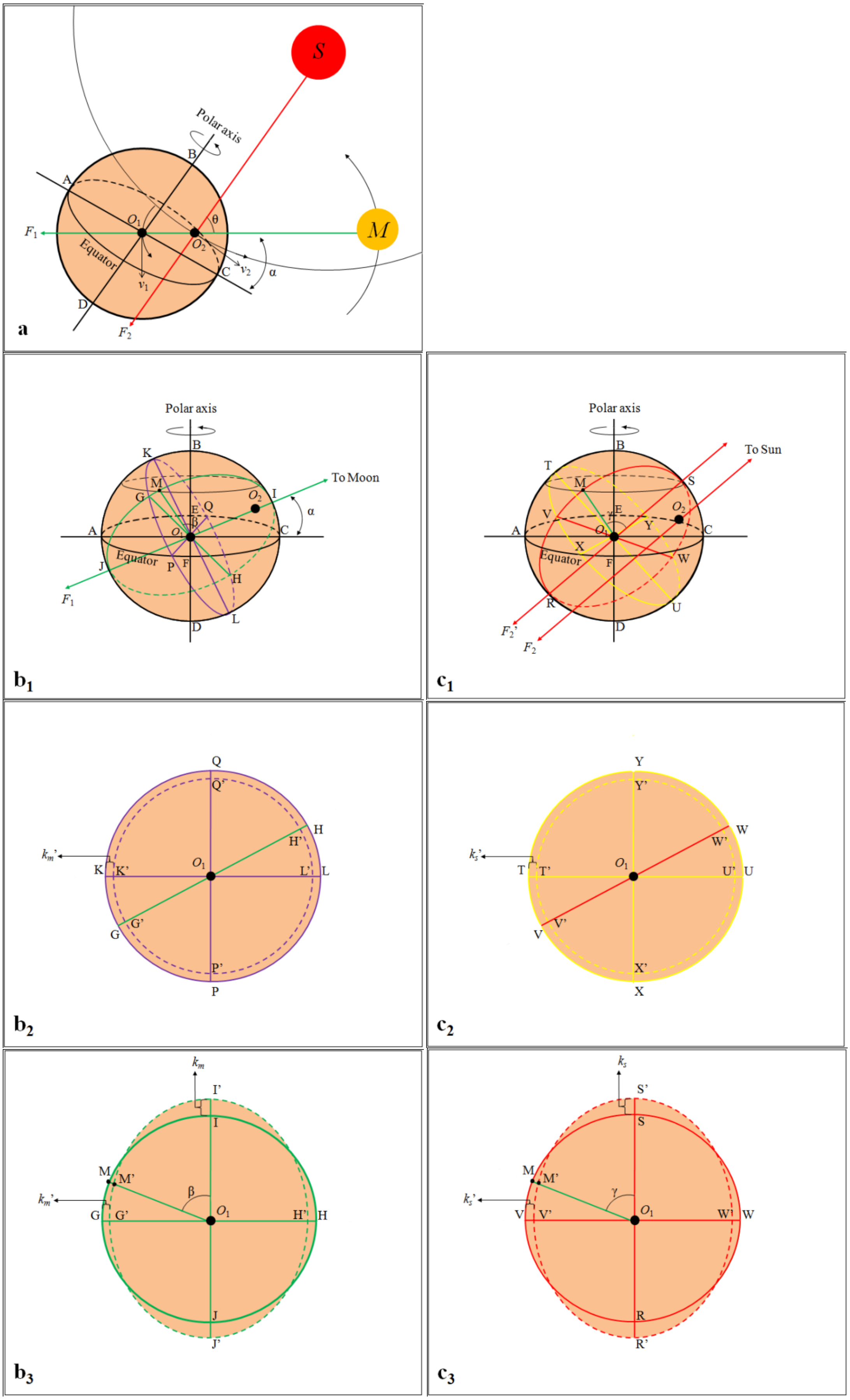



**Figure 2. Combined centrifugal effects for solid Earth and the resulting deformation. a,** the motions of the Earth around the barycenter of the Earth-Moon system and around the Sun. $F_1$ and $F_2$

are the resulting centrifugal effects solid Earth undergoes due to these curved motions. $O_1$, $O_2$, $M$, and $S$ are the Earth's centre, the barycenter of the Earth-Moon system, the Moon, and the Sun, respectively. ϴ is the angle between the Moon and the Sun relative to the barycenter of the Earth-Moon system. Section AECF is the equatorial plane, α is the Moon's declination. ϴ and α may be obtained some ephemeris systems such as NASA's JPL HORIZONS and so on. $v_1$ and $v_2$ are respectively the velocity of the Earth orbiting the barycenter of the Earth-Moon system and the velocity of the Earth-Moon system orbiting the Sun, which generate the centrifugal effects $F_1$ and $F_2$; **$b_1$, $b_2$,** and **$b_3$**, solid Earth under the effect of $F_1$ and the resulting deformations respectively in the direction of section KQLP and in the direction of the Earth-Moon line. Purple (green) real and dashed circles represent the original and disturbed shapes of solid Earth in the related directions; **$c_1$, $c_2$,** and **$c_3$**, solid Earth under the effect of $F_2$ (also $F_2$') and the resulting deformations respectively in the direction of section TYUX and in the direction of the Earth-Sun line. Yellow (red) real and dashed circles represent the original and disturbed shapes of solid Earth in the related directions.

The ratio of centrifugal effect $F_3/F_1$ is about 505:1. A rough evaluation based on this amount is the centrifugal effect $F_1$ may generate an elongation of about 42.00 m for solid Earth if the Earth's oblate spheroid is referred. However, the Earth's oblate spheroid is likely to be resulted from an accumulative effect during a time scale of billions of years, thus, the centrifugal effect $F_1$ at instant could give rise to only a slight amount. Here we assume the elongation of solid Earth in the Earth-Moon line (due to the centrifugal effect $F_1$) at instant to be 0.50 m at the time when the Moon is at perigee and in the Earth-Sun line (due to the centrifugal effect $F_2$) at instance to be 0.30 m, and assume the response of the compression to the elongation to be a factor of 1.0. The Moon's elliptical orbit means a timely changing distance between the Earth and the Moon. As the centrifugal effect $F_1$ is anywhere balanced by the Moon's gravitation and the Moon's gravitation is reversely proportional to the square of distance, the elongation of solid Earth in the Earth-Moon line may thus be approximately expressed as $k_{\mathrm{m}}=0.5R_{\mathrm{m}}^2/R_{\mathrm{peri}}^2$. For the elongation of solid Earth in the Earth-Sun line it may be constant because the Earth's orbit is nearly circular. And then, $k_s$=0.30 m, $k_m'=k_m$, $k_s'=k_s$, where $k_m$ and $k_s$ are respectively the elongation of solid Earth in the Earth-Moon line and in the Earth-Sun line, $k_m'$and $k_m'$ are respectively the compressions to the response of the elongations. Please note, the elongation $k_m$ ($k_s$) and compression $k_m'$($k_s'$) should be obtained by means of measurement.

The formula (1), (2), and (3) indicate that the highest rising and falling of site M occurs at the times of full and new Moon, whereas the lowest rising and falling occurs at first quarter and last quarter. This is because at times of full and new Moon the combined effect becomes maximal, and further the deformation of solid Earth is most serious, but at times of first quarter and last quarter the combined effect turns to minimal, and further the deformation is the slightest. The Moon's declination (α) is not important in determining the deformation.

Here we introduce an idea of oscillating water vessel. As shown in Figure 3(a), we let the right side of box rise, the water then flows towards left side. If line MN represents a reference level, during this course the water level of site M rises whereas the water level of site N falls. We then restore right side to its former level and continue to let the left side rise, the water of left side flows towards right side, the water level of site M correspondingly falls whereas the water level of site N rises. Repeat the rise and fall of the two ends continuously, the water level of sites M and N alternately vary. Compared to sites M and N, another site S, which is in the middle of the vessel, holds the minimal variation of water level. Now we let one end rise and fall (another ideally keeps motionless), repeat it continuously, the water level of sites M and N alternately vary too. The oscillating water vessel indicates the variation of water level of one end is determined by the difference of the rising of one end and the fall of another end. Turn to the Earth, about 71% of the Earth's surface is covered with ocean water [26], each of the ocean basins look likes a water vessel. We cut the Earth (solid Earth + water) to form a latitudinal section A'E'C'F' and a longitudinal section ABCD (Figure 3($b_1$ and $b_2$). Within these sections each ocean basin and its two ends *(a*, *b*, for instance*)* form a water vessel, the bed of the enclosed sea/lake and its two ends (*f*, *j*, for instance) also form a water vessel.

Refer to Figure 2, it can be deduced that, with the Earth's rotation about its axis ($O_1$), the deformed solid Earth makes every part of these water vessels regularly rise and fall, generating water transferring, and also high and low waters for these ends per day.

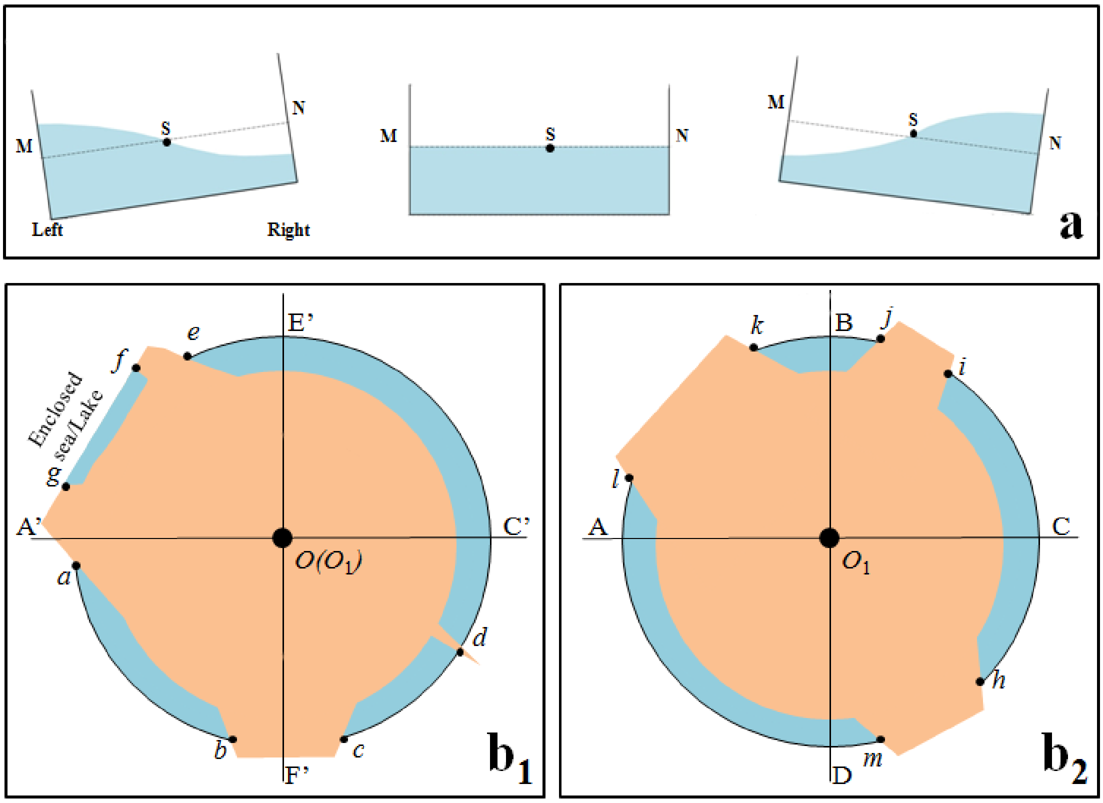
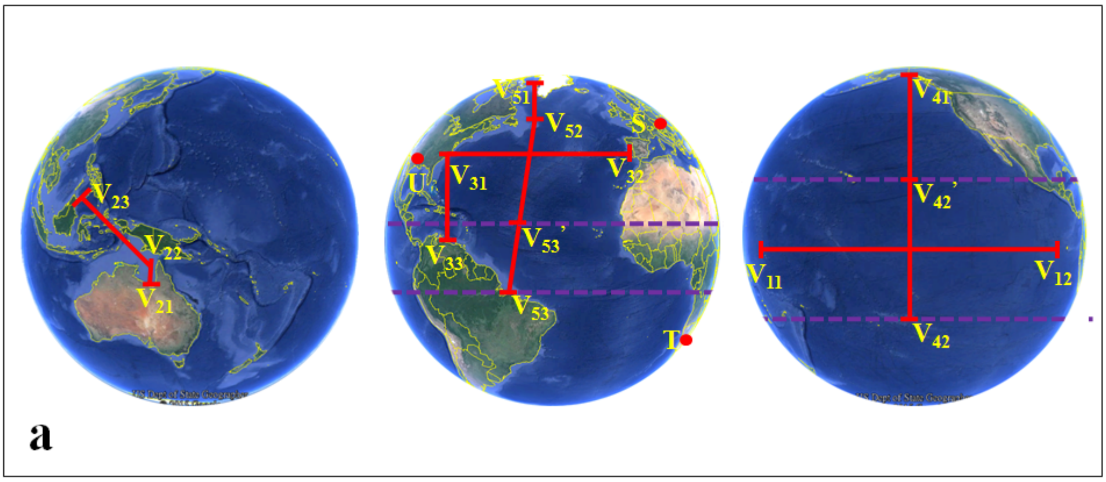
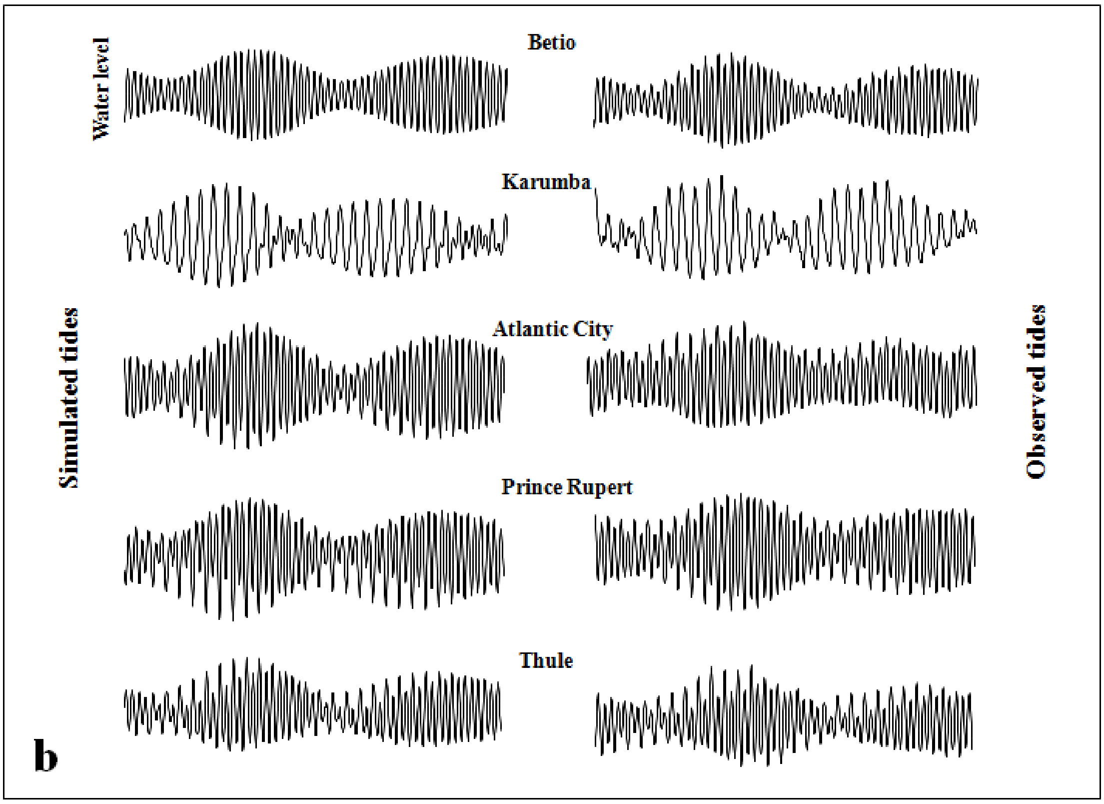



**Figure 3. Modelling oscillating water vessel and ocean vessels on solid Earth. a**, an oscillating water vessel. Line MN is the reference level; $\mathbf{b_1}$, a latitudinal treatment of ocean vessels; $\mathbf{b_2}$, a longitudinal treatment of ocean vessels. Section A'E'C'F' is parallel to the equatorial plane AECF, while section ABCD always passes through polar axis BD. $O_1$ is the Earth's centre. *a*, *b*, *c*, …, represent the ends of these vessels.

Five sites (Betio, Karumba, Atlantic City, Prince Rupert, Thule) are selected to demonstrate the variation of water level. In consideration of the physics of that oscillating vessel, each of these sites acts as one end of a vessel, five partners are needed for acting as another ends of these vessels. The treatments of these vessels are outlined in Figure 4(a). $V_{11}$, $V_{21}$, $V_{31}$, $V_{41}$, and $V_{51}$ represent the five sites, $V_{12}$, $V_{22}$, $V_{32}$, $V_{42}$ ($V_{42}$'), and $V_{52}$ are their partners. The reason for selecting $V_{12}$'s position is because the maximal elongation of solid Earth is always vertical to the maximal compression and the two ends of each water vessel practically shouldn't exceed $90^o$. $V_{23}$ is added as second partner of $V_{21}$ because the rising of $V_{23}$ (at the northwest Celebes Sea) may make water flow towards $V_{22}$, from where the coming water can further deflect to $V_{21}$. $V_{33}$ is added as second partner of $V_{31}$ because the rising of $V_{33}$ may yield a longitudinally water transferring between them. $V_{53}$($V_{53}$') is added as second partner of $V_{51}$ because the rising of $V_{53}$($V_{53}$') may make water flow towards $V_{52}$, form where the coming water can further deflect to $V_{51}$. In other words, $V_{51}$ and $V_{53}$($V_{53}$') may be connected through $V_{52}$. Through Google earth software we figure out the geographic latitudes and longitudes of $V_{11}$, $V_{12}$, $V_{21}$, $V_{22}$, $V_{23}$, $V_{31}$, $V_{32}$, $V_{33}$, $V_{41}$, $V_{51}$, and $V_{52}$. The determination of the geographic latitudes and longitudes of $V_{42}$ ($V_{42}$') and $V_{53}$ ($V_{53}$') becomes relatively complicated. Refer to Figure 2, it can be deduced that the maximal elongation of solid Earth always lies in the Earth-Moon line, this means, for $V_{41}$ there exists another identical longitudinal site that will be passed by the Earth-Moon line as the Earth rotates about its axis, we treat this site as $V_{42}$ ($V_{42}$'), $V_{41}$ and $V_{42}$ ($V_{42}$') may thus form a water vessel. The latitude of $V_{42}$ ($V_{42}$') is equal to the Moon's declination as its hour angle relative to the Moon is between $-90^o$~$90^o$ and equal to the inverse of the Moon's declination as its hour angle is between $90^o \sim 270^o$. For site $V_{52}$ it encounters similar matter with $V_{41}$, a difference is the position of its partner $V_{53}$ ($V_{53}$') may not beyond the brim of South America, at where one end of water vessel is mechanically formed. So, when the Moon declines to the north the latitude of $V_{53}$ ($V_{53}$') is equal to the Moon's declination as its hour angle is between $-90^o \sim 90^o$ and equal to $0^o$ as its hour angle is between $90^o \sim 270^o$; when the Moon declines to the south the latitude of $V_{53}$ ($V_{53}$') is equal to $0^o$ as its hour angle is between $-90^o \sim 90^o$ and equal to the inverse of the Moon's declination as its hour angle is between $90^o \sim 270^o$. This treatment is because $V_{42}$ ($V_{42}$') and $V_{42}$ ($V_{42}$') are timely varying as the Earth-Moon line transfers between the north and the south. After the geographic latitudes and longitudes of these sites are known, the distance between any of these sites and the Earth's centre may be calculated by the formula (1), (2), and (3). And then, $O_1V_{12} - O_1V_{11}$, $O_1V_{23} - O_1V_{22} + O_1V_{22} - O_1V_{21}$, $O_1V_{32} - O_1V_{31} + O_1V_{33} - O_1V_{31}$, $O_1V_{42}$ ($V_{42}$') - $O_1V_{41}$, and $O_1V_{53}$ ($V_{53}$') - $O_1V_{52} + O_1V_{52} - O_1V_{51}$ represent respectively the variation of water level at these sites. We see, $V_{22}$ and $V_{52}$ play a role of tie point for the transferring water. In the experiment of the oscillating water vessel we found that, with the passage of time, the rise or fall of water level at one end cannot timely response to the fall or rise of another end, there is always a time lag between them, so, in the simulation time lag is considered for some of these selected vessels. The reason for this lag could be the inertia of transferring water. In addition, the water depth of the vessel is not critical in determining the size of water level. The deformation of solid Earth we propose here expects gravity variation from site to site. Three sites (Strasbourg, Sutherland, and Apache Point) are selected to verify it. Gravity variation ($g$) is calculated by a formula $g \sim 1/r^2 - 1/a^2$, $r$ is the distance between any of three selected sites and the Earth's centre, which can be known through the formula (1), (2), and (3), $a$ is the mean radius of the Earth. The simulated and observed tides (gravity variations) are shown in Figure 4(b and c). Related parameters for simulation are listed in Table 1. On the whole, the simulated results are morphologically well consistent with the observed ones. But for a more exactly tidal prediction some factors such as shape of ocean basin, orientation of coastline, water depth, Coriolis force, inertia, and so on should be considered.





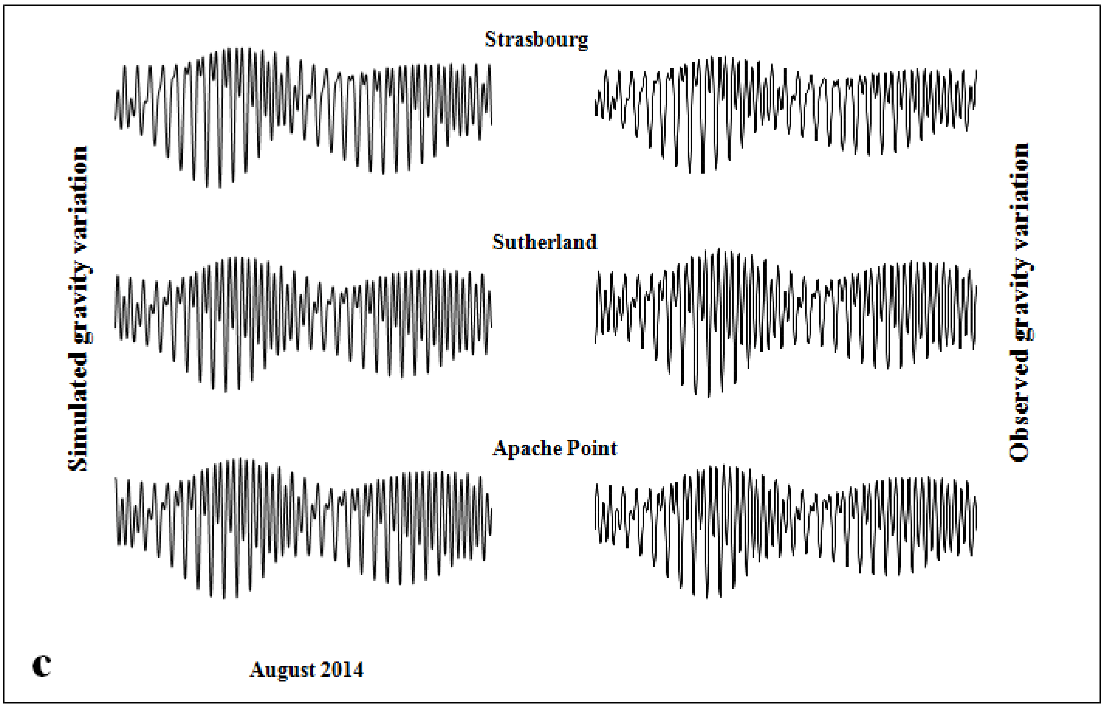




**Figure 4. Treatment of ocean vessels and the comparison of simulation and observation. a**, defining ocean vessels and GGP (Global Geodynamics Project) sites at the surface of solid Earth (base map is from Google Earth). $V_{11}$, $V_{12}$, …, represent the ends of the proposed vessels. S, T, and U represent three gravity observational sites. Please note, vessel $V_{52}V_{53}$($V_{53}$') is slightly mislead, it actually is longitudinally parallel to vessel $V_{31}V_{33}$. Vessels $V_{11}V_{12}$ and $V_{31}V_{32}$ are parallel to equator. Dashed purple lines are the expected boundary of $V_{42}$($V_{42}$') and $V_{53}$($V_{53}$'). **b**, showing a morphological comparison of the simulated and observed tides without scale. In the simulation time lag for Karumba, Atlantic City, Prince Pupert, and Thule is respectively 1.0, 7.0, 7.0, and 12.0 hours; **c**, showing a morphological comparison of the simulated and observed gravity variations. Time span for all simulations is from UTC 2014-08-01 00:00:00 to 2014-08-30 23:00:00. The lunar and solar ephemeris are from JPL HORIZONS system. Water level data is from GLOSS database(University of Hawaii Sea

Level Center) and Bureau National Operations Centre (BNOC) of Australia. Gravity data is from GGP (Global Geodynamics Project).

**Table 1 Astronomical constants**

| | | | |
|---|---|---|---|
| The Moon | | | Symbol |
| | Mass | $7.34*10^{22}$ kg [27] | $M_m$ |
| | Perigee | 362600 km | $R_{mp}$ |
| | Apogee | 405400 km | $R_{ma}$ |
| The Earth | | | |
| | Mass | $5.97*10^{24}$ kg [28] | $M_e$ |
| | Mean radius | 6371 km [29] | $a$ |
| | Mean distance from Sun | 149597870 km [30] | $R_s$ |
| The Sun | | | |
| | Mass | $1.99*10^{30}$ kg [31] | $M_s$ |
| Geographic sites for water vessels | | Latitude, Longitude | |
| | Betio, Kiribati | 1.35°N, 172.92°E | $V_{11}$ |
| | Partner (1) | 1.35°N, 98.00°W | $V_{12}$ |
| | Kuraumba, Austrilia | 17.70°S, 139.20°E | $V_{21}$ |
| | Partner (1) | 13.00°S, 141.00°E | $V_{22}$ |
| | Partner (2) | 5.00°N, 121.00°E | $V_{23}$ |
| | Atlantic City, USA | 39.40°N, 75.00°W | $V_{31}$ |
| | Partner (1) | 39.40°N, 8.70°W | $V_{32}$ |
| | Partner (2) | 10.00°N, 75.00°W | $V_{33}$ |
| | Prince Pupert, Canada | 54.32°N, 130.32°W | $V_{41}$ |
| | Partner (1) | – –, 130.32°W | $V_{42}$($V_{42}$’) |
| | Thule, Greenland | 76.54°N, 68.83°W | $V_{51}$ |
| | Partner (1) | 53.00°N, 49.00°W | $V_{52}$ |
| | Partner (2) | – –, 49.00°W | $V_{53}$($V_{53}$’) |
| Geographic sites for gravity | | Latitude, Longitude | |
| | Strasbourg, France | 48.62°N, 7.68°E | S |
| | Sutherland, South Africa | 32.38°S, 20.82°E | T |
| | Apache Point, New Mexico, USA | 32.78°N, 105.82°W | U |

## 3 Discussion

The mechanism of the oscillating water vessel is also applicable for the matter of the enclosed sea/lake. If we treat Black sea as a water vessel and use formula (1), (2), and (3) to estimate, the west or east end of Black sea may experience a tide of about 12.0 cm. Similarly, a tube of water (20 m in length) horizontally located at equator will experience a tide of about $2.2x10^{-3}$ mm, an imperceptible amount. This means that, any small vessel, such as swimming pool, water cup, water bowl, and so on, because of its short size, will not have an perceptible tide. The regular oscillation of ocean basin makes water transfer back and forth, continental shorelines are long enough to block the transferring water to form large accumulation. In particular, most of the shorelines are concave, this may create an effect of narrow to amplify the coming water. Typical representatives for this amplification are the tides around Qiantang River and Bay of Fundy. In contract, the shorelines of the islands that are isolated in the deep oceans are short, the coming water cannot be effectively accumulated and may bypass. These determine larger tides to occur primarily at the coastal seas and smaller tides to occur primarily at the deep oceans. Solid Earth's deformation results in mainly an oscillation of ocean waters. The transferring water in travel, if constrained by the narrowness of strait, may form swift current, like that in the Cook Strait [32,33]. For the various features of the tides in the Atlantic and in the North West Europe shelf seas, because the Earth is progressively stretched in a manner of from east to west, the rising of east Atlantic firstly makes water flow towards north, west, and south, the succeeding rising of middle Atlantic further makes water flow towards east, north, west, and south, finally, the rising of west Atlantic makes water flow towards north, east, and south. The westerly water may reach the eastern coastline of America nearly at the same time and leaves no difference of tidal phase. The tides from Florida to Nova Scotia are the case. The northerly water may form a progression of tidal phases in the North Atlantic. In particular, a large body of northeasterly water may enter the strait of Gibraltar and cross the Celtic Sea, from where it continues to run into the English Channel and other related regions, a series of progressive tides along the shores of these regions are created, the tides around England shores are the case. As the elongated Earth progressively rolls water to flow from east to west, this generates a generally westward propagation of the tides [1,2,4].

Newton in his book *Mathematical Principles of Natural Philosophy* described a tidal phenomenon (Proposition XXIV. Theorem XIX, translated by Andrew Motte): "*An example of all which Dr. Halley has given us, from the observations of seamen in the port of Batsham, in the Kindom of Tunquin (presently Viet Nam), in the latitude of 25$^{o}$50' north. In that port, on the day which follows after the passage of the moon over the equator, the waters stagnate: when the moon declines to the north, they begin to flow and ebb, not twice, as in other ports, but once only every day: and the flood happens at the setting, and the greatest ebb at the rising of the moon. This tide increases with the declination of the moon till the 7$^{th}$ or 8$^{th}$ day; then for the 7 or 8 days following it decreases at the same rate as it had increased before, and ceases when the moon changes its declination, crossing over the equator to south. After which the flood is immediately changes into an ebb; and thenceforth the ebb happens at the setting and the flood at the rising of the moon; till the moon, again passing the equator, changes its declination. There are two inlets to this port and the neighboring channels, one from the seas of China, between the continent and the islands of Leuconia; the other from the Indian sea, between the continent and the island of Borneo. But whether there be really two tides propagated through the said channels, one from the Indian sea in the space of 12 hours, and once from the sea of China in the space of 6 hours, which therefore happening at the 3d and 9$^{th}$ lunar hours, by being compounded together, produce those motions; or whether there be any other circumstances in the state of those seas, I leave to be determined by observations on the neighbouring shores.*" Newton gave explanations for this tide but without being verified. Here we demonstrate how to generate it. When the Moon declines to the north, the part of the northern hemisphere that nears to the port (marked with Z) is raised at the Moon's rising as the elongation of solid Earth is always in the Earth-Moon line, this leads the waters of the port to ebb. With the Earth's rotation about its axis, the part of the southern hemisphere that nears to the port is gradually raised at the Moon's setting, this lead the waters in the southern hemisphere to flow towards north, generating flood for this port (Figure 5 (a and b)); When the Moon declines to the south, the part of the southern hemisphere that nears to the port is raised at the Moon's rising, this leads the waters in the southern hemisphere to flow towards north, also the port, the flood forms here. With the Earth's rotation about its axis, the part of the northern hemisphere that nears to the port is gradually raised at the Moon's setting, this leads the waters of the port to ebb (Figure 5 (c and d)). This gives the port one high and one low water per day. When the Moon is about the equator, the water transferring between the northern and southern hemispheres becomes slight, the waters of the port therefore is nearly motionless.

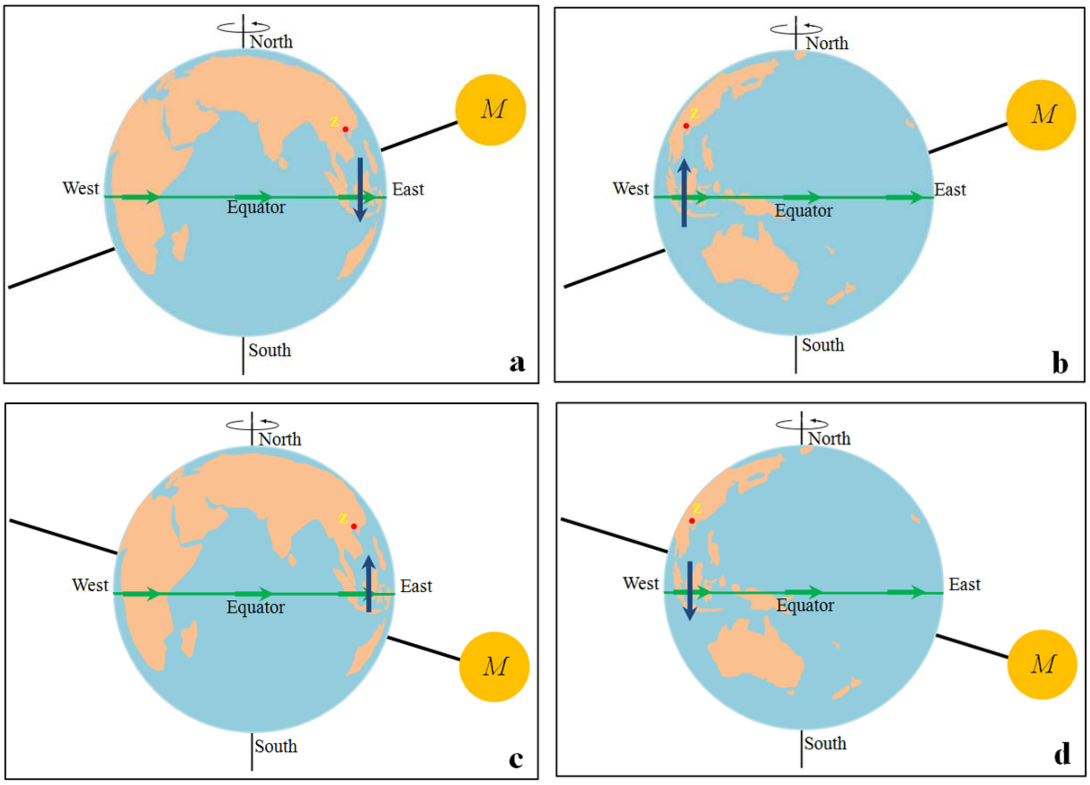



**Figure 5: Modelling the formation of diurnal tide for the port of Batsham (red).** From **a**(**b**) to **c**(**d**) the Moon transfers from the north to the south. Green arrows represent the Earth's rotation from west to east, blue arrow in each diagram represents the direction of water transferring. Please note, refer to Figure 2, the maximal extension of solid Earth is always in the Earth-Moon line.

Galileo in his *Dialogue Concerning the Two Chief World Systems* described the tides of the Mediterranean (translated by Stillman Drake): "*three varieties of these hourly changes are observed: in some places the waters rise a fall without making any forward motions; in others, without rising or falling they move now toward the east and again run back toward the west; and in still others, the height and the course both vary. This occurs here in Venice, where the waters rise in entering and fall in departing. … …, elsewhere the water runs to and fro in its central parts without changing height, as happens notably in the Straits of Messina between Scylla and Charybdis, where the currents are very swift because of the narrowness of the channel. But in the open Mediterranean and around its islands, such as the Balearics, Corsica, Sardinia, Elba, Sicily (on the African side), Malta, Crete, etc., the alterations of height are very small but the currents are quite noticeable, especially where the sea is restrained between islands, or between these and the continent.*" Refer to Figure 3(a), we treat the Mediterranean as a water vessel. With the Earth's rotation around its axis, the deformed Earth makes two ends of this vessel alternately rise and fall, generating water transferring between the two ends, as a result, the greatest alternation of water level occurs at the two ends, whereas the smallest occurs in the open area. The transferring water, when being constrained by the straits, forms swift currents.

The attractive mechanism is widely believed to be competent for tidal prediction. This is actually a misconception. They are completely different, the former provides a reason why the Earth can undergo daily water movement of rise and fall, whereas the latter employs an experienced method to expect when the high and low water to take place. One may see, the daily water movement of a site is continuous, from rise (high) to fall (low), and then from fall (low) to rise (high). This pattern extremely resembles the feature of a sinusoidal or cosine function if one accepted the training of trigonometric knowledge. In practice, the established tidal prediction follows such a path that it employs a harmonic method, considering tide as the sum of a finite number of harmonic constituents, $H_n\cos(\sigma_n t\text{-}g_n)$, a standard cosine function. The attractive mechanism is better than any of earlier ideas in several aspects (for example, defining the two high and two low water per day and exactly connecting them to the Moon and Sun), moreover, its concealment lies at, if we doesn't examine it from a viewpoint of the evolution of high and low water around the globe, we can hardly find its discrepancy with reality. These could be the reason why it keeps unsuspected in the past hundreds of years. In contrast, this proposed theory provides a satisfactory resolution for tide and merits consideration by scientific community.

**Acknowledgements** I am pleased to thank these institutes (U.S. NOAA, NASA's JPL, GLOSS database - University of Hawaii Sea Level Center, Bureau National Operations Centre (BNOC) of Australia, and GGP (Global Geodynamics Project)) for data supporting, and honestly express thanks to these people (Walter Babin, Thierry De Mees, Roger A. Rydin, Duncan Agnew, Wouter Schellart, Jinyun Guo) for helpful comment and suggestive discussion, and thanks Hartmut Wziontek, Calvo Marta, and Mike Davis for the discussion of related data.

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